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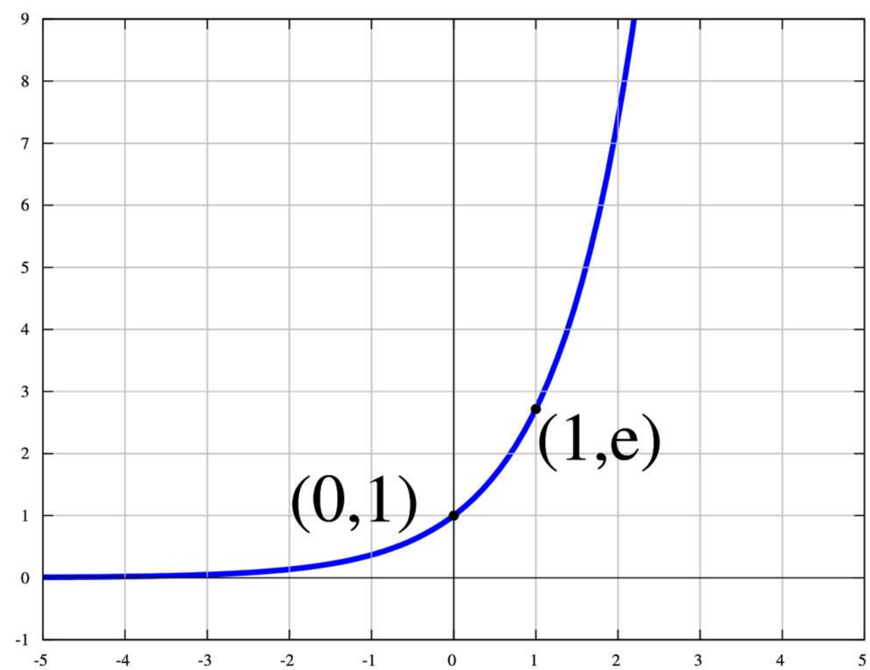
B-MAT-200

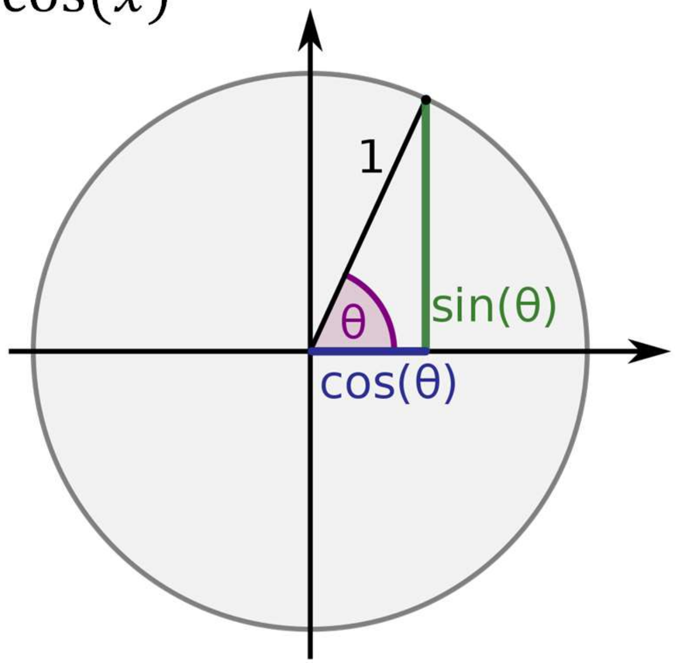
Exponential function

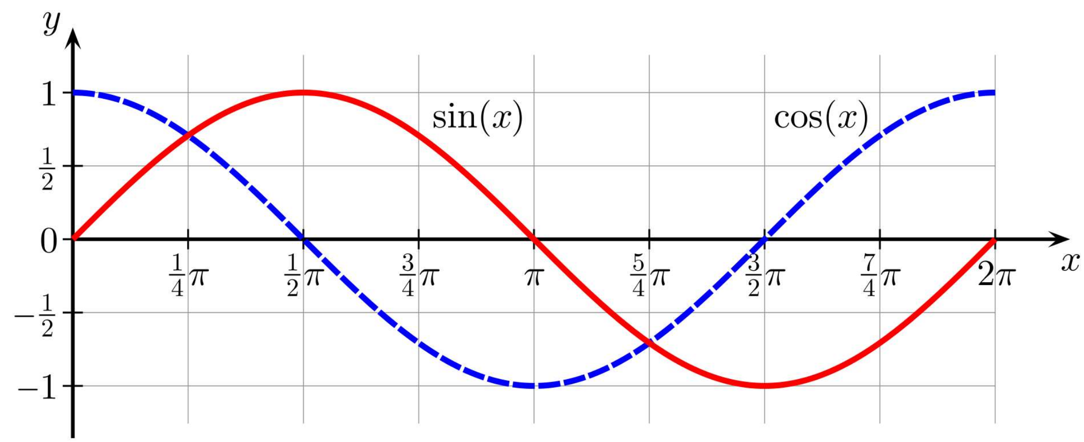
- $\exp(x) = e^x$
- $e \approx 2.71828$
- $e^x > 0$
- Inverse: log
  - $y = e^x$
  - $x = \log(y)$
  - $\log(e) = 1$



Trigonometric functions

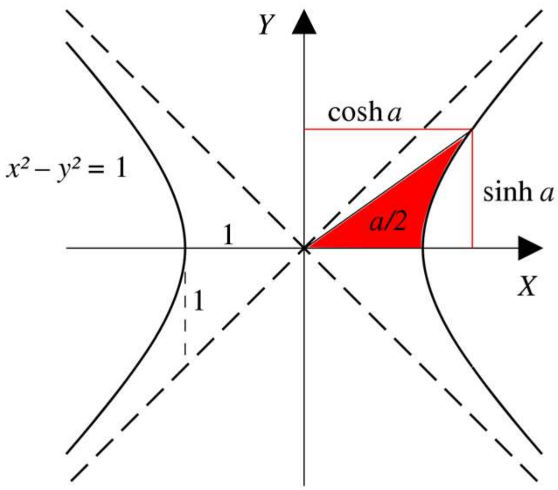
- $\sin(x)$
- $\cos(x)$

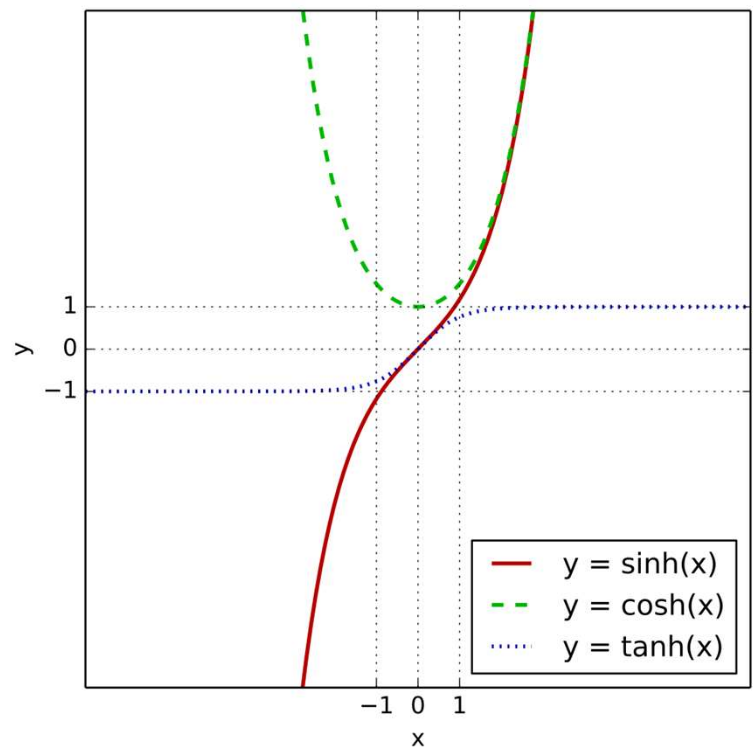




Hyperbolic functions

- $\sinh(x) = \frac{e^x - e^{-x}}{2}$
- $\cosh(x) = \frac{e^x + e^{-x}}{2}$





Taylor series

- Representation of a function as an infinite sum of terms
- General formula:

$$f(x) = f(c) + \frac{f'(c)}{1!}(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots$$
$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

- If $c = 0$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Taylor expansions

- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$
- $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$
- $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$
- $\sinh(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$
- $\cosh(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

Functions on mathematical objects

- Using Taylor series, you can compute functions on various objects:
  - Functions
  - Graphs
  - **Square matrices**
  - …
- Example: If A is a square matrix
  - $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$

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- Goal: Evaluate functions on square matrices using Taylor series
- Inputs:
  - A string representing a function: “EXP”, “SIN”, “COS”, “SINH”, “COSH”…
  - The coefficients of the matrix (line by line)
- Output: Result matrix
- You can add additional functions as a bonus
  - Logarithm
  - Tangent, arcsin…
  - …

Points of attention

- You must decide how many terms of the series you need to compute
- Think about efficient ways to compute:
  - Powers of matrices
  - Factorials

Previously: how to multiply matrices

$$\begin{pmatrix} 3 & 5 \\ 6 & 2 \\ 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \qquad \begin{pmatrix} \boxed{23} & 21 \\ 18 & 18 \end{pmatrix}$$

$$(3 * 3) + (2 * 6) + (1 * 2) = 9 + 12 + 2 = 23$$

Exercises

- Create a function that multiply two matrices
- Start with $e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!} = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$
  - Create a function that computes the n^{th} term of the series: $\frac{A^n}{n!}$
  - Since you need to compute every term of the series, think about what you can do to make it more efficient to compute the next term
- Do the same thing with sin, cos, sinh and cosh.