105torus-bootstrap

B-MAT-100

Exercise 0

- In this project, you need to evaluate a function defined by five coefficients a_0, a_1, a_2, a_3 and a_4:

$$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$$

- Since you cannot pass more than 4 parameters to a function (as per the coding style), how can you represent it?

Exercise 1

- Create a function that takes the definition of f and a number x and returns the value of $f(x)$:

$$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$$

Exercise 2

- Create a function that takes the definition of f and a number x and returns the value of $f'(x)$:

$$f'(x) = 4a_4x^3 + 3a_3x^2 + 2a_2x + a_1$$

Test function

- The functions to be solved in this project verify the following conditions:
  - There is one and only one x in $[0,1]$ where $f(x) = 0$
  - The sign of f changes around its zero

- To test the following exercises, you can use this function:

$$f(x) = x^4 - 5x^3 + 6x^2 - 1$$

Bisection method

- Consider an interval $[a, b]$ that contains the solution
- Compute the midpoint $c = \frac{a+b}{2}$ of the interval
- If $f(c) \leq 10^{-n}$, stop
- If $f(a)f(c) < 0$, then the solution is in $[a, c]$
- If not, the solution is in $[c, b]$
- Start again with the new interval

The interval is initialized with $[0,1]$

Exercise 3

- Create a function that takes the definition of f and an interval $[\mathrm{a}, \mathrm{b}]$, computes the midpoint c, and returns the next interval for the bisection method (according to the sign of $f(a)f(c)$).

Newton's method

- Start with the initial value $x_0 = 0.5$
- For each x_n:
  - Consider the tangent to the graph at the point $(x_n, f(x_n))$
  - This tangent intercept the x-axis in $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
  - If $f(x_{n+1}) \leq 10^{-n}$, stop

Exercise 4

- Create a function that takes the definition of f and a value x_n and returns the next value x_{n+1} for Newton's method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Secant method

- Start with the initial values $x_0 = 0$ and $x_1 = 1$
- For each (x_{n-1}, x_n):
  - Consider the line passing through $(x_{n-1}, f(x_{n-1}))$ and $(x_n, f(x_n))$
  - This line intercept the x-axis in $x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$
  - If $f(x_{n+1}) \leq 10^{-n}$, stop

Exercise 5

- Create a function that takes the definition of f and two values x_{n-1} and x_n and returns the next value x_{n+1} for the secant method.

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$