Optimal Design a Fractional-Order PID Controller using Particle Swarm Optimization Algorithm

Hadi Ramezanian and Saeed Balochian

*Department of Electrical Engineering, Gonabad Branch,
Islamic Azad University, Gonabad, Iran*

hramezanian@gmail.com, saeed.balochian@gmail.com

Abstract

This paper presents a method for optimum tuning of Fractional PID controllers using PSO algorithm which have fractional or integer order transfer functions. Particle swarm optimization (PSO) is a robust stochastic optimization technique based on the movement and intelligence of swarm, applies the concept of social interaction to problem solving. The best parameters of Fractional-Order PID controller consist of proportional gain k_p integral gain k_i , fractional-order of integrator λ, derivative gain k_d and fractional-order of differentiator μ can be determinate by using this technique. The proposed approach with defined cost function has a very easy implementation and ability of fast tuning of optimum fractional-order PID controllers parameters. From the comparison this technique with the other methods, its influence and efficiency are illustrated.

Key words: *Optimal Fractional-Order PID controller design; PSO; fitness function*

1. Introduction

In the last decade, fractional-order dynamic systems and controllers has been studying widely in many areas of engineering and science [1-5].The concept of the fractional-order PID controllers was proposed by Podlubny in 1997 [6]. He also demonstrated the better response of this type of controllers, in comparison with the classical PID controllers, when used for the control of fractional-order systems. He was demonstrated the major role of fractional-order calculus in a smart mechatronic system [7]. Hardware and digital realizations of fractional-order systems can be followed in [8, 9, 10]. A frequency domain approach based on given phase margin and crossover frequency is studied in [11]. In [12] an optimization method is presented such that predefined design specifications are satisfied. A method is presented based on the pole distribution of the characteristic equation in the complex plane [13]. A state-space design approach is presented based on feedback pole placement in [14]. A method is presented based on differential evolution (DE) technique in [15]. Also, a method is presented based on idea of the Ziegler–Nichols and the Astrom–Hagglund methods in [16].A method is presented based on the asymptotic behavior of fractional algebraic equations and applies a delicate property of the root loci of the system in [16]. A fractional order $(PI)^\lambda$ controller is designed to improve the flight control performance of a small fixed-wing unmanned aerial vehicle (UAV) in [17]. In [18] a fractional order controller for AVR system is presented based on a new criterion function with eight terms by use of particle swarm

optimization in [19] by applying Mean of Root of Squared Error (MRSE), Mean of Absolute Magnitude of the Error (MAE) and Mean Minimum Fuel and Absolute Error (MMFAE) as fitness function, Fractional PID Controllers Using PSO Algorithm for Robot Trajectory Control is tuned.

This paper is organized as follows. Section 2 discusses a brief review to fractional calculus especially fractional order PID controller. In Section 3, introduction to particle swarm optimization algorithm is given. Also, the purposed fitness function is illustrated in this section and its application in $PI^\lambda D^\mu$ PSO-controller is discussed. In Section 4, the results are indicated using two examples.

2. Review on Fractional Calculus

2.1. Fractional-order PID controllers (FOPID)

The fractional PID controller is a generalization of the PID controller. The transfer function of this controller is given by the following function:

$$C(s) = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu \qquad (1)$$

where k_p is the proportional constant, k_i is the integration constant, k_d is the differentiation constant and λ and μ are positive real numbers. The most popular definitions of Fractional Derivatives or integrals in Fractional calculus are Grünwald-Letnikov (GL), Riemann-Liouville (RL) and Caputo statements. The Grünwald-Letnikov expressed the Fractional-Order derivative by the following equation [20]:

$$_aD_t^\gamma f(t) = \lim_{h \to 0} h^{-\gamma} \sum_{j=0}^{[t-a/h]} (-1)^j \binom{\gamma}{j} f(t - jh) \qquad (2)$$

where $(-1)^j \binom{r}{j}$ are [21],

$$c_0^{(r)} = 1 \ , c_j^{(r)} = \left(1 - \frac{1+r}{j}\right) c_{j-1}^{(r)} \ , j = 0,1,2, \ldots (3)$$

and used for recursive computation, these are weights.

Reimann-liouville (RL) expression for fractional-order derivative is given by:

$$_aD_t^\gamma f(t) = \frac{1}{\Gamma(n - \gamma)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t - \tau)^{\gamma-n+1}} d\tau \qquad (4)$$

where $\Gamma(.)$ is Euler's Gamma function that generalizes the factorial, and allows operator, to take non-integer values. An another definition is the Caputo definition given by:

$$_aD_t^\gamma f(t) = \frac{1}{\Gamma(\gamma - n)} \int_a^t \frac{f^{(n)}(\tau)}{(t - \tau)^{\gamma-n+1}} d\tau \qquad (5)$$

2.2. Oustaloup approximation algorithm

Oustaloup's approximation method uses a band-pass filter to approximate the fractional – order operator s^λ based on frequency – domain response [22]. The approximate transfer function of a continuous fractional order operator s^λ with Oustaloup Algorithm is as follows:

$$G_f(s) = K \prod_{k=-N}^{N} \frac{s + \omega'_k}{s + \omega_k} \qquad (6)$$

where the zeros, poles and the gain can be evaluated, respectively, as:

$$\omega'_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1-\gamma)}{2N+1}} \qquad (7)$$

$$\omega_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1+\gamma)}{2N+1}} \qquad (8)$$

$$K = \left(\frac{\omega_h}{\omega_b}\right)^{-\frac{\gamma}{2}} \prod_{k=-N}^{N} \frac{\omega_k}{\omega'_k} \qquad (9)$$

In simulation, for approximation of s^λ, frequency range is closed as: $\omega \in [\omega_b, \omega_h]$ and ω_b= 0.001, ω_h=1000, N=2.

More detail is available in [23].

3. Design of $PI^\lambda D^\mu$ pso-controller

3.1. Particle Swarm Optimization (PSO)

PSO was developed in 1995 by James Kennedy (social- psychologist) and Russell Eberhart (electrical engineer)[24]. PSO is the only algorithm that does not implement the survival of the fittest. It uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the best solution. *Jth* particle(k_j) is treated as a point in a N-dimensional space$(k_j = k_{j,1}, k_{j,2}, \ldots, k_{j,N})$ which adjusts its "flying" according to its own flying experience as well as the flying experience of other particles. Each particle keeps track of its coordinates in the solution space which are associated with the best solution (fitness) that has achieved so far by that particle. This value is called personal best, *Pbest*.$pbest_j = (pbest_{j,1}, pbest_{j,2}, \ldots, pbest_{j,N})$ is previous position of the jth particle in a N-dimension space. Another best value that is tracked by the PSO is the best value obtained so far by any particle in the neighborhood of that particle. This value is called *gbest*. The basic concept of PSO lies in accelerating each particle toward its *pbest* and the *gbest* locations, with a random weighted acceleration at each time step. The searching method of the implemented PSO-FOPID controller is given in its flowchart in Figure 1. The modification of the particle's position can be mathematically modeled according the following equations:

$$v_{j,N}^{(t+1)} = \omega . v_{j,N}^{(t)} + c_1 rand_1(\ldots)\left(pbest_{j,N} - K_{j,N}^{(t)}\right)$$
$$+ c_2 rand_2(\ldots)\left(gbest - K_{j,N}^{(t)}\right) \qquad (10)$$

$$K_{j,N}^{(t+1)} = K_{j,N}^{(t)} + v_{j,N}^{(t+1)} \qquad (11)$$

the following constraints for velocity in each iteration is applied as:

$$v_{j,N}^{(t+1)} = \begin{cases} V_N^{max} & v_{j,N}^{(t+1)} > V_N^{max} \\ -V_N^{max} & v_{j,N}^{(t+1)} < -V_N^{max} \end{cases} \qquad (12)$$

where V_N^{max} is the maximum possible magnitude of velocity of any particle in the Nth dimension, j=1,2,…,n,N=1,2,…,m , and

n   number of particles in the population(population size) ;

m   dimension of problem(number of members in a particle) that there is five;

t   pointer of iterations(generations);

$v_{j,N}^{(t)}$ Velocity of particle j at iteration t;

ω   weighting function;

c_1, c_2 acceleration factors;

$rand_1(\dots), rand_1(\dots)$uniformly distributed random numbers between 0 and 1;

$K_{j,N}^{(t)}$Current position of particle j at iteration t;

$pbest_{j,N}$pbest position of particle j;

$gbest$gbest position of swarm.

Changing velocity by this way enables the Jth particle, to search around its local best position, pbest, and global best position, gbest.in many experiences with PSO,V_N^{max} is often set to the maximum dynamic range of the variables on each dimension.

Suitable selection of weighting function ω in (10) provides a balance between global and local explorations, thus requiring less iteration on average to find a sufficiently optimal solution. ω often decreases linearly from (ω$_{max}$) about 0.9 to (ω$_{min}$) about 0.4 during a run. Weighting function ω is set by: $\omega = \omega max - \frac{\omega max - \omega min}{iter_{max}} \times iter$ (13)
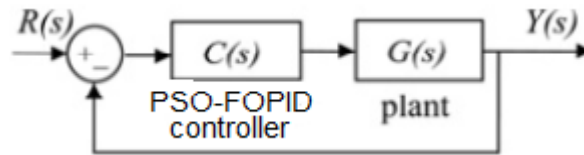
where $iter_{max}$ is the maximum number of iterations and iter is the current number of iterations.
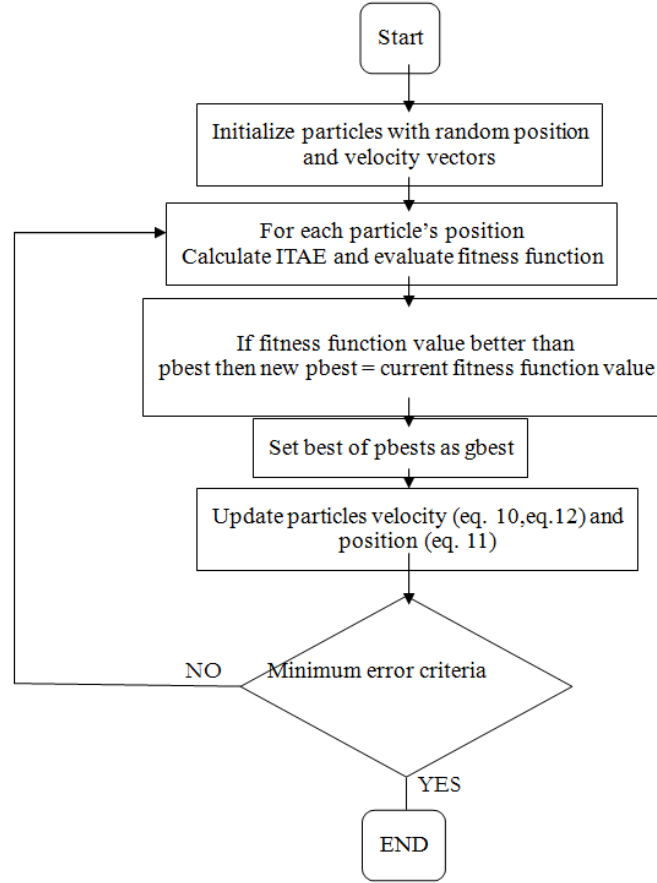
Figure 1. The PSO-FOPID flowchart

3.2. Purposed fitness function

The most difficult step in applying PSO is to choose the best fitness function which is used to evaluate fitness of each particle. Unity feedback closed loop system is shown in Figure 2.

Figure 2. Unity feedback closed loop system

In this paper transfer function of the plant can be fractional or integer order. Controller of the system is stated in equation (1). In [25] is calculated the following equations:

$$f_1(\lambda, \mu) = k_p + k_i \omega_{cp}^{-\lambda} \cos(\frac{\pi}{2}\lambda) + k_d \omega_{cp}^{\mu} \cos\left(\frac{\pi}{2}\mu\right) - k_c(\cos \phi_{pm}) = 0 \quad (14)$$

$$f_2(\lambda, \mu) = -k_i \omega_{cp}^{-\lambda} \sin(\frac{\pi}{2}\lambda) + k_d \omega_{cp}^{\mu} \sin\left(\frac{\pi}{2}\mu\right) - k_c(\sin \phi_{pm}) = 0 \quad (15)$$

where ϕ_{pm} is the required phase margin and ω_{cp} is frequency of the critical point on the Nyquist curve of G(s) as:

$$\arg\left(G(j\omega_{cp})\right) = -180^0 \qquad (16)$$

And k_c is gain margin as:

$$\frac{1}{\left|G(j\omega_{cp})\right|} = k_c \qquad (17)$$

The fitness functions defined as:

$$J(k_p, k_i, \lambda, k_d, \mu) = |f_1| + |f_2| + ITAE \qquad (18)$$

where f_1, f_2 are equations (14), (15), $|.|$ is absolute operator, ITAE is integral of the absolute magnitude of error criterion given by:

$$ITAE = \int_0^{t_{sim}} t|e(t)|\,dt \qquad (19)$$

where t_{sim} is total simulation time and $e(t)=r(t)-y(t)$ is the tracking error. The *ITAE* performance indice has excellences of smaller overshoot and oscillation than the IAE(integral of the absolute error) or the ISE(integral square error) performance indices and it is the most sensitive then *ITAE* has the best selectivity the purposed fitness function have a suitable structure of time and frequency domain criterion that provide good control performance. The goal is obtaining the best step response, by minimization of $J(k_p, k_i, \lambda, k_d, \mu)$ using PSO algorithm and obtain optimum parameters of $PI^\lambda D^\mu$pso-controller. The following limits are fixed for every parameter by using the practical assumption of the FOPID controller design in [26]:

$$1 \le k_p \le 1000, 1 \le k_i, \lambda \ge 0, k_d \le 500, \mu \le 2 \; ; \qquad (20)$$

With this suitable fitness function and assumption, the optimized parameters of the fractional order controller are calculated with least run of PSO algorithm.

4. Results

A. Example 1

Consider the following fraction order plant for Figure 2 given in [27, 16, 28]:

$$G(s) = \frac{1}{0.8s^{2.2}+0.5s^{0.9}+1}(21)$$

The unit step response of open loop system is shown in Figure 3. Phase crossover frequency and gain margin of the system can be calculated as:

ω_{cp} =1.63 rad/s

k_c =1.39dB

And specified phase margin is ϕ_m as:

$$\phi_m = \frac{\pi}{3} \text{rad}$$

In this paper PSO parameters is selected as:

$c_1 = c_2 = 2$, $\omega_{max} = 0.9, \omega_{min} = 0.4$, maxIter=100 , population size=100.

Optimal $PI^\lambda D^\mu, PD^\mu$ controller using this method is designed as:

$$c_{pso-PI^\lambda D^\mu}(s) = 82.3532 + \frac{40.888}{s^{1.5733}} + 27.1914s^{1.8797} \qquad (22)$$

$$c_{pso-PD^\mu}(s) = 109.7672 + 40.1513s^{1.8663} \qquad (23)$$

The PD^μ controllers were designed in [27, 16] as:

$$c_1(s) = 20.5 + 3.7343s^{1.15} \qquad (24)$$

$$c_2(s) = 20.5 + 20.5s^{1.2} (25)$$

The $PI^\lambda D^\mu$ controller was designed in [28] as:

$$c_3(s) = 233.4234 + \frac{22.3972}{s^{0.1}} + 18.5274s^{1.15} \qquad (26)$$

The close loop step response of the system with $c_{pso-PI^\lambda D^\mu}(s), c_3(s)$ controllers is shown in Figure 4. The close loop step response of the system with $c_1(s), c_2(s), c_{pso-PD^\mu}(s)$ controllers is shown in Figure 5 where obviously illustrate that the proposed method has better response than the others. Step response specifications $c_3(s), c_{pso-PI^\lambda D^\mu}(s)$, $c_2(s), c_{pso-PD^\mu}(s)$ are shown in Table 1. The convergence characteristic of the PSO-fractional PID controller is shown in Figure 8. The response of the system to a square wave with period of four seconds and sample time 0.01 seconds is shown in Figure 6.
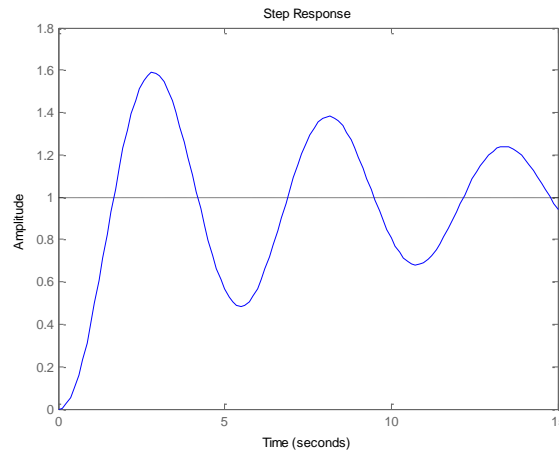
Figure 3. The unit step response of open loop fractional order system
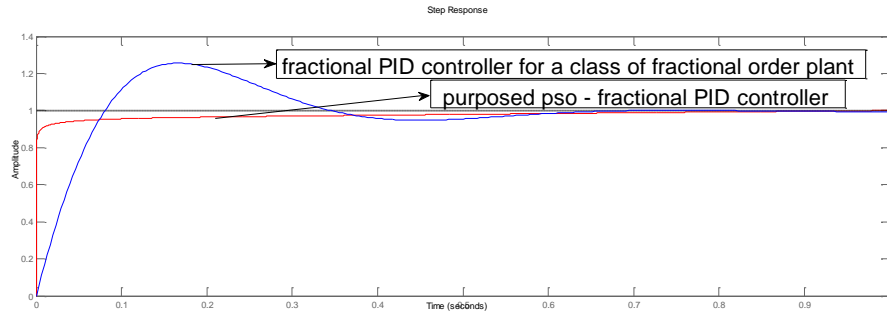
Figure 4. Comparison Close Loop step response of the system with $c_{pso-PI^\lambda D^\mu}(s), c_3(s)$ **controllers**
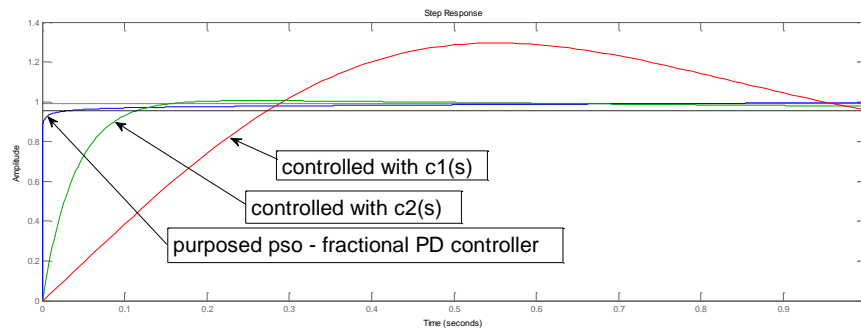

Figure 5. The unit step response of close loop fractional order system with $c_1(s), c_2(s), c_{pso-PD^\mu}(s)$ **controllers**

Table 1.Step Response Specifications $c_{`3}(s), c_{pso-PI^\lambda D^\mu}(s), c_2(s), c_{pso-PD^\mu}(s)$ **Controllers**

Step response specifications	fractional PID for a class of fractional order plants	Proposed $PI^\lambda D^\mu$ controller with PSO algorithm	PD^μ controller $c_2(s)$	Proposed PD^μ controller with PSO algorithm
max. overshoot,%	26.1	0	5.49	0
peak time, s	0.168	-	0.2	-
rise time, s	0.06	0.006	0.07	0
settling time (%5)	0.31	0.07	0.04	0.01
settling time (%2)	0.57	0.516	0.124	0.124
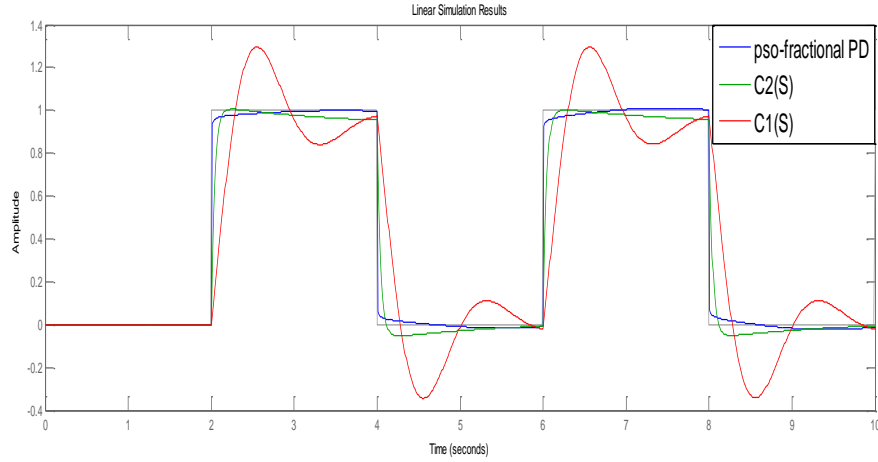
Figure 6. The Response of the System to a Square Wave

B. Example 2

Consider the following plant for Figure 2 given in [25]:

$$G(s) = \frac{1}{s(s+3)(s+4)} \qquad (27)$$

Phase crossover frequency and gain margin of the system can be calculated as:

$$\omega_{cp} = 3.46 \ rad/s$$

$$k_c = 38.5 \ dB$$

And specified phase margin is ϕ_m as:

$$\phi_m = \frac{\pi}{6} rad$$

Optimal $PI^\lambda D^\mu$ controller using this method with proposed fitness function is designed as:

$$c_{pso}(s) = 39 + \frac{39}{s^{0.0934}} + 39s^{1.1388} \qquad (28)$$

Using the Ziegler–Nichols and Astrom–Hagglund methods, the $PI^\lambda D^\mu$ controller were designed in [25] as:

$$c_1(s) = 50.4 + \frac{55.6}{s^{.7569}} + 22s^{0.8564} \qquad (29)$$

Using Simulink by considering least square method for optimization, the $PI^\lambda D^\mu$ controller were designed in [25] as:

$$c_{2opt}(s) = 42.4580 + \frac{87.2733}{s^{0.5030}} + 53.1352s^{0.9623} \qquad (30)$$

The close loop step response of the system with $c_1(s), c_{2opt}(s), c_{pso}(s)$ controllers is shown in Figure 7. Step response specifications $c_1(s), c_{2opt}(s), c_{pso}(s)$ are shown in Table 2. Obviously the proposed method influence and efficiency are illustrated. The convergence characteristic of the PSO-fractional PID controller is shown in Figure 9. The response of the system to a square wave with period of four seconds and sample time 0.01 seconds is shown in Figure 10.
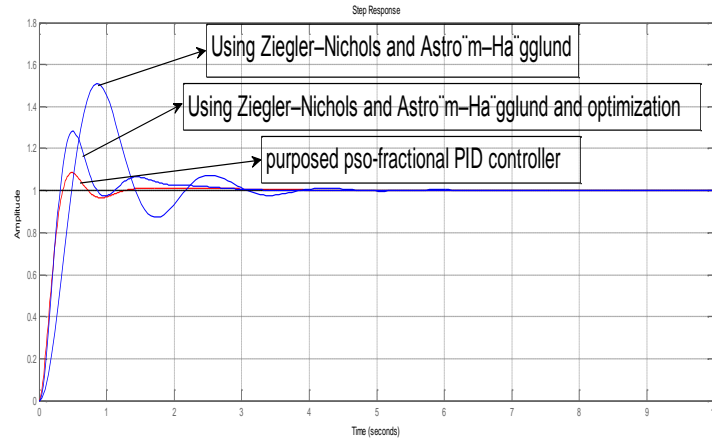
Figure 7. Comparison close loop step response with $c_1(s), c_{2opt}(s), c_{pso}(s)$ Controllers
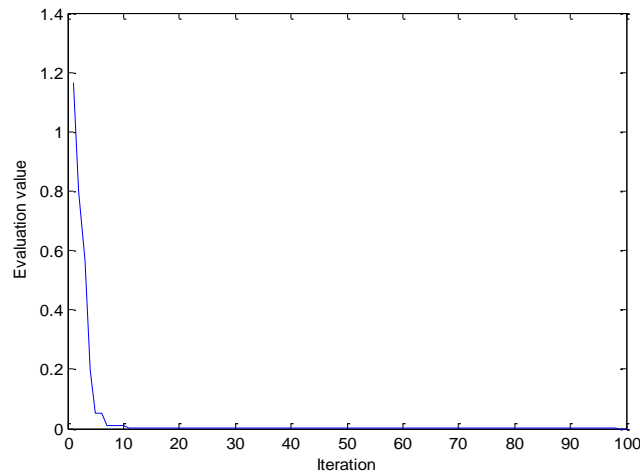

Figure 8. The Convergence Characteristic of the PSO-fractional PID Controller
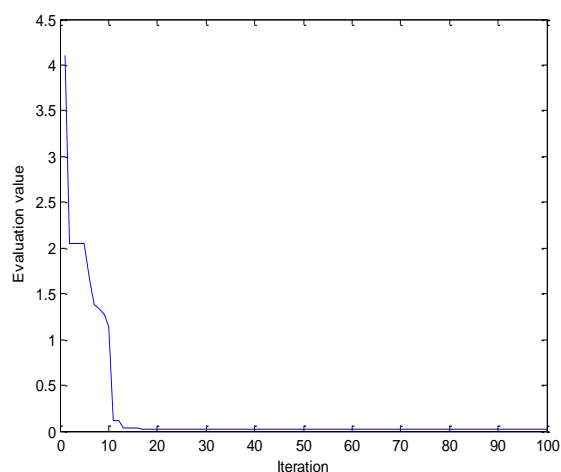
Figure 9. The Convergence Characteristic of the PSO-fractional PID Controller

Table 2. Step Response Specifications $c_{\text{`}1}(s), c_{2opt}(s), c_{pso}(s)$ Controllers

Step response specifications	fractional PID	fractional PID with optimized values	Proposed $PI^\lambda D^\mu$ controller with PSO algorithm
max. overshoot, %	52.5	31.5	8.6
peak time, s	0.83	0.47	0.45
rise time, s	0.47	0.31	0.24
settling time (%5)	2.75	1.60	0.6
settling time (%2)	2.98	1.83	1.09
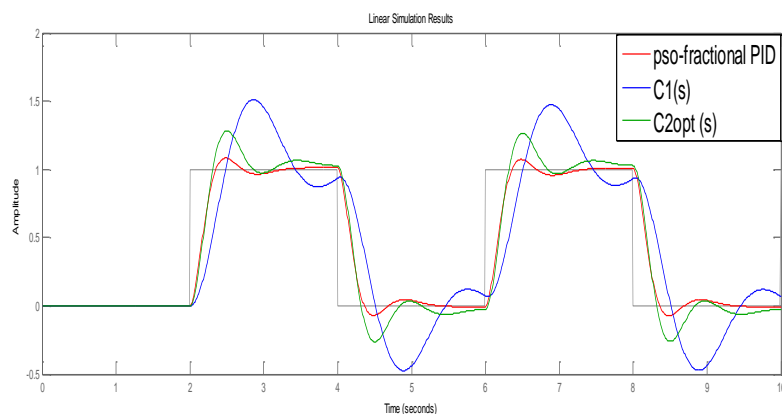
Figure 10. The response of the system to a square wave

5. Conclusions

In this paper, a method of tuning the optimal parameters of fractional order PID using PSO algorithm is proposed for the fractional or integer order systems. Using purposed fitness function, optimum parameters can be found in less number of iterations. The simulation results illustrate that this method has better control performance than the other methods. The proposed technique may apply as an efficient method for the optimal design of fractional-order controllers. The proposed method of fractional PID controller design can apply for practical systems.

References

[1] K. B. Oldhamand and J. Spanier, "The Fractional Calculus", Academic Press, New York, **(1974)**.

[2] C. H. Lubich, "Discretized fractional calculus", SIAM Journal on Mathematical Analysis, **(1986)**, pp. 704-719.

[3] K. S. Miller and B. Ross, "An Introduction to the Fractional Calculus and Fractional Differential Equations", Wiley, New York, **(1993)**.

[4] A. Oustaloup, "Fractional order sinusoidal oscillators: optimization and their use in highly linear FM modulators", IEEE Transactions on Circuits and Systems, **(1981)**, pp.1007–1009.

[5] M. Chengbin and Y. Hori, "The application of fractional order PID controller for robust two-inertia speed control", Proceedings of the 4th International Power Electronics and Motion Control Conference, Xi'an, **(2004)** August.

[6] I. Podlubny, L. Dorcak and I. Kostial, "On Fractional Derivatives, Fractional-Order Dynamic Systems and $PI^\lambda D^\mu$ controllers", Proceedings of the 36th Conference on Decision & Control, San Diego, California, USA, **(1997)** December.

[7] I. Podlubny, "Fractional-Order Systems and $PI^\lambda D^\mu$ Controllers", IEEE Transactions on Automatic Control, vol. 44, no. 1, **(1999)** January, pp. 208-214.

[8] Y. Q. Chen, D. Xue and H. Dou, "Fractional calculus and biomimetic control", Proceedings of the First IEEE International Conference on Robotics and Biomimetics (RoBio04), Shengyang, China, **(2004)** August.

[9] M. Nakagawa and K. Sorimachi, "Basic characteristics of a fractance device", IEICE Transactions on Fundamentals E75-A, no. 12, **(1992)**, pp.1814–1819.

[10] D.Valerio and J. Sada Costa, "Introduction to single-input, single-output fractional control", IET Control Theory Appl., vol. 5, Issue 8, **(2011)**, pp. 1033–1057.

[11] B. M. Vinagre, I. Podlubny, L. Dorcak and V. Feliu, "On fractional PID controllers: a frequency domain approach", Proceedings of IFAC Workshop on Digital Control-PID'00, Terrassa, Spain, **(2000)**.

[12] C. A. Monje, B. M. Vinagre, Y. Q. Chen, V. Feliu, P. Lanusse and J. Sabatier, "Proposals for fractional $PI^\lambda D^\mu$ tuning", Proceedings of Fractional Differentiation and its Applications, Bordeaux, **(2004)**.

[13] I. Petras, "The fractional order controllers: methods for their synthesis and application", Journal of Electrical Engineering, **(1999)**, pp. 284–288.

[14] L. Dorcak, I. Petras, I. Kostial and J. Terpak, "State-space controller design for the fractional-order regulated system", Proceedings of the ICCC, Krynica, **(2001)**, pp. 15–20.

[15] A. Biswas, S. Das, A. Abraham and S. Dasgupta, "Design of fractional-order $PI^\lambda D^\mu$ controllers with an improved differential evolution", Engineering Applications of Artificial Intelligence, vol. 22, **(2009)**, pp. 343–350.

[16] F. Merrikh-Bayat, M. Karimi-Ghartemani, "Method for designing $PI^\lambda D^\mu$ stabilisers for minimum-phase fractional-order systems", IET Control Theory Appl., vol. 4, Issue 1, **(2010)**, pp. 61–70.

[17] Y. Luo, H. Chao, L. Di and Y. Q. Chen, "Lateral directional fractional order (PI) a control of a small fixed-wing unmanned aerial vehicles: controller designs and flight tests", IET Control Theory Appl., vol. 5, Issue 18, **(2011)**, pp. 2156–2167.

[18] M. Zamani, M. Karimi-Ghartemani, N. Sadati and M. Parniani, "Design of a fractional order PID controller for anAVR using particle swarm optimization", Control Engineering Practice, vol. 17, no. 12, **(2009)**, pp. 1380–1387.

[19] Z. Binguland and O. Karahan, "Tuning of Fractional PID Controllers Using PSO Algorithm for Robot Trajectory Control", Proceedings of the IEEE International Conference on Mechatronics, Istanbul, Turkey, **(2011)** April 13-15.

[20] R. Caponetto, G. Dongola, L. Fortuna and I. Petras, "Fractional order systems, modeling and control applications, world scientific series on nonlinear science, series A", World Scientific Publishing Co. Pvt. Ltd., vol. 72, **(2010)**.

[21] L. Dorcak, "Numerical models for simulation the fractional-order control systems", UEF-04-94 (The Academy of Sciences, Institute of Experimental Physics, Kosice, Slovakia, **(1994)**.

[22] A. Oustaloup, X. Moreau and M.Nouillant, "The CRONE suspension", Control Eng. Pract., **(1996)**, pp. 1101-1108.

[23] S. Das, "Functional fractional calculus", Berlin: Springer, **(2011)**.

[24] J. Kennedy and R. C. Eberhart, "Particle swarm optimization", Proceedings of IEEE International Conference on Neural Networks, Perth, Australia, **(1995)**, pp. 1942–1948.

[25] C. Yeroglu and N. Tan, "Note on fractional-order proportional integral differential controller design", IET Control Theory Appl., vol. 5, Issue 17, **(2011)**, pp. 1978–1989.

[26] K. Astrom and T. Hagglund, "PID Controllers; Theory, Design and Tuning", Instrument Society of America, Research Triangle Park, **(1995)**.

[27] I. Podlubny, "Fractional-order Systems and Fractional-order Controllers", The Academy of Sciences Institute of Experimental Physics, UEF-03-94, Kosice, Slovak Republic, **(1994)**.

[28] C. Zhao, D. Xue and Y. Q. Chen, "A Fractional Order PID Tuning Algorithm for A Class of Fractional Order Plants", Proceedings of the IEEE International Conference on Mechatronics & Automation, Niagara Falls, Canada, **(2005)** July, pp. 216-221.

Authors

Hadi Ramezanian received the M.S degree in control engineering from2011 to 2013. He is member of young researcher club IAU in Iran. His research interests focus on design of fractional controllers, fractional and fuzzy systems and particle swarm optimization algorithm.

Saeed Balochian received the B.S. degree in communication system engineering 2005 and M.S. degree in control and automation engineering in 2007, and Ph.D. at the Islamic Azad University, science and research branch of Tehran. Currently he is assistant professor at Islamic Azad University of Gonabad branch. His research interests focus on fuzzy systems, Fractional derivative systems control.