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Impact of Fractional Order Integral Performance Indices in LQR Based PID Controller Design via Optimum Selection of Weighting Matrices

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Abstract—**A continuous and discrete time Linear Quadratic Regulator (LQR) based technique has been used in this paper for the design of optimal analog and discrete PID controllers respectively. The PID controller gains are selected as the optimal state-feedback gains corresponding to the standard quadratic cost function. Genetic Algorithm (GA) has been used next to optimally find out the weighting matrices, associated with the respective optimal state-feedback regulator designs while minimizing another integral performance index which comprises of a weighted sum of Integral of Time multiplied Squared Error (ITSE) and the controller effort. Next, the proposed methodology is applied with fractional order (FO) integral performance indices. The impact of these FO objective functions on the LQR tuned PID control loops is also highlighted, along with the achievable cost of control.**

Keywords-fractional calculus; integral performance index; LQR; optimal control; PID controller tuning

I. INTRODUCTION

Classical optimal control theory has evolved over decades for the design of Linear Quadratic Regulators which minimizes the deviation in state trajectories while requiring minimum controller effort [1]. This typical behaviour of LQR technique has motivated control designers to use it for the tuning of PID controllers [2]-[3]. PID controllers are most common in process industries due to its simplicity, ease of implementation and robustness. For the design of an optimal quadratic regulator the Algebraic Riccati Equations (ARE) are conventionally used to calculate the state feedback gains for a chosen set of weighting matrices. These weighting matrices regulate the penalties on the deviation in the trajectories of the state variables (x) and control signal (u). Indeed, with an arbitrary choice of weighting matrices, the classical state-feedback optimal regulators seldom show good set-point tracking performance due to the absence of integral term unlike the PID controllers. Thus, combining the tuning philosophy of PID controllers with the concept of LQR allows the designer to enjoy both optimal set-point tracking and optimal cost of control within the same design framework.

Optimal control theory has been extended to tune PID controllers in few of the recent literatures. In Choi and Chung [4], an inverse optimal PID controller is designed considering the error and its integro-differential as the state variables, similar to the present approach. In Arruda *et al.* [5], a custom cost function has been minimized with GA to design multi-loop PID controllers as the weighted sum of ITSE and variance of the manipulated variable and controlled variable. PID controller tuning with state-space approach using the error and its first and second order derivative has been investigated in [6]-[7]. The LQR-PID method of He *et al.* [2]-[3] has been extended for first and second order systems with zeros in the process model in Ghartemani *et al.* [8]. Ochi and Kondo [9] have shown that the integral type optimal servo for second order system can be reduced to a LQR problem and an optimal I-PD controller can be designed with this technique. Several classical optimal and robust control approaches of PID controller can be cast into a Linear Matrix Inequality (LMI) problem as in Ge *et al.* [10] which consider the controlled variable, its rate and integral of error as the state variables.

Genetic algorithm and other stochastic global optimization techniques have also been employed for various optimal control problems. Wang *et al.* [11] used GA to optimally find out the weighting matrices of LQR i.e. Q and R with a specified structure. The concept of GA based optimum selection of weighting matrices has been extended for LQR as well as pole placement problems in Poodeh *et al.* [12]. GA based optimal time domain [13] and frequency domain loop-shaping [14] based PID tuning problems are also popular in the contemporary research community. The mixed H_2/H_∞ optimal PID controller tuning of Chen *et al.* [14] has been improved with GA as a single objective disturbance rejection PID controller in Krohling and Rey [15] and as multi-objective loop-shaping based design in Lin *et al.* [16]. A wide class of standard optimal control problems has been solved using evolutionary and swarm intelligence based global optimization techniques in Ghosh *et al.* [17], [18].

For optimum set-point tracking control of PID/FOPID controllers, time domain performance index optimization based tuning techniques are more popular among the research community and have been applied in Cao *et al.* [19], Das *et al.* [20] and Pan *et al.* [21], [22]. The impact of choosing the weighting matrices of LQR are discussed by Saif [23] in a detailed manner. The present methodology selects the weighting matrices for the quadratic regulator design similar to that in [11], [12], using Genetic Algorithm while minimizing a suitable time domain performance index. Then a new arbitrary (fractional) order integral performance index has been used as the objective function of GA, as suggested by Romero *et al.* [24]. The impact of these new FO integral indices based PID design on the closed loop control performance as well as the corresponding optimality of the quadratic regulators are also highlighted. An analog PID controller and its analogous digital PID both have been tuned with the proposed optimum weight selection based LQR technique for second order systems with very low and high damping as two illustrative examples.

The rest of the paper is organized as follows. Section II discusses about the theoretical framework for LQR based optimal analog and digital PID controller design. Section III proposes the GA based optimum weight selection methodology for LQR tuning of PID controllers. Section IV validates the proposed argument with two classes of second order systems as two illustrative examples. The paper ends with the conclusions in section V, followed by the references.

II. Design of LQR Based Optimal PID Controller for Second Order Systems

A. *Tuning of PID Controller as a Continuous Time Linear Quadratic Regulator*

He *et al.* [2]-[3] has given a formulation for tuning over-damped or critically-damped second order systems (having two real open loop process poles) which has been extended in this sub-section for lightly damped processes as well. Also, in [2], it has been suggested that one of the real poles needs to be cancelled out by placing one of the controller zeros at the same position on the negative real axis of complex *s*-plane. Thus the second order plant to be controlled with a PID controller can be reduced to a first order process to be controlled by a PI controller. Indeed, this approach of He *et al.* [2] does not hold for lightly damped processes having oscillatory open loop dynamics. With the approach of optimal PID tuning for second order processes in [2], the provision of simultaneously and optimally finding the three parameters of a PID controller (i.e. K_p, K_i, K_d) is also lost which has been addressed in the present paper. The presented approach assumes the error, its rate and integral as the state variables and designs the optimal state-feedback controller gains as the PID controller parameters (Fig. 1).

In Fig. 1, if the system is excited with an external input $r(t)$ to have a control signal $u(t)$ and output $y(t)$, then let us define the state variables as:

$$x_1 = \int e(t)dt, \quad x_2 = e(t), \quad x_3 = \frac{de(t)}{dt} \tag{1}$$

From the block diagram presented in Fig. 1, it is clear that

$$\frac{Y(s)}{U(s)} = \frac{K}{s^2 + 2\xi^{ol}\omega_n^{ol}s + \left(\omega_n^{ol}\right)^2} = \frac{-E(s)}{U(s)} \tag{2}$$

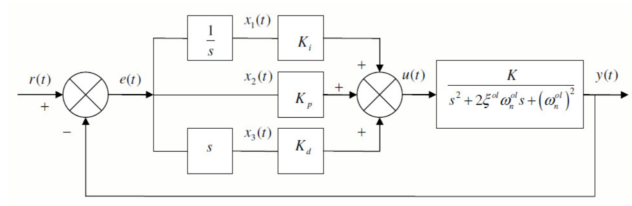

Figure 1. LQR Formulation of PID controller for second order processes.

In the case of feedback design, the external set-point does not affect the controller design i.e. $r(t) = 0$. In (2), the relation $y(t) = -e(t)$ is valid for standard regulator problem as in He *et al.* [2], when there is no change in the set point. Thus, equation (2) turns out to be

$$\left[s^2 + 2\xi^{ol}\omega_n^{ol}s + \left(\omega_n^{ol}\right)^2\right]E(s) = -KU(s) \tag{3}$$

$$\Rightarrow \quad \ddot{e} + 2\xi^{ol}\omega_n^{ol}\dot{e} + \left(\omega_n^{ol}\right)^2 e = -Ku \tag{4}$$

Using (1), equation (4) can be re-written as:

$$\dot{x}_3 + 2\xi^{ol}\omega_n^{ol}x_3 + \left(\omega_n^{ol}\right)^2 x_2 = -Ku \tag{5}$$

Using (1) and (5) the state space formulation becomes:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -\left(\omega_n^{ol}\right)^2 & -2\xi^{ol}\omega_n^{ol}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -K\end{bmatrix}u \tag{6}$$

Comparing (6) with the standard state-space representation i.e.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{7}$$

we get the system matrices as:

$$A = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -\left(\omega_n^{ol}\right)^2 & -2\xi^{ol}\omega_n^{ol}\end{bmatrix}, \quad B = \begin{bmatrix}0\\ 0\\ -K\end{bmatrix} \tag{8}$$

In order to have a LQR formulation with the system (7), the following quadratic cost function (J) is minimized

$$J = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Ru(t)\right]dt \tag{9}$$

It has been shown in [25] that minimization of cost function (9) gives the state feedback control signal as:

$$u(t) = -R^{-1}B^T Px(t) = -Fx(t) \tag{10}$$

where, P is the symmetric positive definite solution of the Continuous Algebraic Riccati Equation (CARE) given by (11)

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{11}$$

Here, the weighting matrix Q is symmetric positive semi-definite and the weighting factor R is a positive number. It is a common practice in optimal control to design regulators by varying Q , while keeping R fixed [23]. Also, they can be designed with user specified closed loop performance specifications [2]. Here P and Q are given by:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12} & P_{22} & P_{23} \\ P_{13} & P_{23} & P_{33} \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1 & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q_3 \end{bmatrix} \tag{12}$$

If it is now considered that the unique solution of the CARE (11) be P , the state feedback gain matrix becomes (13), corresponding to the optimal control signal involving the states as the loop error and its integro-differentials (1).

$$\begin{aligned} F &= R^{-1}B^T P = R^{-1}\begin{bmatrix} 0 & 0 & -K \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12} & P_{22} & P_{23} \\ P_{13} & P_{23} & P_{33} \end{bmatrix} \\ &= -R^{-1}K\begin{bmatrix} P_{13} & P_{23} & P_{33} \end{bmatrix} \\ &= -\begin{bmatrix} K_i & K_p & K_d \end{bmatrix} \end{aligned} \tag{13}$$

Using (10), the corresponding expression for the state feedback control signal can be derived as the output of a PID controller:

$$\begin{aligned} u(t) &= -Fx(t) \\ &= -\begin{bmatrix} -K_i & -K_p & -K_d \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \\ &= K_i \int e(t)dt + K_p e(t) + K_d \frac{de(t)}{dt} \end{aligned} \tag{14}$$

The above formulation clearly shows that with judicious choice of weighting matrices Q, R a PID controller can easily be tuned which preserves the achievable performance of an LQR i.e. minimum deviation in the state trajectories with minimum controller effort. The GA based choice of Q, R are discussed in the next section.

B. *Extension of Discrete Time Quadratic Regulator Theory in Digital PID Controller Design*

It is well known that discrete time realization of PID controllers are now more preferred than their continuous time counterpart [13], [26] since the gains of a digital PID controller can be changed, switched or scheduled online so as to control complicated time varying processes over the fixed gain, lossy analog realization. In order to do so, the system matrices (8) needs to be discretized with a specified sampling-time (T_s) for designing a digital PID controller using the discrete time optimal regulator theory. Infact, the continuous time LQR based analog PID may not remain optimal upon discretization with arbitrary sampling time. For this reason the basics of discrete time optimal quadratic control is first introduced [26]. For the continuous time system governed by (8), the task is to design an optimal state feedback controller which minimizes the infinite horizon quadratic optimal cost

$$J = \sum_{k=0}^{\infty} \left[x^T(k)Qx(k) + u^T(k)Ru(k) \right] \tag{15}$$

Minimization of the quadratic cost given in (15) leads to the solution of the Discrete Algebraic Riccati Equation (DARE) given by (16)

$$P = Q + G^T PG - G^T PH\left(R + H^T PH\right)^{-1} H^T PG \tag{16}$$

In (16), Q, R are the positive semi-definite weighting matrices and P is the positive definite solution of the discrete time Riccati equation. The discretized system matrices can be obtained from (8) using the specified sampling time T_s as:

$$\left.\begin{aligned} G &= e^{AT_s} \\ H &= \left(\int_0^{T_s} e^{A\lambda} d\lambda\right) B = \left(e^{AT_s} - I\right) A^{-1} B \end{aligned}\right\} \tag{17}$$

Matrix P in (16) produces the optimal state-feedback gain matrix F which minimizes the discrete time quadratic cost function (15) using the following relation, similar to the continuous time treatments (13):

$$F = \left(R + H^T PH\right)^{-1} H^T PG \tag{18}$$

Thus the optimal control law is given by:

$$\begin{aligned} u(k) &= -Fx(k) \\ &= -\left(R + H^T PH\right)^{-1} H^T PGx(k) \end{aligned} \tag{19}$$

To demonstrate the need of discrete LQR based design for the present problem a second order system of the structure (2) has been considered with $K = 1, \xi^{ol} = 0.2, \omega_n^{ol} = 1$. For the continuous time regulator design with (11) the weighting matrices have been considered as $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = 1$.

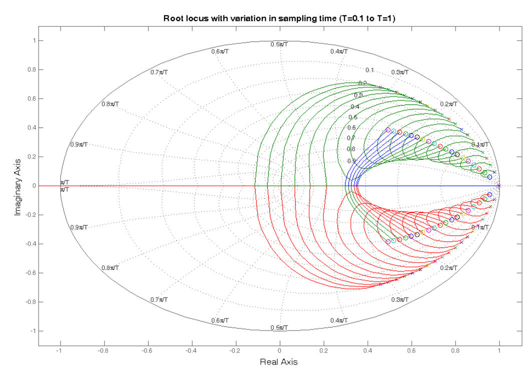

Figure 2. Root locus of the discretized open loop system.

Next, the control system with the obtained controller using (13) is discretized with sampling time $T_s \in [0.1, 1]$. The open loop root locus and closed loop pole locations are shown in Fig. 2-3 respectively. It is clear that the dominant complex poles shift towards high frequencies thus losing its dominant dynamic behavior. Thus, for a specific sampling time the optimal controller needs to be derived using the discrete version of the LQR formulation.

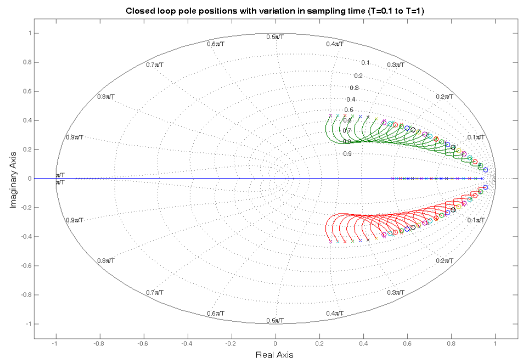

Figure 3. Shifting of closed loop pole locations with higher sampling time.

III. LQR Based PID Controller Design with Optimal Selection of Weighting Matrices

A. Effect of Weighting Matrices on the Control Performance

Fig. 4 shows the variation in time domain performances for the above example with change in the weighting matrices Q, R . With the variation in elements of Q matrix, the overshoot slightly increases with fall in rise time. For high value of R , the time response becomes sluggish. Similar observations can be found from control signal point of view.

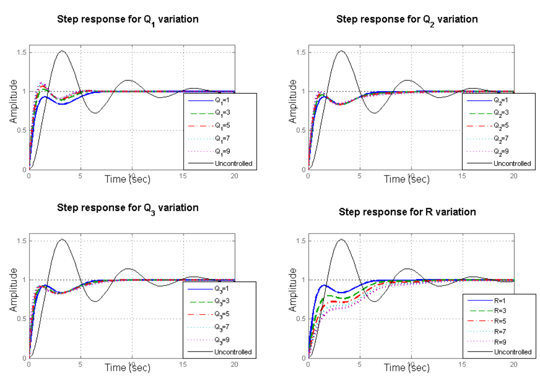

Figure 4. Effect of weighting matrices *Q* and *R* on the time response.

B. Optimum Selection of Weighting Matrices Using Genetic Algorithm Minimizing Another Integral Performance Index

It is well known [25], the above LQR based PID controller is the most optimal for a specific choice of the weighting matrices Q, R . Indeed, the time domain performance is heavily affected for any arbitrary choice of the weighting matrices (Fig. 4) although the optimality in terms of the trade-off between deviation in states trajectories and controller effort is preserved. Therefore, it is logical to choose the weighting matrices also optimally with respect to another time domain performance index as they determine the state feedback gains (PID controller gains in this case), indirectly monitoring the closed loop performance. Thus, a GA based stochastic optimization is formed by minimizing the cost function $\mathcal{J}$ (20) as a weighted sum of ITSE and Integral Squared Controller Output (ISCO) as in [21]. This tunes the elements of the weighting matrices i.e. $\{Q_1, Q_2, Q_3, R\}$ of LQR producing time domain optimal PID.

$$\mathcal{J} = \int_0^\infty \left[w_1 \cdot t \cdot e^2(t) + w_2 \cdot u^2(t) \right] dt \qquad (20)$$

Here, w_1, w_2 are the corresponding weights of ITSE and ISCO and are considered to be same, so as to put equal penalties on the loop error index and control signal. The rationale for using both these parameters in the objective function is to get a good time domain response and at the same time to limit the controller output to avoid actuator saturation and integral wind-up [21]. At a first glance this might seem as a redundant repetition since the LQR methodology already gives optimal values of the controller gains with the lowest cost. However, this is actually obtained for a specified value of the weighting matrices. When Q, R are varied, for each choice of weighting matrices the LQR would give an optimal gain with the lowest possible cost, but that does not necessarily imply a good time domain performance. Also, for an optimal choice of weighting matrices, the PID tuning problem becomes optimal due to the introduction of time domain performance index (20) as well as the continuous/discrete time optimal regulator (LQR) based approach (9) and (15) respectively involving the state variables.

C. Fractional Order Integral Performance Indices and Their Impact on the LQR Based PID Design

Fractional calculus is a 300 year's old subject and has found wide application in many branches of engineering and science [27]. The fractional order integral of any arbitrary function $f(t)$ can be represented by the left sided Riemann-Liouville definition as:

$${}_0I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t f(\tau)(t-\tau)^{\alpha-1} d\tau, \quad t \geq 0 \qquad (21)$$

Podlubny [28] has shown that for FO integral (21), the function $f(t)$ changes its shape with time unlike evaluation of the area, under a constant curve for integer order integrals. Now, the cost function (20) is generalized with any arbitrary order (Λ) [24], taking the order of cost function as an extra design tool.

$$\mathcal{J}^* = \frac{d^{-\Lambda}}{dt^{-\Lambda}} \left(w_1 \cdot t \cdot e^2(t) + w_2 \cdot u^2(t) \right) \qquad (22)$$

To show the flexibility of fractional order integral in the error index in controller design an oscillatory system is considered with $K = 1, \xi = 0.2, \omega_n^{ol} = 1$ with a badly tuned PID controller setting $K_p = 2, K_i = 2, K_d = 1$ (as a guess solution in the optimization process) and the time evolution of ITSE has been shown in Fig. 5. Fig. 5 also shows that the ITSE which is based on a first order integration approaches towards a steady value monotonically as time increases whereas with a low order of fractional integration (Λ) the integration no longer remains monotonic function which justifies the fact that the integrand in (22) is changing its shape over time [27]-[28].

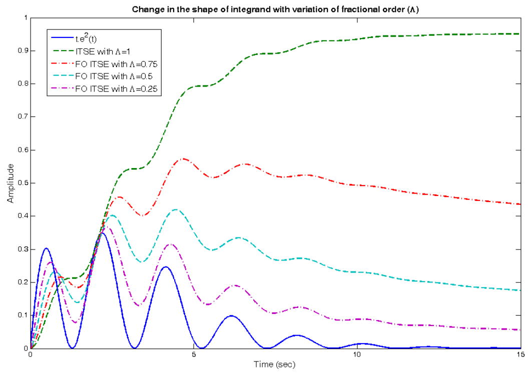

Figure 5. Changing shape of integrand with variation in FO of integration.

IV. SIMULATIONS AND RESULTS

To show the effectiveness of the proposed methodology a heavily oscillatory and sluggish system has been considered of the structure (2) with parameters $K = 1, \xi^{ol} = \{0.2, 5\}, \omega_n^{ol} = 1$ respectively, excluding the time delay [29]. GA based selection of weighting matrices yields the PID controller gains as the optimal state-feedback gains for the continuous and discrete LQR formulation and has been reported in Table I and II respectively along with the minima of the cost function ($J_{\min}$).

TABLE I. OPTIMUM PID CONTROLLER PARAMETERS WITH CARE

Process	FO (Λ)	J_{min}	K_p	K_i	K_d
Oscillatory	0.5	14.163	1.366073	0.260574	1.776595
	1.0	96.855	2.291051	0.234846	3.793601
	1.5	842.254	1.802627	0.246183	2.408207
Sluggish	0.5	16.366	1.641867	0.172061	0.168591
	1.0	118.156	1.732432	0.16095	0.173052
	1.5	999.961	1.892469	0.252889	0.198146

TABLE II. OPTIMUM PID CONTROLLER PARAMETERS WITH DARE

Process	FO (Λ)	J_{min}	K_p	K_i	K_d
Oscillatory	0.5	14.85861	0.098233	0.171755	0.198904
	1.0	103.9139	0.097949	0.170926	0.198634
	1.5	940.4062	0.101853	0.175119	0.204367
Sluggish	0.5	16.65402	1.351018	0.16446	0.134639
	1.0	120.8922	1.527714	0.14705	0.151819
	1.5	1173.427	1.389176	0.150055	0.138047

Fig. 6 shows that with the designed controllers, the set-point tracking performance is good and control signals are significantly low. Here, all simulations have been run for a finite time horizon of 100 seconds.

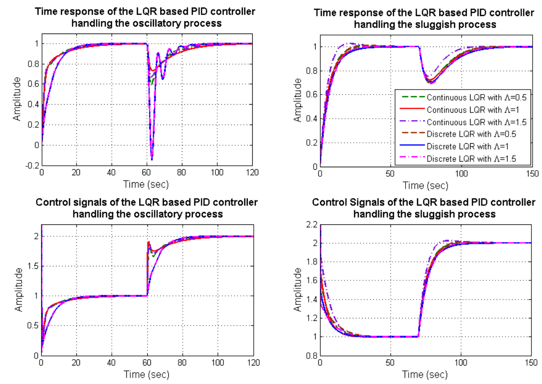

Figure 6. Time response and control signal with LQR based PID.

It is also shown in [15] that the infinite time performance index (15) can be calculated from the Riccati solution (P matrix) using the initial values of the state variables i.e.

$$J = x^T(0)Px(0) = \sum_{k=0}^{\infty}\left[x^T(k)Qx(k) + u^T(k)Ru(k)\right] \quad (23)$$

For a PID controller as in our case, initial values of the state variables (i.e. error, its rate and integral) can not be calculated directly to find out the optimal control cost (23) since with a step-input excitation the initial value of the error rate will tend to infinity and initial value of integral error will tend to zero with the initial value of error signal remaining one. To overcome this problem the following methodology has been adopted. Here, eigen-values of the GA based differential P matrices (for DARE) are evaluated, corresponding to the gains in Table II. For the oscillatory system with $\xi = 0.2$,

$$\begin{aligned} eig\left(P_{\Lambda=1.0} - P_{\Lambda=0.5}\right) &= \left[1.9362 \quad 2.9975 \quad 41.0457\right]^T \\ eig\left(P_{\Lambda=1.5} - P_{\Lambda=1.0}\right) &= \left[0.7795 \quad 0.9049 \quad 11.1710\right]^T \end{aligned} \quad (24)$$

Similarly, for the sluggish system with $\xi = 5$,

$$\begin{aligned} eig\left(P_{\Lambda=0.5} - P_{\Lambda=1.0}\right) &= \left[6 \quad 88.1 \quad 7128.4\right]^T \\ eig\left(P_{\Lambda=1.0} - P_{\Lambda=1.5}\right) &= \left[0.0601 \quad 2.6516 \quad 213.4166\right]^T \end{aligned} \quad (25)$$

Clearly, the eigen-values in (24) and (25) are positive which indicates that the differential matrices in left hand side of (24) and (25) are positive definite. Now, it is well known that for any two Riccati solutions P_1, P_2 (where, $P_1 > P_2$), considering initial value of the state variables as $x(0)$, pre-multiplication with $x^T(0)$ and post-multiplication with $x(0)$ yields the comparison of costs:

$$x^T(0)P_1x(0) > x^T(0)P_2x(0) \quad \Rightarrow J_1 > J_2 \quad (26)$$

From (24)-(26) it is clear that while designing an optimum discrete time LQR based digital PID controller to control an oscillatory system, the cost of control increases for higher values of the fractional order integral performance index whereas, the converse is true for discrete time LQR-PID control of a sluggish system. Therefore, it is recommended to set low values of the FO ($\Lambda < 1$) for controlling oscillatory processes and high values of FO ($\Lambda > 1$) for the control of sluggish processes in the arbitrary order cost function (22).

V. Conclusion

GA based optimum selection of weighting matrices is done for designing a discrete time LQR. Conventional PID controller design has been generalized as a LQR problem for the control of second order sluggish and oscillatory systems. The cost of control for the LQR-PID design has been shown to be dependent on the process characteristics, especially the damping and the fractional order of the cost function. Future scope of research can be directed towards extension of the proposed methodology to Linear Quadratic Gaussian (LQG) problems considering noisy measurements and disturbances.

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