Question 1

implement a FA function with parameters: the dataset and the desired numbers of factors. These part is a bit like the PCA function implemented in the last homework.

```
from google.colab import drive
drive.mount('/content/drive')
```

```
Drive already mounted at /content/drive; to attempt to forcibly
remount, call drive.mount("/content/drive", force_remount=True).
```

```
import numpy as np
def manual_fa(X, num_factors):
    # Compute the covariance matrix
    cov_mx = np.cov(X.T)

    # Compute the eigenvectors and eigenvalues
    eigvals, eigvecs = np.linalg.eig(cov_mx)

    # Sort the eigenvectors and eigenvalues in descending order
    sort_idx = np.argsort(eigvals)[::-1]
    eigvals = np.real(eigvals[sort_idx])
    eigvecs = np.real(eigvecs[:, sort_idx])

    # Compute the loading matrix A
    loading_matrix = eigvecs[:, :num_factors]

    # Compute the factor matrix F
    factor_matrix = np.dot(X,loading_matrix)

    # Compute the communality factor of (h_i)^2
    communality = np.sum(loading_matrix**2, axis=1)

    # Compute the uniqueness vector of ψ_i
    uniqueness = 1 - communality

    # Compute the vector of the proportions of total variance
contributed by the ith factor
    prop_var = eigvals[:num_factors] / np.sum(eigvals)

    return loading_matrix, factor_matrix, communality, uniqueness,
prop_var

# Just a test case.

from sklearn.datasets import load_iris
iris = load_iris()
X = iris.data
loading_matrix, factor_matrix, communality, uniqueness, prop_va =
```

```
manual_fa(X,2)
print("Loading Matrix:\n", loading_matrix)
print("\nfactor_matrix:\n", factor_matrix)
print("\ncommunality:\n", communality)
print("\nuniqueness:\n", uniqueness)
print("\nprop_va:\n", prop_va)
```

```
Loading Matrix:
 [[ 0.36138659 -0.65658877]
 [-0.08452251 -0.73016143]
 [ 0.85667061  0.17337266]
 [ 0.3582892   0.07548102]]

factor_matrix:
 [[ 2.81823951 -5.64634982]
 [ 2.78822345 -5.14995135]
 [ 2.61337456 -5.18200315]
 [ 2.75702228 -5.0086536 ]
 [ 2.7736486  -5.65370709]
 [ 3.2215055  -6.06828303]
 [ 2.68182738 -5.23749119]
 [ 2.87622016 -5.49033754]
 [ 2.6159824  -4.74864082]
 [ 2.82960933 -5.21317833]
 [ 2.99541804 -5.97202148]
 [ 2.8896099  -5.34168252]
 [ 2.71625587 -5.09184058]
 [ 2.27856139 -4.81555799]
 [ 2.85761474 -6.50571721]
 [ 3.1163261  -6.66501491]
 [ 2.87883726 -6.13763209]
 [ 2.85406843 -5.63880172]
 [ 3.30254481 -6.19979162]
 [ 2.91437873 -5.84051289]
 [ 3.19210892 -5.71829851]
 [ 2.9586599  -5.75994864]
 [ 2.28642572 -5.46042065]
 [ 3.19963195 -5.42566143]
 [ 3.14661108 -5.28967072]
 [ 2.99569623 -5.1809357 ]
 [ 3.03354506 -5.45790407]
 [ 2.94004523 -5.69467143]
 [ 2.86283042 -5.63899256]
 [ 2.87037575 -5.12999135]
 [ 2.91496666 -5.12263409]
 [ 3.09243264 -5.73787684]
 [ 2.8535028  -6.1403164 ]
 [ 2.90362838 -6.42009834]
 [ 2.86543825 -5.20563023]
 [ 2.63612348 -5.39631705]
```

```
[ 2.87712708 -5.9263226 ]
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[ 2.52186309 -4.83899423]
[ 2.91235882 -5.55599641]
[ 2.73226271 -5.59048011]
[ 2.65299643 -4.385992  ]
[ 2.50495859 -4.98502652]
[ 3.09675065 -5.51582401]
[ 3.29287589 -5.76361572]
[ 2.78791371 -5.07674437]
[ 2.96421687 -5.83072372]
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[ 2.95927938 -5.9063626 ]
[ 2.79900535 -5.43465866]
[ 6.78719082 -6.01211305]
[ 6.43485366 -5.64528622]
[ 6.96666745 -5.83121539]
[ 5.68568285 -4.49899357]
[ 6.59046839 -5.40154325]
[ 6.14403422 -4.90870571]
[ 6.5974258  -5.61042085]
[ 4.75324246 -4.32206162]
[ 6.54649696 -5.55531448]
[ 5.49361973 -4.60387067]
[ 4.99452425 -4.06098139]
[ 6.01406369 -5.22297134]
[ 5.76734164 -4.77691611]
[ 6.48729964 -5.20213472]
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[ 6.43022591 -5.79413207]
[ 6.16264889 -4.97398291]
[ 5.73847013 -4.99334181]
[ 6.44709886 -4.78380703]
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[ 6.09037158 -4.84266516]
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```

```
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[ 5.74832281 -5.05842818]
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[ 6.39318033 -5.29248813]
[ 5.73291316 -4.92256673]
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[ 5.85934663 -4.82204248]
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[ 5.87858078 -5.03373365]
[ 6.14494114 -5.34469077]
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[ 5.80136597 -4.97805477]
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[ 6.91760101 -4.75203623]
[ 8.11904115 -5.67085573]
[ 7.47389619 -5.14722467]
[ 7.85237105 -5.28669163]
[ 8.89940387 -5.87778925]
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[ 7.82359395 -5.08312107]
[ 8.4191161  -6.10974453]
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[ 6.79349719 -5.21028393]
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[ 7.66180278 -5.109675  ]
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[ 8.57886506 -6.01503825]
```

```
 [ 7.64660845 -5.46701678]
 [ 7.40746328 -5.3762531 ]
 [ 6.67169147 -5.16196231]
 [ 7.60997628 -5.69924045]
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 [ 7.92111132 -5.63175077]
 [ 7.44647493 -5.51448488]
 [ 7.02953175 -4.95163559]
 [ 7.26671085 -5.40581143]
 [ 7.40330675 -5.44358054]
 [ 6.89255399 -5.04429164]]

communality:
 [0.56170908 0.54027978 0.76394261 0.13406853]

uniqueness:
 [0.43829092 0.45972022 0.23605739 0.86593147]

prop_va:
 [0.92461872 0.05306648]
```

question b

On the AutoMPG dataset, compare with the PCA results in HW05.

```
import pandas as pd
df = pd.read_csv('/content/drive/MyDrive/autompg.csv')
df.replace('?', np.nan, inplace=True)
df.dropna(inplace=True)
autompg_variables = df.drop(['mpg','car name'], axis=1)
autompg_mpg = df['mpg']
autompg_variables = pd.DataFrame(autompg_variables, dtype=float)

loading_matrix, factor_matrix, communality, uniqueness, prop_va =
manual_fa(autompg_variables, 2)
print("Loading Matrix:\n", loading_matrix)
print("\nfactor_matrix:\n", factor_matrix)
print("\ncommunality:\n", communality)
print("\nuniqueness:\n", uniqueness)
print("\nprop_va:\n", prop_va)
```

```
Loading Matrix:
 [[ 1.79262233e-03 -1.33244815e-02]
 [ 1.14341275e-01 -9.45778439e-01]
 [ 3.89670355e-02 -2.98248416e-01]
 [ 9.92673415e-01  1.20752748e-01]
 [-1.35283460e-03  3.48258394e-02]
 [-1.33684138e-03  2.38516836e-02]
```

```
 [-5.51538021e-04  3.24298106e-03]]

factor_matrix:
 [[3518.40010719   95.97553085]
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```

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```

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[2658.52248922  176.13771674]
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[2792.71925133  191.12655805]
[3408.72059144  220.27516606]
[1988.24476452  137.28057521]
[2133.09172864  147.23490994]
[3252.00303393  141.41317856]
[2994.15393469  148.98532414]
[2888.14065015  192.75731609]
[3270.99287968  150.43617784]
[3365.27797038  162.50744836]
[3851.69306073  138.77308422]
[3737.15633286  127.9524502 ]
[3971.42491457  106.69123792]
[3843.4478689   123.77222978]
[4374.00344598  151.53086035]
[4069.85396264  117.49107478]
[3613.87554581  147.81308075]
[3958.03175982   92.78199459]
[1923.72087328  129.42415499]
[1972.77554303  140.13365221]
[1915.17392434  117.03107317]
[2667.27138995  186.46738966]
[3527.93712605  232.73876565]
[3916.20128213  105.01911313]
```

```
[3185.38387839  233.37202203]
[3428.05698854  142.78371196]
[2198.49818946  147.76585056]
[2148.86221891  141.78741706]
[2017.32609408  146.65260769]
[2127.36872476  152.90778647]
[2671.08994573  155.14637818]
[2600.13906714  117.63625558]
[2704.36761113  130.37101551]
[2557.9289644   141.2830525 ]
[2142.3379739   145.91398864]
[1965.96881073  138.09151444]
[2118.27947901  144.83003087]
[2016.45021303  145.51241584]
[2679.02931978  156.15366477]
[2868.28476275  190.39397347]
[3001.64330931  195.52368104]
[3385.34059358  171.10521034]
[2185.97770882  151.61469961]
[2709.84399055  176.18839905]
[2539.89419443  173.56546062]
[2433.23694608  156.31265456]
[2263.55467482  151.38619114]
[2106.7814645   156.55315471]
[2801.29452922  161.60974738]
[2106.66536454  157.54420674]
[2081.7549654   154.95054052]
[2329.92061337  185.20837931]
[2944.70664992  224.33980543]
[3245.36284316  237.00066258]
[1849.34156379  119.68926514]
[2142.86058683  149.78292383]
[1843.95320127  122.51729613]
[2912.91935075  155.3658607 ]
[2414.05012154  198.50553215]
[2498.94096338  162.63823391]
[2288.1378327   156.30822004]
[2490.34320066  150.37008459]
[2636.10935919  152.77115618]
[2622.1054063   143.77533375]
[2728.98734349  134.91833192]
[2386.11628007  137.59353365]
[1753.30926016  122.38314283]
[1873.46604053  128.4396342 ]
[1758.58032612  120.47099219]
[2063.44564739  140.13959046]
[1972.65284617  141.27340255]
[2047.44665721  147.36835591]
[1983.38712871  135.79310629]
[2213.11054793  151.77178799]
```

```
 [2043.63185391  137.31304658]
 [2376.17136003  177.92193359]
 [2188.7227454   145.4511016 ]
 [2208.84304117  145.68655942]
 [2347.92841368  161.72974778]
 [2613.22150412  175.79980122]
 [2632.17143584  185.145427  ]
 [3225.44479708  235.4123659 ]
 [3156.26382253  224.31504378]
 [2902.3661838   158.99595056]
 [2929.78512292  182.27445602]
 [3420.55946441  163.49402158]
 [3741.69925565   89.95577518]
 [3063.75678925  156.55266623]
 [3468.53182934  182.69040071]
 [2602.02004925  184.97623355]
 [2636.7649716   189.16775391]
 [2393.56079673  159.56243503]
 [2572.12754535  182.12998849]
 [2525.08502748  154.63073022]
 [2735.60700148  163.13651356]
 [2863.47688995  188.58571551]
 [1980.25850338  120.15518453]
 [2023.08975222  140.72368476]
 [1968.49352612  134.06138809]
 [2123.76887434  140.92092714]
 [2123.23773729  145.54418449]
 [2161.20090636  123.50390715]
 [2203.87820197  145.11012995]
 [2243.50139785  150.56928554]
 [1963.49470938  133.66532559]
 [1963.49376239  133.68970367]
 [1993.27328841  137.32969905]
 [2948.28374728  154.07389894]
 [3026.05754199   93.39564368]
 [2587.36036466  139.57632182]
 [2845.00131059   91.90069547]
 [2665.55771929  159.37847976]
 [2371.22470672  135.80957669]
 [2949.03273259  189.07397639]
 [2788.79366391  181.2908892 ]
 [2127.37492922  152.71966976]
 [2296.7760946   126.70436438]
 [2622.43889726  182.47447091]
 [2716.74434921  194.02487588]]

communality:
 [1.80755303e-04 9.07570783e-01 9.04705474e-02 9.99981734e-01
 1.21466925e-03 5.70689956e-04 1.08211203e-05]
```

```
uniqueness:
 [9.99819245e-01 9.24292173e-02 9.09529453e-01 1.82657685e-05
 9.98785331e-01 9.99429310e-01 9.99989179e-01]

prop_va:
 [0.99756151 0.0020628 ]
```

We can see that the structure of two model are different. The FA use X=FA, and the PCA use Z=XV.

Question 2:

Transpose the ORL face dataset to be a data matrix of 2576x400. Perform the factore matrix. We take n=400 and see how many variables are needed to get the desired explanation.

```python
from PIL import Image

def edge(datas):
    criteria = []
    element = 0
    for i, nums in enumerate(datas):
        element += nums
        if element >= 0.5 and len(criteria) == 0:
            criteria.append(i+1)
        if element >= 0.6 and len(criteria) == 1:
            criteria.append(i+1)
        if element >= 0.7 and len(criteria) == 2:
            criteria.append(i+1)
        if element >= 0.8 and len(criteria) == 3:
            criteria.append(i+1)
        if element >= 0.9 and len(criteria) == 4:
            criteria.append(i+1)
            break
    return criteria

ORLface_data = []
for i in range(1, 41):
    for j in range(1,11):
        image_dir = f"/content/drive/MyDrive/DA/ORL Faces/{i}_{j}.png"
        img = Image.open(image_dir)
        img_array = np.asarray(img)
        ORLface_data.append(img_array.flatten())
ORLface_data = np.array(ORLface_data)

loading_matrix, factor_matrix, communality, uniqueness, prop_va =
manual_fa(ORLface_data, 400)
print("Loading Matrix:\n", loading_matrix)
print("\nfactor_matrix:\n", factor_matrix)
print("\ncommunality:\n", communality)
```

```python
print("\nuniqueness:\n", uniqueness)
print("\nprop_va:\n", prop_va)

criteria = edge(prop_va)
print("50% explanable:", criteria[0], " principal components")
print("60% explanable:", criteria[1], " principal components")
print("70% explanable:", criteria[2], " principal components")
print("80% explanable:", criteria[3], " principal components")
print("90% explanable:", criteria[4], " principal components")
```

```
Loading Matrix:
 [[ 4.18418191e-03  2.92082392e-02  3.98717037e-02 ...  1.08463577e-02
  -3.88942168e-03 -2.43641364e-03]
 [ 4.34579327e-03  2.92119864e-02  3.97740019e-02 ...  9.40164343e-03
  -1.01740124e-03 -2.45015920e-03]
 [ 3.69863744e-03  2.93623995e-02  4.00714546e-02 ...  1.27420187e-02
   4.05565911e-03  5.34254516e-03]
 ...
 [ 1.53271003e-02 -1.27914953e-02  2.41267237e-02 ... -3.11265398e-02
  -2.33792628e-02 -2.19128626e-06]
 [ 1.33515940e-02 -1.78069997e-02  2.89114944e-02 ... -1.43982079e-03
  -4.05094260e-03  2.61679055e-03]
 [ 1.38193377e-02 -1.75111834e-02  3.08289732e-02 ... -2.35419739e-03
  -4.65625113e-02 -7.79406816e-03]]

factor_matrix:
 [[-3735.25330174  3663.54695712   761.4230817  ...   -15.44696504
     32.79484107    10.95171292]
 [-4020.91757202  3539.56387202   398.21358833 ...   -14.493715
     28.63463334    10.95171292]
 [-4201.5410653   2920.14627233   298.77109127 ...    -8.79447361
     31.76165768    10.95171292]
 ...
 [-4339.84011576  3889.9586488    488.35937662 ...   -10.55792251
     30.33874686    10.95171292]
 [-4618.60206838  4054.94613442   121.88426316 ...    -6.52740545
     28.16675215    10.95171292]
 [-4184.84387619  3653.11249044   746.73153846 ...   -15.23761258
     28.07977455    10.95171292]]

communality:
 [0.04367717 0.03707898 0.0410595  ... 0.23481048 0.21846609
0.24361547]

uniqueness:
 [0.95632283 0.96292102 0.9589405  ... 0.76518952 0.78153391
0.75638453]

prop_va:
 [1.87166976e-01 1.36991737e-01 7.25151882e-02 5.91353234e-02
```

5.41577962e-02	3.55799201e-02	2.58114751e-02	2.46087008e-02
2.06763355e-02	1.89634592e-02	1.51035765e-02	1.46901473e-02
1.17120662e-02	1.10960257e-02	1.01470442e-02	9.49254659e-03
9.08642298e-03	8.71692121e-03	7.62451171e-03	7.18196305e-03
6.89473181e-03	6.62338845e-03	6.08294048e-03	5.56952482e-03
5.44672720e-03	5.01839357e-03	4.90268374e-03	4.85857277e-03
4.53852438e-03	4.41693873e-03	4.30960519e-03	4.02032884e-03
3.97989405e-03	3.82763871e-03	3.72423522e-03	3.50839081e-03
3.41460546e-03	3.35659372e-03	3.16588723e-03	2.99127509e-03
2.93549888e-03	2.86920867e-03	2.79085774e-03	2.74853383e-03
2.61556413e-03	2.53969984e-03	2.50963275e-03	2.42690473e-03
2.40341348e-03	2.32714174e-03	2.28969785e-03	2.12752835e-03
2.07420534e-03	2.07037068e-03	2.03197694e-03	1.94762992e-03
1.91939862e-03	1.88601335e-03	1.82784190e-03	1.79435399e-03
1.75263679e-03	1.70687597e-03	1.67031461e-03	1.61999760e-03
1.56815209e-03	1.54401167e-03	1.50579786e-03	1.46628926e-03
1.43069616e-03	1.39858747e-03	1.38167165e-03	1.33385796e-03
1.32021295e-03	1.30045584e-03	1.28221922e-03	1.26744540e-03
1.26552851e-03	1.23452170e-03	1.21874792e-03	1.19265317e-03
1.17628157e-03	1.16492010e-03	1.14998815e-03	1.12392815e-03
1.09078804e-03	1.08120539e-03	1.06870472e-03	1.04075104e-03
1.02100457e-03	1.01327209e-03	1.00368742e-03	9.75710965e-04
9.59024976e-04	9.52584603e-04	9.34109333e-04	9.17643277e-04
8.98946096e-04	8.90368890e-04	8.64845611e-04	8.45556465e-04
8.38812067e-04	8.21580423e-04	8.17613011e-04	8.02030653e-04
7.77691813e-04	7.70682752e-04	7.63704629e-04	7.57768260e-04
7.42395766e-04	7.37199973e-04	7.29475848e-04	7.23809968e-04
7.11401667e-04	7.03844439e-04	6.99238028e-04	6.76665053e-04
6.68075522e-04	6.58481962e-04	6.47069048e-04	6.41235270e-04
6.37868534e-04	6.30332123e-04	6.22901494e-04	6.18033899e-04
6.13921219e-04	6.05718474e-04	5.99875068e-04	5.91114546e-04
5.84656392e-04	5.78804064e-04	5.73220570e-04	5.65188325e-04
5.54828264e-04	5.49324542e-04	5.41018661e-04	5.38277396e-04
5.30728101e-04	5.23739359e-04	5.23037633e-04	5.09879057e-04
5.06888086e-04	4.95835576e-04	4.89012683e-04	4.84900865e-04
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4.64069630e-04	4.59749330e-04	4.55778752e-04	4.51134277e-04
4.45782598e-04	4.43629525e-04	4.41204688e-04	4.37462328e-04
4.33272230e-04	4.25936125e-04	4.19596922e-04	4.16877592e-04
4.10023341e-04	4.07051525e-04	4.05540451e-04	4.00485202e-04
3.98948109e-04	3.94399572e-04	3.90904504e-04	3.87769664e-04
3.85354591e-04	3.80802935e-04	3.71901974e-04	3.66689950e-04
3.63714840e-04	3.59378275e-04	3.57043532e-04	3.55517427e-04
3.52230448e-04	3.47496624e-04	3.45124406e-04	3.42008445e-04
3.40172257e-04	3.36067757e-04	3.32425529e-04	3.28885542e-04
3.23583664e-04	3.19732934e-04	3.18067938e-04	3.15875599e-04
3.09023633e-04	3.06823798e-04	3.05022495e-04	3.00638867e-04
2.99301057e-04	2.97530018e-04	2.93887210e-04	2.88334570e-04
2.85485856e-04	2.84622246e-04	2.81273883e-04	2.79222826e-04
2.76653917e-04	2.73420666e-04	2.70228697e-04	2.67891771e-04

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 2.57426267e-04 2.54480386e-04 2.54204181e-04 2.51678211e-04
 2.51111998e-04 2.45870456e-04 2.44001696e-04 2.41976543e-04
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 1.24449613e-04 1.23281931e-04 1.22368208e-04 1.20580431e-04
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 1.11360312e-04 1.11010614e-04 1.09863889e-04 1.08062207e-04
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 8.62616016e-05 8.46162655e-05 8.38471703e-05 8.32517943e-05
 8.29109130e-05 8.24954665e-05 8.09449541e-05 8.00407434e-05
 7.93364437e-05 7.85626600e-05 7.75538776e-05 7.65576759e-05
 7.58030936e-05 7.50981106e-05 7.45724805e-05 7.37126933e-05
 7.26998536e-05 7.23732672e-05 7.13899133e-05 7.07091910e-05
 7.01600931e-05 6.82024657e-05 6.73119491e-05 6.69535230e-05
 6.66839428e-05 6.58472168e-05 6.54152509e-05 6.51298885e-05
 6.37861779e-05 6.32658311e-05 6.26770140e-05 6.17638636e-05
 6.11019405e-05 5.99386287e-05 5.94646804e-05 5.82988907e-05
 5.78406156e-05 5.71973132e-05 5.54883035e-05 5.52375174e-05
 5.46075120e-05 5.35365577e-05 5.30118055e-05 5.23294732e-05
 5.17212997e-05 5.10679332e-05 5.04610091e-05 5.03765335e-05
 4.88577530e-05 4.84496082e-05 4.78408641e-05 4.70591770e-05
 4.59865381e-05 4.53896259e-05 4.44145408e-05 4.31185222e-05
 4.25026905e-05 4.19901001e-05 4.07482349e-05 4.05317183e-05
 3.96772063e-05 3.89453214e-05 3.75388229e-05 3.48953150e-05
 3.37625881e-05 3.15625035e-05 2.96045980e-05 2.88188348e-05
 2.60902740e-05 2.46645951e-05 2.35903444e-05 6.97012414e-18]
50% explanable: 5  principal components
```

```
60% explanable: 9  principal components
70% explanable: 16  principal components
80% explanable: 32  principal components
90% explanable: 76  principal components
```

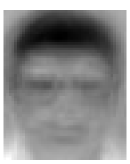
- 50% explanable: 5 principal components
- 60% explanable: 9 principal components
- 70% explanable: 16 principal components
- 80% explanable: 32 principal components
- 90% explanable: 76 principal components
- If we want to plot an image with 80% explaining of the total variance, we take n=32.

```
loading_matrix, factor_matrix, communality, uniqueness, prop_va = manual_fa(ORLface_data, 32)


first_PC = loading_matrix[:, 0]

min_value = np.min(first_PC)
max_value = np.max(first_PC)
scaled_pc = (first_PC - min_value) * (255 / (max_value - min_value))

scaled_pc_2D = scaled_pc.reshape(56,46)
img = Image.fromarray(scaled_pc_2D.astype('uint8'))
img.save('r.png', 'PNG')
img.show()
```



Question 3

a

find a oackage to perform PLSR on the AutoMPG dataset. Take 300 cars randomly to build the model and the rest 92 cars to test. In PLSR, we can see the relationship of multiple x and multiple y altogether, and see if the relashionship are strong and give us high R^2 score.

```
from sklearn.cross_decomposition import PLSRegression
from sklearn.model_selection import train_test_split

df = pd.read_csv('/content/drive/MyDrive/autompg.csv')
df.replace('?', np.nan, inplace=True)
df.dropna(inplace=True)
data = pd.DataFrame(df)

# Split data into training and testing sets
```

```python
train, test = train_test_split(data, test_size=92)
X_train = train.drop(['mpg', 'car name'], axis=1)
y_train = train['mpg']
X_test = test.drop(['mpg', 'car name'], axis=1)
y_test = test['mpg']

# Fit PLSR model
plsr = PLSRegression(n_components=2)
plsr.fit(X_train, y_train)

# Predict on test set
y_pred = plsr.predict(X_test)
# print(y_pred)
# Print R^2 score
score = plsr.score(X_test, y_test)
print(f'R^2 score: {score:.3f}')
```

```
R^2 score: 0.792
```

We can see that the R^2 is 0.792, which is really a nice number! However, if we take the model year as a part of y, things will be different.

```python
# Split data into training and testing sets
train, test = train_test_split(data, test_size=92, random_state=42)
X_train = train.drop(['mpg', 'car name', 'model year'], axis=1)
y_train = train[['mpg', 'model year']]
X_test = test.drop(['mpg', 'car name', 'model year'], axis=1)
y_test = test[['mpg', 'model year']]

# Fit PLSR model
plsr = PLSRegression(n_components=2)
plsr.fit(X_train, y_train)

# Predict on test set
y_pred = plsr.predict(X_test)
# print(y_pred)
# Print R^2 score
score = plsr.score(X_test, y_test)
print(f'R^2 score: {score:.3f}')
```

```
R^2 score: 0.355
```

We can see the R^2 score drop dramatically. In my opinion, the model year shouldn't be in the dependent part, since it makes the result worse.