Checking for Feasibility

Before we discuss model checking response properties we discuss the problem of checking whether a given FDS is feasible.

A run of an FDS is an infinite sequence of states which satisfies the requirements of initiality and consecution but not necessarily any of the fairness requirements.

A state s of an FDS $\mathcal{D}$ is called reachable if it participates in some run of $\mathcal{D}$.

A state s is called feasible if it participates in some computation. The FDS is called feasible if it has at least one computation.

A set of states S is defined to be an F-set if it satisfies the following requirements:

F1. All states in S are reachable.

F2. Each state $s \in S$ has a ρ-successor in S.

F3. For every state $s \in S$ and every justice requirement $J \in \mathcal{J}$, there exists an S-path leading from s to some J-state.

F4. For every state $s \in S$ and every compassion requirement $(p, q) \in \mathcal{C}$, either there exists an S-path leading from s to some q-state, or s satisfies $\neg p$.

F-Sets Imply Feasibility

Claim 3. [F-sets]
A reachable state s is feasible iff it has a path leading to some F-set.

Proof:
Assume that s is a feasible state. Then it participates in some computation σ. Let S be the (finite) set of all states that appear infinitely many times in σ. We will show that S is an F-set. It is not difficult to see that there exists a cutoff position $t \geq 0$ such that S contains all the states that appear at positions beyond t.

Obviously all states appearing in σ are reachable. If $s \in S$ appears in σ at position $i > t$ then it has a successor $s_{i+1} \in \sigma$ which is also a member of S.

Let $s = s_i \in \sigma$, $i > t$ be a member of S and $J \in \mathcal{J}$ be some justice requirement. Since σ is a computation it contains infinitely many J-positions. Let $k \geq i$ one of the J-positions appearing later than i. Then the path $s_i, \ldots, s_k$ is an S-path leading from s to a J-state.

Let $s = s_i \in \sigma$, $i > t$ be a member of S and $(p, q) \in \mathcal{C}$ be some compassion requirement. There are two possibilities by which σ may satisfy (p, q). Either σ contains only finitely many p-positions, or σ contains infinitely many q positions. It follows that either S contains no p-states, or it contains some q-states which appear infinitely many times in σ. In the first case, s satisfies $\neg p$. In the second case, there exists a path leading from s_i to s_k, a q-state such that $k \geq i$.

Proof Continued

In the other direction, assume the existence of an F-set S and a reachable state s which has a path leading to some state $s_1 \in S$. We will show that there exists a computation σ which contains s.

Since s is reachable and has a path leading to state $s_1 \in S$, there exists a finite sequence of states π leading from an initial state to s_1 and passing through s. We will show how π can be extended to a computation by an infinite repetition of the following steps. At any point in the construction, we denote by *end*(π) the state which currently appears last in π.

- We know that *end*$(\pi) \in S$ has a successor $s \in S$. Append s to the end of π.
- Consider in turn each of the justice requirements $J \in \mathcal{J}$. We append to π the S-path π_J connecting *end*(π) to a J-state.
- Consider in turn each of the compassion requirements $(p, q) \in \mathcal{C}$. If there exists an S-path π_q, connecting *end*(π) to a q-state, we append π_q to the end of π. Otherwise, we do not modify π. We observe that if there does not exist an S-path leading from *end*(π) to a q-state, then *end*(π) and all of its progeny within S must satisfy $\neg p$.

It is not difficult to see that the infinite sequence constructed in this way is a computation. ∎

Computing F-Sets

Assume an assertion φ which characterizes an F-set. Translating the requirements 1–4 into formulas, we obtain the following requirements:

φ	$\rightarrow$	$\mathit{reachable}_{\mathcal{D}}$	
φ	$\rightarrow$	$\rho \diamond \varphi$	Every φ-state has a φ-successor
φ	$\rightarrow$	$(\varphi \wedge \rho)^* \diamond (\varphi \wedge J)$	For every $J \in \mathcal{J}$
φ	$\rightarrow$	$\neg p \ \vee \ (\varphi \wedge \rho)^* \diamond (\varphi \wedge q)$	For every $(p, q) \in \mathcal{C}$

This can be summarized as

$$\varphi \quad \rightarrow \quad \left[\begin{array}{lcl} \mathit{reachable}_{\mathcal{D}} & \wedge & \rho \diamond \varphi \qquad \wedge \\ \bigwedge\limits_{J \in \mathcal{J}} (\varphi \wedge \rho)^* \diamond (\varphi \wedge J) & \wedge & \bigwedge\limits_{(p,q) \in \mathcal{C}} \neg p \ \vee \ (\varphi \wedge \rho)^* \diamond (\varphi \wedge q) \end{array}\right]$$

Since we are interested in a maximal F-set, the computation can be expressed as:

$$\nu\varphi. \left[\begin{array}{lcl} \mathit{reachable}_{\mathcal{D}} & \wedge & \rho \diamond \varphi \qquad \wedge \\ \bigwedge\limits_{J \in \mathcal{J}} (\varphi \wedge \rho)^* \diamond (\varphi \wedge J) & \wedge & \bigwedge\limits_{(p,q) \in \mathcal{C}} \neg p \ \vee \ (\varphi \wedge \rho)^* \diamond (\varphi \wedge q) \end{array}\right]$$

Algorithmic Interpretation

Computing the maximal fix-point as a sequence of iterations, we can describe the computational process as follows:

Start by letting $\varphi := \mathit{reachable}_{\mathcal{D}}$. Then repeat the following steps:

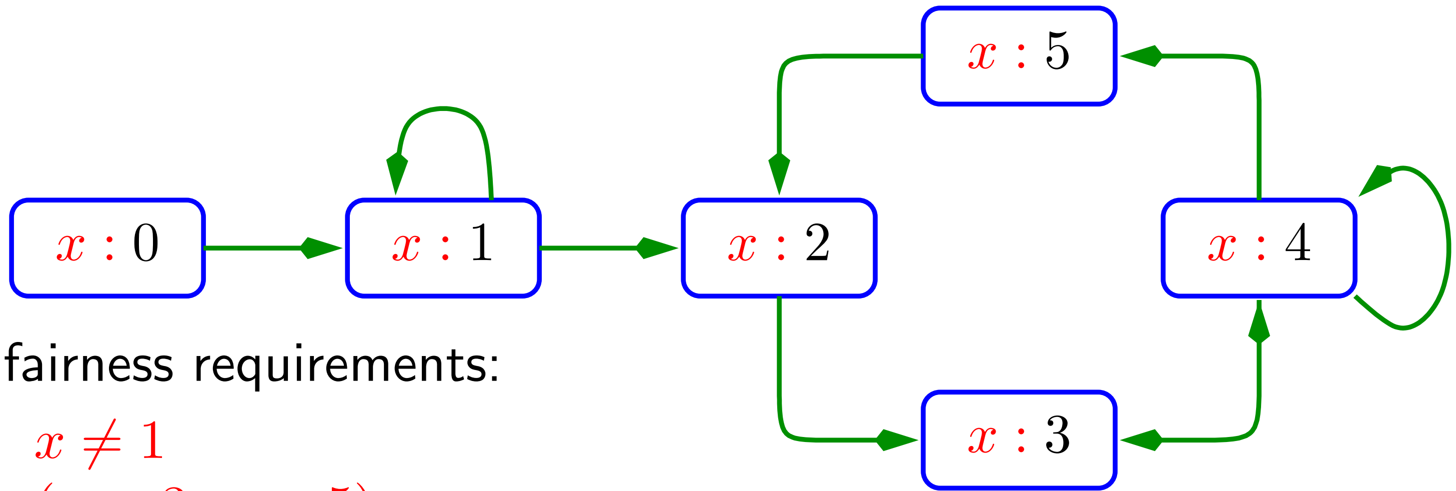
- Remove from φ all states which do not have a φ-successor.
- For each $J \in \mathcal{J}$, remove from φ all states which do not have a φ-path leading to a J-state.
- For each $(p, q) \in \mathcal{C}$, remove from φ all p-states which do not have a φ-path leading to a q-state.

until no further change.

To check whether an FDS $\mathcal{D}$ is feasible, we compute for it the maximal F-set and check whether it is empty. $\mathcal{D}$ is feasible iff the maximal F-set is not-empty.

Example

As an example, consider the following FDS:



with the fairness requirements:

$J_1: \quad x \neq 1$
$C_1: \quad (x=3,\ x=5)$
$C_2: \quad (x=2,\ x=1)$

We set $\varphi_0 : \{0..5\}$ and then proceed as follows:

- Removing from φ_0 all $(x=2)$-states which do not have a φ_0-path leading to an $(x=1)$-state, we are left with $\varphi_1 : \{0,1,3,4,5\}$.
- Successively removing from φ_1 all states without successors, leaves $\varphi_2 : \{3,4\}$.
- Removing from φ_2 all $(x=3)$-states which do not have a φ_2-path leading to a $(x=5)$-state, we are left with $\varphi_3 : \{4\}$.
- No reasons to remove any further states from $\varphi_3 : \{4\}$, so this is our final set.

We conclude that the above FDS is feasible.

Verifying Response Properties Through Feasibility Checking

Let $\mathcal{D} : \langle V, \Theta, \rho, \mathcal{J}, \mathcal{C} \rangle$ be an FDS and $p \Rightarrow \Diamond q$ be a response property we wish to verify over $\mathcal{D}$. Let *reachable*$_{\mathcal{D}}$ be the assertion characterizing all the reachable states in $\mathcal{D}$.

We define an auxiliary FDS $\mathcal{D}_{p,q} : \langle V, \Theta_{p,q}, \rho_{p,q}, \mathcal{J}, \mathcal{C} \rangle$, where

$$\begin{array}{ll} \Theta_{p,q}: & \textit{reachable}_{\mathcal{D}} \ \wedge \ p \ \wedge \ \neg q \\ \rho_{p,q}: & \rho \ \wedge \ \neg q' \end{array}$$

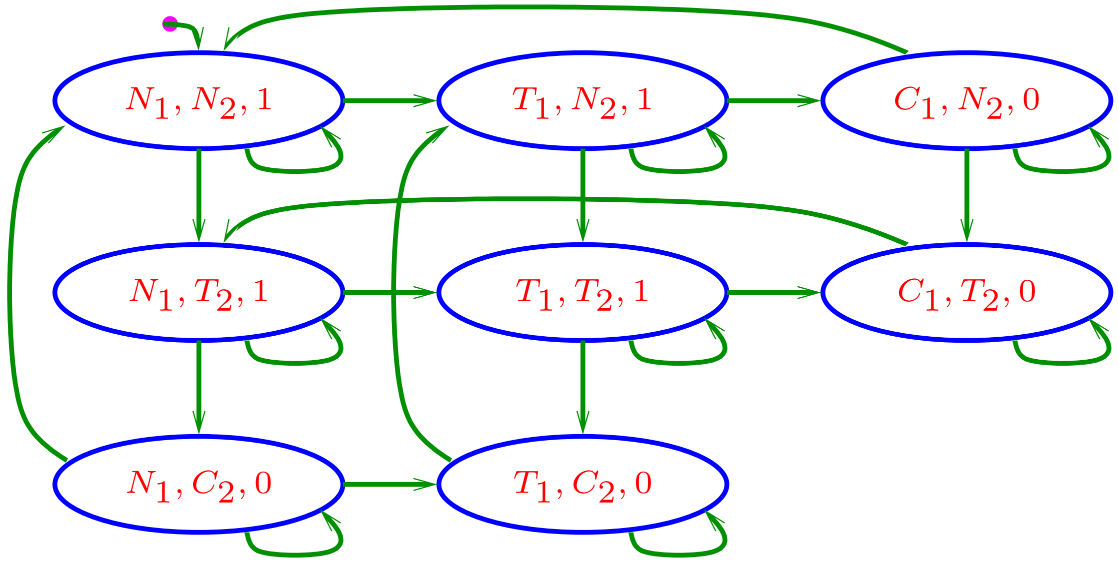
Thus, $\Theta_{p,q}$ characterizes all the $\mathcal{D}$-reachable p-states which do not satisfy q, while $\rho_{p,q}$ allows any ρ-step as long as the successor does not satisfy q.

Claim 4. [Model Checking Response]
$\mathcal{D} \models p \Rightarrow \Diamond q$ *iff* $\mathcal{D}_{p,q}$ *is unfeasible.*

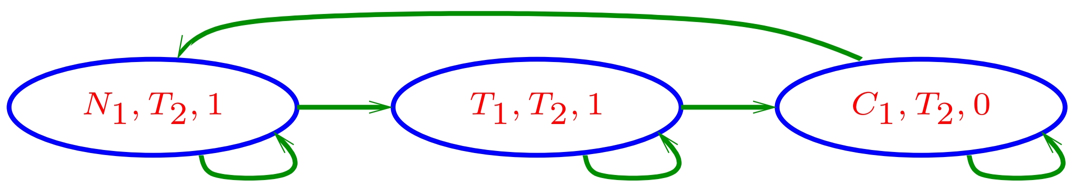
Proof: The claim is justifed by the observation that every computation of $\mathcal{D}_{p,q}$ can be extendable to a computation of $\mathcal{D}$ which violates the reponse property $p \Rightarrow \Diamond q$. Indeed, let $\sigma : s_k, s_{k+1}, \ldots$ be a computation of $\mathcal{D}_{p,q}$. By the definition of $\Theta_{p,q}$, we know that s_k is a $\mathcal{D}$-reachable p-state. Thus, there exists, a finite sequence $s_0, \ldots, s_k$, such that s_0 is $\mathcal{D}$-initial. The infinite sequence $s_0, \ldots, s_{k-1}, s_k, s_{k+1}, \ldots$ is a computation of $\mathcal{D}$ which contains a p-state at position k, and has no following q-state. This sequence violates $p \Rightarrow \Diamond q$. ∎

Example: MUX-SEM

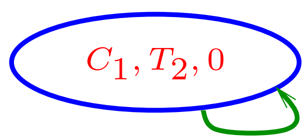
Following is the set of all reachable states of program MUX-SEM.



Assume we wish to verify the property $T_2 \Rightarrow \Diamond C_2$. We start by forming $\text{MUX-SEM}_{T_2,C_2}$, whose set of reachable states is given by:



First, we eliminate all $(T_2 \wedge y = 1)$-states which do not have a path leading to a C_2-state. This leaves us with:



Next, we eliminate all states which do not have a path leading to a $\neg C_1$-state. This leaves us with nothing. We conclude that MUX-SEM $\models T_2 \Rightarrow \Diamond C_2$.

Demonstrating what can be achieved by Formal Verification

We will illustrate how formal verification (when it works) can aid us in the development of reliable programs.

Consider the following program TRY-1 which attempts to solve the mutual exclusion problem by shared variables:

$$\textbf{local}\quad y_1, y_2\quad : \textbf{boolean where } y_1 = y_2 = 0$$

$$P_1 :: \left[\begin{array}{l} \ell_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} \ell_1 : \textbf{Non-Critical} \\ \ell_2 : \textbf{await } \neg y_2 \\ \ell_3 : y_1 := 1 \\ \ell_4 : \textbf{Critical} \\ \ell_5 : y_1 := 0 \end{array}\right] \end{array}\right] \quad \| \quad P_2 :: \left[\begin{array}{l} m_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} m_1 : \textbf{Non-Critical} \\ m_2 : \textbf{await } \neg y_1 \\ m_3 : y_2 := 1 \\ m_4 : \textbf{Critical} \\ m_5 : y_2 := 0 \end{array}\right] \end{array}\right]$$

Variables y_1 and y_2 signify whether processes P_1 and P_2 are interested in entering their critical sections.

Program Properties: Invariance

For program TRY-1, the property of mutual exclusion can be specified by requiring that the assertion

$$\varphi_{exclusion}: \quad \neg(at\_\ell_4 \ \wedge \ at\_m_4)$$

be an invariant of TRY-1. This implies that no execution of TRY-1 can ever get to a state in which both processes execute their critical sections at the same time.

Invoking TLV

To check whether assertion $\varphi_{exclusion}$ is an invariant of program TRY-1, we invoke the model checking tool TLV, a model checker based on the SMV tool developed in CMU by Ken McMillan and Ed Clarke.

We prepare two input files: `try1.spl` which contains the SPL representation of TRY-1, and `try1.pf`, a proof script file. The proof script file contains some printing commands, definition of the assertion $\varphi_{exclusion}$ and a command to check its invariance over the program.

We will present each of these input files.

File try1.spl

```
local y1 : bool where y1 = F;
      y2 : bool where y2 = F;

P1:: [l_0: loop forever do [
        l_1: noncritical;
        l_2: await !y2;
        l_3: y1 := T;
        l_4: critical;
        l_5: y1 := F       ]
     ]
||
P2:: [m_0: loop forever do [
        m_1: noncritical;
        m_2: await !y1;
        m_3: y2 := T;
        m_4: critical;
        m_5: y2 := F       ]
     ]
```

File try1.pf

```
Print "Check for Mutual Exclusion\n";

Let exclusion := !(at_l_4 & at_m_4);
Call Invariance(exclusion);
```

The call to procedure `Invariance` invokes the process which checks whether any reachable state violates the assertion `exclusion`.

Results of Verifying TRY-1

The results of model-checking TRY-1 are

```
>> Load "try1.pf";
Check for Mutual Exclusion
Model checking Invariance Property
*** Property is NOT VALID ***
Counter-Example Follows:
---- State no. 1 =
pi1 = l_0,   pi2 = m_0,   y1 = 0,       y2 = 0,
---- State no. 2 =
pi1 = l_1,   pi2 = m_0,   y1 = 0,       y2 = 0,
---- State no. 3 =
pi1 = l_1,   pi2 = m_1,   y1 = 0,       y2 = 0,
---- State no. 4 =
pi1 = l_1,   pi2 = m_2,   y1 = 0,       y2 = 0,
---- State no. 5 =
pi1 = l_1,   pi2 = m_3,   y1 = 0,       y2 = 0,
---- State no. 6 =
pi1 = l_2,   pi2 = m_3,   y1 = 0,       y2 = 0,
---- State no. 7 =
pi1 = l_3,   pi2 = m_3,   y1 = 0,       y2 = 0,
---- State no. 8 =
pi1 = l_3,   pi2 = m_4,   y1 = 0,       y2 = 1,
---- State no. 9 =
pi1 = l_4,   pi2 = m_4,   y1 = 1,       y2 = 1,
```

Expressed in a More Readable Form

$$\textbf{local} \quad y_1, y_2 \quad : \textbf{boolean where } y_1 = y_2 = 0$$

$$P_1 :: \left[\begin{array}{l} \ell_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} \ell_1 : \textbf{Non-Critical} \\ \ell_2 : \textbf{await } \neg y_2 \\ \ell_3 : y_1 := 1 \\ \ell_4 : \textbf{Critical} \\ \ell_5 : y_1 := 0 \end{array}\right] \end{array}\right] \quad \| \quad P_2 :: \left[\begin{array}{l} m_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} m_1 : \textbf{Non-Critical} \\ m_2 : \textbf{await } \neg y_1 \\ m_3 : y_2 := 1 \\ m_4 : \textbf{Critical} \\ m_5 : y_2 := 0 \end{array}\right] \end{array}\right]$$

The counter example is:

$$\begin{array}{l} \langle \ell_0,\, m_0,\, y_1 : 0,\, y_2 : 0 \rangle, \langle \ell_1,\, m_0,\, y_1 : 0,\, y_2 : 0 \rangle, \langle \ell_1,\, m_1,\, y_1 : 0,\, y_2 : 0 \rangle, \\ \langle \ell_1,\, m_2,\, y_1 : 0,\, y_2 : 0 \rangle, \langle \ell_1,\, m_3,\, y_1 : 0,\, y_2 : 0 \rangle, \langle \ell_2,\, m_3,\, y_1 : 0,\, y_2 : 0 \rangle, \\ \langle \ell_3,\, m_3,\, y_1 : 0,\, y_2 : 0 \rangle, \langle \ell_3,\, m_4,\, y_1 : 0,\, y_2 : 1 \rangle, \langle \ell_4,\, m_4,\, y_1 : 1,\, y_2 : 1 \rangle \end{array}$$

reaching the state $\langle \ell_4,\, m_4,\, y_1 : 1,\, y_2 : 1 \rangle$ which violates mutual exclusion!

Obviously, the problem is that the processes test each other's y value first and only later set their own y.

Second Attempt: Set first and Test Later

The following program TRY-1 interchange the order of testing and setting:

$$\textbf{local} \quad y_1, y_2 \quad : \textbf{boolean where } y_1 = y_2 = 0$$

$$P_1 :: \left[\begin{array}{l} \ell_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} \ell_1 : \textbf{Non-Critical} \\ \ell_2 : y_1 := 1 \\ \ell_3 : \textbf{await } \neg y_2 \\ \ell_4 : \textbf{Critical} \\ \ell_5 : y_1 := 0 \end{array}\right] \end{array}\right] \quad \| \quad P_2 :: \left[\begin{array}{l} m_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} m_1 : \textbf{Non-Critical} \\ m_2 : y_2 := 1 \\ m_3 : \textbf{await } \neg y_1 \\ m_4 : \textbf{Critical} \\ m_5 : y_2 := 0 \end{array}\right] \end{array}\right]$$

Let us see whether the program is now correct.

Program Properties: Absence of Deadlock

A state s is said to be a deadlock state if no process can perform any action. In our FDS model, the idling transition is always enabled. Therefore, we define s to be a deadlock state if it has no $\mathcal{D}$-successor different from itself.

Mathematically, we can characterize all deadlock states by the assertion

$$\delta : \quad \neg\exists V' \neq V : \rho(V, V')$$

and then check for the invariance of the assertion $\neg\delta$.

To check for the interesting properties of program TRY-2, we prepare the following script file:

```
Print "Check for Mutual Exclusion\n";
Let exclusion := !(at_l_4 & at_m_4);
Call Invariance(exclusion);
Run check_deadlock;
```

Model Checking TRY-2

We obtain the following results:

```
>> Load "try2.pf";
Check for Mutual Exclusion
Model checking Invariance Property
*** Property is VALID ***
 Check for the absence of Deadlock.
Model checking Invariance Property
*** Property is NOT VALID ***
Counter-Example Follows:
---- State no. 1 =
pi1 = l_0,    pi2 = m_0,    y1 = 0,    y2 = 0,
---- State no. 2 =
pi1 = l_1,    pi2 = m_0,    y1 = 0,    y2 = 0,
---- State no. 3 =
pi1 = l_1,    pi2 = m_1,    y1 = 0,    y2 = 0,
---- State no. 4 =
pi1 = l_1,    pi2 = m_2,    y1 = 0,    y2 = 0,
---- State no. 5 =
pi1 = l_1,    pi2 = m_3,    y1 = 0,    y2 = 1,
---- State no. 6 =
pi1 = l_2,    pi2 = m_3,    y1 = 0,    y2 = 1,
---- State no. 7 =
pi1 = l_3,    pi2 = m_3,    y1 = 1,    y2 = 1,
```

In a More Readable Form

$$\textbf{local}\quad y_1, y_2 \quad : \textbf{boolean where } y_1 = y_2 = 0$$

$$P_1 :: \left[\begin{array}{l} \ell_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} \ell_1 : \textbf{Non-Critical} \\ \ell_2 : y_1 := 1 \\ \ell_3 : \textbf{await } \neg y_2 \\ \ell_4 : \textbf{Critical} \\ \ell_5 : y_1 := 0 \end{array}\right] \end{array}\right] \quad \| \quad P_2 :: \left[\begin{array}{l} m_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} m_1 : \textbf{Non-Critical} \\ m_2 : y_2 := 1 \\ m_3 : \textbf{await } \neg y_1 \\ m_4 : \textbf{Critical} \\ m_5 : y_2 := 0 \end{array}\right] \end{array}\right]$$

The counter example is:

$$\langle \ell_0, m_0, y_1 : 0, y_2 : 0\rangle, \langle \ell_1, m_0, y_1 : 0, y_2 : 0\rangle, \langle \ell_1, m_1, y_1 : 0, y_2 : 0\rangle,$$
$$\langle \ell_1, m_2, y_1 : 0, y_2 : 0\rangle, \langle \ell_1, m_3, y_1 : 0, y_2 : 1\rangle, \langle \ell_2, m_3, y_1 : 0, y_2 : 1\rangle,$$
$$\langle \ell_3, m_3, y_1 : 1, y_2 : 1\rangle$$

reaching the deadlock state $\langle \ell_3, m_3, y_1 : 1, y_2 : 1\rangle$!

Try a Different Approach

The following program TRY-3 uses a variable *turn* to indicate which process has the higher priority.

$$\textbf{local} \quad turn \quad : [1..2] \ \textbf{where} \ turn = 0$$

$$P_1 :: \left[\begin{array}{l} \ell_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} \ell_1 : \textbf{Non-Critical} \\ \ell_2 : \textbf{await} \ turn = 1 \\ \ell_3 : \textbf{Critical} \\ \ell_4 : turn := 2 \end{array}\right] \end{array}\right] \quad \| \quad P_2 :: \left[\begin{array}{l} m_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} m_1 : \textbf{Non-Critical} \\ m_2 : \textbf{await} \ turn = 2 \\ m_3 : \textbf{Critical} \\ m_4 : turn := 1 \end{array}\right] \end{array}\right]$$

Program Properties: Response

This property refers to two assertions p and q. Written $p \Rightarrow \diamondsuit q$, it means

Every occurrence of a p-state must be followed by an occurence of a q-state

The response construct can be used to specify the property of accessibility. For example, the response property

$$at\_\ell_2 \Rightarrow \diamondsuit at\_\ell_3$$

requires for program TRY-3 that every visit to ℓ_2 must be followed by a visit to ℓ_3.

To model check this property, we prepare the following file try3.pf:

```
Print "Check for Mutual Exclusion\n";
Let exclusion := !(at_l_3 & at_m_3);
Call Invariance(exclusion);
Run check_deadlock;
Print "\n Check Accessibility for P1\n";
Call Temp_Entail(at_l_2,at_l_3);
Print "\n Check Accessibility for P2\n";
Call Temp_Entail(at_m_2,at_m_3);
```

Model Checking TRY-3

We obtain the following results:

```
>> Load "try3.pf";
Check for Mutual Exclusion
Model checking Invariance Property
*** Property is VALID ***
 Check for the absence of Deadlock.
Model checking Invariance Property
*** Property is VALID ***
 Check Accessibility for P1
Model checking...
*** Property is NOT VALID ***
Counter-Example Follows:
---- State no. 1 : pi1 = l_0,    pi2 = m_0,    turn = 1,
---- State no. 2 : pi1 = l_1,    pi2 = m_0,    turn = 1,
---- State no. 3 : pi1 = l_2,    pi2 = m_0,    turn = 1,
---- State no. 4 : pi1 = l_3,    pi2 = m_0,    turn = 1,
---- State no. 5 : pi1 = l_4,    pi2 = m_0,    turn = 1,
---- State no. 6 : pi1 = l_0,    pi2 = m_0,    turn = 2,
---- State no. 7 : pi1 = l_1,    pi2 = m_0,    turn = 2,
---- State no. 8 : pi1 = l_2,    pi2 = m_0,    turn = 2,

Loop back to state 8
```

In a More Readable Form

$$\textbf{local} \quad turn \quad : [1..2] \ \textbf{where} \ turn = 0$$

$$P_1 :: \left[\begin{array}{l} \ell_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} \ell_1 : \textbf{Non-Critical} \\ \ell_2 : \textbf{await} \ turn = 1 \\ \ell_3 : \textbf{Critical} \\ \ell_4 : turn := 2 \end{array}\right] \end{array}\right] \quad \| \quad P_2 :: \left[\begin{array}{l} m_0 : \textbf{loop forever do} \\ \quad \left[\begin{array}{l} m_1 : \textbf{Non-Critical} \\ m_2 : \textbf{await} \ turn = 2 \\ m_3 : \textbf{Critical} \\ m_4 : turn := 1 \end{array}\right] \end{array}\right]$$

The counter example is:

$$\begin{array}{lll} \langle \ell_0, m_0, turn : 1 \rangle, & \langle \ell_1, m_0, turn : 1 \rangle, & \langle \ell_2, m_0, turn : 1 \rangle \\ \langle \ell_3, m_0, turn : 1 \rangle, & \langle \ell_4, m_0, turn : 1 \rangle, & \langle \ell_0, m_0, turn : 2 \rangle \\ \langle \ell_1, m_0, turn : 2 \rangle, & \langle \ell_2, m_0, turn : 2 \rangle & \end{array}$$

Finally a good program for Mutual Exclusion

Following is a good shared variables solution to the mutual exclusion problem.

Peterson's for 2 Processes:

$$
\begin{array}{lll}
\textbf{local} & y_1, y_2 & : \textbf{boolean where } y_1 = y_2 = 0 \\
 & s & : \{1,2\} \textbf{ where } s = 1
\end{array}
$$

$$
\left[\begin{array}{l}
\ell_0 : \textbf{loop forever do} \\
\left[\begin{array}{l}
\ell_1 : \textbf{Non-Critical} \\
\ell_2 : (y_1, s) := (1, 1) \\
\ell_3 : \textbf{await } y_2 = 0 \ \vee \ s \neq 1 \\
\ell_4 : \textbf{Critical} \\
\ell_5 : y_1 := 0
\end{array}\right]
\end{array}\right]
\quad \| \quad
\left[\begin{array}{l}
m_0 : \textbf{loop forever do} \\
\left[\begin{array}{l}
m_1 : \textbf{Non-Critical} \\
m_2 : (y_2, s) := (1, 2) \\
m_3 : \textbf{await } y_1 = 0 \ \vee \ s \neq 2 \\
m_4 : \textbf{Critical} \\
m_5 : y_2 := 0
\end{array}\right]
\end{array}\right]
$$

$$- \quad P_1 \quad - \qquad\qquad\qquad - \quad P_2 \quad -$$

Variables y_1 and y_2 signify whether processes P_1 and P_2 are interested in entering their critical sections. Variable s serves as a tie-breaker. It always contains the signature of the last process to enter the waiting location (ℓ_3, m_3). Model checking this program, we find that it satisfies the three properties of (invariance of) mutual exclusion, absence of deadlock, and accessibility.