Physics 3020 – Methods of Computational Physics – Fall 2015

Example Problem

A very simple model of population growth is given by the model of Robert May, George Oster and Jim Yorke, which states that the population of a k-th generation is given by

$$N^k = \lambda N^{k-1}(1 - N^{k-1}). \tag{1}$$

Here N^k can take values between 0 and 1, $0 \leq N^k \leq 1$, and provides some measure of the size of a given generation. As long as a population is sufficiently small, the size of the next generation should be proportional to the size of the previous generation. That's taken care of by the term N^{k-1} in (1). When the size of the population reaches the maximum of what can possibly be supported, $N = 1$ in this model, over-population should lead to a decrease in the size of the next generation. That's accomplished by the term $(1 - N^{k-1})$ in (1). The factor λ is a constant of proportionality, the size of which may lead to completely different behavior of the population development. In particular,

- $\lambda < \lambda_1$ leads to extinction (i.e. $N^k \to 0$ as $k \to \infty$),
- for $\lambda_1 < \lambda < \lambda_2$ the population settles down to a constant value (greater than zero),
- for $\lambda_2 < \lambda < \lambda_3$ the population goes through repeating cycles,
- and for $\lambda > \lambda_3$ we find *chaotic* behavior (at least for some values for λ).

Chaotic behavior means that an arbitrarily small difference between two initial values N^0 leads to an arbitrarily large difference between the populations after a finite number of steps.

1. What is the maximum value that λ may take to insure that $N^k \leq 1$?
2. Write a computer program that computes population size sequences N^k. To search for chaotic behavior, you may want to evolve two sequences simultaniously, starting one with N_0, say, and the other with $N_0(1 + \delta)$, where $\delta \ll 1$ is a small number.
3. Run your program from part (b) for various values of λ (it's ok to always start with the same starting value N_0, for example $N_0 = 1/2$) and describe the different kinds of behavior that you find. Locate the transition from one behavior to another by "bracketing" (i.e. find two very similar values of λ so that the smaller value leads to one behavior and the larger value to another) and determine λ_1, λ_2, and λ_3.