Two Way Deterministic Finite Automata

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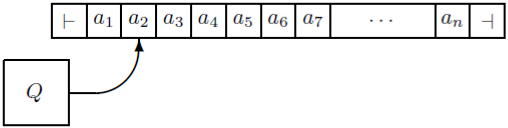
November 29, 2013

Overview

- Introduction.
- Formal Construction.
- Example.
- Configuration and Acceptance.
- 2DFA vs DFA.

2-way Deterministic Finite Automata

1. Generalised version of DFA.
2. Process the input in either direction.
   - Have read only head which can move in both direction over the input string.
   - Revisit the characters again and again.
3. Like a Turing Machine but.
   - Have read only head.
   - Have finite memory like DFA.



2-way Deterministic Finite Automata

4. 2DFA has finite set of states Q like DFA.
5. Input string
   - Input string is stored on finite tape.
   - One character per cell.
   - Input string is stored in between two extra symbol called left endmarker($\vdash$) and right endmarker($\dashv$).
6. At any time instance the machine is in state p and scan some symbol $a_i \in \Sigma$ or an endmarkers $\{\vdash, \dashv\}$, based on p and current symbol it will move its head one cell in direction $d \in \{L, R\}$ and enter in new state q .
7. Machine head never go outside the endmarkers.

8. Accept and reject states.
    - 2DFA needs only single accept and single reject state.
    - It will accept the input string by entering in a special accept state t.
    - It will reject the input string by entering in a special reject state r.
    - Accept and reject states are like sink state.
9. The machine action on a present state and head symbol is depend on transition function δ.
10. Transition function take present state and head symbol as input argument and return next state and direction of movement of head.

Formal definition of 2DFA

2DFA is represented by octuple.

$$M = \{Q, \Sigma, \delta, s, t, r, \vdash, \dashv\}$$

where

- Q is a finite set of states.
- Σ is a finite set of input symbol.
- $\delta : Q \times (\Sigma \cup \{\vdash, \dashv\}) = Q \times \{L, R\}$ is a transition function.
- $s \in Q$ is a start state.
- $t \in Q$ is a accept state.
- $r \in Q$ is a reject state.
- $\vdash$ is left endmarker.
- $\dashv$ is right endmarker.

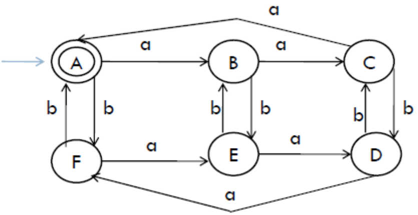
Some properties of transition function

1. Input is endmarker.
   - $\delta(p, \vdash) = (q, R)$
   - $\delta(p, \dashv) = (q, L)$
2. Accept and reject states are t, r respectively and current input symbol is $a \in \Sigma \cup \{\vdash\}$.
   - $\delta(t, a) = (t, R)$ and $\delta(t, \dashv) = (t, L)$
   - $\delta(r, a) = (r, R)$ and $\delta(r, \dashv) = (r, L)$
3. In general
   $\delta(p, a) = (q, d)$ where $p, q \in Q$ and $d \in \{L, R\}$

Example (Constructing a normal DFA)

Construct the DFA to accept the language
$L = \{x \in \Sigma^* | \#a(x) \text{ } \textit{are multiple of } 3, \#b(x) \text{ } \textit{are multiple of } 2\}$
Construct a normal DFA

1. DFA M_1 accepting $L_1 = \{x \in \Sigma^* | \#a(x) \text{ } \textit{are multiple of } 3\}$
   $M_1 = \{Q_1, \Sigma, \delta_1, s_1, F_1\}$
2. DFA M_2 accepting $L_2 = \{x \in \Sigma^* | \#b(x) \text{ } \textit{are multiple of } 2\}$
   $M_2 = \{Q_2, \Sigma, \delta_2, s_2, F_2\}$
3. DFA M accepting L such that $M = M_1 \times M_2$
   $M = \{Q, \Sigma, \delta, s, F\}$



Example(Constructing a 2DFA)

Construct a 2DFA accepting the set
$L = \{x \in \Sigma^* | \#a(x) \textit{ are multiple of } 3, \#b(x) \textit{ are multiple of } 2\}$

1. Machine start scanning from the left endmarker.
2. Scan input string from left to right consider only a and ignore b.
   if $\#a(x)$ are not multiple of 3 then rejects x and enters in state r.
3. if $\#a(x)$ are multiple of 3 then start scanning from right consider only b and ignore a.
   if $\#b(x)$ are not multiple of 2 then enters in t otherwise enters in state r.

Example (Formal construction of 2DFA)

$M = \{Q, \Sigma, \delta, s, t, r, \vdash, \dashv\}$
where $\Sigma = \{a, b\}$, $Q = \{q_0, q_1, q_2, p_0, p_1, t, r\}$ and the transition function δ is given by following table.

states	$\vdash$	a	b	$\dashv$
q_0	(q_0, R)	(q_1, R)	(q_0, R)	(p_0, L)
q_1	-	(q_2, R)	(q_1, R)	(r, L)
q_2	-	(q_0, R)	(q_2, R)	(r, L)
p_0	(t, R)	(p_0, L)	(p_1, L)	-
p_1	(r, R)	(p_1, L)	(p_0, L)	-
t	(t, R)	(t, R)	(t, R)	(t, L)
r	(r, R)	(r, R)	(r, R)	(r, L)

Configuration and Acceptance

Let we have input string $x \in \Sigma^*$ such that $x = a_1 a_2 \cdots a_{n-1} a_n$,$|x| = n$ and let $a_0 = \vdash$, $a_{n+1} = \dashv$ then machine head will scan $\vdash x \dashv$.

1. Configuration for the input x is pair (p, j) such that $p \in Q$ and $0 \leq j \leq n + 1$.
2. In pair (p, j) p is current state and j is current position of head.
3. Initial configuration of machine is $(s, 0)$ this mean initially machine is in state s and scanning left endmarker.
4. The relation $\xrightarrow[x]{1}$ describes one step of the machine on input x.
   - $\delta(p, a_j) = (q, L) \Rightarrow (p, j) \xrightarrow[x]{1} (q, j - 1)$
   - $\delta(p, a_j) = (q, R) \Rightarrow (p, j) \xrightarrow[x]{1} (q, j + 1)$
   - $(p, j) \xrightarrow[x]{0} (p, j)$
   - $(p, i) \xrightarrow[x]{n} (q, j)$ and $(q, j) \xrightarrow[x]{1} (u, k)$ then $(p, i) \xrightarrow[x]{n+1} (u, k)$

5. $(p,j) \xrightarrow[x]{*} (q,k) \overset{def}{\Longleftrightarrow} \exists n \geq 0 \; such \; that \; (p,j) \xrightarrow[x]{n} (q,k)$
6. Machine accept input string x if $(s,0) \xrightarrow[x]{*} (t,k)$ for some k.
7. Machine reject input string x if $(s,0) \xrightarrow[x]{*} (r,k)$ for some k.
8. It is possible that machine neither accept nor reject input string x then machine go in loop.
9. Language accepted by machine M is $L(M) = \{x \in \Sigma^* | (s,0) \xrightarrow[x]{*} (t,k)\}$

2DFA vs DFA