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How does aversion to intertemporal variation affect hedging behavior?

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Abstract

Standard models of hedging behavior assume that hedgers wish to minimize net price variation. They treat variation as risk and fail to distinguish random variation at a specific point in time from pre-determined intertemporal variation. Recursive utility differentiates between random and nonrandom variation by allowing the elasticity of intertemporal substitution (EIS) to be imperfect. The inverse of the EIS measures the hedger's aversion to intertemporal variation and is distinct from aversion to risk. Optimal hedge ratios decline toward zero as aversion to intertemporal variation increases. These results may help explain why hedgers commonly hedge less than recommended by standard models.

JEL classification:

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1. Introduction

Not every source of variation involves uncertainty, and not every source of uncertainty involves risk. The distinctions among these three concepts have often been drawn, but their implications for risk management and price analysis have seldom been clear. Variation aversion is the tendency to prefer a steady stream over a lump sum. Uncertainty aversion is the tendency to prefer a guaranteed value over one that is unknown. Using the curvature of a von Neumann–Morganstern (1944) utility function to measure risk aversion can easily confuse these concepts. This article reviews the distinctions and explains their relevance to commodity price hedging models.

One way to account for the distinctions is known as recursive utility, which parameterizes aversion to temporal variation as distinct from risk and uncertainty. Recursive utility accounts for risk aversion within a single time period by using a typical sort of von Neumann–Morgenstern (1944) utility function, such as the constant relative risk aversion (CRRA) utility function or power utility function, $U(x) = x^{\alpha}$. Time preferences (regarding the time value of money) are handled by multiplying $U(x)$ by a discount factor at each time period. Aversion to temporal variation is handled by allowing utility to be aggregated over time nonlinearly, typically by using a constant elasticity of substitution (CES) aggregator function. Applying the CES aggregator to utility in different time periods enables the model to capture the agent's preferences for spreading value out over time versus clumping it together.

Recursive utility was developed by Epstein and Zin (1989, 1991) and Weil (1990) based on the work of Kreps and Porteus (1978, 1979). It has been used in agricultural contexts such as resource management (Knapp and Olson, 1996) and farm finance (Lence, 2000) and is also known as generalized expected utility. The objective function's value at time t is defined recursively as the sum of expected utility at time t and the discounted value of expected utility at time $t + 1$. However, the summation is not time-separable—the values are raised to a power before being added, as in the CES utility function. The exponent depends on the elasticity of intertemporal substitution between payments separated by time, which is a parameter to be determined outside the model. The usefulness of the recursive utility model relies upon knowledge of the elasticity of intertemporal substitution.

After a thorough discussion of the issues involved, the article turns to an empirical application to hedging. Expected utility, recursive utility, and an alternative objective function are considered using a simple two-period discrete-time framework. Empirical examples demonstrate the differences among the approaches and highlight the distinctions among risk aversion, uncertainty aversion, and variation aversion with an eye toward practical applications.

This work is important because the distinctions among risk, variation, and uncertainty are so very important for understanding hedging and intertemporal decision making in general. The particular mathematical form of the objective function may

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not be of long-standing interest to industry participants, but the insights garnered from a rigorous dissection of temporal decision making can be critically important for making good decisions in the future. Therefore, the presentation of this work will focus on the distinctions among risk, variation, and uncertainty and on practical conclusions drawn from the empirical analysis.

2. Literature review

Static choice under uncertainty is typically modeled using a von Neumann–Morgenstern (1944) expected utility function or some variant (see Starmer, 2000 for examples). An Arrow–Pratt coefficient of risk aversion measures attitudes toward risk, which is measured by variance at a point in time. Several other methods exist to measure and elicit static risk attitudes, as demonstrated by Pennings and Smidts (2000) and Pennings and Garcia (2001), among others.

Intertemporal choice typically includes a discount factor on future utility to measure time preferences, which may be modeled as endogenous or time-varying (Becker and Mulligan, 1997; Frederick et al., 2002). Time preferences are different from attitudes toward intertemporal variation. Time preferences measure a decision maker's time value of money, or impatience, so one unit of utility today may be worth the same as 1.01 units of utility tomorrow. Attitudes toward variation aversion measure a decision maker's desire to smooth out the stream of utility over time, so one unit of discounted utility in each period (e.g., 1.00 units today and 1.01 units tomorrow) is better than 2 units of discounted utility in a single period (e.g., 2.00 units of utility today or 2.02 units of utility tomorrow). In empirical work, the discount factor is usually assumed to be a constant $\beta = 1/(1 + \delta) < 1$, where $\delta > 0$ is the rate of time preference.[1]

Intertemporal choice under uncertainty, however, is more complicated since decision makers consider not only risk and time preferences, but also the timing of events. Kreps and Porteus (1978, 1979) developed the foundation for representing the individual's utility when timing matters. Their preference functional is defined recursively by $U_t = V(y_t, E_t U_{t+1})$, where $V(\cdot)$ is an aggregator function, y_t is a control vector and E is the expectation operator. Kreps and Porteus showed that individuals prefer uncertainty to be resolved earlier rather than later (as most people in fact do) if $V(\cdot)$ is convex in the second argument.

The aggregator function need not be linear, which means that U_t is not necessarily separable in time. For many decision makers, payoffs in different times are treated as if they are separate goods with convex indifference curves to represent the substitutability of one good for another. Orange juice and housing are not additive, and for these decision makers neither is income today additive with income next June. The two may be close substitutes after discounting, but cash flow budgeting requires a steady stream of income. It becomes more and more difficult to manage one's money as variation in payments over time increases. Most people prefer a stable source of income to a turbulent one. Even when payments are deterministic and known in advance, there is an incentive to even-out the payments as much as possible and eliminate the variation. Therefore, substitution between time periods is imperfect.

Substitutability between periods is quantified using the elasticity of intertemporal substitution. This elasticity measures the straightness (lack of curvature) of the indifference curve representing the tradeoff between income in consecutive periods, analogous to the elasticity of substitution in consumer theory and the elasticity of technical substitution in producer theory.

The conventional time-additive expected utility function implies the restriction that the elasticity of intertemporal substitution is equal to the reciprocal of the coefficient of relative risk aversion (Epstein and Zin, 1989, 1991; see next section for further discussion). Hence, if an agent is highly risk-averse, then he must have a low elasticity of intertemporal substitution. As a result, preferences regarding intertemporal variation and static uncertainty are erroneously equated.

There is little evidence in favor of this restriction. Hall (1988) estimated a representative consumer's utility function, concentrating on the elasticity of intertemporal substitution. He determined empirically that its value can sometimes be very close to zero. His result coupled with the time-additivity restriction would imply that relative risk aversion must be nearly infinite, contradicting common sense and all previous work on the subject.

It was clear from Hall's work that a new functional form for utility must be designed for modeling time preferences and risk aversion separately. Weil (1989, 1990) and Epstein and Zin (1989, 1991) developed an isoelastic utility function to fit the need. Their utility function is a nonexpected utility function with a constant coefficient of relative risk aversion and a separate elasticity of intertemporal substitution. It is represented by

$$U_t = \left[(1-\beta)y_t^{\rho} + \beta\left(E_t U_{t+1}^{\alpha}\right)^{\rho/\alpha}\right]^{1/\rho}. \quad (1)$$

Here U_{t+1} reflects random future utility, y_t represents wealth (or income, depending on the model) at time t, and E_t is the conditional expectation given the information available to the agent at t. The parameter $\beta = 1/(1+\delta)$, where δ is the rate of time preference, and ρ is equal to one minus the reciprocal of the elasticity of intertemporal substitution. Attitudes toward risk are modeled by the parameter α, which equals one minus the coefficient of relative risk aversion. The parameter β lies between zero and one, but it should be very close to one in empirical applications. The parameter ρ is less than or equal to one, and the parameter α is less than or equal to one for decision makers who are averse to risk. Equation (1) is a specific example of a recursive utility function and has come to dominate the literature on the topic and to represent the entire class of recursive utility functions.

Epstein and Zin (1991) derived first-order conditions (Euler Equations) from (1) that can be written in terms of observable

[1] Lence (2000) calls β the discount factor and calls δ the rate of time preference. Some authors call β the rate of time preference and δ the discount rate.

variables, and estimated them using the Generalized Method of Moments (GMM). They used a set of monthly U.S. data from 1959 to 1986, which includes four different measures of consumption per capita, and returns for stocks and bonds. The empirical results show that the standard time-additive expected utility function is rejected in favor of expected utility. They conclude that the elasticity of intertemporal substitution is finite and the coefficient of relative risk aversion is near one for their dataset.

Recursive utility has also been used in agricultural contexts, such as resource management and farm finance. Knapp and Olson (1996) used it to study rangeland and groundwater management under uncertainty. They determined that the optimal decision rule for their management problem will be "rotated" when the elasticity of intertemporal substitution is finite. The rotation of optimal decision rules smoothes the evolution of state and control variables over time and dampens the variation in payoffs.

Lence (2000) applied recursive utility to U.S. aggregate annual farm data on consumption and asset returns between 1934 and 1994. His results showed that the empirical performance of the recursive utility model was better than that of the time-additive expected utility model for his data set. He estimated a rate of time preference between 2.9% and 5.1% per year, and he rejected the hypothesis that the elasticity of intertemporal substitution is less than one. His estimate of ρ was 0.863 with a 95% confidence interval of [0.699, 1].

Barry et al. (1996) relaxed time separability by allowing more general time patterns and developing explicit measures of changes in time attitudes. They introduced the concept of constant, increasing or decreasing absolute time aversion, which is analogous to Arrow–Pratt risk attitudes addressing the same sort of issues that Kreps and Porteus (1978, 1979) discussed, but without using recursive utility.

Recursive utility has also been used in other areas, especially in solving consumption, portfolio choice, and asset-pricing problems. Kandel and Stambaugh (1991) determined separate roles for risk aversion and intertemporal substitution in determining the mean and volatility of equity returns in an equilibrium risk pricing model. Increasing risk aversion raises the equity premium, while equity volatility decreases in the level of intertemporal substitution. Hung (1994) also used the recursive utility model to determine the influence of preference parameters on the equity premium and risk-free rate. He claimed that the equity premium puzzle can be resolved if nonexpected utility is combined with asymmetric market fundamentals. Campbell and Viceira (2001) used recursive utility to explain the demand for long-term bonds. Koskievic's (1999) consumption-leisure choice model yielded an estimate of ρ equal to 0.685.

In summary, Weil (1989, 1990) and Epstein and Zin (1991) have successfully developed a recursive, isoelastic utility function that can be implemented empirically based on Kreps and Porteus's (1978, 1979) insights. The utility function is useful to analyze problems involving intertemporal choice under uncertainty. The advantage of this line of empirical work is that it distinguishes the elasticity of intertemporal substitution from the coefficient of relative risk aversion. The two parameters can be estimated from financial data and analyzed empirically.

Still, some issues remain unresolved. This line of research has done an excellent job of developing the elasticity of intertemporal substitution and identifying it as a variation-aversion parameter. It has not addressed the role of the so-called risk aversion parameter and its suitability to measure risk aversion over time. The parameter α has been described as one minus the coefficient of relative risk aversion, but the concept of risk across time periods has not been adequately defined. The next section starts from the simplest specification of an objective function and develops an argument to show that the literature has misinterpreted the role of α and that an alternative utility function is required to model intertemporal risk aversion properly.

3. Discussion

In this article, it is assumed that a decision maker perceives the actual distribution of prices. It is also assumed that there is an objective function that characterizes decision making at a point in time and that it can be approximated mathematically. If a decision maker perceives no risk inherent in a particular distribution of prices, then his objective function ought to incorporate his perceptions and he ought not to hedge. In this case, the decision maker may be "risk averse" but facing no perceived risk.

Pennings and Wansink (2005) emphasize the importance of incorporating risk perception into models of hedging and contracting in the agricultural industry. Their results show that the way risk is perceived can be just as important as the risk itself. This is an important distinction to be made, surely, but it is a difficult one to model analytically. To consider risk perception separately from risk aversion is to consider a person who thinks about risky prospects in two stages by mapping price distributions to risk measurements and then mapping risk measurements to decisions. Is it better to use a single analytical representation to combine both stages of the decision-making process? Or, would it be preferable to model both stages separately? The approach used in this article is the former one. The two mappings are combined into a single function mapping price distributions to decisions. The two thought processes—risk perception and risk aversion—are combined into a single analytical function determining behavior directly from price distributions.

There are many different ways of specifying objective functions for decision making over time. A simple one is

$$U_t = E_t \sum_{s=0}^{T} y_{t+s}. \tag{2}$$

Equation (2) restricts the objective function to have perfect dollar-for-dollar intertemporal substitution. The decision maker with this objective function is indifferent between income now and income later. A more reasonable specification is

$$U_t = E_t \sum_{s=0}^{T} \left(y_{t+s}\beta^s\right). \quad (3)$$

Equation (3) is the Net Present Value rule with a constant discount rate, $\beta < 1$. Equation (3) improves over (2) because intertemporal substitution is no longer dollar-for-dollar. Income now is valued more highly than income later. However, the decision maker is now indifferent between income now and discounted income later. In some periods he may expect very high income and in some other periods he may expect very low income, but Eq. (3) is too simple to account for such preferences. There is no way in (3) to account for his preferences against intertemporal variation in y_t.

We can use a concave function $V(\cdot)$ to add this feature

$$U_t = E_t \sum_{s=0}^{T} [V(y_{t+s})\beta^s]. \quad (4)$$

If $V(\cdot)$ is concave, then the decision maker is averse to variation in y_t over time and to uncertainty within a single period of time. He would prefer two average income years to one high income year and one low income year. He would also prefer an average income year to an equal chance of a high income year and a low income year. Equation (4) is called discounted expected utility because $\underline{V}(\cdot)$ is like a von Neumann–Morgenstern expected utility function. The objective function can be rewritten to emphasize that it is the sum of discounted expected utilities into the future

$$U_t = \sum_{s=0}^{T} \{[E_t V(y_{t+s})]\beta^s\}. \quad (5)$$

If $V(\cdot)$ is concave, then the decision maker is averse to uncertainty in y_t at each time t. Unlike in (3), he would prefer to know the outcomes for certain than to wait for the uncertainty to be resolved.

The problem with (4) and (5) is that the function $V(\cdot)$ serves two roles—it embodies aversion to variation through time and to uncertainty at each time period. It seems unlikely that a decision maker will feel equally averse to both phenomena. Note also that $V(\cdot)$ is often said to embody risk aversion, but risk is hard to define in an intertemporal context. Risk aversion is not the same as aversion to pre-determined intertemporal variation.

One could easily imagine a decision maker facing known temporal variation and no uncertainty. He would face no risk but would nevertheless require a concave $V(\cdot)$ function. The decision maker may face no real risk and yet $V(\cdot)$ may rightly be concave to capture his preferences toward temporal variation. One could also imagine a decision maker facing uncertainty at each time period without any uncertainty on the whole, over the longer planning horizon. He may feel that he faces no risk at all because losses in one period are matched by gains in another. He faces no risk, yet a concave $V(\cdot)$ function may be appropriate if he is averse to period-by-period uncertainty. The three phenomena—aversion to pre-determined intertemporal variation, aversion to uncertainty within a single period, and whole-horizon risk aversion—are different conceptually, and therefore (4) and (5) are oversimplifications that may fail to capture decision maker behavior in many situations.

Many authors have tried to capture these effects using an objective function (more precisely, a dynamic utility aggregator functional) based on recursive utility. The recursive utility objective function based on a power utility or CRRA function can be written as

$$U_t = \left[y_t^\rho + \beta\left(E_t U_{t+1}^\alpha\right)^{\rho/\alpha}\right]^{1/\rho}, \quad (6)$$

which is equivalent to Eq. (1). Utility is defined recursively with U_t depending on the expectation of future U_{t+1}. The expression can be written more simply as

$$U_t = \left[y_t^\rho + \beta\chi(U_{t+1})^\rho\right]^{1/\rho}, \quad (7)$$

where $\chi(U_{t+1})$ represents the certainty equivalent of future utility. For the power utility function, $V(y_t) = {y_t}^\alpha$ and $\chi(y_t) = [E_t({y_t}^\alpha)]^{1/\alpha}$.

Equation (7) makes it clear that recursive utility does not depend on the power utility specification. Utility can be specified as a negative exponential or constant absolute risk aversion (CARA) function or in any other desired form. The following discussion will continue using power utility because the literature has focused there exclusively to date, but the empirical application below will also use the negative exponential utility function to make the results more comparable to previous work on hedging.

From Eq. (7) it is clear that ρ measures the extent to which the decision maker is averse to intertemporal variation, after the appropriate discount factor is applied. The factor $(1-\rho)^{-1}$ is called the elasticity of intertemporal substitution, and corresponds to the elasticity of substitution from consumer theory (the reciprocal of the derivative of the marginal rate of substitution with respect to the quantity ratio). If $\rho = \alpha$, then (7) reduces to the expected utility model (5) when $V(\cdot)$ is the power utility function. If $\rho = 1$, then intertemporal substitution is infinite or perfect. If there is no uncertainty in this case, then the objective function reduces to Eq. (3). However, if $\rho = 1$ and there is uncertainty, then recursive utility does not reduce to Eqs. (4) or (5). To see why more clearly, consider the two-period case.

In the two-period case, Eq. (6) becomes

$$U_t = \left\{y_1^\rho + \beta\left[E_1\left[(y_2^{\rho\alpha})^{1/\rho}\right]\right]^{\rho/\alpha}\right\}^{1/\rho} = \left\{y_1^\rho + \beta\left[E_1\left(y_2^\alpha\right)\right]^{\rho/\alpha}\right\}^{1/\rho}. \quad (8)$$

If y_2 were known with certainty, then

$$U_t = \left(y_1^\rho + \beta y_2^\rho\right)^{1/\rho}. \quad (9)$$

If $\rho = 1$, then

$$U_t = y_1 + \beta y_2, \quad (10)$$

which is the same as Eq. (3). On the other hand, if $\rho = 1$ and y_2 is uncertain, then

$$U_t = y_1 + \beta\left(E_1 y_2^{\alpha}\right)^{1/\alpha} = y_1 + \beta\chi(y_2), \tag{11}$$

which is the sum of discounted certainty equivalents. A quick inspection reveals that (11) is not the same as (4) or (5). Equation (11) is the sum of discounted certainty equivalents, while (4) and (5) are the sum of discounted expected utilities.

Recursive utility, in general, is the sum of discounted certainty equivalents with imperfect intertemporal substitution. There are three parameters—β, α, and ρ—which correspond to the time discount rate, the aversion to uncertainty within a single period, and the aversion to intertemporal variation. There are several other ways to write an intertemporal objective function with three parameters, and recursive utility is just the one most favored to date.

A related objective function is

$$U_t = \left\{E_t\left[\left(\sum_{s=0}^{T} y_{t+s}^{\rho}\beta^{s}\right)^{\alpha/\rho}\right]\right\}^{1/\alpha}. \tag{12}$$

This function represents the certainty equivalent of the sum of the discounted values, with imperfect intertemporal substitution between values. It also accounts for three different kinds of behavior, but the difference between (12) and recursive utility is somewhat subtle. In (12), the discounted values are each raised to the power ρ and discounted, then summed. The sum is raised to the power $1/\rho$ to capture intertemporal substitution effects. This new value is uncertain, so it is raised to the power α and the expectation operator is applied. Finally, the expectation is raised to the power $1/\alpha$ to capture whole-horizon risk aversion.

The distinction is made between aversion to uncertainty within a period and aversion to whole-horizon risk. The treatment of risk and uncertainty is the distinguishing difference between the two objective functions. It can be shown that the two functions are the same if $\rho = \alpha$ or if there is no uncertainty, but optimization of the different functions will otherwise yield different results.

To make the distinction clearer, consider a situation in which the decision maker faces a set of uncertain payoffs, but he is guaranteed a fixed known net present value—he is just not sure when he will be paid. There are two future payments to be received. He might receive all his payments immediately so $(y_1, y_2) = (200, 0)$, or he could receive them all later so $(y_1, y_2) = (0, 210)$, or he could have them split evenly so $(y_1, y_2) = (100, 105)$. All three sets of payments are equivalent in net present value if his discount rate is 0.05. Assume the probability of each outcome is one-third.

Recursive utility and the alternative both capture the decision maker's aversion to intertemporal variation through the parameter ρ, so focus on what happens when $\rho = 1$ (perfect intertemporal substitution). If there is no risk involved over the whole planning horizon, then the decision maker is indifferent toward the timing of payments. Equation (12) captures this aspect of the decision maker's behavior; the value of the objective function is 200 for all values of α.

In recursive utility, the uncertainty within each period drags down the value of the objective function. Recursive utility is distinctive in this way because it treats each period individually. In the example, the value of the objective function depends on the level of risk aversion: $U = 200(1 + 2^{\alpha}/3)^{1/\alpha}$. Recursive utility falls as the level of "risk aversion" (measured by $1-\alpha$) increases, even though only the timing of payments is uncertain.

The problem is that recursive utility does not recognize that future outcomes may be correlated. A decision maker may be indifferent among sets of uncertain outcomes that all yield the same net present value, but he may still behave in a risk-averse manner when net present value over the whole horizon is uncertain. These effects are not captured adequately in recursive utility because the parameter α is applied on a single period basis and not over total net present value. In recursive utility, the parameter α captures along some of the decision maker's aversion to intertemporal variation that ought to be captured entirely by ρ. The alternative utility function (12) handles aversion to intertemporal variation and whole-horizon risk aversion separately by applying the parameter α to net present value over the whole planning horizon. The two behaviors are conceptually distinct and are distinguished in the objective function.

The next section turns to an empirical examination of their use as a tool for managing risk in commodity markets. A two-period hedging model is calibrated using data for the corn market in Southeastern Pennsylvania. Each objective function implies a range of optimal hedge ratios, depending on the range of parameter values and transaction costs. The goal will be to compute the hedge ratios implied by the objective functions and compare them to each other and to those implied by expected utility.

4. Empirical performance

4.1. Data

The dataset is the same one used by Frechette (2000, 2001). It consists of (1) weekly corn cash prices collected by the Pennsylvania Department of Agriculture (PDA), and (2) the nearby corn futures price in Chicago. Local cash prices were collected through surveys and phone calls for five regions: Southeastern, Central, South Central, Western, and the Lehigh Valley. Only the Southeastern region was used in this analysis. The prices were collected and reported by PDA on Monday mornings before the market opened and the futures price that corresponds most closely is the previous Friday's settlement price for the nearby futures contract. If the Chicago Board of Trade was closed due to a holiday, then the closest day was used, matching the information sets as closely as possible in each case. All prices are reported in cents per bushel, for the years 1997–1998.

4.2. Procedures

The example hedger is a livestock farmer purchasing corn for feed, which results in an input cost hedge. The quantity of corn to be hedged is treated as pre-determined by the number of animals in the herd, flock, etc. The ratio of corn to other ingredients in the feed are assumed to be fixed and do not vary with market conditions. These assumptions eliminate the need for modeling any additional sources of uncertainty. The hedge ratio is assumed to be fixed over the hedging period due to the transaction costs associated with frequent updating. This model can be understood as representing a single hedging decision or a series of hedging decisions with updates at fixed intervals.

Variable profits, π, are treated as a random variable:

$$\pi_{t+1} = -p_{t+1} + h(f_{t+1} - f_t) - \tau|h|, \tag{13}$$

where p_{t+1} represents the local commodity price at the time the corn is purchased, f_t represents the futures price when the hedge is placed, f_{t+1} represents the futures price when the hedge is lifted, τ represents the marginal transaction cost of hedging, and h is the hedge ratio. All money values are adjusted by appropriate discount rates, suitably defined.

Estimates of basis risk and expected basis depend on the structural forecasting model. There are many such models in use, such as naïve expectations, adaptive expectations, and rational expectations. The results depend on the model chosen, and yet there is no clear consensus in the literature to guide this choice. Each hedger has a unique perception of market structure, and no single model has come to dominate the literature.

To proceed, a general AR specification is adopted:

$$E_t p_{t+1} = \alpha_0 + \alpha_1 p_t + \alpha_2 p_{t-1} + \cdots. \tag{14}$$

In practice, (14) is truncated at a lag length sufficient to balance accuracy against degrees of freedom, and an error term is appended. If the error term satisfies standard assumptions, then ordinary least squares can be used to get estimates of the α_i, which generate corresponding estimates of $E_t p_{t+1}$. The lag length affects the results of forecasting models like this one. The forecast means are often very different for different specifications of the lag length. However, for this dataset, the variance estimates changed very little when the lag length was changed. The lag length is chosen by maximizing the Aikaike Information Criterion and testing the standard OLS assumptions. The conditional covariance matrix is estimated by substituting expected local price minus expected futures price for expected basis. The conditional covariance matrix is assumed to be constant and to represent a bivariate normal distribution. These statistics represent actual results for the sample period, and therefore the results represent optimal *ex post* behavior in the sense that hedgers are assumed to have known the covariance matrix before the sample period began. Individual hedgers' expectations will depend on the sample period and available information.

The results will depend on the parameter values chosen. These parameters include the risk aversion coefficient, the elasticity of intertemporal substitution, and transaction costs. Each parameter is allowed to vary over a range of possibilities, and the results are compared.

A range of risk aversion coefficients was selected to span a range of possible farmer risk preferences. Reasonable values were chosen to be 2.00 for high risk aversion, 0.20 for moderate risk aversion, and 0.02 for low risk aversion. Frechette (2000) varied the absolute risk aversion parameter, and each value yielded an optimal hedge ratio for each possible level of marginal transaction costs. Simplistic intertemporal aggregation restricted the hedger's preferences toward intertemporal substitution to be infinitely elastic.

In the results to follow, this restriction is relaxed. The elasticity of intertemporal substitution was allowed to vary indirectly by using a range of values for ρ from 0.1 through 1.0, which results in a range of elasticities from 1.11 through infinity (perfect substitution). Each combination of γ and ρ yields an optimal hedge ratio for each level of marginal transaction costs and each discount rate. The optimal hedge ratios under recursive utility are compared with those under the alternative utility function. Negative exponential utility is embedded within each specification.

Let $m_{t+1} = -p_{t+1} + h(f_{t+1} - f_t)$ be the net gain in the spot and futures markets in the second period from procuring corn and hedging a fraction h of the amount to be procured. Let $\tau|h|$ be the transaction cost paid in the first period. Transaction costs enter into the objective functions with a negative sign because they represent a net loss to the hedger. Specifically, the objective functions are

$$\left(-\frac{\beta}{\gamma}\log\{-E_t[-\exp(-\gamma m_{t+1})]\}^{\rho} - (\tau|h|)^{\rho}\right)^{1/\rho}, \tag{14}$$

or using $\chi(\cdot)$ to represent the certainty equivalent,

$$[-\beta\chi(m_{t+1})^{\rho} - (\tau|h|)^{\rho}]^{1/\rho}, \tag{14a}$$

and

$$-\log[-E_t\left(-\exp\{-\gamma[\beta(m_{t+1})^{\rho} - (\tau|h|)^{\rho}]^{1/\rho}\}\right)]/\gamma, \tag{15}$$

or

$$\chi\left(\left[\beta m_{t+1}^{\rho} - (\tau|h|)^{\rho}\right]^{1/\rho}\right). \tag{15a}$$

The other notation in (14) and (15) is defined as follows: β is the discount factor, e.g., 1.000 or 0.985; h is the hedge ratio; τ is the marginal transaction cost of hedging; E_t is the mathematical expectations operator; ρ is the intertemporal substitution parameter; γ is the coefficient of absolute risk aversion; and $|\cdot|$ is the absolute value operator.

Expressions (14) and (15) are maximized over the control variable, h. That is, h is chosen to maximize the objective function. Expectations over bivariate price risk are computed using trapezoidal integration in the Matlab computing language. Optimization proceeds using the simplex method. Further details of the optimization routine are available from the author.

4.3. Results

The results are shown in Tables 1, 2, and 3. In Table 1, recursive utility hedge ratios are shown to vary little for high levels of risk aversion, regardless of the value of ρ. The highly risk-averse hedger ($\gamma = 2.00$) faces an optimal hedge ratio of about 58% when $\rho = 0.1$ and 59% when $\rho = 1.0$. Alternative utility hedge ratios are always equal to recursive utility hedge ratios when $\rho = 1.0$ because both objective functions reduce to simple expected utility when $\rho = 1.0$. However, the two objective functions prescribe substantially different optimal hedge ratios for the risk-averse hedger if ρ is less than 1.0. The alternative utility hedge ratio drops to 44% when $\rho = 0.5$ and then to zero for $\rho = 0.4$ or lower.

The reason for the divergence between the two functions and for the differences between them and the expected utility result can be explained. A decision maker with aversion to intertemporal variation is someone who perceives that money now is different from money later. He treats money values received at two different times as fundamentally different goods. He is averse to trading a sure thing for a gamble, and he is also averse to trading money now for money later, even after factoring in the time value of money. He is willing to trade a sure thing now for a reduction in a gamble now, but he is less interested in trading a sure thing now for a reduction in a gamble later. Therefore, he hedges less. His reduction in hedging is magnified by using the alternative functional form, in which a certainty equivalent now is being traded for a certainty equivalent later.

The results for the moderately risk averse hedger ($\gamma = 0.20$) follow. When $\rho = 1.0$, the optimal (expected utility) hedge ratio is 55% (Table 1). Both recursive utility and the alternative yield optimal hedge ratios that fall as ρ falls, with the alternative utility hedge ratios falling more dramatically than the recursive utility hedge ratios. Both hedge ratios become zero for low values of ρ. The effect is magnified further for low risk aversion hedgers ($\gamma = 0.02$). Both hedge ratios are 18% for $\rho = 1.0$, but for $\rho = 0.9$ or lower both are zero.

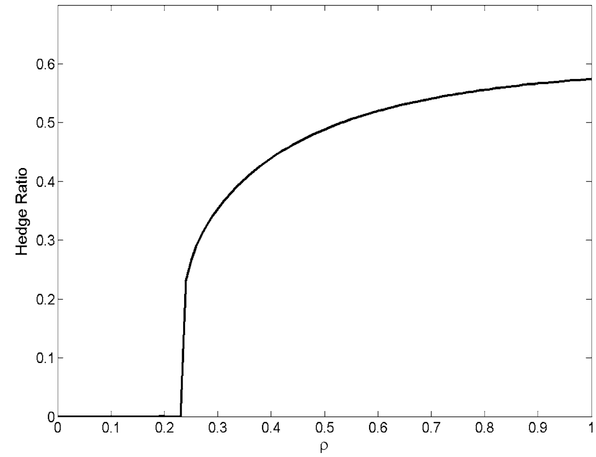

Fig. 1. The hedge ratio depends on aversion to intertemporal variation (CARA = 0.40).

The hedge ratio changes substantially as the parameters ρ and γ are varied. Fig. 1 illustrates the effect of a changing ρ for the negative exponential recursive utility function with $\gamma = 0.40$ and marginal transaction costs = 0.50 cents per bushel. As ρ falls, the optimal hedge ratio falls slowly at first and then drops precipitously to zero when ρ falls below 0.25, which corresponds to an elasticity of intertemporal substitution of 1.33.

Varying the marginal transaction costs does not change the basic story. As shown in Table 2, hedge ratios are lower when

Table 1
Comparison of optimal hedge ratios with different objective functions (transaction costs = 0.5 cents/bu; discount rate = 0%)

ρ	Elasticity of intertemporal substitution	Coefficient of absolute risk aversion (γ)					
		2.00		0.20		0.02	
		Recursive utility	Alternative utility	Recursive utility	Alternative utility	Recursive utility	Alternative utility
0.1	1.11	0.5779	0	0	0	0	0
0.2	1.25	0.5805	0	0	0	0	0
0.3	1.43	0.5826	0	0	0	0	0
0.4	1.67	0.5845	0	0	0	0	0
0.5	2.00	0.5860	0.4407	0.3164	0	0	0
0.6	2.50	0.5873	0.5252	0.4228	0	0	0
0.7	3.33	0.5884	0.5611	0.4783	0	0	0
0.8	5.00	0.5894	0.5782	0.5134	0.4184	0	0
0.9	10.00	0.5901	0.5868	0.5370	0.5111	0	0
1.0	Infinite	0.5908	0.5908	0.5535	0.5535	0.1802	0.1802

Source: Own simulations.

Table 2
Comparison of optimal hedge ratios with different objective functions (transaction costs = 1.0 cents/bu; discount rate = 0%)

ρ	Elasticity of intertemporal substitution	Coefficient of absolute risk aversion (γ)					
		2.00		0.20		0.02	
		Recursive utility	Alternative utility	Recursive utility	Alternative utility	Recursive utility	Alternative utility
0.1	1.11	0.5767	0	0	0	0	0
0.2	1.25	0.5782	0	0	0	0	0
0.3	1.43	0.5797	0	0	0	0	0
0.4	1.67	0.5811	0	0	0	0	0
0.5	2.00	0.5822	0.3172	0	0	0	0
0.6	2.50	0.5833	0.4859	0.3019	0	0	0
0.7	3.33	0.5843	0.5394	0.3955	0	0	0
0.8	5.00	0.5852	0.5657	0.4494	0.2554	0	0
0.9	10.00	0.5859	0.5794	0.4858	0.4359	0	0
1.0	Infinite	0.5867	0.5867	0.5120	0.5120	0	0

Source: Own simulations.

marginal transaction costs are higher, but they still differ between recursive utility and the alternative. Inelastic intertemporal substitution reduces hedge ratios and results in no hedging for low ρ/low γ combinations.

Experimentation with discount rates also results in little change to the pattern of hedge ratios. Increasing the discount rate from 0% to 10% per year (β = 1–0.9982 for weekly data) has very small effects on the optimal hedge ratios for both objective functions. As shown in Table 3 (compare Table 1), the effects are significant only in the 3rd and 4th decimal places throughout the range of parameters considered in the experiments.

Results were also computed for the power utility function, also known as the constant relative risk aversion utility function. The power utility function is the one used by Epstein and Zin (1989, 1991) and Weil (1990), and it has been applied broadly in studies of intertemporal choice. The objective functions are

$$\text{Recursive}: \quad \{\beta[E_t(m_{t+1}{}^{\alpha})]^{\rho/\alpha} - (\tau|h|)^{\rho}\}^{1/\rho} \tag{16}$$

and

$$\text{Alternative}: \quad \{E_t[\beta m_{t+1}{}^{\rho} - (\tau|h|)^{\rho}]^{\alpha/\rho}\}^{1/\alpha}. \tag{17}$$

Optimal hedge ratios vary widely as shown in Table 4. They are comparable to those for the negative exponential, and the same qualitative conclusion can be drawn: Hedging demand declines when intertemporal substitution is imperfect.

The results differ between recursive utility and the alternative, except when the coefficient of relative risk aversion is very high (represented by CRRA = 100 in the table). The two specifications do not generate equal optimal hedge ratios when ρ = 1.0 because they do not reduce to the same expression mathematically for power utility. The alternative specification yields lower hedge ratios over a broad range of parameter values, as

Table 3
Comparison of optimal hedge ratios with different objective functions (transaction costs = 0.5 cents/bu; discount rate = 10%/year)

ρ	Elasticity of intertemporal substitution	Coefficient of absolute risk aversion (γ)					
		2.00		0.20		0.02	
		Recursive utility	Alternative utility	Recursive utility	Alternative utility	Recursive utility	Alternative utility
0.1	1.11	0.5779	0	0	0	0	0
0.2	1.25	0.5805	0	0	0	0	0
0.3	1.43	0.5826	0	0	0	0	0
0.4	1.67	0.5845	0	0	0	0	0
0.5	2.00	0.5859	0.4397	0.3156	0	0	0
0.6	2.50	0.5873	0.5248	0.4224	0	0	0
0.7	3.33	0.5884	0.5609	0.4780	0	0	0
0.8	5.00	0.5894	0.5781	0.5133	0.4176	0	0
0.9	10.00	0.5901	0.5866	0.5369	0.5107	0	0
1.0	Infinite	0.5908	0.5908	0.5534	0.5533	0.1794	0.1787

Source: Own simulations.

Table 4
Comparison of optimal hedge ratios with different objective functions (power utility) (transaction costs = 0.5 cents/bu; discount rate = 0%)

ρ	Elasticity of intertemporal substitution	Coefficient of relative risk aversion					
		100.00		20.0		4.0	
		Recursive utility	Alternative utility	Recursive utility	Alternative utility	Recursive utility	Alternative utility
0.1	1.11	0	0	0	0	0	0
0.2	1.25	0	0	0	0	0	0
0.3	1.43	0	0	0	0	0	0
0.4	1.67	0	0	0	0	0	0
0.5	2.00	0	0	0	0	0	0
0.6	2.50	0	0	0	0	0	0
0.7	3.33	0.3693	0.3692	0	0	0	0
0.8	5.00	0.4936	0.4937	0.3832	0	0	0
0.9	10.00	0.5455	0.5457	0.4960	0.3392	0	0
1.0	Infinite	0.5704	0.5705	0.5462	0.5462	0.3527	0

Source: Own simulations.

shown in the table. This result indicates that hedgers who hedge less than standard models recommend may do so because they are averse to intertemporal variation and that aversion to uncertainty within a single period does little to explain observed hedging behavior.

Perhaps the most striking result is displayed in the center of all four tables. Agents with moderate risk aversion (α) and moderate aversion to intertemporal variation (ρ) do not hedge. This result holds for the recursive utility function with $\rho < 0.5$ and for the alternative with $\rho < 0.8$ for negative exponential and power utility functions. This result implies that a perhaps large set of potential hedgers will optimally choose not to hedge.

5. Conclusions

The main conclusions of this research are twofold. The first conclusion is that optimal hedge ratios are lower when intertemporal substitution is imperfect, in some cases much lower. The alternative utility function yields hedge ratios that drop off considerably and fall all the way to zero for hedgers who are highly averse to intertemporal substitution. The recursive utility function prescribes higher hedge ratios that the alternative does, but its optimal hedge ratios also fall quickly in some cases. Moderately risk-averse and even highly risk-averse hedgers may not hedge at all if they are averse to intertemporal variation.

The effect does not work the other way around. That is, hedgers who are nearly risk neutral will not hedge more than recommended by expected utility due to alterations in the objective function of this sort. Expected utility represents an extreme for both recursive utility and the alternative.

The intuition behind the effect of variation aversion is important to understand. Variation aversion is the desire for a steady stream of utility. A hedger who pays transaction costs in the first period experiences a net loss of utility in exchange for a futures position that minimizes the expected net loss of utility in the second period. The tendency to prefer a steady stream of utility to a jagged one will lead the hedger to smooth out his utility stream by reducing the larger of the two losses and increasing the smaller one. In the situations described here, the larger expected net loss is associated with the transaction costs in the first period. The transaction costs can be reduced by hedging less, and the result is a smoother stream of utility for the hedger.

The result is that hedgers who are averse to intertemporal variation may hedge considerably less than the minimum variance hedge ratio or expected utility hedge ratio. Zero is an optimal hedge ratio for many people, according to the models discussed here. It would be inappropriate to conclude that hedgers with a zero hedge ratio are somehow “uneducated,” “untrusting,” or “fearful” with respect to price risk management. This research may help explain hedgers’ seemingly paradoxical behavior in the complex field of choice under uncertainty.

The second main conclusion is that recursive utility and the alternative suggested above differ in their prescriptions for optimal hedge ratios. The two differ most when the elasticity of intertemporal substitution is low. They coincide when it is perfect, if the negative exponential is used, but the alternative specification yields a smaller hedge ratio when the power function is used. There is little difference between them at low levels of risk aversion because optimal hedge ratios are already very low or zero.

The difference between the two objective functions at high and moderate levels of risk aversion implies that careful attention must be paid to the choice of aggregator functional. Further research is needed to investigate the properties of the two objective functions discussed here more fully. In addition, incorporating risk perception into this model seems an especially promising avenue for better understanding hedging behavior.

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