Question 1

implement a FA function with parameters: the dataset and the desired numbers of factors. These part is a bit like the PCA function implemented in the last homework.

```
from google.colab import drive
drive.mount('/content/drive')
```

```
Drive already mounted at /content/drive; to attempt to forcibly
remount, call drive.mount("/content/drive", force_remount=True).
```

```
import numpy as np
def manual_fa(X, k):
    U,S,VT = np.linalg.svd(X)
    SD = np.eye(k) * S[:k]
    factor_matrix = (len(U)**0.5)*np.dot(U[:len(U), :k],SD)
    loading_matrix = VT[:k, :len(VT)]/(len(U)**0.5)
    print(loading_matrix.shape)
    communality = np.sum(loading_matrix**2, axis=0)
    cov_mx = np.cov(X.T)
    r = []
    for i in range(loading_matrix.shape[1]):
        total = 0
        for j in cov_mx[:,i]:
            total += j #**2
        r.append(total/len(cov_mx[:,i]))
    uniqueness = r - communality
    prop_va = S[:k] / np.sum(S[:k])
    return factor_matrix, loading_matrix, communality, uniqueness,
prop_va

def edge(datas):
    element = 0
    criteria = []
    for i, nums in enumerate(datas):
        element += nums
        if element >= 0.5 and len(criteria) == 0:
            criteria.append(i+1)
        if element >= 0.6 and len(criteria) == 1:
            criteria.append(i+1)
        if element >= 0.7 and len(criteria) == 2:
            criteria.append(i+1)
        if element >= 0.8 and len(criteria) == 3:
            criteria.append(i+1)
        if element >= 0.9 and len(criteria) == 4:
            criteria.append(i+1)
            break
    return criteria
```

```python
from sklearn.datasets import load_iris
iris = load_iris()
X = iris.data
loading_matrix, factor_matrix, communality, uniqueness, prop_va =
manual_fa(X,2)
print("Loading Matrix:\n", loading_matrix)
print("\nfactor_matrix:\n", factor_matrix)
print("\ncommunality:\n", communality)
print("\nuniqueness:\n", uniqueness)
print("\nprop_va:\n", prop_va)
```

```
(2, 4)
Loading Matrix:
 [[-7.24160674e+01  2.81940338e+01]
 [-6.82486927e+01  2.41498376e+01]
 [-6.67115731e+01  2.56609325e+01]
 [-6.65827586e+01  2.29074016e+01]
 [-7.19616601e+01  2.85156145e+01]
 [-7.93340520e+01  2.84710312e+01]
 [-6.75566233e+01  2.53632934e+01]
 [-7.16589479e+01  2.63084362e+01]
 [-6.31836047e+01  2.17400063e+01]
 [-6.91368623e+01  2.43724359e+01]
 [-7.67351371e+01  2.97094708e+01]
 [-7.04474211e+01  2.47444194e+01]
 [-6.71231329e+01  2.42227049e+01]
 [-6.06386387e+01  2.50862960e+01]
 [-7.99264115e+01  3.57143052e+01]
 [-8.31647327e+01  3.45991356e+01]
 [-7.68208322e+01  3.19427643e+01]
 [-7.26217113e+01  2.77731247e+01]
 [-8.14226472e+01  2.92664430e+01]
 [-7.46465420e+01  2.89140590e+01]
 [-7.65952213e+01  2.59647367e+01]
 [-7.43866773e+01  2.78235274e+01]
 [-6.57687767e+01  3.05951806e+01]
 [-7.39868968e+01  2.29882617e+01]
 [-7.23323360e+01  2.21406196e+01]
 [-7.04252185e+01  2.27620128e+01]
 [-7.26985406e+01  2.45986848e+01]
 [-7.39642882e+01  2.76741422e+01]
 [-7.28704746e+01  2.78724530e+01]
 [-6.85964881e+01  2.30571326e+01]
 [-6.90508953e+01  2.27355518e+01]
 [-7.57498991e+01  2.68587851e+01]
 [-7.65516959e+01  3.21127864e+01]
 [-7.93542910e+01  3.42735584e+01]
 [-6.93425062e+01  2.39515269e+01]
 [-6.88430158e+01  2.75729910e+01]
 [-7.54674259e+01  3.04541341e+01]
```

```
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[-6.30208084e+01  2.32775621e+01]
[-7.25788637e+01  2.66564780e+01]
[-7.10734904e+01  2.82930162e+01]
[-6.08878080e+01  1.85173372e+01]
[-6.39518255e+01  2.46168072e+01]
[-7.35753370e+01  2.44264892e+01]
[-7.73654058e+01  2.50214168e+01]
[-6.75344207e+01  2.33808868e+01]
[-7.50692031e+01  2.84670348e+01]
[-6.64199622e+01  2.44449574e+01]
[-7.58152213e+01  2.93614290e+01]
[-7.05651343e+01  2.65067470e+01]
[-1.11699734e+02 -8.94747409e-01]
[-1.05129273e+02 -1.66804043e+00]
[-1.11776563e+02 -4.06918730e+00]
[-8.91076398e+01 -5.64553442e+00]
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[-7.90385867e+01  2.74097712e-01]
[-1.05789596e+02 -3.00693966e+00]
[-8.77872655e+01 -3.56414537e+00]
[-7.93530781e+01 -3.79221719e+00]
[-9.77137611e+01 -2.14369441e+00]
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[-1.08626173e+02 -5.13725429e+00]
[-1.10510816e+02 -7.14464473e+00]
[-1.00053083e+02 -5.06907502e+00]
[-8.85855408e+01  2.66181023e+00]
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[-1.03097540e+02 -1.20368288e+01]
[-9.49990967e+01 -6.48770302e+00]
[-1.02586270e+02 -2.14187146e+00]
[-1.08680122e+02 -3.02940425e+00]
```

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[-9.00386570e+01 -4.30628937e+00]
[-9.28117415e+01 -6.68749091e+00]
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[-9.30582693e+01 -2.17163253e+00]
[-7.94929940e+01 -4.74830578e-02]
[-9.31462000e+01 -4.35486913e+00]
[-9.52569977e+01 -1.57705074e+00]
[-9.49971330e+01 -2.66758233e+00]
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[-7.96646560e+01  3.82269452e+00]
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[-1.11412456e+02 -1.64192510e+01]
[-1.19931191e+02 -1.19269931e+01]
[-1.23341852e+02 -1.64366817e+01]
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[-1.06117152e+02 -9.90217196e+00]
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[-1.13902524e+02 -1.40122847e+01]
```

```
 [-1.11563745e+02 -1.22797229e+01]
 [-1.03020438e+02 -8.26597951e+00]
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 [-1.19250357e+02 -1.57942118e+01]
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 [-1.13001289e+02 -1.14059656e+01]
 [-1.04914897e+02 -1.27267425e+01]
 [-1.10544525e+02 -1.08393220e+01]
 [-1.11520354e+02 -1.22035509e+01]
 [-1.03985438e+02 -1.12178211e+01]]

factor_matrix:
 [[-0.06132772 -0.03103391 -0.041887   -0.01370959]
 [ 0.02320278  0.0446415  -0.05786222 -0.0280606 ]]

communality:
 [0.00429946 0.00295597 0.00510256 0.00097535]

uniqueness:
 [ 0.60416195 -0.07889355  1.33403402  0.56683639]

prop_va:
 [0.84381916 0.15618084]
```

question b

On the AutoMPG dataset, compare with the PCA results in HW05.

```python
import pandas as pd
df = pd.read_csv('/content/drive/MyDrive/autompg.csv')
df.replace('?', np.nan, inplace=True)
df.dropna(inplace=True)
autompg_variables = df.drop(['mpg','car name'], axis=1)
autompg_mpg = df['mpg']
autompg_variables = pd.DataFrame(autompg_variables, dtype=float)

loading_matrix, factor_matrix, communality, uniqueness, prop_va = 
manual_fa(autompg_variables, 2)
print("Loading Matrix:\n", loading_matrix)
print("\nfactor_matrix:\n", factor_matrix)
print("\ncommunality:\n", communality)
print("\nuniqueness:\n", uniqueness)
print("\nprop_va:\n", prop_va)
```

```
(2, 7)
Loading Matrix:
 [[-6.96908877e+04 -1.30385708e+03]
```

```
[-7.35036008e+04 -1.97650470e+03]
[-6.83778400e+04 -1.65695060e+03]
[-6.82996570e+04 -1.40010884e+03]
[-6.86055510e+04 -1.31399331e+03]
[-8.64217775e+04 -2.74039141e+03]
[-8.67276552e+04 -3.25817671e+03]
[-8.58762136e+04 -3.03637494e+03]
[-8.81337174e+04 -3.20274639e+03]
[-7.66735202e+04 -2.60581341e+03]
[-7.09864945e+04 -2.76652715e+03]
[-7.18285014e+04 -1.88585562e+03]
[-7.49029709e+04 -2.77234717e+03]
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[-5.62766725e+04 -3.84027804e+00]
[-5.51151260e+04 -1.01931078e+02]
[-5.14178949e+04 -3.13561110e+02]
[-4.22610258e+04  1.01244210e+03]
[-3.64107083e+04  7.89617972e+02]
[-5.29741377e+04  1.45283486e+03]
[-4.81962384e+04  1.19331333e+03]
[-4.71105534e+04  1.16966507e+03]
[-4.43633285e+04  6.14629466e+02]
[-5.26237058e+04 -2.36927201e+02]
[-9.17471323e+04 -1.17165633e+03]
[-8.69481119e+04 -4.38909023e+02]
[-8.70883483e+04 -6.69188441e+02]
[-9.39647650e+04  8.66366971e+01]
[-4.22614895e+04  1.01840011e+03]
[-4.49659033e+04  3.84404125e+02]
[-4.42220637e+04  8.21089105e+02]
[-5.23996531e+04 -8.93733119e+02]
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[-6.61395399e+04 -3.59630666e+02]
[-6.55983396e+04 -3.54993229e+02]
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[-8.14029298e+04 -8.28329894e+02]
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[-9.43556276e+04 -1.60303362e+03]
[-1.02134316e+05 -1.12983592e+03]
[-5.89148605e+04 -9.98303442e+02]
[-4.77953521e+04  6.26281565e+02]
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[-4.40701493e+04  6.72827141e+02]
[-4.21505796e+04  6.50908111e+02]
[-4.11197452e+04  1.34711870e+03]
```

```
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[-4.53829523e+04  1.21198233e+03]
[-4.97195600e+04  1.03718589e+03]
[-4.29287123e+04  1.07382826e+03]
[-4.16701196e+04  9.90483450e+02]
[-8.15438547e+04 -1.48266828e+03]
[-7.30174272e+04 -1.08639164e+03]
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[-7.51086647e+04 -1.21324875e+03]
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[-8.87775965e+04 -1.87759050e+03]
[-8.67233840e+04 -1.12063096e+03]
[-8.41864909e+04 -6.38857758e+02]
[-9.42253194e+04 -2.48919865e+03]
[-9.85152853e+04 -2.53023474e+03]
[-7.60515069e+04 -2.01728967e+03]
[-6.20053200e+04 -1.57841010e+02]
[-6.51342663e+04 -4.06816325e+02]
[-5.85381339e+04 -4.90705477e+02]
[-6.00541063e+04 -6.90215903e+02]
[-5.76792240e+04  1.03011736e+02]
```

```
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[-9.75119459e+04 -1.38212018e+03]
[-9.24866222e+04 -9.63974404e+02]
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[-5.54595368e+04 -6.86040444e+02]
[-4.52032022e+04  1.22285223e+03]
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[-3.27172547e+04  9.45333337e+02]
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[-3.96583984e+04  1.27678011e+03]
[-6.48211773e+04  6.25428617e+01]
```

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[-9.28181837e+04 -1.67686905e+03]
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[-8.93379920e+04 -3.02947160e+02]
[-9.25191685e+04 -7.11118912e+02]
[-7.75292577e+04  7.10509163e+02]
[-7.73540785e+04  3.58444590e+02]
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[-5.23632579e+04  8.98526748e+02]
[-5.79273073e+04 -5.17271564e+02]
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[-5.36070974e+04  1.04097987e+03]
[-4.40773716e+04  1.34427664e+03]
[-5.04894476e+04  1.12617214e+03]
[-5.92233424e+04  7.04626831e+02]
[-3.84325161e+04  9.88220015e+02]
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[-5.34227714e+04  1.38783299e+03]
[-5.86149222e+04  1.64747479e+03]
[-5.83862289e+04  1.57808416e+03]
[-5.29908948e+04  1.18221722e+03]
[-3.56201108e+04  8.53658797e+02]
[-4.88672754e+04  1.28566868e+03]
[-4.40610795e+04  8.34606356e+02]
[-5.10476346e+04  7.89163771e+02]
[-4.47259140e+04  1.21834765e+03]
[-4.36866786e+04  1.08058445e+03]
[-8.37288969e+04 -3.78631897e+02]
[-8.32602440e+04 -6.81852126e+02]
[-7.87209459e+04 -6.09099041e+02]
[-8.37999332e+04 -1.26773753e+03]
[-6.42133237e+04  1.31113889e+01]
[-6.66188736e+04 -3.17628583e+02]
[-5.98049816e+04  2.65175239e+02]
[-6.12954487e+04 -2.64553284e+02]
[-4.03482816e+04  1.27906039e+03]
[-4.29172583e+04  1.17320009e+03]
[-3.84329984e+04  9.94514636e+02]
[-3.56205651e+04  8.59448507e+02]
[-7.24623499e+04  5.35870236e+02]
[-7.09618332e+04  5.34981681e+01]
[-7.23848525e+04  3.18117199e+01]
[-6.34656556e+04 -6.26829843e+02]
```

```
[-3.62328397e+04  7.19495496e+02]
[-3.94723580e+04  1.15773830e+03]
[-4.27482540e+04  1.12375555e+03]
[-5.08954048e+04  8.41876506e+02]
[-6.24473305e+04  1.66139762e+03]
[-7.83267628e+04 -9.91855145e+02]
[-6.47925845e+04  2.05022491e+03]
[-5.81455281e+04  8.88156370e+02]
[-7.57336223e+04  1.73467042e+03]
[-8.70741742e+04 -1.13462615e+03]
[-8.06361666e+04 -1.42695312e+03]
[-7.69097809e+04 -7.16264521e+02]
[-7.46760475e+04 -1.22016004e+03]
[-4.05746463e+04  1.00046345e+03]
[-4.27711178e+04  8.51718321e+02]
[-3.62002950e+04  1.11040790e+03]
[-4.56587763e+04  7.78613364e+02]
[-3.85847730e+04  1.10749858e+03]
[-7.71219167e+04 -8.05268053e+02]
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[-7.00091149e+04 -6.83266598e+01]
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[-8.28247775e+04 -1.36494324e+03]
[-8.60643850e+04 -2.15315106e+03]
[-8.61665397e+04 -1.10041679e+03]
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[-5.46574084e+04  1.03334366e+03]
[-4.06894001e+04  1.02076256e+03]
[-4.11768841e+04  9.85113623e+02]
[-3.93883495e+04  9.44118690e+02]
[-4.34412800e+04  1.15949057e+03]
[-5.58551802e+04  9.69566007e+02]
[-5.15871502e+04  1.12071622e+03]
[-5.38994077e+04  2.02937884e+03]
[-3.93664706e+04  1.14825127e+03]
[-3.57384831e+04  7.01225154e+02]
[-3.93527284e+04  1.35415941e+03]
[-4.10521544e+04  1.27165587e+03]
[-3.57249589e+04  8.53631718e+02]
[-6.68738917e+04 -4.90154436e+02]
[-7.42752349e+04 -1.20262251e+03]
[-7.09965900e+04 -1.10907432e+03]
[-7.01859404e+04  2.79465917e+02]
```

```
[-6.26377062e+04  4.11102220e+02]
[-5.88810373e+04  2.02999906e+02]
[-5.39664463e+04  9.98360416e+02]
[-6.81020167e+04  2.72604513e+02]
[-6.37630887e+04 -9.81406592e+01]
[-6.71268535e+04  8.13330136e+01]
[-6.09531911e+04  3.34857285e+02]
[-7.18586017e+04  4.79192091e+02]
[-6.77660723e+04 -4.29570970e+02]
[-6.81434759e+04 -1.36295003e+03]
[-6.84512514e+04 -3.27885649e+01]
[-6.37935310e+04 -1.56486304e+03]
[-8.10835114e+04 -7.73512310e+02]
[-4.27456457e+04  1.13908279e+03]
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[-4.56557943e+04  8.36042350e+02]
[-4.42399361e+04  1.07712063e+03]
[-4.99176532e+04  8.32087328e+02]
[-5.44935916e+04  6.81664610e+02]
[-5.66436327e+04  9.75777119e+02]
[-4.77278829e+04  9.66721602e+02]
[-5.61354938e+04  1.25464070e+03]
[-6.23118214e+04  9.73805830e+02]
[-5.54395068e+04  1.35772240e+03]
[-6.76457946e+04  1.28723924e+03]
[-3.94792290e+04  1.08871291e+03]
[-4.23509927e+04  1.11494595e+03]
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[-5.93750746e+04  2.44031177e+02]
[-5.73218670e+04  1.21854715e+03]
[-6.48490155e+04 -2.22248883e+01]
[-6.67280297e+04  1.58862510e+02]
[-7.63232765e+04 -7.90432905e+02]
[-7.40488942e+04 -8.77900818e+02]
[-7.86607422e+04 -1.52743034e+03]
[-7.61471223e+04 -1.05947182e+03]
[-8.66636830e+04 -1.05737902e+03]
[-8.06173048e+04 -1.41540255e+03]
[-7.16304836e+04 -3.64245945e+02]
[-7.83853580e+04 -1.75096610e+03]
[-3.81968988e+04  1.01246789e+03]
[-3.91754410e+04  1.15136842e+03]
[-3.80181361e+04  8.05426259e+02]
[-5.29486979e+04  1.31835430e+03]
[-7.00028342e+04  1.25772102e+03]
[-7.75653205e+04 -1.52881104e+03]
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[-6.79463793e+04 -3.41055144e+02]
[-4.36447659e+04  1.05954749e+03]
[-4.26582244e+04  1.00034073e+03]
```

```
[-4.00624814e+04  1.23245334e+03]
[-4.22437618e+04  1.23750027e+03]
[-5.29966499e+04  7.33493396e+02]
[-5.15636713e+04  1.45970266e+02]
[-5.36358904e+04  2.79005772e+02]
[-5.07467129e+04  5.87266016e+02]
[-4.25349283e+04  1.10913934e+03]
[-3.90386970e+04  1.11511853e+03]
[-4.20571869e+04  1.09950921e+03]
[-4.00443125e+04  1.21396585e+03]
[-5.31550322e+04  7.50223798e+02]
[-5.69277252e+04  1.20102443e+03]
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[-6.71291326e+04  2.58013630e+02]
[-4.34033614e+04  1.17774306e+03]
[-5.37830170e+04  1.10438530e+03]
[-5.04186053e+04  1.20453723e+03]
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[-4.94194390e+04  8.36403092e+02]
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[-4.73471080e+04  7.01347938e+02]
[-3.48206201e+04  1.04474471e+03]
[-3.72023640e+04  1.04382572e+03]
[-3.49233374e+04  1.00626279e+03]
[-4.09690546e+04  1.06961568e+03]
[-3.91753974e+04  1.18884484e+03]
[-4.06586058e+04  1.21398436e+03]
[-3.93826907e+04  1.07504779e+03]
[-4.39369720e+04  1.11520163e+03]
[-4.05741732e+04  1.02944101e+03]
[-4.71854825e+04  1.45286979e+03]
[-4.34512494e+04  1.04827594e+03]
[-4.38493803e+04  1.03353797e+03]
```

```
 [-4.66137264e+04  1.19287583e+03]
 [-5.18755000e+04  1.23557410e+03]
 [-5.22537070e+04  1.33046753e+03]
 [-6.40284262e+04  1.66494796e+03]
 [-6.26496998e+04  1.51525057e+03]
 [-5.75774936e+04  6.28489250e+02]
 [-5.81424457e+04  1.06184744e+03]
 [-6.78224271e+04  1.26757053e+02]
 [-7.40989765e+04 -1.66784969e+03]
 [-6.07593749e+04  3.31452427e+02]
 [-6.87835638e+04  3.80608925e+02]
 [-5.16611340e+04  1.40427060e+03]
 [-5.23517307e+04  1.44603511e+03]
 [-4.75168449e+04  1.14089245e+03]
 [-5.10667014e+04  1.37084604e+03]
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 [-3.89876826e+04  1.05767510e+03]
 [-3.89877479e+04  1.05885323e+03]
 [-3.95798145e+04  1.09693551e+03]
 [-5.84798445e+04  4.74867808e+02]
 [-5.99541756e+04 -8.55561423e+02]
 [-5.13287204e+04  5.44510243e+02]
 [-5.63798280e+04 -6.13974860e+02]
 [-5.28938468e+04  8.52512630e+02]
 [-4.70515710e+04  6.88853878e+02]
 [-5.85236819e+04  1.10113577e+03]
 [-5.53483025e+04  1.11578319e+03]
 [-4.22423754e+04  1.21513830e+03]
 [-4.55713905e+04  5.93395735e+02]
 [-5.20603336e+04  1.30809729e+03]
 [-5.39358712e+04  1.43632588e+03]]

factor_matrix:
 [[-9.23880562e-05 -3.47563545e-03 -1.78196007e-03 -5.03418354e-02
  -2.37824279e-04 -1.18279036e-03 -2.25437295e-05]
 [-2.64527112e-04 -4.71725213e-02 -8.11099434e-03  3.16673089e-03
   4.29346790e-03  1.51990141e-02  6.65853186e-04]]

communality:
 [7.85101460e-08 2.23732680e-03 6.89636109e-05 2.54432857e-03
```

```
 1.84904270e-05 2.32409024e-04 4.43868685e-07]

uniqueness:
 [ 2.17582219e+02  1.39016915e+04  4.75289071e+03  1.18805075e+05
 -1.71191036e+02 -1.64850222e+02 -6.64715746e+01]

prop_va:
 [0.98145224 0.01854776]
```

We can see that the structure of two model are different. The FA use X=FA, and the PCA use Z=XV.

Question 2:

Transpose the ORL face dataset to be a data matrix of 2576x400. Perform the factore matrix. We take n=400 and see how many variables are needed to get the desired explanation.

```python
from PIL import Image
ORLface_data = []
for i in range(1, 41):
    for j in range(1,11):
        image_dir = f"/content/drive/MyDrive/DA/ORL Faces/{i}_{j}.png"
        img = Image.open(image_dir)
        img_array = np.asarray(img)
        ORLface_data.append(img_array.flatten())
ORLface_data = np.array(ORLface_data)

loading_matrix, factor_matrix, communality, uniqueness, prop_va =
manual_fa(ORLface_data, 400)
print("Loading Matrix:\n", loading_matrix)
print("\nfactor_matrix:\n", factor_matrix)
print("\ncommunality:\n", communality)
print("\nuniqueness:\n", uniqueness)
print("\nprop_va:\n", prop_va)

criteria = edge(prop_va)
print("50% explanable:", criteria[0], " principal components")
print("60% explanable:", criteria[1], " principal components")
print("70% explanable:", criteria[2], " principal components")
print("80% explanable:", criteria[3], " principal components")
print("90% explanable:", criteria[4], " principal components")
```

```
(400, 2576)
Loading Matrix:
 [[-1.18892763e+05 -6.38993275e+03 -1.90629505e+04 ... -4.33302144e+01
   1.06380416e+01 -5.13136303e+01]
 [-1.23185712e+05 -1.11326275e+03 -1.18429329e+04 ... -1.14157214e+00
  -7.21565692e-01  2.59104622e+01]
 [-1.16932006e+05  1.06274678e+04 -9.64696329e+03 ...  4.65476422e+01
```

```
  -1.21891467e+02 -2.13761848e+01]
 ...
 [-1.24976775e+05 -1.18503705e+02 -1.83944122e+04 ...  7.43332747e+01
  -7.79625810e+01 -9.53746388e+00]
 [-1.35310568e+05  2.78513619e+02 -1.02079445e+04 ...  3.32538543e+01
  -1.60903178e+02  4.72475286e+01]
 [-1.21840430e+05  5.60352656e+02 -2.01118519e+04 ... -6.37431055e+01
   2.84059236e+00  6.09980723e+01]]

factor_matrix:
 [[-7.22081129e-04 -7.23468559e-04 -7.25482736e-04 ... -6.17906910e-04
  -6.11483923e-04 -6.06204228e-04]
 [-1.17594051e-03 -1.18281076e-03 -1.16182564e-03 ... -3.64767970e-04
  -1.30441628e-04 -1.52842266e-04]
 [-2.07413198e-03 -2.06789095e-03 -2.08809780e-03 ... -6.04518506e-04
  -7.88263084e-04 -8.97290148e-04]
 ...
 [ 3.65108346e-04 -1.93124573e-04  2.30523456e-04 ...  3.39713315e-04
  -4.85693019e-05  1.00238880e-03]
 [-5.62456218e-04 -5.06450188e-04 -6.60392053e-04 ...  1.60871870e-03
   7.10744707e-05  1.30939029e-04]
 [ 2.98442145e-04  1.64977133e-04 -1.37403649e-04 ...  1.11100238e-03
   2.39512793e-04  2.58187438e-03]]

communality:
 [1.10273381e-04 9.37126111e-05 1.02726074e-04 ... 5.88326549e-04
 5.48392666e-04 6.21440009e-04]

uniqueness:
 [ 132.11248455  130.60007157  137.90564271 ... -142.29266099 -
151.83753976
 -158.93115901]

prop_va:
 [0.21734064 0.02824963 0.01911292 0.01806289 0.0171627  0.01418227
 0.01238431 0.01116304 0.01081492 0.00975075 0.00903707 0.00857003
 0.00773795 0.0074704  0.00711729 0.00691439 0.00673192 0.0066135
 0.00625661 0.00613726 0.00598432 0.0058279  0.00555529 0.00542818
 0.00522281 0.00509411 0.00497247 0.0049285  0.00483009 0.0047485
 0.00463617 0.00449128 0.00447736 0.00437413 0.00432861 0.00418925
 0.00418246 0.0040972  0.00406768 0.0039702  0.00382861 0.00379803
 0.00375968 0.00371926 0.00362618 0.00361077 0.00354295 0.00350142
 0.00346544 0.00341686 0.00338065 0.00326679 0.00322612 0.00321362
 0.00320027 0.00313133 0.00311531 0.00309295 0.00302453 0.00299261
 0.00296845 0.00293269 0.00291516 0.00284248 0.00280547 0.00278269
 0.00274618 0.00271475 0.00268172 0.0026672  0.00262738 0.00261398
 0.0025678  0.00256558 0.00253503 0.00252241 0.00251418 0.00248664
 0.00247309 0.00244214 0.00242362 0.00241147 0.00239795 0.00236764
 0.00233645 0.00232863 0.00232213 0.00227894 0.00227749 0.00225651
 0.00224009 0.00220952 0.00218873 0.00217982 0.00215914 0.00213944
```

0.00212593	0.00211029	0.00209645	0.00206238	0.00204551	0.00202444
0.00202122	0.00201127	0.00197049	0.00196459	0.00195407	0.00194496
0.00193173	0.00192272	0.00191335	0.0019026	0.00189237	0.00188166
0.00186746	0.00184332	0.00183688	0.00182252	0.00179968	0.00179147
0.00178564	0.0017804	0.00176744	0.00175812	0.00175002	0.00174497
0.00173132	0.00171704	0.00171008	0.00170351	0.00169111	0.00168321
0.00166706	0.00166224	0.00165185	0.00163885	0.00162871	0.00161844
0.00161575	0.0016082	0.00159384	0.00157905	0.00157213	0.00156117
0.00155302	0.00155032	0.00153729	0.00153291	0.00152808	0.00152131
0.00150786	0.00150003	0.00149702	0.00149106	0.00148737	0.00147874
0.00147159	0.00145935	0.00145512	0.00144195	0.00143519	0.00142805
0.00142308	0.0014164	0.00141276	0.00140352	0.00139633	0.00139324
0.00138734	0.00137956	0.00136707	0.00135388	0.00134865	0.00134015
0.00133757	0.00133259	0.00132943	0.00131762	0.00131456	0.00130688
0.00130252	0.00129745	0.00128761	0.00128125	0.0012781	0.00126453
0.00125967	0.00125562	0.00124422	0.00123761	0.0012335	0.00122542
0.00122187	0.00121819	0.00121074	0.00120941	0.0011989	0.00119309
0.00118844	0.0011804	0.00117576	0.00116776	0.00116516	0.00115654
0.00115545	0.00115212	0.00114397	0.00114178	0.00113422	0.00113215
0.00112656	0.00112221	0.00111989	0.00111473	0.00110525	0.0011024
0.00109364	0.00108616	0.00108537	0.00107903	0.00107814	0.00106685
0.0010621	0.001059	0.00105529	0.00104972	0.00104684	0.00104122
0.00104048	0.00103328	0.00102861	0.00102684	0.00101579	0.00101159
0.00100833	0.0009984	0.00099545	0.00098618	0.00098288	0.00097719
0.00097589	0.00097126	0.00096806	0.00096606	0.00095998	0.00095594
0.00095242	0.00094622	0.00094534	0.00094033	0.00093571	0.00093184
0.00092926	0.00092693	0.00092194	0.00091683	0.00091551	0.00090835
0.00090359	0.00089641	0.00089308	0.00089095	0.00088672	0.00088371
0.00087901	0.00087741	0.00087467	0.00087046	0.00086795	0.00086344
0.0008597	0.00085447	0.00085198	0.0008435	0.00084179	0.0008382
0.00083411	0.0008273	0.00082386	0.00081993	0.0008179	0.00081318
0.00081044	0.00080449	0.00080001	0.00079744	0.00079519	0.00079137
0.00078931	0.00078783	0.00078121	0.00077998	0.00077483	0.00077259
0.00076818	0.00076382	0.00076243	0.00075598	0.0007536	0.00075254
0.00075031	0.00074419	0.00074287	0.00073824	0.00073413	0.00073268
0.00072638	0.00072575	0.00071897	0.00071624	0.00071367	0.00071035
0.00070835	0.00070429	0.00069849	0.00069531	0.00069379	0.00069175
0.00068489	0.00068096	0.00067852	0.00067698	0.00067349	0.00067052
0.00067009	0.00066701	0.00066057	0.00065831	0.00065597	0.00065339
0.0006491	0.00064496	0.00064311	0.00064231	0.00063603	0.00063187
0.00063088	0.00062736	0.00062393	0.00062053	0.0006165	0.00061414
0.00061188	0.00060908	0.00060508	0.00060215	0.00060074	0.00059612
0.00059271	0.00059154	0.00058164	0.00057856	0.00057741	0.00057521
0.00057305	0.00057077	0.00056972	0.00056337	0.00056133	0.000559
0.00055484	0.00055203	0.00054675	0.0005433	0.00053909	0.0005344
0.00053227	0.00052581	0.00052441	0.00052119	0.00051631	0.00051418
0.00051083	0.00050734	0.00050465	0.00050167	0.00050097	0.00049218
0.0004915	0.0004868	0.00048276	0.00047888	0.00047576	0.00047063
0.00046251	0.00045787	0.00045762	0.00045078	0.00044929	0.00044447
0.00044065	0.00043239	0.00041633	0.00040845	0.00039519	0.00038425

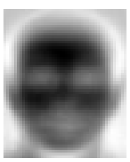
```
 0.00037911 0.00036069 0.00035053 0.0003421 ]
50% explanable: 34  principal components
60% explanable: 62  principal components
70% explanable: 104  principal components
80% explanable: 164  principal components
90% explanable: 251  principal components
```

```
loading_matrix, factor_matrix, communality, uniqueness, prop_va =
manual_fa(ORLface_data, 164)
first_PC = factor_matrix[0:1,:].T


min_value = np.min(first_PC)
max_value = np.max(first_PC)
scaled_pc = (first_PC - min_value) * (255 / (max_value - min_value))

scaled_pc_2D = scaled_pc.reshape(56,46)
img = Image.fromarray(scaled_pc_2D.astype('uint8'))
img.save('r.png', 'PNG')
img.show()
```

```
(164, 2576)
```



Question 3

a

find a oackage to perform PLSR on the AutoMPG dataset. Take 300 cars randomly to build the model and the rest 92 cars to test. In PLSR, we can see the relationship of multiple x and multiple y altogether, and see if the relashionship are strong and give us high R^2 score.

```
from sklearn.cross_decomposition import PLSRegression
from sklearn.model_selection import train_test_split

df = pd.read_csv('/content/drive/MyDrive/autompg.csv')
df.replace('?', np.nan, inplace=True)
df.dropna(inplace=True)
data = pd.DataFrame(df)

# Split data into training and testing sets
train, test = train_test_split(data, test_size=92)
X_train = train.drop(['mpg', 'car name'], axis=1)
y_train = train['mpg']
X_test = test.drop(['mpg', 'car name'], axis=1)
y_test = test['mpg']
```

```python
# Fit PLSR model
plsr = PLSRegression(n_components=2)
plsr.fit(X_train, y_train)

# Predict on test set
y_pred = plsr.predict(X_test)
# print(y_pred)
# Print R^2 score
score = plsr.score(X_test, y_test)
print(f'R^2 score: {score:.3f}')
```

```
R^2 score: 0.771
```

We can see that the R^2 is 0.771, which is really a nice number! However, if we take the model year as a part of y, things will be different.

```python
# Split data into training and testing sets
train, test = train_test_split(data, test_size=92, random_state=42)
X_train = train.drop(['mpg', 'car name', 'model year'], axis=1)
y_train = train[['mpg', 'model year']]
X_test = test.drop(['mpg', 'car name', 'model year'], axis=1)
y_test = test[['mpg', 'model year']]

# Fit PLSR model
plsr = PLSRegression(n_components=2)
plsr.fit(X_train, y_train)

# Predict on test set
y_pred = plsr.predict(X_test)
# print(y_pred)
# Print R^2 score
score = plsr.score(X_test, y_test)
print(f'R^2 score: {score:.3f}')
```

```
R^2 score: 0.355
```

We can see the R^2 score drop dramatically. In my opinion, the model year shouldn't be in the dependent part, since it makes the result worse.