STA2201_Lab7

Alice Huang

15/03/2023

Lip cancer

Here is the lip cancer data given to you in terribly unreproducible and error-prone format.

- `aff.i` is proportion of male population working outside in each region
- `observe.i` is observed deaths in each region
- `expect.i` is expected deaths, based on region-specific age distribution and national-level age-specific mortality rates.

```
observe.i <- c(
  5,13,18,5,10,18,29,10,15,22,4,11,10,22,13,14,17,21,25,6,11,21,13,5,19,18,14,17,3,10,
  7,3,12,11,6,16,13,6,9,10,4,9,11,12,23,18,12,7,13,12,12,13,6,14,7,18,13,9,6,8,7,6,16,4,6,12,5,5,
  17,5,7,2,9,7,6,12,13,17,5,5,6,12,10,16,10,16,15,18,6,12,6,8,33,15,14,18,25,14,2,73,13,14,6,20,8,
  12,10,3,11,3,11,13,11,13,10,5,18,10,23,5,9,2,11,9,11,6,11,5,19,15,4,8,9,6,4,4,2,12,12,11,9,7,7,
  8,12,11,23,7,16,46,9,18,12,13,14,14,3,9,15,6,13,13,12,8,11,5,9,8,22,9,2,10,6,10,12,9,11,32,5,11,
  9,11,11,0,9,3,11,11,11,5,4,8,9,30,110)
expect.i <- c(
    6.17,8.44,7.23,5.62,4.18,29.35,11.79,12.35,7.28,9.40,3.77,3.41,8.70,9.57,8.18,4.35,
    4.91,10.66,16.99,2.94,3.07,5.50,6.47,4.85,9.85,6.95,5.74,5.70,2.22,3.46,4.40,4.05,5.74,6.36,5.13,
    16.99,6.19,5.56,11.69,4.69,6.25,10.84,8.40,13.19,9.25,16.98,8.39,2.86,9.70,12.12,12.94,9.77,
    10.34,5.09,3.29,17.19,5.42,11.39,8.33,4.97,7.14,6.74,17.01,5.80,4.84,12.00,4.50,4.39,16.35,6.02,
    6.42,5.26,4.59,11.86,4.05,5.48,13.13,8.72,2.87,2.13,4.48,5.85,6.67,6.11,5.78,12.31,10.56,10.23,
    2.52,6.22,14.29,5.71,37.93,7.81,9.86,11.61,18.52,12.28,5.41,61.96,8.55,12.07,4.29,19.42,8.25,
    12.90,4.76,5.56,11.11,4.76,10.48,13.13,12.94,14.61,9.26,6.94,16.82,33.49,20.91,5.32,6.77,8.70,
    12.94,16.07,8.87,7.79,14.60,5.10,24.42,17.78,4.04,7.84,9.89,8.45,5.06,4.49,6.25,9.16,12.37,8.40,
    9.57,5.83,9.21,9.64,9.09,12.94,17.42,10.29,7.14,92.50,14.29,15.61,6.00,8.55,15.22,18.42,5.77,
    18.37,13.16,7.69,14.61,15.85,12.77,7.41,14.86,6.94,5.66,9.88,102.16,7.63,5.13,7.58,8.00,12.82,
    18.75,12.33,5.88,64.64,8.62,12.09,11.11,14.10,10.48,7.00,10.23,6.82,15.71,9.65,8.59,8.33,6.06,
    12.31,8.91,50.10,288.00)
aff.i <- c(0.2415,0.2309,0.3999,0.2977,0.3264,0.3346,0.4150,0.4202,0.1023,0.1752,
        0.2548,0.3248,0.2287,0.2520,0.2058,0.2785,0.2528,0.1847,0.3736,0.2411,
        0.3700,0.2997,0.2883,0.2427,0.3782,0.1865,0.2633,0.2978,0.3541,0.4176,
        0.2910,0.3431,0.1168,0.2195,0.2911,0.4297,0.2119,0.2698,0.0874,0.3204,
        0.1839,0.1796,0.2471,0.2016,0.1560,0.3162,0.0732,0.1490,0.2283,0.1187,
        0.3500,0.2915,0.1339,0.0995,0.2355,0.2392,0.0877,0.3571,0.1014,0.0363,
        0.1665,0.1226,0.2186,0.1279,0.0842,0.0733,0.0377,0.2216,0.3062,0.0310,
        0.0755,0.0583,0.2546,0.2933,0.1682,0.2518,0.1971,0.1473,0.2311,0.2471,
        0.3063,0.1526,0.1487,0.3537,0.2753,0.0849,0.1013,0.1622,0.1267,0.2376,
        0.0737,0.2755,0.0152,0.1415,0.1344,0.1058,0.0545,0.1047,0.1335,0.3134,
        0.1326,0.1222,0.1992,0.0620,0.1313,0.0848,0.2687,0.1396,0.1234,0.0997,
```

```
        0.0694,0.1022,0.0779,0.0253,0.1012,0.0999,0.0828,0.2950,0.0778,0.1388,
        0.2449,0.0978,0.1144,0.1038,0.1613,0.1921,0.2714,0.1467,0.1783,0.1790,
        0.1482,0.1383,0.0805,0.0619,0.1934,0.1315,0.1050,0.0702,0.1002,0.1445,
        0.0353,0.0400,0.1385,0.0491,0.0520,0.0640,0.1017,0.0837,0.1462,0.0958,
        0.0745,0.2942,0.2278,0.1347,0.0907,0.1238,0.1773,0.0623,0.0742,0.1003,
        0.0590,0.0719,0.0652,0.1687,0.1199,0.1768,0.1638,0.1360,0.0832,0.2174,
        0.1662,0.2023,0.1319,0.0526,0.0287,0.0405,0.1616,0.0730,0.1005,0.0743,
        0.0577,0.0481,0.1002,0.0433,0.0838,0.1124,0.2265,0.0436,0.1402,0.0313,
        0.0359,0.0696,0.0618,0.0932,0.0097)
```

Question 1

Explain a bit more what the `expect.i` variable is. For example, if a particular area has an expected deaths of 6, what does this mean?

Solution

Expected deaths is the implied number of lip cancer deaths for a particular region given that region's age structure and the national level age-specific mortality rates for lip cancer. For example, an expected number of deaths being 6 would mean that for that particular region, we would expect 6 lip cancer deaths if this region were to experience the same age specific mortality rates as at the national level.

Question 2

Run three different models in Stan with three different set-up's for estimating θ_i, that is the relative risk of lip cancer in each region:

1. Intercept α_i is same in each region $= \alpha$
2. α_i is different in each region and modeled separately (with covariate)
3. α_i is different in each region and the intercept is modeled hierarchically (with covariate)

Solution

$$y_i|\theta_i \sim \text{Poisson}(\theta_i \cdot e_i)$$

Look at 3 models for $\log(\theta_i)$

$$\log(\theta_i) = \alpha + \beta x_i$$

Model 1

```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.2 --
## v ggplot2 3.4.0     v purrr   0.3.5
## v tibble  3.1.8     v dplyr   1.1.0
## v tidyr   1.2.1     v stringr 1.4.1
```

```
## v readr   2.1.3     v forcats 0.5.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```r
library(rstan)
```

```
## Loading required package: StanHeaders
##
## rstan version 2.26.13 (Stan version 2.26.1)
##
## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)
## For within-chain threading using 'reduce_sum()' or 'map_rect()' Stan functions,
## change 'threads_per_chain' option:
## rstan_options(threads_per_chain = 1)
##
## Do not specify '-march=native' in 'LOCAL_CPPFLAGS' or a Makevars file
##
## Attaching package: 'rstan'
##
## The following object is masked from 'package:tidyr':
##
##     extract
```

```r
library(tidybayes)

stan_data <- list(y = observe.i, log_e = log(expect.i), N=length(observe.i), x=aff.i-mean(aff.i))
```

```r
mod1 <- stan(data=stan_data, file = "lab9_1.stan")
```

```
## Warning in readLines(file, warn = TRUE): incomplete
## final line found on 'C:\Users\Alice\Desktop\GRAD
## SCHOOL\STA2201\STA2201AppliedStatAliceHuang\lab9_1.stan'
```

```
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.000129 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 1.29 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
```

```
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.148 seconds (Warm-up)
## Chain 1:                0.127 seconds (Sampling)
## Chain 1:                0.275 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 3.1e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.31 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.135 seconds (Warm-up)
## Chain 2:                0.12 seconds (Sampling)
## Chain 2:                0.255 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 2.6e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.26 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
```

```
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.133 seconds (Warm-up)
## Chain 3:                0.169 seconds (Sampling)
## Chain 3:                0.302 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 4.2e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.42 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.173 seconds (Warm-up)
## Chain 4:                0.126 seconds (Sampling)
## Chain 4:                0.299 seconds (Total)
## Chain 4:
```

```
mod1
```

```
## Inference for Stan model: anon_model.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##                  mean se_mean   sd  2.5%   25%   50%   75% 97.5%
## alpha           -0.01    0.00 0.02 -0.05 -0.02 -0.01  0.00  0.03
## beta             2.42    0.00 0.18  2.06  2.30  2.42  2.54  2.76
## log_theta[1]     0.17    0.00 0.02  0.12  0.15  0.17  0.19  0.22
## log_theta[2]     0.15    0.00 0.02  0.10  0.13  0.15  0.16  0.19
## log_theta[3]     0.55    0.00 0.05  0.46  0.52  0.55  0.59  0.64
## log_theta[4]     0.31    0.00 0.03  0.24  0.29  0.31  0.33  0.37
## log_theta[5]     0.38    0.00 0.04  0.31  0.35  0.38  0.40  0.45
## log_theta[6]     0.40    0.00 0.04  0.32  0.37  0.40  0.42  0.47
## log_theta[7]     0.59    0.00 0.05  0.49  0.56  0.59  0.62  0.69
## log_theta[8]     0.60    0.00 0.05  0.50  0.57  0.60  0.64  0.70
## log_theta[9]    -0.17    0.00 0.02 -0.21 -0.18 -0.17 -0.15 -0.12
## log_theta[10]    0.01    0.00 0.02 -0.03  0.00  0.01  0.02  0.05
## log_theta[11]    0.20    0.00 0.03  0.15  0.19  0.20  0.22  0.25
## log_theta[12]    0.37    0.00 0.04  0.30  0.35  0.37  0.40  0.44
## log_theta[13]    0.14    0.00 0.02  0.09  0.12  0.14  0.16  0.19
```

```
## log_theta[14]     0.20    0.00 0.03    0.14    0.18    0.20    0.21    0.25
## log_theta[15]     0.08    0.00 0.02    0.04    0.07    0.08    0.10    0.13
## log_theta[16]     0.26    0.00 0.03    0.20    0.24    0.26    0.28    0.32
## log_theta[17]     0.20    0.00 0.03    0.15    0.18    0.20    0.22    0.25
## log_theta[18]     0.03    0.00 0.02   -0.01    0.02    0.03    0.05    0.08
## log_theta[19]     0.49    0.00 0.04    0.41    0.46    0.49    0.52    0.57
## log_theta[20]     0.17    0.00 0.02    0.12    0.15    0.17    0.19    0.22
## log_theta[21]     0.48    0.00 0.04    0.40    0.45    0.48    0.51    0.56
## log_theta[22]     0.31    0.00 0.03    0.25    0.29    0.31    0.33    0.37
## log_theta[23]     0.28    0.00 0.03    0.22    0.26    0.28    0.30    0.34
## log_theta[24]     0.17    0.00 0.03    0.12    0.16    0.17    0.19    0.22
## log_theta[25]     0.50    0.00 0.04    0.42    0.47    0.50    0.53    0.59
## log_theta[26]     0.04    0.00 0.02    0.00    0.02    0.04    0.05    0.08
## log_theta[27]     0.22    0.00 0.03    0.17    0.21    0.22    0.24    0.28
## log_theta[28]     0.31    0.00 0.03    0.24    0.29    0.31    0.33    0.37
## log_theta[29]     0.44    0.00 0.04    0.36    0.42    0.44    0.47    0.52
## log_theta[30]     0.60    0.00 0.05    0.50    0.56    0.60    0.63    0.69
## log_theta[31]     0.29    0.00 0.03    0.23    0.27    0.29    0.31    0.35
## log_theta[32]     0.42    0.00 0.04    0.34    0.39    0.42    0.44    0.49
## log_theta[33]    -0.13    0.00 0.02   -0.18   -0.15   -0.13   -0.12   -0.09
## log_theta[34]     0.12    0.00 0.02    0.07    0.10    0.12    0.13    0.16
## log_theta[35]     0.29    0.00 0.03    0.23    0.27    0.29    0.31    0.35
## log_theta[36]     0.63    0.00 0.05    0.52    0.59    0.63    0.66    0.73
## log_theta[37]     0.10    0.00 0.02    0.05    0.08    0.10    0.11    0.14
## log_theta[38]     0.24    0.00 0.03    0.18    0.22    0.24    0.26    0.29
## log_theta[39]    -0.20    0.00 0.03   -0.25   -0.22   -0.20   -0.19   -0.15
## log_theta[40]     0.36    0.00 0.03    0.29    0.34    0.36    0.39    0.43
## log_theta[41]     0.03    0.00 0.02   -0.01    0.02    0.03    0.05    0.07
## log_theta[42]     0.02    0.00 0.02   -0.02    0.01    0.02    0.04    0.06
## log_theta[43]     0.18    0.00 0.03    0.13    0.17    0.18    0.20    0.23
## log_theta[44]     0.07    0.00 0.02    0.03    0.06    0.07    0.09    0.12
## log_theta[45]    -0.04    0.00 0.02   -0.08   -0.05   -0.04   -0.02    0.01
## log_theta[46]     0.35    0.00 0.03    0.28    0.33    0.35    0.38    0.42
## log_theta[47]    -0.24    0.00 0.03   -0.29   -0.25   -0.24   -0.22   -0.18
## log_theta[48]    -0.05    0.00 0.02   -0.09   -0.07   -0.05   -0.04   -0.01
## log_theta[49]     0.14    0.00 0.02    0.09    0.12    0.14    0.15    0.19
## log_theta[50]    -0.13    0.00 0.02   -0.17   -0.14   -0.13   -0.11   -0.08
## log_theta[51]     0.43    0.00 0.04    0.36    0.41    0.43    0.46    0.51
## log_theta[52]     0.29    0.00 0.03    0.23    0.27    0.29    0.31    0.35
## log_theta[53]    -0.09    0.00 0.02   -0.13   -0.10   -0.09   -0.08   -0.05
## log_theta[54]    -0.17    0.00 0.02   -0.22   -0.19   -0.17   -0.16   -0.13
## log_theta[55]     0.16    0.00 0.02    0.11    0.14    0.16    0.17    0.20
## log_theta[56]     0.17    0.00 0.02    0.12    0.15    0.17    0.18    0.21
## log_theta[57]    -0.20    0.00 0.03   -0.25   -0.22   -0.20   -0.19   -0.15
## log_theta[58]     0.45    0.00 0.04    0.37    0.42    0.45    0.48    0.53
## log_theta[59]    -0.17    0.00 0.02   -0.21   -0.18   -0.17   -0.15   -0.12
## log_theta[60]    -0.33    0.00 0.03   -0.39   -0.35   -0.33   -0.31   -0.26
## log_theta[61]    -0.01    0.00 0.02   -0.05   -0.02   -0.01    0.00    0.03
## log_theta[62]    -0.12    0.00 0.02   -0.16   -0.13   -0.12   -0.10   -0.07
## log_theta[63]     0.12    0.00 0.02    0.07    0.10    0.12    0.13    0.16
## log_theta[64]    -0.10    0.00 0.02   -0.15   -0.12   -0.10   -0.09   -0.06
## log_theta[65]    -0.21    0.00 0.03   -0.26   -0.23   -0.21   -0.19   -0.16
## log_theta[66]    -0.24    0.00 0.03   -0.29   -0.25   -0.24   -0.22   -0.18
## log_theta[67]    -0.32    0.00 0.03   -0.38   -0.34   -0.32   -0.30   -0.26
```

```
## log_theta[68]     0.12    0.00 0.02   0.08   0.11   0.12   0.14   0.17
## log_theta[69]     0.33    0.00 0.03   0.26   0.31   0.33   0.35   0.39
## log_theta[70]    -0.34    0.00 0.03  -0.40  -0.36  -0.34  -0.32  -0.28
## log_theta[71]    -0.23    0.00 0.03  -0.28  -0.25  -0.23  -0.21  -0.18
## log_theta[72]    -0.27    0.00 0.03  -0.33  -0.29  -0.27  -0.25  -0.22
## log_theta[73]     0.20    0.00 0.03   0.15   0.19   0.20   0.22   0.25
## log_theta[74]     0.30    0.00 0.03   0.23   0.28   0.30   0.32   0.36
## log_theta[75]    -0.01    0.00 0.02  -0.05  -0.02  -0.01   0.01   0.03
## log_theta[76]     0.20    0.00 0.03   0.14   0.18   0.20   0.21   0.25
## log_theta[77]     0.06    0.00 0.02   0.02   0.05   0.06   0.08   0.11
## log_theta[78]    -0.06    0.00 0.02  -0.10  -0.07  -0.06  -0.04  -0.02
## log_theta[79]     0.15    0.00 0.02   0.10   0.13   0.15   0.16   0.19
## log_theta[80]     0.18    0.00 0.03   0.13   0.17   0.18   0.20   0.23
## log_theta[81]     0.33    0.00 0.03   0.26   0.31   0.33   0.35   0.39
## log_theta[82]    -0.04    0.00 0.02  -0.09  -0.06  -0.04  -0.03   0.00
## log_theta[83]    -0.05    0.00 0.02  -0.10  -0.07  -0.05  -0.04  -0.01
## log_theta[84]     0.44    0.00 0.04   0.36   0.42   0.44   0.47   0.52
## log_theta[85]     0.25    0.00 0.03   0.20   0.23   0.25   0.27   0.31
## log_theta[86]    -0.21    0.00 0.03  -0.26  -0.22  -0.21  -0.19  -0.16
## log_theta[87]    -0.17    0.00 0.02  -0.21  -0.18  -0.17  -0.15  -0.12
## log_theta[88]    -0.02    0.00 0.02  -0.06  -0.04  -0.02  -0.01   0.02
## log_theta[89]    -0.11    0.00 0.02  -0.15  -0.12  -0.11  -0.09  -0.06
## log_theta[90]     0.16    0.00 0.02   0.11   0.15   0.16   0.18   0.21
## log_theta[91]    -0.24    0.00 0.03  -0.29  -0.25  -0.24  -0.22  -0.18
## log_theta[92]     0.25    0.00 0.03   0.20   0.23   0.25   0.27   0.31
## log_theta[93]    -0.38    0.00 0.03  -0.44  -0.40  -0.38  -0.36  -0.31
## log_theta[94]    -0.07    0.00 0.02  -0.11  -0.09  -0.07  -0.06  -0.03
## log_theta[95]    -0.09    0.00 0.02  -0.13  -0.10  -0.09  -0.07  -0.05
## log_theta[96]    -0.16    0.00 0.02  -0.20  -0.17  -0.16  -0.14  -0.11
## log_theta[97]    -0.28    0.00 0.03  -0.34  -0.30  -0.28  -0.26  -0.23
## log_theta[98]    -0.16    0.00 0.02  -0.21  -0.18  -0.16  -0.14  -0.11
## log_theta[99]    -0.09    0.00 0.02  -0.13  -0.10  -0.09  -0.08  -0.05
## log_theta[100]    0.35    0.00 0.03   0.28   0.32   0.35   0.37   0.41
## log_theta[101]   -0.09    0.00 0.02  -0.14  -0.11  -0.09  -0.08  -0.05
## log_theta[102]   -0.12    0.00 0.02  -0.16  -0.13  -0.12  -0.10  -0.07
## log_theta[103]    0.07    0.00 0.02   0.03   0.05   0.07   0.08   0.11
## log_theta[104]   -0.26    0.00 0.03  -0.32  -0.28  -0.26  -0.25  -0.21
## log_theta[105]   -0.10    0.00 0.02  -0.14  -0.11  -0.10  -0.08  -0.05
## log_theta[106]   -0.21    0.00 0.03  -0.26  -0.23  -0.21  -0.19  -0.16
## log_theta[107]    0.24    0.00 0.03   0.18   0.22   0.24   0.26   0.29
## log_theta[108]   -0.08    0.00 0.02  -0.12  -0.09  -0.08  -0.06  -0.03
## log_theta[109]   -0.11    0.00 0.02  -0.16  -0.13  -0.12  -0.10  -0.07
## log_theta[110]   -0.17    0.00 0.02  -0.22  -0.19  -0.17  -0.16  -0.12
## log_theta[111]   -0.25    0.00 0.03  -0.30  -0.26  -0.25  -0.23  -0.19
## log_theta[112]   -0.17    0.00 0.02  -0.21  -0.18  -0.17  -0.15  -0.12
## log_theta[113]   -0.23    0.00 0.03  -0.28  -0.24  -0.23  -0.21  -0.17
## log_theta[114]   -0.35    0.00 0.03  -0.42  -0.37  -0.35  -0.33  -0.29
## log_theta[115]   -0.17    0.00 0.02  -0.21  -0.18  -0.17  -0.15  -0.12
## log_theta[116]   -0.17    0.00 0.02  -0.22  -0.19  -0.17  -0.16  -0.12
## log_theta[117]   -0.21    0.00 0.03  -0.26  -0.23  -0.21  -0.20  -0.16
## log_theta[118]    0.30    0.00 0.03   0.24   0.28   0.30   0.32   0.36
## log_theta[119]   -0.23    0.00 0.03  -0.28  -0.24  -0.23  -0.21  -0.17
## log_theta[120]   -0.08    0.00 0.02  -0.12  -0.09  -0.08  -0.06  -0.04
## log_theta[121]    0.18    0.00 0.03   0.13   0.16   0.18   0.20   0.23
```

```
## log_theta[122]   -0.18    0.00 0.02   -0.22   -0.19   -0.18   -0.16   -0.13
## log_theta[123]   -0.14    0.00 0.02   -0.18   -0.15   -0.14   -0.12   -0.09
## log_theta[124]   -0.16    0.00 0.02   -0.21   -0.18   -0.16   -0.15   -0.12
## log_theta[125]   -0.02    0.00 0.02   -0.06   -0.04   -0.02   -0.01    0.02
## log_theta[126]    0.05    0.00 0.02    0.01    0.04    0.05    0.07    0.09
## log_theta[127]    0.24    0.00 0.03    0.19    0.22    0.24    0.26    0.30
## log_theta[128]   -0.06    0.00 0.02   -0.10   -0.07   -0.06   -0.04   -0.02
## log_theta[129]    0.02    0.00 0.02   -0.02    0.00    0.02    0.03    0.06
## log_theta[130]    0.02    0.00 0.02   -0.02    0.01    0.02    0.03    0.06
## log_theta[131]   -0.05    0.00 0.02   -0.10   -0.07   -0.05   -0.04   -0.01
## log_theta[132]   -0.08    0.00 0.02   -0.12   -0.09   -0.08   -0.06   -0.04
## log_theta[133]   -0.22    0.00 0.03   -0.27   -0.24   -0.22   -0.20   -0.17
## log_theta[134]   -0.26    0.00 0.03   -0.32   -0.28   -0.26   -0.25   -0.21
## log_theta[135]    0.05    0.00 0.02    0.01    0.04    0.05    0.07    0.10
## log_theta[136]   -0.10    0.00 0.02   -0.14   -0.11   -0.10   -0.08   -0.05
## log_theta[137]   -0.16    0.00 0.02   -0.21   -0.18   -0.16   -0.14   -0.11
## log_theta[138]   -0.24    0.00 0.03   -0.30   -0.26   -0.24   -0.23   -0.19
## log_theta[139]   -0.17    0.00 0.02   -0.22   -0.19   -0.17   -0.16   -0.12
## log_theta[140]   -0.06    0.00 0.02   -0.11   -0.08   -0.06   -0.05   -0.02
## log_theta[141]   -0.33    0.00 0.03   -0.39   -0.35   -0.33   -0.31   -0.27
## log_theta[142]   -0.32    0.00 0.03   -0.38   -0.34   -0.32   -0.30   -0.26
## log_theta[143]   -0.08    0.00 0.02   -0.12   -0.09   -0.08   -0.06   -0.04
## log_theta[144]   -0.29    0.00 0.03   -0.35   -0.31   -0.30   -0.28   -0.24
## log_theta[145]   -0.29    0.00 0.03   -0.34   -0.31   -0.29   -0.27   -0.23
## log_theta[146]   -0.26    0.00 0.03   -0.31   -0.28   -0.26   -0.24   -0.20
## log_theta[147]   -0.17    0.00 0.02   -0.21   -0.18   -0.17   -0.15   -0.12
## log_theta[148]   -0.21    0.00 0.03   -0.26   -0.23   -0.21   -0.19   -0.16
## log_theta[149]   -0.06    0.00 0.02   -0.10   -0.07   -0.06   -0.05   -0.02
## log_theta[150]   -0.18    0.00 0.02   -0.23   -0.20   -0.18   -0.17   -0.13
## log_theta[151]   -0.23    0.00 0.03   -0.28   -0.25   -0.23   -0.22   -0.18
## log_theta[152]    0.30    0.00 0.03    0.24    0.28    0.30    0.32    0.36
## log_theta[153]    0.14    0.00 0.02    0.09    0.12    0.14    0.15    0.18
## log_theta[154]   -0.09    0.00 0.02   -0.13   -0.10   -0.09   -0.07   -0.04
## log_theta[155]   -0.19    0.00 0.02   -0.24   -0.21   -0.19   -0.18   -0.14
## log_theta[156]   -0.11    0.00 0.02   -0.16   -0.13   -0.11   -0.10   -0.07
## log_theta[157]    0.02    0.00 0.02   -0.03    0.00    0.02    0.03    0.06
## log_theta[158]   -0.26    0.00 0.03   -0.32   -0.28   -0.26   -0.25   -0.21
## log_theta[159]   -0.23    0.00 0.03   -0.29   -0.25   -0.23   -0.22   -0.18
## log_theta[160]   -0.17    0.00 0.02   -0.22   -0.19   -0.17   -0.16   -0.12
## log_theta[161]   -0.27    0.00 0.03   -0.33   -0.29   -0.27   -0.25   -0.21
## log_theta[162]   -0.24    0.00 0.03   -0.29   -0.26   -0.24   -0.22   -0.19
## log_theta[163]   -0.26    0.00 0.03   -0.31   -0.27   -0.26   -0.24   -0.20
## log_theta[164]   -0.01    0.00 0.02   -0.05   -0.02   -0.01    0.01    0.04
## log_theta[165]   -0.12    0.00 0.02   -0.17   -0.14   -0.12   -0.11   -0.08
## log_theta[166]    0.01    0.00 0.02   -0.03    0.00    0.01    0.03    0.06
## log_theta[167]   -0.02    0.00 0.02   -0.06   -0.03   -0.02    0.00    0.02
## log_theta[168]   -0.08    0.00 0.02   -0.13   -0.10   -0.08   -0.07   -0.04
## log_theta[169]   -0.21    0.00 0.03   -0.26   -0.23   -0.21   -0.20   -0.16
## log_theta[170]    0.11    0.00 0.02    0.07    0.10    0.11    0.13    0.16
## log_theta[171]   -0.01    0.00 0.02   -0.05   -0.03   -0.01    0.00    0.03
## log_theta[172]    0.08    0.00 0.02    0.03    0.06    0.08    0.09    0.12
## log_theta[173]   -0.09    0.00 0.02   -0.14   -0.11   -0.09   -0.08   -0.05
## log_theta[174]   -0.29    0.00 0.03   -0.34   -0.31   -0.29   -0.27   -0.23
## log_theta[175]   -0.34    0.00 0.03   -0.41   -0.37   -0.34   -0.32   -0.28
```

```
## log_theta[176]   -0.32    0.00 0.03   -0.37   -0.34   -0.32   -0.30   -0.26
## log_theta[177]   -0.02    0.00 0.02   -0.06   -0.04   -0.02   -0.01    0.02
## log_theta[178]   -0.24    0.00 0.03   -0.29   -0.25   -0.24   -0.22   -0.18
## log_theta[179]   -0.17    0.00 0.02   -0.22   -0.19   -0.17   -0.15   -0.12
## log_theta[180]   -0.23    0.00 0.03   -0.29   -0.25   -0.23   -0.22   -0.18
## log_theta[181]   -0.27    0.00 0.03   -0.33   -0.29   -0.27   -0.26   -0.22
## log_theta[182]   -0.30    0.00 0.03   -0.35   -0.32   -0.30   -0.28   -0.24
## log_theta[183]   -0.17    0.00 0.02   -0.22   -0.19   -0.17   -0.16   -0.12
## log_theta[184]   -0.31    0.00 0.03   -0.37   -0.33   -0.31   -0.29   -0.25
## log_theta[185]   -0.21    0.00 0.03   -0.26   -0.23   -0.21   -0.19   -0.16
## log_theta[186]   -0.14    0.00 0.02   -0.19   -0.16   -0.14   -0.13   -0.10
## log_theta[187]    0.13    0.00 0.02    0.09    0.12    0.13    0.15    0.18
## log_theta[188]   -0.31    0.00 0.03   -0.37   -0.33   -0.31   -0.29   -0.25
## log_theta[189]   -0.07    0.00 0.02   -0.12   -0.09   -0.07   -0.06   -0.03
## log_theta[190]   -0.34    0.00 0.03   -0.40   -0.36   -0.34   -0.32   -0.27
## log_theta[191]   -0.33    0.00 0.03   -0.39   -0.35   -0.33   -0.31   -0.27
## log_theta[192]   -0.25    0.00 0.03   -0.30   -0.26   -0.25   -0.23   -0.19
## log_theta[193]   -0.26    0.00 0.03   -0.32   -0.28   -0.26   -0.25   -0.21
## log_theta[194]   -0.19    0.00 0.02   -0.24   -0.20   -0.19   -0.17   -0.14
## log_theta[195]   -0.39    0.00 0.03   -0.46   -0.41   -0.39   -0.37   -0.32
## lp__           3710.75    0.02 1.03 3708.04 3710.32 3711.09 3711.49 3711.75
##                n_eff Rhat
## alpha           3809    1
## beta            3838    1
## log_theta[1]    3892    1
## log_theta[2]    3881    1
## log_theta[3]    3884    1
## log_theta[4]    3911    1
## log_theta[5]    3877    1
## log_theta[6]    3868    1
## log_theta[7]    3883    1
## log_theta[8]    3882    1
## log_theta[9]    3709    1
## log_theta[10]   3817    1
## log_theta[11]   3904    1
## log_theta[12]   3879    1
## log_theta[13]   3878    1
## log_theta[14]   3902    1
## log_theta[15]   3851    1
## log_theta[16]   3922    1
## log_theta[17]   3902    1
## log_theta[18]   3827    1
## log_theta[19]   3886    1
## log_theta[20]   3891    1
## log_theta[21]   3887    1
## log_theta[22]   3909    1
## log_theta[23]   3923    1
## log_theta[24]   3893    1
## log_theta[25]   3886    1
## log_theta[26]   3829    1
## log_theta[27]   3911    1
## log_theta[28]   3911    1
## log_theta[29]   3887    1
## log_theta[30]   3882    1
```

```
## log_theta[31]   3920    1
## log_theta[32]   3873    1
## log_theta[33]   3793    1
## log_theta[34]   3868    1
## log_theta[35]   3920    1
## log_theta[36]   3881    1
## log_theta[37]   3859    1
## log_theta[38]   3916    1
## log_theta[39]   3725    1
## log_theta[40]   3884    1
## log_theta[41]   3826    1
## log_theta[42]   3821    1
## log_theta[43]   3897    1
## log_theta[44]   3846    1
## log_theta[45]   3801    1
## log_theta[46]   3889    1
## log_theta[47]   3740    1
## log_theta[48]   3798    1
## log_theta[49]   3878    1
## log_theta[50]   3793    1
## log_theta[51]   3882    1
## log_theta[52]   3919    1
## log_theta[53]   3793    1
## log_theta[54]   3712    1
## log_theta[55]   3886    1
## log_theta[56]   3889    1
## log_theta[57]   3725    1
## log_theta[58]   3888    1
## log_theta[59]   3710    1
## log_theta[60]   3767    1
## log_theta[61]   3809    1
## log_theta[62]   3793    1
## log_theta[63]   3867    1
## log_theta[64]   3793    1
## log_theta[65]   3729    1
## log_theta[66]   3740    1
## log_theta[67]   3767    1
## log_theta[68]   3870    1
## log_theta[69]   3901    1
## log_theta[70]   3767    1
## log_theta[71]   3737    1
## log_theta[72]   3753    1
## log_theta[73]   3904    1
## log_theta[74]   3917    1
## log_theta[75]   3810    1
## log_theta[76]   3901    1
## log_theta[77]   3841    1
## log_theta[78]   3797    1
## log_theta[79]   3881    1
## log_theta[80]   3897    1
## log_theta[81]   3901    1
## log_theta[82]   3799    1
## log_theta[83]   3797    1
## log_theta[84]   3887    1
```

```
## log_theta[85]   3920    1
## log_theta[86]   3728    1
## log_theta[87]   3710    1
## log_theta[88]   3806    1
## log_theta[89]   3793    1
## log_theta[90]   3888    1
## log_theta[91]   3739    1
## log_theta[92]   3920    1
## log_theta[93]   3768    1
## log_theta[94]   3795    1
## log_theta[95]   3793    1
## log_theta[96]   3721    1
## log_theta[97]   3756    1
## log_theta[98]   3714    1
## log_theta[99]   3793    1
## log_theta[100]  3892    1
## log_theta[101]  3793    1
## log_theta[102]  3793    1
## log_theta[103]  3844    1
## log_theta[104]  3750    1
## log_theta[105]  3793    1
## log_theta[106]  3728    1
## log_theta[107]  3915    1
## log_theta[108]  3794    1
## log_theta[109]  3793    1
## log_theta[110]  3712    1
## log_theta[111]  3743    1
## log_theta[112]  3709    1
## log_theta[113]  3735    1
## log_theta[114]  3767    1
## log_theta[115]  3711    1
## log_theta[116]  3712    1
## log_theta[117]  3730    1
## log_theta[118]  3915    1
## log_theta[119]  3735    1
## log_theta[120]  3794    1
## log_theta[121]  3895    1
## log_theta[122]  3714    1
## log_theta[123]  3778    1
## log_theta[124]  3708    1
## log_theta[125]  3805    1
## log_theta[126]  3835    1
## log_theta[127]  3917    1
## log_theta[128]  3797    1
## log_theta[129]  3820    1
## log_theta[130]  3821    1
## log_theta[131]  3797    1
## log_theta[132]  3794    1
## log_theta[133]  3733    1
## log_theta[134]  3750    1
## log_theta[135]  3837    1
## log_theta[136]  3793    1
## log_theta[137]  3716    1
## log_theta[138]  3742    1
```

```
## log_theta[139]  3712    1
## log_theta[140]  3796    1
## log_theta[141]  3767    1
## log_theta[142]  3767    1
## log_theta[143]  3794    1
## log_theta[144]  3760    1
## log_theta[145]  3758    1
## log_theta[146]  3748    1
## log_theta[147]  3710    1
## log_theta[148]  3729    1
## log_theta[149]  3796    1
## log_theta[150]  3716    1
## log_theta[151]  3738    1
## log_theta[152]  3916    1
## log_theta[153]  3877    1
## log_theta[154]  3793    1
## log_theta[155]  3722    1
## log_theta[156]  3793    1
## log_theta[157]  3819    1
## log_theta[158]  3750    1
## log_theta[159]  3739    1
## log_theta[160]  3712    1
## log_theta[161]  3752    1
## log_theta[162]  3741    1
## log_theta[163]  3747    1
## log_theta[164]  3811    1
## log_theta[165]  3793    1
## log_theta[166]  3819    1
## log_theta[167]  3807    1
## log_theta[168]  3793    1
## log_theta[169]  3730    1
## log_theta[170]  3865    1
## log_theta[171]  3809    1
## log_theta[172]  3847    1
## log_theta[173]  3793    1
## log_theta[174]  3758    1
## log_theta[175]  3767    1
## log_theta[176]  3767    1
## log_theta[177]  3805    1
## log_theta[178]  3740    1
## log_theta[179]  3711    1
## log_theta[180]  3739    1
## log_theta[181]  3753    1
## log_theta[182]  3761    1
## log_theta[183]  3712    1
## log_theta[184]  3767    1
## log_theta[185]  3729    1
## log_theta[186]  3764    1
## log_theta[187]  3876    1
## log_theta[188]  3767    1
## log_theta[189]  3794    1
## log_theta[190]  3767    1
## log_theta[191]  3767    1
## log_theta[192]  3743    1
```

```
## log_theta[193]  3750    1
## log_theta[194]  3719    1
## log_theta[195]  3769    1
## lp__            1873    1
##
## Samples were drawn using NUTS(diag_e) at Wed Mar 15 17:09:25 2023.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

Model 2

$$\log(\theta_i) = \alpha_i + \beta x_i$$

where

$$\alpha_i \sim N(0, 1)$$

```
mod2 <- stan(data=stan_data, file = "lab9_2.stan")
```

```
## Warning in readLines(file, warn = TRUE): incomplete
## final line found on 'C:\Users\Alice\Desktop\GRAD
## SCHOOL\STA2201\STA2201AppliedStatAliceHuang\lab9_2.stan'
```

```
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 6.9e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.69 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.746 seconds (Warm-up)
## Chain 1:                0.642 seconds (Sampling)
## Chain 1:                1.388 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 3.1e-05 seconds
```

```
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.31 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.693 seconds (Warm-up)
## Chain 2:                0.638 seconds (Sampling)
## Chain 2:                1.331 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 5.5e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.55 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.64 seconds (Warm-up)
## Chain 3:                0.574 seconds (Sampling)
## Chain 3:                1.214 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 4.8e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.48 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
```

```
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 1.069 seconds (Warm-up)
## Chain 4:                0.848 seconds (Sampling)
## Chain 4:                1.917 seconds (Total)
## Chain 4:
```

Model 3

$$\log \theta_i = \alpha_i + \beta x_i$$

with

$$\alpha_i \sim N(\mu, \sigma^2)$$

$$\mu \sim N(0, 1)$$

$$\sigma \sim N(0, 1)$$

```
mod3 <- stan(data=stan_data, file = "lab9_3.stan")
```

```
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 6.8e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.68 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
```

```
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.443 seconds (Warm-up)
## Chain 1:                0.422 seconds (Sampling)
## Chain 1:                0.865 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 4.3e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.43 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.748 seconds (Warm-up)
## Chain 2:                0.531 seconds (Sampling)
## Chain 2:                1.279 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 2.6e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.26 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.536 seconds (Warm-up)
## Chain 3:                0.464 seconds (Sampling)
```

```
## Chain 3:                 1 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 3.5e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.35 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.617 seconds (Warm-up)
## Chain 4:                0.453 seconds (Sampling)
## Chain 4:                1.07 seconds (Total)
## Chain 4:
```

Question 3

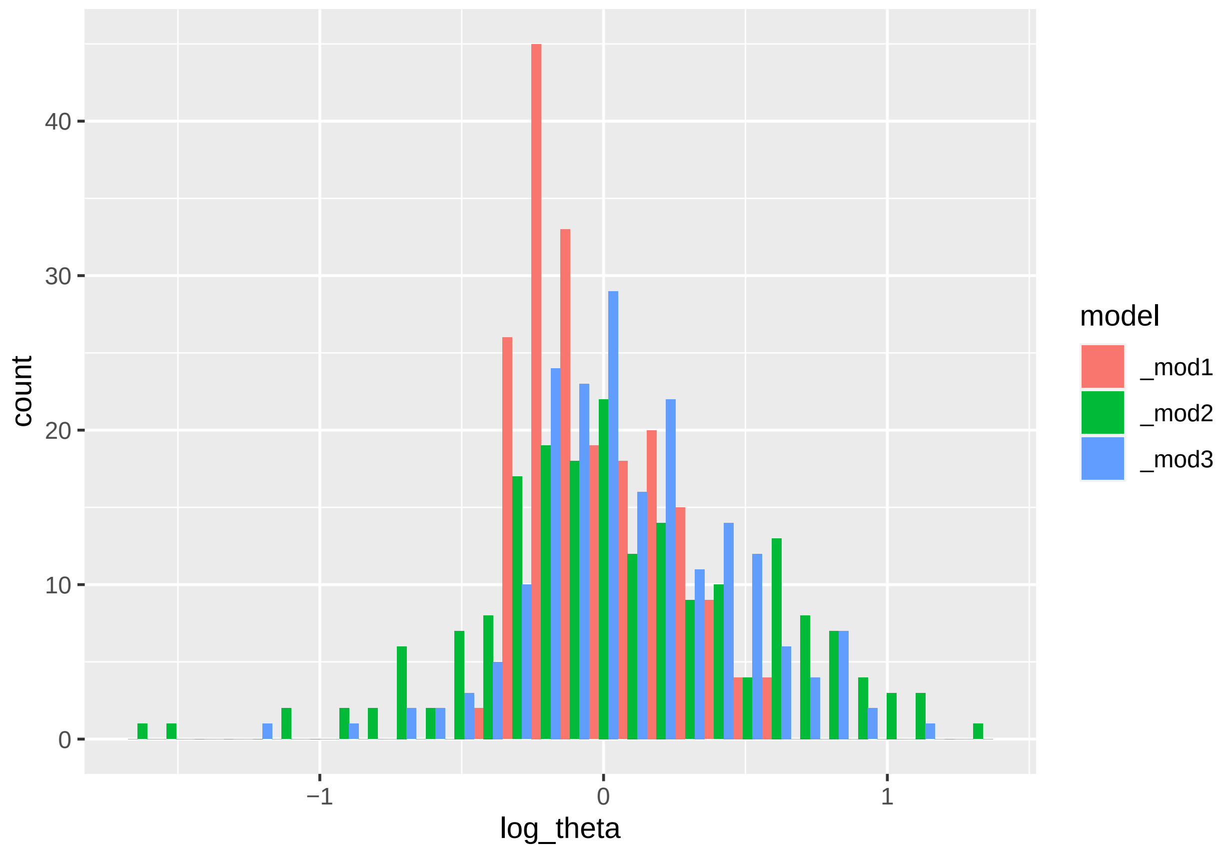
Make two plots (appropriately labeled and described) that illustrate the differences in estimated θ_i's across regions and the differences in θs across models.

```
mod1 %>% gather_draws(log_theta[i]) %>%
  median_qi() %>%
  rename(median_mod1 = .value, lower_mod1 = .lower, upper_mod1 = .upper) %>%
  select(i, median_mod1:upper_mod1) -> res_mod1

mod2 %>% gather_draws(log_theta[i]) %>%
  median_qi() %>%
  rename(median_mod2 = .value, lower_mod2 = .lower, upper_mod2 = .upper) %>%
  select(i, median_mod2:upper_mod2) -> res_mod2


mod3 %>% gather_draws(log_theta[i]) %>%
  median_qi() %>%
  rename(median_mod3 = .value, lower_mod3 = .lower, upper_mod3 = .upper) %>%
  select(i, median_mod3:upper_mod3) -> res_mod3

res <- res_mod1 %>% left_join(res_mod2) %>% left_join(res_mod3)
```

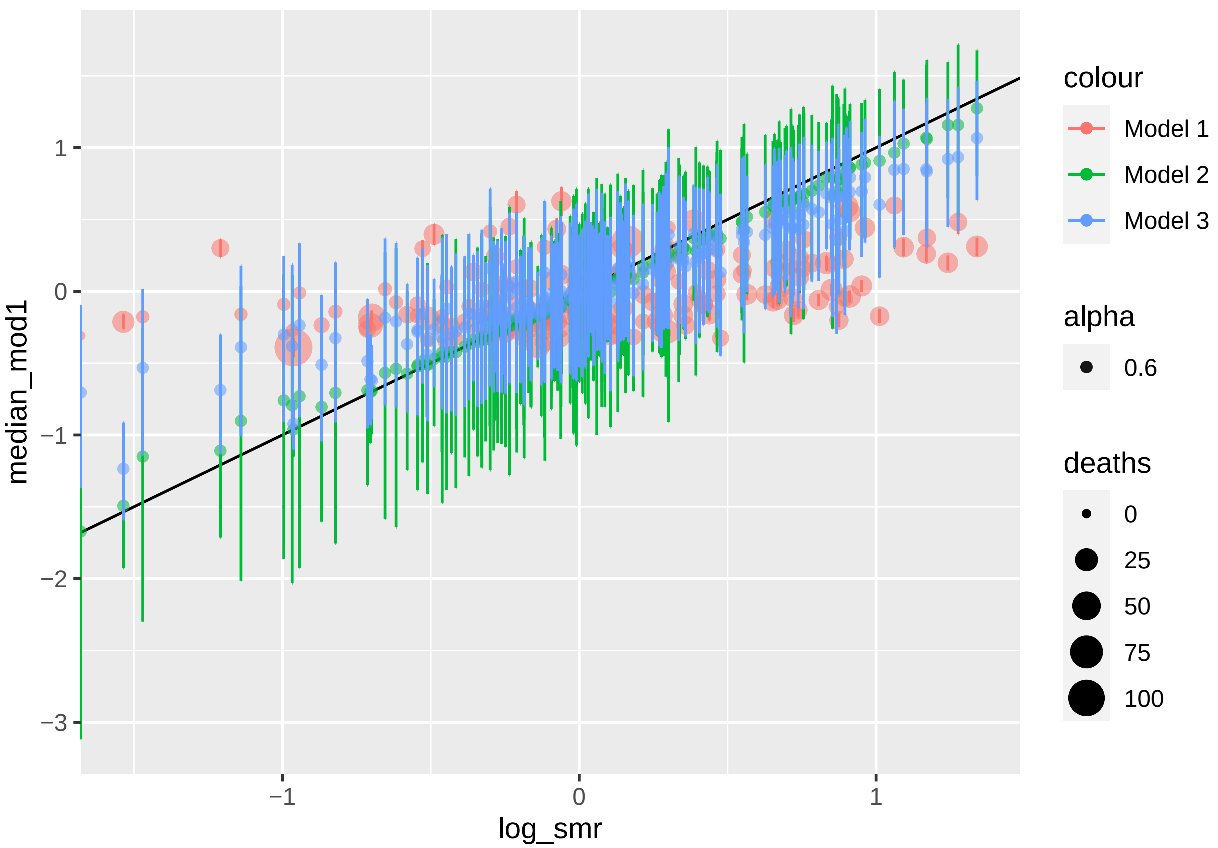
```
## Joining with `by = join_by(i)`
## Joining with `by = join_by(i)`
```

```
res %>% select(median_mod1, median_mod2, median_mod3) %>%
  pivot_longer(median_mod1:median_mod3, names_to = "model", values_to = "log_theta") %>%
  mutate(model = str_remove(model, "median")) %>%
  ggplot(aes(log_theta, fill = model)) + geom_histogram(position = "dodge")
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```
res %>% mutate(deaths = observe.i) %>%
  mutate(log_smr = log(observe.i/expect.i)) -> res
res %>%
  ggplot(aes(log_smr, median_mod1, colour = "Model 1")) +
  geom_point(aes(size = deaths, alpha = 0.6)) +
  geom_errorbar(aes(ymin = lower_mod1, ymax = upper_mod1, color="Model 1")) +
  geom_abline(slope = 1, intercept =0) +
  geom_point(aes(log_smr, median_mod2, color = "Model 2", alpha = 0.6)) +
  geom_errorbar(aes(ymin = lower_mod2, ymax = upper_mod2, color="Model 2"))+
  geom_point(aes(log_smr, median_mod3, color = "Model 3", alpha = 0.6)) +
  geom_errorbar(aes(ymin = lower_mod3, ymax = upper_mod3, color="Model 3"))
```



Model 2 and Model 3 look similar, but there is more uncertainty around Model 2 (bias variance tradeoff).