Process Algebra (2IMF10) — Assignment 2

Deadline: Friday May 26, 2023

This is the second assignment for the course on *Process Algebra* (2IMF10). Please submit your solutions via Canvas. The only accepted format for your document is PDF.

Minimal Process Theory with Disrupt and Recursion in Bisimulation Semantics

The starting point of this assignment is the process theory MPT+D(A), which is the extension of the process theory MPT(A) with the binary operation $\blacktriangleright$ that was also considered in Assignment 1.

By a *recursive specification* over MPT+D(A), we mean a recursive specification over the signature of MPT+D(A) and some set of recursion variables V_R in the sense of Definition 5.2.1 of [1]. We denote by $(\text{MPT+D})_{\text{rec}}(A)$ the extension of MPT(A) with disrupt and recursion. That is, the syntax of $(\text{MPT+D})_{\text{rec}}(A)$ is given by the following grammar:

$$p ::= 0 \;\mid\; a.p \;\mid\; p + p \;\mid p \blacktriangleright p \;\mid\; \mu X.E \;\;,$$

with a ranging over A, E ranging over recursive specifications over MPT+D(A), and X a recursion variable defined in E.

The theory $(\text{MPT+D})_{\text{rec}}(A)$ has the following axioms:

$x + y = y + x$	A1	
$(x + y) + z = x + (y + z)$	A2	
$x + x = x$	A3	
$x + 0 = x$	A6	
$\mu X.E = \mu t_X.E$	Rec	(where $(X = t_X) \in E$)
$0 \blacktriangleright x = x$	D1	
$a.x \blacktriangleright y = a.(x \blacktriangleright y) + y$	D2	
$(x + y) \blacktriangleright z = (x \blacktriangleright z) + (y \blacktriangleright z)$	D3	

Recall that $\mu t_X.E$ is the term obtained from t_X by replacing every occurrence of a recursion variable Y by $\mu Y.E$ (i.e., $\mu 0.E \equiv 0$, $\mu(a.t).E \equiv a.\mu t.E$, $\mu(t_1 + t_2).E \equiv \mu t_1.E + \mu t_2.E$, and $\mu(t_1 \blacktriangleright t_2).E \equiv \mu t_1.E \blacktriangleright \mu t_2.E$). To derive the equivalence of closed $(\text{MPT+D})_{\text{rec}}(A)$-terms, we may also use the recursion princple RSP: every guarded recursive specification over MPT(A) (without disrupt) has at most one solution. We write $(\text{MPT+D})_{\text{rec}}(A) + \text{RSP} \vdash p = q$ if the equation $p = q$ can be derived using the axioms of $(\text{MPT+D})_{\text{rec}}(A)$, the rules of equational logic, and the recursion principle RSP.

Whenever it is somehow clear from the context what is the recursive specification E in which recursion variable X is defined, it is fine to write just X instead of

$\mu X.E$. We shall not do so below not to avoid confusion, but feel free to do so in your solutions to the assignment.

The term deduction system for $(\text{MPT+D})_{\text{rec}}(A)$ consists of the rules for $\text{MPT}(A)$, the rules for recursion (see Table 5.2 in [1]), and the following rules for $\blacktriangleright$:

$$\frac{x \xrightarrow{a} x'}{x \blacktriangleright y \xrightarrow{a} x' \blacktriangleright y} \qquad \frac{y \xrightarrow{a} y'}{x \blacktriangleright y \xrightarrow{a} y'}$$

The relation $\leftrightarrow$ (see Definition 3.1.10 in [1]) is a congruence on the algebra of closed $(\text{MPT+D})_{\text{rec}}(A)$-terms; we refer to the quotient algebra $\mathbb{P}((\text{MPT+D})_{\text{rec}}(A))/\underline{\leftrightarrow}$ as the *term model* of $(\text{MPT+D})_{\text{rec}}(A)$. The theory $(\text{MPT+D})_{\text{rec}}(A) + \text{RSP}$ is sound for $\mathbb{P}((\text{MPT+D})_{\text{rec}}(A))/\underline{\leftrightarrow}$.

1. Consider the recursive specification E consisting of the following equation:

$$X = a.X \blacktriangleright b.Y$$
$$Y = b.Y \quad .$$

   (a) Give three formal derivations of transitions with $(\mu X.E) \blacktriangleright b.(\mu Y.E)$ as source.

   (b) Sketch the transition system associated with $\mu X.E$.

   (c) Give a finite, guarded recursive specification F over $\text{MPT}(A)$ (i.e., *not containing* $\blacktriangleright$) including a variable Z such that

$$(\text{MPT+D})_{\text{rec}}(A) + \text{RSP} \vdash \mu X.E = \mu Z.F \quad ,$$

   and **prove that your answer is correct**.

   (d) Formally explain how it follows from the result in (c) that the process denoted by $\mu X.E$ in $\mathbb{P}((\text{MPT+D})_{\text{rec}}(A))/\underline{\leftrightarrow}$ is regular.

2. Consider the recursive specification G consisting of the following equation:

$$X = a.X \blacktriangleright b.Y$$
$$Y = c.Y \quad .$$

   Prove that the process denoted by $\mu X.G$ in $\mathbb{P}((\text{MPT+D})_{\text{rec}}(A))/\underline{\leftrightarrow}$ is *not* regular.

References

[1] J. C. M. Baeten, T. Basten, and M. A. Reniers. *Process Algebra (Equational Theories of Communicating Processes)*. Cambridge University Press, 2010.