103cipher-bootstrap

B-MAT-100

Exercise 1

- Create a function that takes a string and returns the smallest square matrix that contains the string

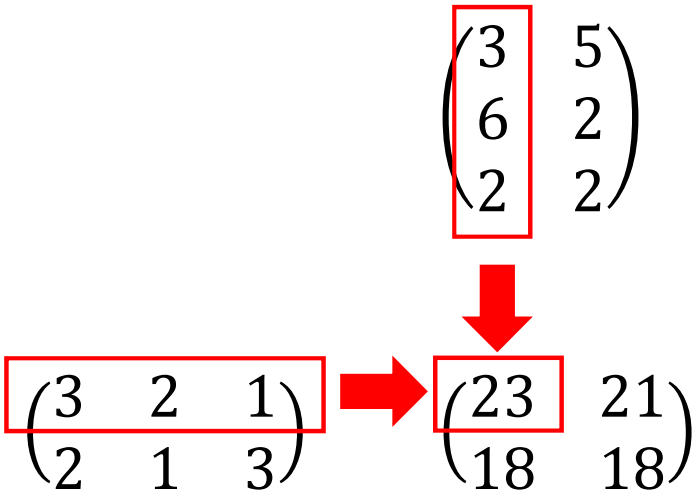
"abcdefg" → 97, 98, 99, 100, 101, 102, 103 → $\mathrm{K} = \begin{pmatrix} 97 & 98 & 99 \\ 100 & 101 & 102 \\ 103 & 0 & 0 \end{pmatrix}$

Exercise 2

- Create a function that takes a string and a number n and returns a matrix with n columns containing the string

Exercise 3

- Create a function that multiplies two matrices



$$(3 * 3) + (2 * 6) + (1 * 2) = 9 + 12 + 2 = 23$$

Exercise 4

- Create a function that takes a matrix and returns its transpose

$$A = \begin{pmatrix} 3 & 4 & 1 \\ 2 & 1 & 3 \end{pmatrix} \qquad A^T = \begin{pmatrix} 3 & 2 \\ 4 & 1 \\ 1 & 3 \end{pmatrix}$$

Exercise 5

- Create a function that takes a 2x2 matrix and returns its determinant

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Exercise 6

- Create a function that takes a 2x2 matrix and compute its inverse

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\det(A) = ad - bc \neq 0 \, ?$$

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Cofactors

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = \mathrm{a}_{22}\mathrm{a}_{33} - \mathrm{a}_{23}\mathrm{a}_{32}$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = -(\mathrm{a}_{21}\mathrm{a}_{33} - \mathrm{a}_{23}\mathrm{a}_{31})$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = \mathrm{a}_{21}\mathrm{a}_{32} - \mathrm{a}_{22}\mathrm{a}_{31}$$

Exercise 7

- Create a function that takes a 3x3 matrix and two integers i and j, and returns its cofactor (i, j)

Exercise 8

- Create a function that takes a 3x3 matrix and returns its determinant

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\det(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

Exercise 9

- Create a function that takes a 3x3 matrix and return its inverse

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^{T}$$