On Fractional PI^λ Controllers: Some Tuning Rules for Robustness to Plant Uncertainties

C. A. Monje (cmonje@unex.es)
University of Extremadura, Spain

Blas M. Vinagre (bvinagre@unex.es)
University of Extremadura, Spain

YangQuan Chen (yqchen@ece.usu.edu)
CSOIS, Utah State University, USA

Vicente Feliu (vicente.feliu@uclm.es)
Universidad de Castilla-La Mancha, Spain

Abstract. The objective of this work is to find out optimum settings for a fractional PI^λ controller in order to fulfil three different robustness specifications of design for the compensated system, taking advantage of the fractional order, λ. Since this fractional controller has one parameter more than the conventional PI controller, one more specification can be fulfilled, improving the performance of the system and making it more robust to plant uncertainties, such as gain and time constant changes. For the tuning of the controller, an iterative optimization method has been used based on a nonlinear parametric minimization routine. Two real life examples of application are presented and simulation results are presented to illustrate the effectiveness of this kind of unconventional controllers.

Keywords: Fractional differentiator, fractional-order dynamic systems, proportional and integral (PI) controller, PID, controller tuning.

1. Introduction

The PID controller is by far the most dominating form of feedback in use today. Due to its functional simplicity and performance robustness, the proportional-integral-derivative controller has been widely used in the process industries. Design and tuning of PID controllers have been a large research area ever since Ziegler and Nichols presented their methods in 1942 (see [1]). Specifications, stability, design, applications and performance of the PID controller have been widely treated since then (see [2] and [3] for additional references).

On the other hand, in recent years we find an increasing number of studies related with the application of fractional controllers in many areas of science and engineering. This fact is due to a better understanding of the fractional calculus (FC) potentialities revealed by many phenomena such as viscoelasticity and damping, chaos, diffusion and wave propagation, percolation and irreversibility.

In what concerns automatic control theory the FC concepts were adapted to frequency-based methods. The frequency response and the transient response of the non-integer integral (in fact Bode's ideal transfer function) and its application to control systems was introduced by Manabe (see [4]) and more recently in [5]. Oustaloup (see [6]) studied the fractional order algorithms for the control of dynamic systems and demonstrated the superior performance of the $CRONE$ (Commande Robuste d'Ordre Non Entier) method over the PID controller. More recently, Podlubny (see [7]) proposed a generalization of the PID controller, namely the $PI^{\lambda}D^{\mu}$ controller, involving an integrator of order λ and a differentiator of order μ. He also demonstrated the better response of this type of controller, in comparison with the classical PID controller, when used for the control of fractional order systems. A frequency domain approach by using fractional PID controllers is also studied in [8].

Further research activities are running in order to develop new tuning rules for fractional controllers, studying previously the effects of the non integer order of the derivative and integral parts to design a more effective controller to be used in real-life models (see [9] and [10]). Some of these technics are based on an extension of the classical PID control theory. To this respect, in [11] the extension of derivation and integration order from integer to non integer numbers provides a more flexible tuning strategy and therefore an easier achieving of control requirements with respect to classical controllers. In [12] an optimal fractional order PID controller based on specified gain margin and phase margin with a minimum ISE criterion has been designed by using a differential evolution algorithm.

A more extensive work has been developed in [13], where a fractional PID control has been applied for active reduction of vertical tail buffeting. In this work PID algorithms are adequate for a linear description of the damping, for instance in conventional, metallic aerospace structures, where damping was modelled as linearly proportional to velocity. However, for non-linear damped structures such as composites, which exhibits viscoelastic behavior and/or non-linear forcing functions such as buffeting, the design of control system using PID algorithms is not accurate and a better tool to model and control those non-linear damping situations is needed. Fractional order calculus has proven to be a very powerful tool to model and control viscoelastic damped structures, as commented above.

A fractional order control strategy has also been successfully applied in the control of a power electronic buck converter (see [14], [15], [16] and [17]), more concretely a fractional sliding mode control. The control strategy is based on the use of a fractional order controller with a Smith

Predictor structure and Bode's ideal transfer function has been used as reference system.

Another approach is the use of a new control strategy to control first-order systems with delay (see [18]) based on a $D^{\beta}I^{\alpha}$ controller with fractional order integral and derivative parts. Besides, it is being developed another method for plants with long dead-time based on the use of a PI^{α} controller with a fractional integral part of order α (see [19] and [20]). From the results obtained, it can be concluded that the system controlled with this type of controller is more robust to gain changes.

In this work it is studied the problem of designing a non integer order controller of the form:

$$C(s) = k_p(1 + \frac{1}{T_i s^{\lambda}}) \tag{1}$$

The interest of this kind of controller is justified by a better flexibility, since it exhibits a fractional integral part of order λ. Thus, three parameters can be tuned in this structure (k_p, T_i and λ), that is, one more parameter than in the case of a conventional PI controller ($\lambda = 1$). We can take advantage of the fractional order λ to fulfil an additional specification of design and make the system more robust to plant uncertainties, such as gain and time constant changes.

The paper is organized as follows. Section 2 proposes different design specifications to make the system more robust to gain changes and time constant changes, respectively. The compensation problem using this fractional PI^{λ} controller is formulated in both cases. In section 3, the optimization method used for the tuning of the fractional controller is commented, describing shortly the problem of nonlinear minimization. In section 4 two illustrative examples of application are presented, applying this kind of controllers to two real cases: the model of an open irrigation canal and the pH dynamic model in a sugar cane raw juice neutralization process. Finally, some concluding remarks are cited in section 5.

2. Robustness Design Specifications

In this section several interesting design specifications are remarked to be met by the fractional compensated system in order to be more robust to gain changes and time constant changes.

2.1. PI^λ Controller Robust to Gain Changes

Let us comment three interesting specifications for robustness to plant gain changes:

- Gain and phase margins have always served as important measures of robustness. It is known from classical control that the phase margin is related to the damping of the system and therefore can also serve as a performance measure (see [21]). This way, the **phase margin (ϕ_m) and phase crossover frequency (ω_{cp}) specifications** have been taken into account for the robustness of the system. Thus, the next conditions must be fulfilled:

$$\begin{aligned} Arg(F(j\omega_{cp})) &= Arg(C(j\omega_{cp})G(j\omega_{cp})) = -\pi + \phi_m & (2) \\ |F(j\omega_{cp})|_{dB} &= |C(j\omega_{cp})G(j\omega_{cp})|_{dB} = 0dB & (3) \end{aligned}$$

  where $F(s)$ is the open-loop transfer function of the system.

- **Robustness to plant gain variations.** To this respect, the next constraint must be fulfilled (see [2] and [20]):

$$\left(\frac{d(Arg(C(j\omega)G(j\omega)))}{d\omega}\right)_{\omega=\omega_{cp}} = 0 \qquad (4)$$

  With this condition the phase is forced to be flat at ω_{cp} and so, to be almost constant within an interval around ω_{cp}. It means that the system is more robust to gain changes and the overshoot of the response is almost constant within the interval.

To meet these three specifications (2, 3, 4) a set of three nonlinear equations and three unknown variables (k_p, T_i, λ) must be solved, since the fractional PI^α controller has three parameters to tune. The complexity of this set of nonlinear equations is very significant, specially when fractional orders of the Laplace variable s are introduced.

The optimization method proposed deals with this kind of problem, solving the set of equations above and obtaining optimum settings for the fractional PI^λ controller. This method will be explained in section 3.

The condition of no steady-state error is fulfilled just with the introduction of the fractional integrator, without any other requirements, since a fractional integrator of order $k + \lambda, k \in N, 0 < \lambda < 1$, properly implemented, is for steady-state error cancellation as efficient as an integer order integrator of order $k + 1$ (see [22]).

2.2. PI^λ Controller Robust to Time Constant Changes

For the purpose of robustness to time constant variations, the gain and phase margins have been taken as the main indicators for the reasons commented before. Thus, the specifications to meet in this case are the ones in equations (2) and (3), referred to phase margin (ϕ_m) and phase crossover frequency (ω_{cp}) specifications, and the one referring to gain margin (M_g), that is:

$$Arg(F(j\omega_{cg})) = Arg(C(j\omega_{cg})G(j\omega_{cg})) = -\pi \tag{5}$$

The relation between the gain margin (M_g) and the gain crossover frequency (ω_{cg}) is given by:

$$|F(j\omega_{cg})|_{dB} = |C(j\omega_{cg})G(j\omega_{cg})|_{dB} = 1/M_g \tag{6}$$

Thus, it has to be solved a set of four nonlinear equation (2,3,5,6) with four unknown variables $(k_p, T_i, \lambda, \omega_{cg})$, by using the optimization method proposed before. Again, the condition of no steady-state error is already fulfilled with the introduction of the fractional integrator.

3. The Problem of Nonlinear Minimization

This paper deals with the problem of compensating a general system $G(s)$ by using a fractional PI^λ controller of the form in equation (1), so that the design specifications mentioned above are met.

This way, a set of nonlinear equations must be solved. The solution for this kind of systems is not a trivial problem. Thus, the optimization toolbox of Matlab has been used to reach out the better solution with the minimum error. The function used for this purpose is called FMINCON, which finds the constrained minimum of a function of several variables. It solves problems of the form MIN$_X$F(X) subject to: C(X) ¡=0, Ceq(X)=0, LB ¡=X ¡=UB, where F is the function to minimize; C and C_{eq} represent the nonlinear inequalities and equalities, respectively (nonlinear constraints); X is the minimum we are looking for; and LB and LU define a set of lower and upper bounds on the design variables, X.

In our case, to solve the set of three or four nonlinear equations, depending on the specifications required, the following parameters for the function FMINCON have been considered:

```
[parameters,error]=
=FMINCON('main_fun',init_cond,[],[],[],[],lb,ub,'constraint_fun',options)
```

where:

- *main_fun* is the function corresponding to the specification of magnitude in equation (3), the one we want to minimize.

- *init_cond* is the set of initial conditions for the parameters of the controller.

- *lb* is the set of lower bounds on the parameters of the controller *(parameters)*.

- *ub* is the set of upper bounds on the parameters of the controller *(parameters)*.

- *constraint_fun* is the function corresponding to the set of equations obtained from specifications in (2) and (4), for the case of robustness to gain changes, and from the specifications in (2), (5) and (6), for the case of robustness to time constant changes.

- *options* is the structure in which the optimization parameters are defined, such as the maximum number of function evaluations allowed, the termination tolerance on the value of the function *main_fun,* the termination tolerance on the violation of the constraints defined in the function *constraint_fun,* and others.

- *parameters* is the set of the parameters of the controller that minimizes the function *main_fun.* In fact, the solution we are looking for.

- *error* is the value of the objective function *main_fun* at the solution *parameters.*

Thus, the specification in equation (3) is taken as the main function to minimize, and the rest of specifications are taken as constrains for the minimization, all of them subjected to the optimization parameters defined in *options.*

4. Application Examples

This section shows the results obtained when using the fractional PI^{λ} controller to compensate a second-order plant with a time delay and a first-order plant with a time delay. These two models correspond to two real plants which will be detailed in next subsections.

4.1. Compensation of an Open Irrigation Canal

The dynamic behavior of the real single canal pool to be controlled can be represented by a second order transfer function with a time delay :

$$G_1(s) = \frac{K_1}{(T_1 s + 1)(T_2 s + 1)} e^{-\tau_1 s} \tag{7}$$

where K_1 is the static gain; T_1, T_2 are the time constants; and τ_1 is the time delay. In this real plant, $K_1 = 1.25, T_1 = 300\,\mathrm{sec}, T_2 = 60\,\mathrm{sec}$ and $\tau_1 = 600\,\mathrm{sec}$ (see[23]).

The design specifications required for the compensated system are:

– Phase Crossover frequency, ω_{cp}=0.0011rad/sec.

– Phase margin, $\phi_m = 1.31 rad \approx 75^\circ\,\mathrm{deg}$.

– Gain margin, $M_g = 2.4$.

For these design specifications, the fractional PI^λ parameters obtained by applying the proposed optimization method are $k_p = 0.1435$, $T_i = 10.5269$, $\lambda = 0.5883$ and $\omega_{cg} = 0.0027$. Therefore, the fractional PI^λcontroller will be given by expression:

$$C_3'(s) = 0.1435(1 + \frac{1}{10.5269 s^{0.5883}}) \tag{8}$$

Though the final value theorem states that the fractional controller exhibits null steady state error if $\lambda > 0$, the fact of being $\lambda < 1$ makes the output converge to its final value more slowly than in the case of an integer controller. In order to avoid this problem the fractional integrator must be implemented as $\frac{1}{s^\lambda} = \frac{1}{s} s^{1-\lambda}$, ensuring this way the effect of an integer integrator $1/s$. In this particular example of application the fractional part $s^{1-0.5883}$ has been implemented by the Oustaloup continuous approximation of a fractional differentiator (see [24]).

Figure 1 shows the bode plots of the compensated system obtained with the fractional PI^λ controller.

As it can be observed, the frequency specifications are met, that is, a phase margin $\phi_m = 1.31 rad \approx 75^o$ at the frequency $\omega_{cp} = 0.0011 rad/\sec$ and a gain margin $M_g = 2.4$ at the frequency $\omega_{cg} = 0.0027 rad/\sec$.

Now we will show and compare the step responses obtained when compensating the open irrigation canal described above using a conventional PID controller, a fractional $D^{1-\lambda} I^\lambda$ controller (see[23]) and this fractional PI^λ controller.

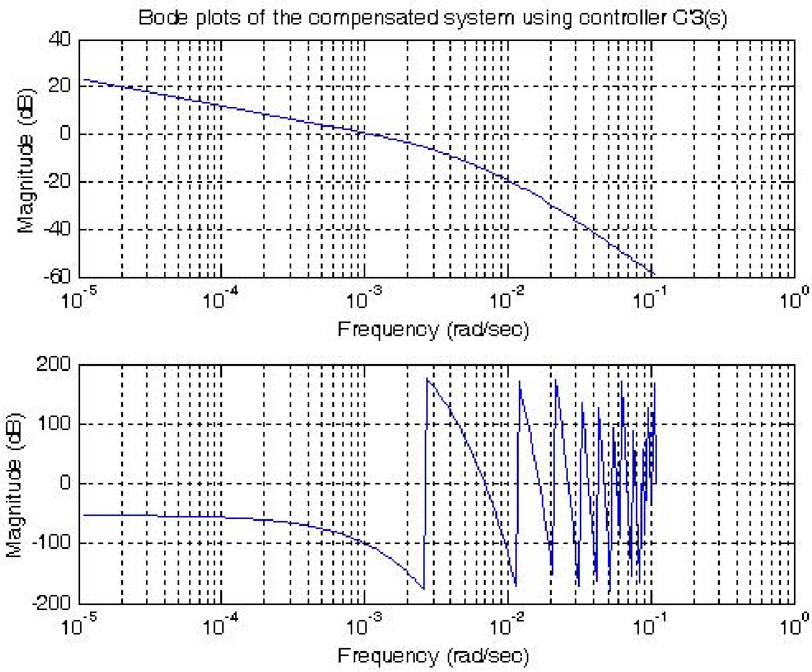

Figure 1. Bode plots of the compensated system using controller $C_3^{'}(s)$

- The conventional PID controller that fulfills the given specifications is $C_1'(s) = 0.5511 + 80.1334s + \frac{0.0008}{s}$.
- The fractional $D^{1-\lambda}I^{\lambda}$ controller to compensate this system is $C_2'(s) = \frac{0.0089+1.9964s}{s^{0.66}}$.
- The fractional PI^{λ} controller is the one proposed in equation (8).

Figure 2 shows the dynamic behavior of the compensated system using the conventional controller $C_1'(s)$, considering values of the time constant T_1 in the range [6sec,10000sec].

Figure 3 shows the dynamic behavior of the compensated system using the fractional $D^{1-\lambda}I^{\lambda}$ controller $C_2'(s)$, considering decreasing values of the time constant T_1. This behavior is compared in the same figure with the one obtained using the proposed fractional PI^{λ} controller $C_3'(s)$.

Figure 4 shows the dynamic behavior of the compensated system using controller $C_2'(s)$ and $C_3'(s)$, considering values of the time constant T_1 in the range $[300\,\text{sec} \leq T_1 \leq 10000\,\text{sec}]$.

The comparative Table I shows the overshoot of these three compensated systems for $6\,\text{sec} \leq T_1 \leq 300\,\text{sec}$, since in this range the differences

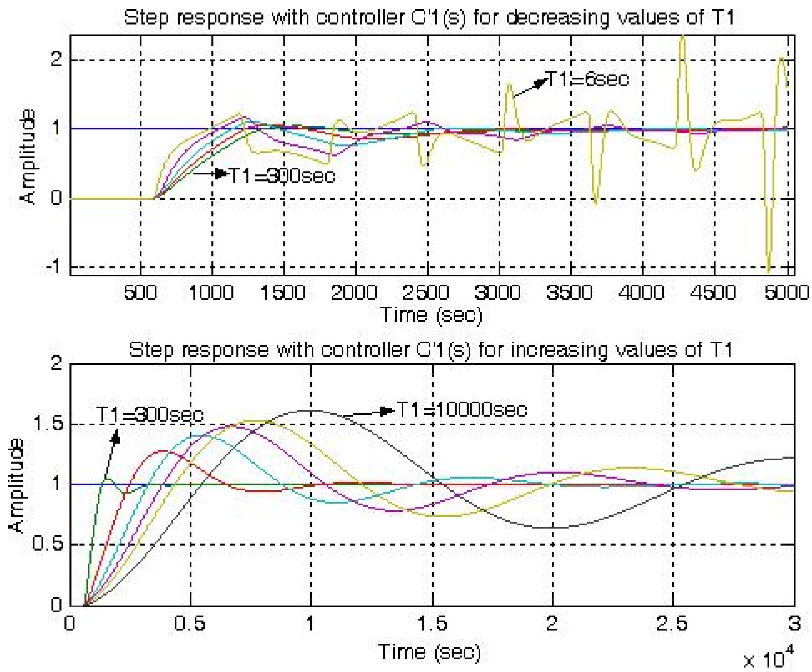

Figure 2. Step responses of the compensated system using controller $C_1'(s)$, for $6\,\text{sec} \leq T_1 \leq 10000\,\text{sec}$

Table I. Overshoot for Decreasing Values of T_1

T_1 (sec)	PID Controller	$D^{1-\lambda}$ I^{λ} Controller	PI^{λ} Controller
300	4.4%	9.4%	19.4%
225	6.8%	6.4%	18.0%
150	11.9%	3.0%	16.3%
75	17.3%	—	13.9%
6	Unstable	—	9.5%

among the controllers are more significant, as it can be observed in the graphs.

From the results obtained it can be concluded that:

– **For decreasing values of the time constant $\mathbf{T}_1$** ($6\,\text{sec} \leq T_1 \leq 300\,\text{sec}$). The performance of the compensated systems using the

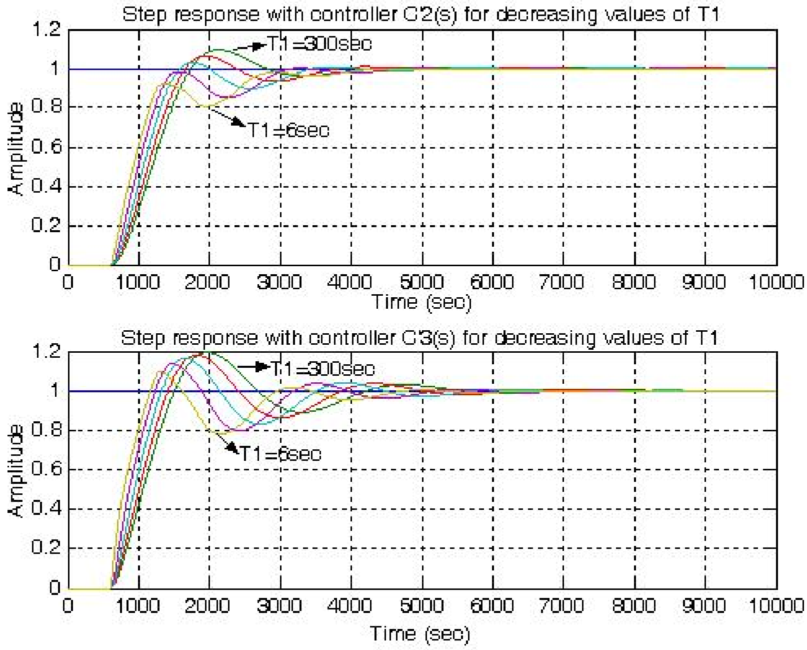

Figure 3. Step responses of the compensated system using controllers $C_2'(s)$ and $C_3'(s)$, for $6\,\mathrm{sec} \leq T_1 \leq 300\,\mathrm{sec}$

proposed fractional structures is much better than in the case of a conventional PID controller. The response provided by the fractional controllers is more damped than the one provided by the PID controller, which becomes unstable for decreasing values of the time constant T_1. Besides, with the fractional PI^λ controller the compensated system is more robust to changes of the time constant than with the fractional $D^{1-\lambda}I^\lambda$ controller, since variations of the overshoot for different values of T_1 are lower for the PI^λ structure, as it can be observed in the graphs and Table I.

– **For increasing values of the time constant $\mathbf{T}_1$** ($300\,\mathrm{sec} \leq T_1 \leq 10000\,\mathrm{sec}$)**.** The responses with these controllers are very similar, presenting almost the same overshoot rate and similar settling time. Then, the behavior in this range of variation of the time constant is not significant for the selection of one controller or another.

In short, it can be said that the use of the fractional structures provide a better response than the one obtained with the use of the conventional PID controller. With the fractional $D^{1-\lambda}I^\lambda$ controller

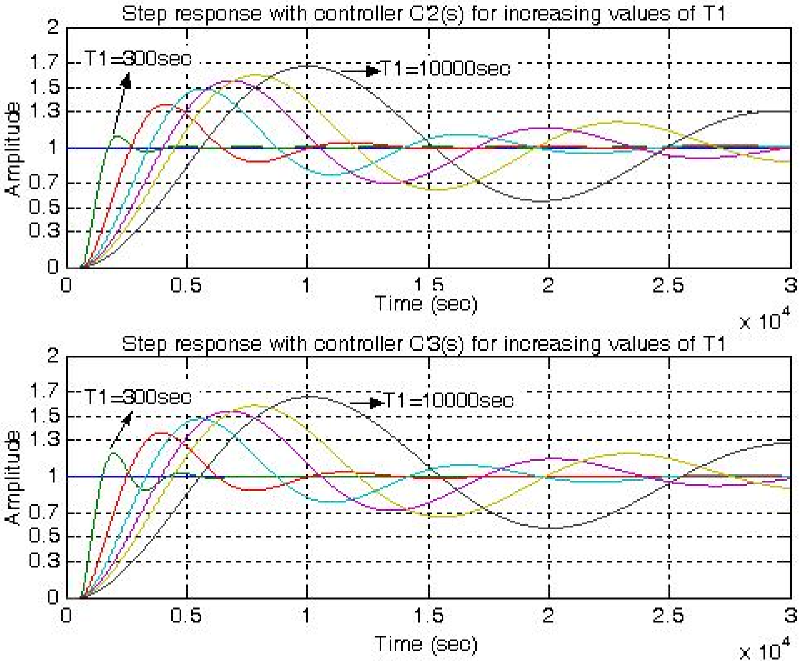

Figure 4. Step responses of the compensated system using controllers $C_2'(s)$ and $C_3'(s)$, for $300\,\text{sec} \le T_1 \le 10000\,\text{sec}$

the overshoot for the nominal plant and for decreasing values of T_1 is lower than with the PI^λ controller and the correspondence frequency domain-time domain is better. However, the PI^λ structure is more robust than the others, as it can be concluded from Table I.

4.2. Compensation of a Sugar Cane Raw Juice Neutralization Process

This section will deal with the problem of compensating the pH dynamic model of a real sugar cane raw juice neutralization process (see [25]). After several experiments carried out and assuming for the process a first order model with dead time of the form:

$$G_2(s) = \frac{K_2}{1 + Ts} e^{-\tau_2 s}$$

the characteristic parameters obtained for the plant are: time constant $T = 62\,\text{sec}$, dead time $\tau_2 = 10\,\text{sec}$, nominal gain $K_2 = 0.55$. The variations of the dead time and the time constant were insignificant when this model was fitted. However, in all the cases large changes were observed in the gain of the transfer function, with a variation range $K_2 \in [0.15, 0.94]$. For that reason, it is important to obtain a controller very robust to these gain changes.

The design specifications required for the compensated system are:

- Phase Crossover frequency, ω_{cp}=0.02rad/sec.
- Phase margin, $\phi_m = 1.13rad \approx 65°$ deg .
- Robustness to variations in the gain of the plant must be fulfilled, with the condition formulated in equation (4).

For these design specifications, the fractional PI^λ parameters obtained when applying the proposed optimization method are $k_p = 2.2326, T_i = 78.4142$ and $\lambda = 1.1274$. Therefore, the fractional PI^λcontroller will be given by expression:

$$C_4(s) = 2.2326(1 + \frac{1}{78.4142s^{1.1274}}) \tag{9}$$

As commented before, the fractional integrator will be implemented ensuring an integer integrator $\frac{1}{s}$. In this particular case, the corresponding fractional part $s^{1-1.1274}$ is implemented by the Grünwald-Letnikov definition of the fractional differentiator (see [26]).

Now we will show and compare the results obtained when compensating this process using the next types of controllers (see [20]):

- A conventional PI controller ($\lambda = 1$) based on these frequency specifications: $C_1(s) = 1.7662(1 + \frac{1}{38.47s})$.
- A conventional PI controller to be used with a Smith predictor structure, to compensate the effects of the delay: $C_2(s) = 1.2749(1 + \frac{1}{24.51s})$.
- A fractional $D^{1-\lambda}I^\lambda$ controller that fulfills the given specifications with plant pole cancellation: $C_3(s) = 0.0202(\frac{1+62s}{s^{1.1505}})$.
- The fractional PI^λcontroller $C_4(s)$ in equation (9).

Figures 5, 6, 7 and 8 show the bode plots of the compensated systems obtained with these controllers. Figure 9 shows the step responses of the close-loop systems for values of gains $K_2 = 0.15$, $K_2 = 0.55$ and $K_2 = 94$.

From the results obtained it is concluded that:

- With the controller $C_1(s)$ the frequency requirements are fulfilled, but the system is not robust to gain changes.

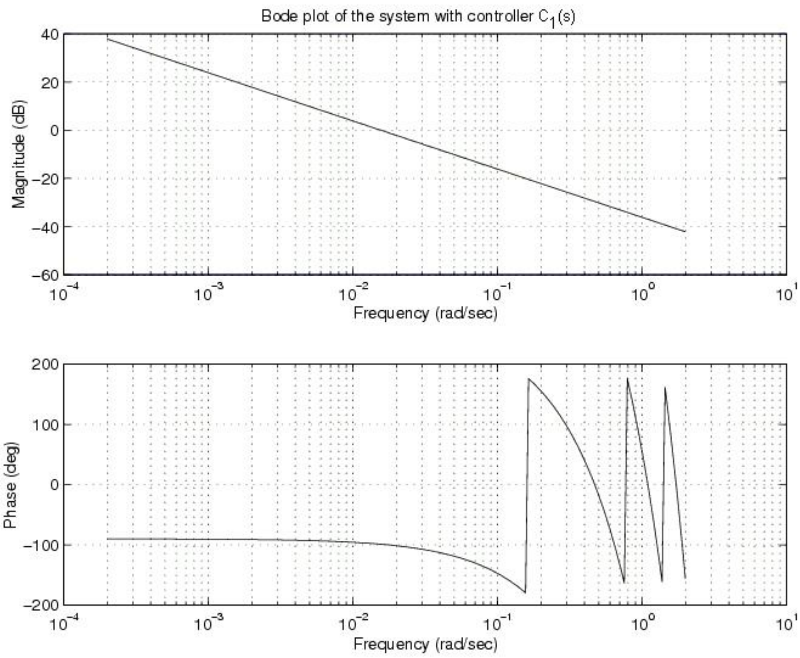

Figure 5. Bode plot of the system with controller $C_1(s)$

- With the Smith Predictor structure, the controller $C_2(s)$ can be designed without taking into account the dead time of the plant, but variations in the gain of the plant make the system behave poorly, since the appearance of mismatches between the real plant and the model make this structure not exclude the dead time effects from the control loop.

- With the controller $C_3(s)$ the frequency requirements are fulfilled but the fact of cancelling the pole of the plant is not trivial, since it is difficult to know where exactly the pole is and the control efforts can be important. However, the system seems to be more robust than with the controllers $C_1(s)$ and $C_2(s)$.

- With the controller $C_4(s)$ the phase of the system is forced to be flat at ω_c and so, to be almost constant within an interval around ω_c, as it can be observed in the Bode plot of Figure 8. It means that the systems is more robust to gain changes and the overshoot of the response is almost constant within this interval.

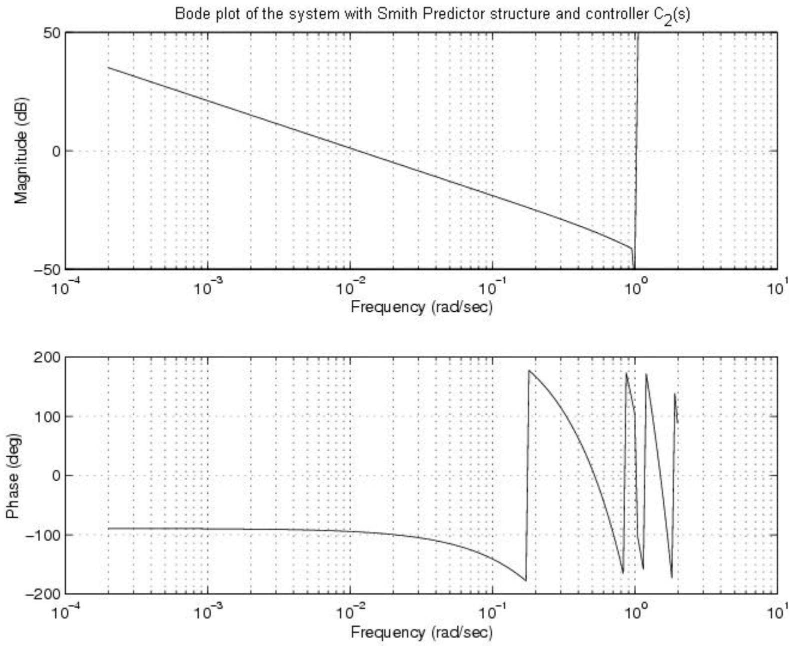

Figure 6. Bode plot of the system with controller $C_2(s)$ and Smith predictor structure

Therefore, with the fractional PI^{λ} controller proposed $C_4(s)$, the robustness and performance of the system are better than in any other cases.

5. Concluding Remarks

In this paper, a fractional PI^{λ} controller has been proposed to fulfil three different robustness design specifications for the compensated system, that is, one more specification than in the case of a conventional PI controller. An optimization method to tune the controller has been used, based on a nonlinear function minimization subject to some given nonlinear constraints. This controller has been applied and compared with other structures in two real cases of application. From the simulation results, it can be concluded that the specifications required in each case are totally fulfilled with the fractional PI^{λ} structure, given a higher robustness to the system and obtaining a better performance than with the rest of controllers tested here. The goodness of this

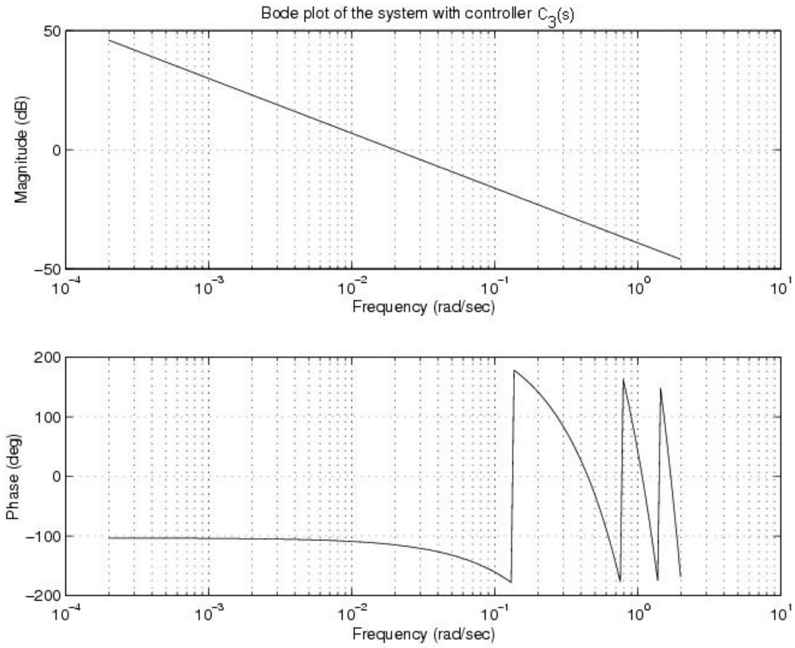

Figure 7. Bode plot of the system with controller $C_3(s)$

fractional structure and the accuracy of the tuning method proposed are then remarkable.

Further investigations include the tuning of a fractional $PI^\lambda D^\mu$ controller in order to fulfil two more design requirements (a total of five), due to the introduction of two more parameters (k_d and μ). This way, the compensated system could meet not only robustness specifications, but also noise rejection and sensibility constraints, improving the performance of the system.

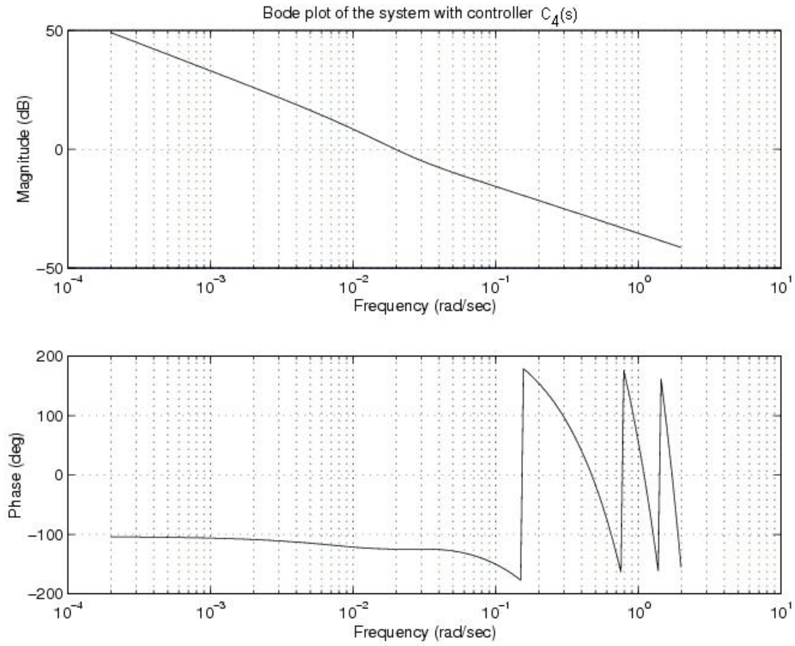

Figure 8. Bode plot of the system with controller $C_4(s)$

References

1. J. G. Ziegler and N. B. Nichols, "Optimum Settings for Automatic Controllers," *Transactions of the ASME*, no. 64, pp. 759–768, 1942.
2. Y. Chen, C. Hu, and K. L. Moore, "Relay Feedback Tuning of Robust PID Controllers with Iso-Damping Property." Accepted to present at the IEEE 2003 Conference on Decision and Control, December 9-12, 2003, Maui, HI, USA.
3. K. J. Åström and T. Hägglund, "The Future of PID Control," in *IFAC Workshop on Digital Control. Past, Present and Future of PID Control*, (Terrassa, Spain), pp. 19–30, April 2000.
4. S. Manabe, "The Non-integer Integral and its Application to Control Systems," *ETJ of Japan*, vol. 6, no. 3/4, pp. 83–87, 1961.
5. R. S. Barbosa, J. A. T. Machado, and I. M. Ferreira, "A Fractional Calculus Perspective of PID Tuning." Preprint.
6. A. Oustaloup, *La Commade CRONE: Commande Robuste d'Ordre Non Entier.* Hermès, Paris, 1991.
7. I. Podlubny, "Fractional-Order Systems and PID-Controllers," *IEEE Trans. on Automatic Control*, vol. 44, no. 1, pp. 208–214, 1999.
8. B. M. Vinagre, I. Podlubny, L. Dorčák, and V. Feliu, "On Fractional PID Controllers: A Frequency Domain Approach," in *IFAC Workshop on Digital Control. Past, Present and Future of PID Control*, (Terrassa (Spain)), pp. 53–58, April 2000.

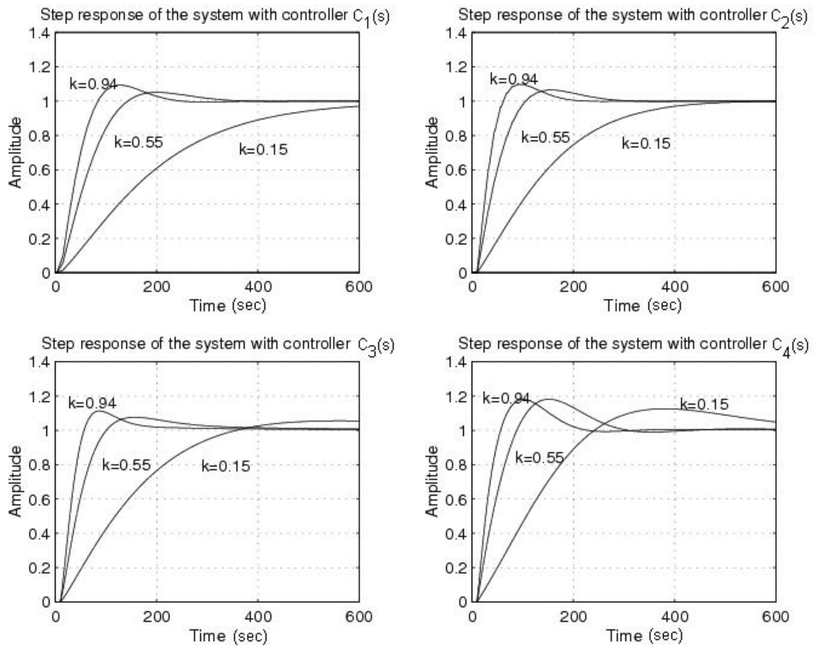

Figure 9. Step responses of the system with the considered controllers

9. B. M. Vinagre, C. A. Monje, and A. J. Calderón, "Fractional Order Systems and Fractional Order Control Actions," in *41st Conference on Decision and Control. Tutorial Workshop 2: Fractional Calculus Applications in Automatic Control and Robotics*, (Las Vegas), December 2002.
10. Blas Manuel Vinagre Jara, *Modelado y Control de Sistemas Dinmicos Caracterizados por Ecuaciones Integro-Diferenciales de Orden Fraccional*. PhD thesis, Escuela de Ingenierias Industriales, Universidad de Extremadura, Badajoz, 2001.
11. R. Caponetto, L. Fortuna, and D. Porto, "Parameter Tuning of a Non Integer Order PID Controller." August, 2002.
12. J.-F. Leu, S.-Y. Tsay, and C. Hwang, "Design of Optimal Fractional-Order PID Controllers." Preprint.
13. Y. Sánchez, "Fractional-PID Control for Active Reduction of Vertical Tail Buffeting." 1999.
14. A. J. Calderón, C. A. Monje, and B. M. Vinagre, "Fractional Order Control of a Power Electronic Buck Converter," in *5th Portuguese Conference on Automatic Control*, (Aveiro, Portugal), September 2002.
15. A. J. Calderón, B. M. Vinagre, and V. Feliu, "Fractional Sliding Mode Control of a DC-DC Buck Converter with Application to DC Motor Drives ," in *ICAR 2003: The 11th International Conference on Advanced Robotics*, (University of Coimbra, Portugal), June 30-July 3 2003.

16. A. J. Calderón, B. M. Vinagre, and V. Feliu, “Linear Fractional Order Control of a DC-DC Buck Converter,” in *ECC 03: European Control Conference 2003*, (University of Cambridge, UK), 1-4 September 2003.
17. Antonio Jos Calderón Godoy, *Control Fraccionario de Convertidores Electrnicos de Potencia tipo Buck*. PhD thesis, Escuela de Ingenierias Industriales, Universidad de Extremadura, Badajoz, 2003.
18. C. A. Monje, A. J. Calderón, and B. M. Vinagre, “PI Vs Fractional DI Control: First Results,” *5th Portuguese Conference on Automatic Control*, September 2002.
19. Y. Q. Chen, C. A. Monje, and B. M. Vinagre, “Une proposition pour la synthèse de correcteurs PI d'ordre non entier ,” 2003.
20. Y. Chen, C. Hu, B. Vinagre, and C. Monje, “Robust PI^{α} Controller Tuning Rule with Iso-Damping Property.” Submitted to the 2004 American Control Conferemce (ACC2004), June 30-July 2 2004.
21. G. Franklin, J. Powell, and A. Naeini, *Feedback Control of dynamic Systems*. Addison-Wesley, 1986.
22. M. Axtel and E. Bise, “Fractional Calculus Applications in Control Systems,” in *Proceedings of the IEEE Nat. Aerospace and Electronics Conf.*, (New York, USA), pp. 563–566, 1990.
23. V. Feliu, R. Rivas, L. Gorostiaga, and L. Sánchez, “Fractional Control for an Open Irrigation Canal.” Internal report, September 2003.
24. A. Oustaloup, *La Dérivation non Entière*. Paris: HERMES, 1995.
25. V. Feliu, L. Gorostiaga, B. Vinagre, and C. Monje, “Robust Smith Predictor for First Order Proceses with Dead Time Based on a Fractional Controller.” Internal report, December 2002.
26. K. Oldham and J. Spanier, *The Fractional Calculus*. New York: Academic Press, 1974.