Applying Dupeire's Local Volatility Model to CVaR minimization

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Stock Price movement

Equation

$$dS_t = rS_t dt + \sigma(t, S_t) S_t d\tilde{W}_t \tag{1}$$

- S_t is the stock price at time t
- r is the risk-free rate
- $\sigma(t, S_t)$ is the volatility as a function of the time and stock price.
- $\tilde{W}_t$ is the Brownian Motion under the risk-neutral measure.

The Dupire Equation

Equation

$$\frac{\partial C(T,K)}{\partial T} + rK\frac{\partial C(T,K)}{\partial K} = \frac{1}{2}\sigma^2(T,K)K^2\frac{\partial^2 C(T,K)}{\partial K^2} \quad (2)$$

- C is the price of the call option
- T is the time to maturity
- K is the strike price.

Solving For The volatility

$$\sigma(T,K) = \frac{1}{K}\sqrt{\frac{2\partial C/\partial T(T,K) + 2rK\partial C(T,K)/\partial K}{\partial^2 C/\partial K^2(T,K)}} \quad (3)$$

Steps(Taken from the net)

- Collect option prices for various strikes and maturities.
- Interpolate/Extrapolate the option prices/ Black-Scholes Volatilities to get a smooth volatility surface.
- Use the dupeire equation to get the volatility function.
- Can calculate prices of other options using finite difference and monte-carlo simulations.

Integrating into our CVaR minimization Problem(Doubtful??)

- Once we have the volatility function, we can integrate it into our pricing model and get the estimated **value** of the options at maturity using simulations.
- Run the fast gradient descent algorithm.

References:

- Online Resource
- Shreve, Stochastic Calculus for Finance II: Continuous-Time Models