ΚΒΑΝΤΙΚΗ ΠΛΗΡΟΦΟΡΙΑ ΚΑΙ ΕΠΕΞΕΡΓΑΣΙΑ

BANTIS PETER

MSC IN COMPUTATIONAL PHYSICS

25/6/2022

Aristotle University of Thessaloniki

Theoretical background

- Hydrostatic equation: $\frac{d}{dr}\left[\frac{r^2}{\rho(r)}\frac{dp(r)}{dr}\right] = -4\pi G r^2 \rho(r)$
- General polytropic equation of state: $p = K\rho^{\gamma}$
- The solution of the above equation is achieved for the following initial conditions: $\begin{cases} \rho(0) = \rho_0 \\ \rho'(0) = 0 \end{cases}$
- With a new variable $\rho(r) = \rho_0 \theta(\xi)^{1/(\gamma-1)}$ and ξ=r/a

with $a^2 = \frac{K\gamma}{4\pi G(\gamma-1)}\rho_0^{(\gamma-2)}$

- So, the first equations become the Lane-Emden equation with boundary conditions $\theta(0) = 1$ and $\theta'(0) = 0$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\xi^2\frac{d\theta}{d\xi} + \theta^{1/(\gamma-1)} = 0$$

- Where $\rho(R) = 0 \rightarrow \theta(\xi_R) = 0$ with $\xi_R \equiv R/a$
- The k-integration is in the interval $k \in [k_{min} = \frac{\pi}{R}, \infty)$
- $M = 4\pi \int_0^R \rho(r) r^2 dr = 4\pi\rho_0\alpha^3 \int_0^{\xi_R} \theta^{\frac{1}{\gamma-1}}(\xi)\xi^2 d\xi \propto \rho_0\alpha^3 \propto \rho_0^{(3\gamma-4)/2}$
- Using dimensionless variables in the Fourier transform of the energy density, the modal fraction becomes:

$$\tilde{f}(k) = \frac{h(ak)}{h(\frac{\alpha\pi}{R})} = \frac{h(ak)}{h(\frac{\pi}{\xi_R})} = \frac{h(ak)}{\mathrm{C}(\gamma)}\,,\ \tilde{f}(\kappa) = \frac{h(k)}{h(\kappa_{min})}$$

Where C(γ) independent of ρ_0, and $h(ak)$ is

$$h(ak) = \left| \int_0^{\xi_R} \theta^{\frac{1}{\gamma-1}}(\xi) e^{iak*\xi} \xi^2 d\xi \right|^2, \mathrm{h}(\kappa) = \frac{4\pi\rho_0\alpha^3}{\kappa_{min}} \int_0^{\xi_R} \theta^{\frac{1}{\gamma-1}}(\xi) \sin(\kappa_{min}\xi)\, \xi d\xi^{\ 2}$$

- The configurational entropy is then

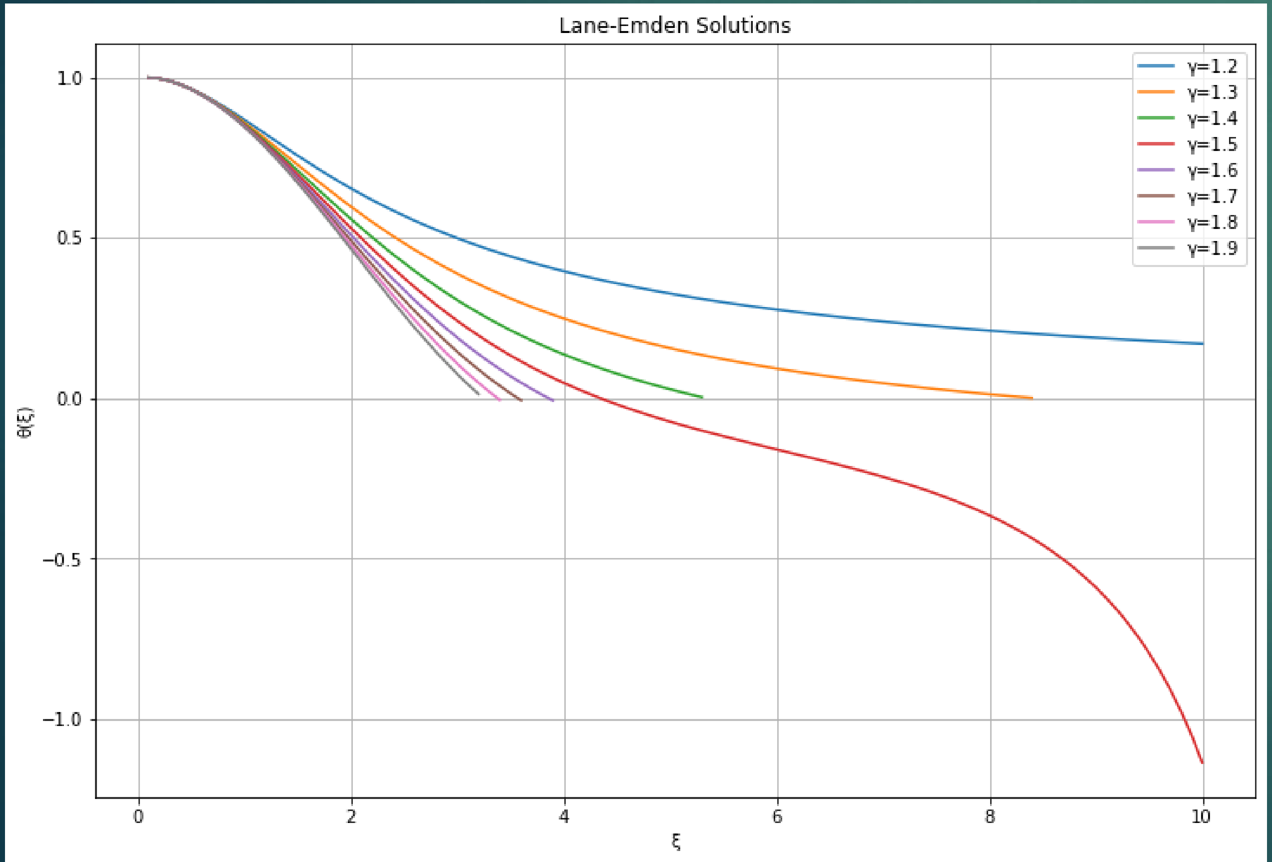
$$S = -4\pi \int_{k_{min}}^{\infty} \frac{h(ak)}{C(\gamma)} \log\left(\frac{h(ak)}{C(\gamma)}\right) k^2 dk = -4\pi a^{-3} \int_{k_{min}}^{\infty} \frac{h(k)}{C(\gamma)} \log\left(\frac{h(k)}{C(\gamma)}\right) \kappa^2 d\kappa$$

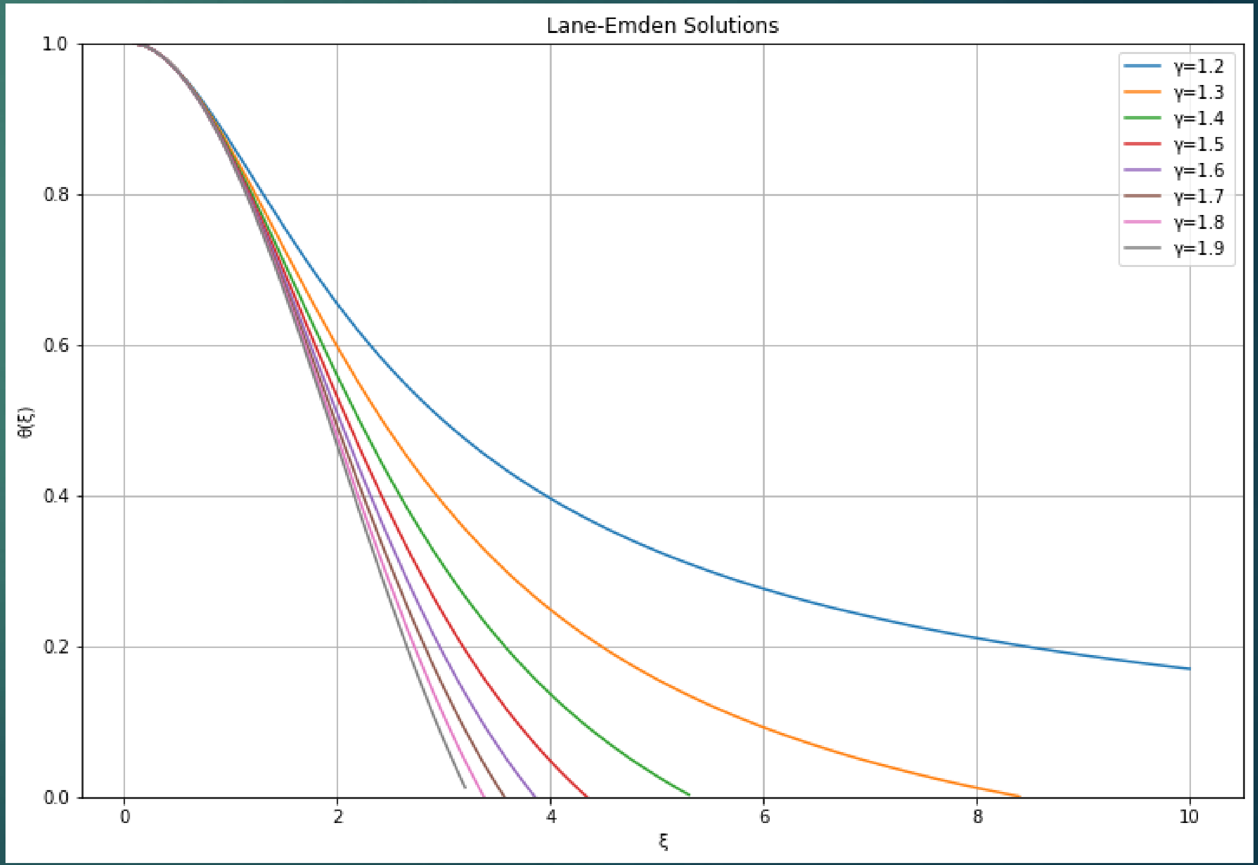
$$= -4\pi a^{-3} \int_{k_{min}}^{\infty} \tilde{f}(\kappa) \log\left(\tilde{f}(\kappa)\right) \kappa^2 d\kappa$$

where $\kappa = ak$, so $\kappa_{min} = \pi/\xi_R$, $\xi_R = R/\alpha$ with $S\rho_0^{-1} \propto \alpha^{-3}\rho_0^{-1} \propto \rho_0^{(4-3\gamma)/2}$.

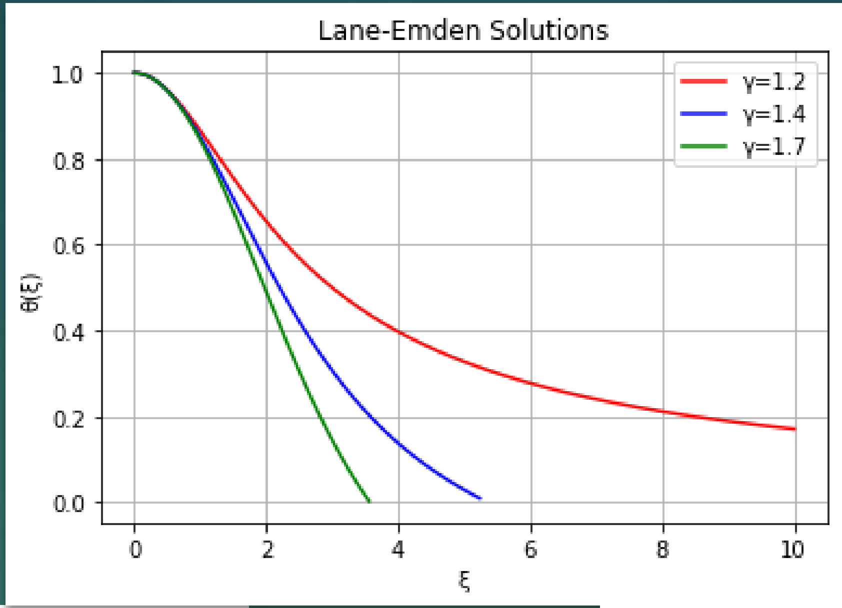
1st CODE AND FIGURES

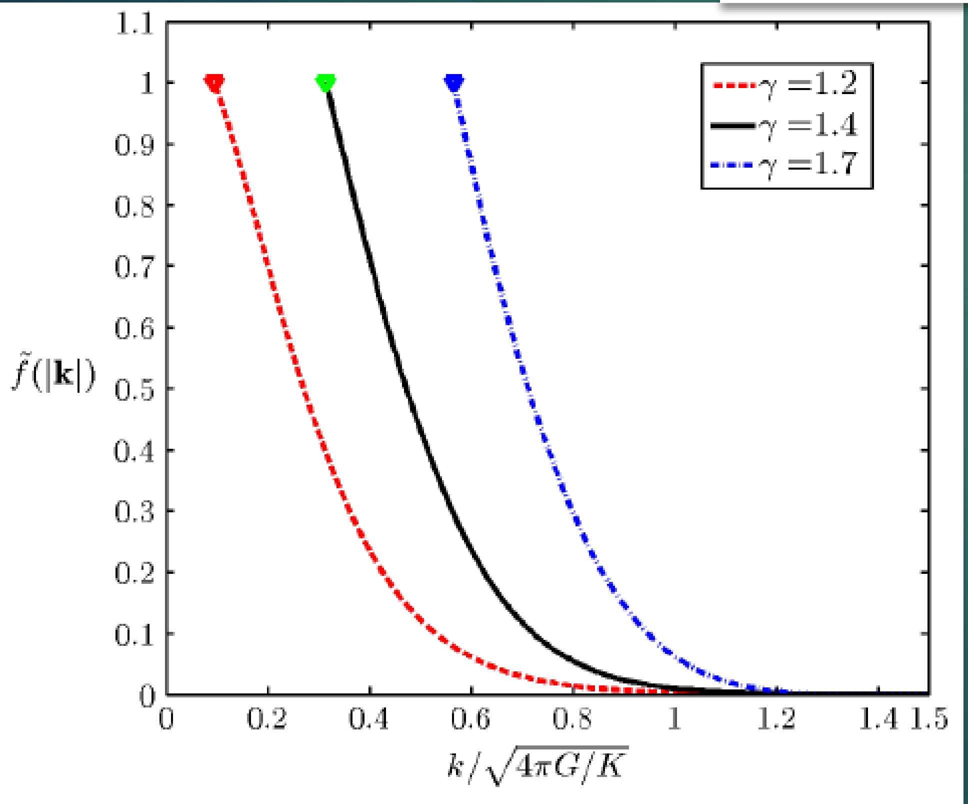
- Line 1- Line 7 : libraries used
- Line 11 – Line 179 : Uses RK4 to solve RK4 with γ=1.2 - 1.9

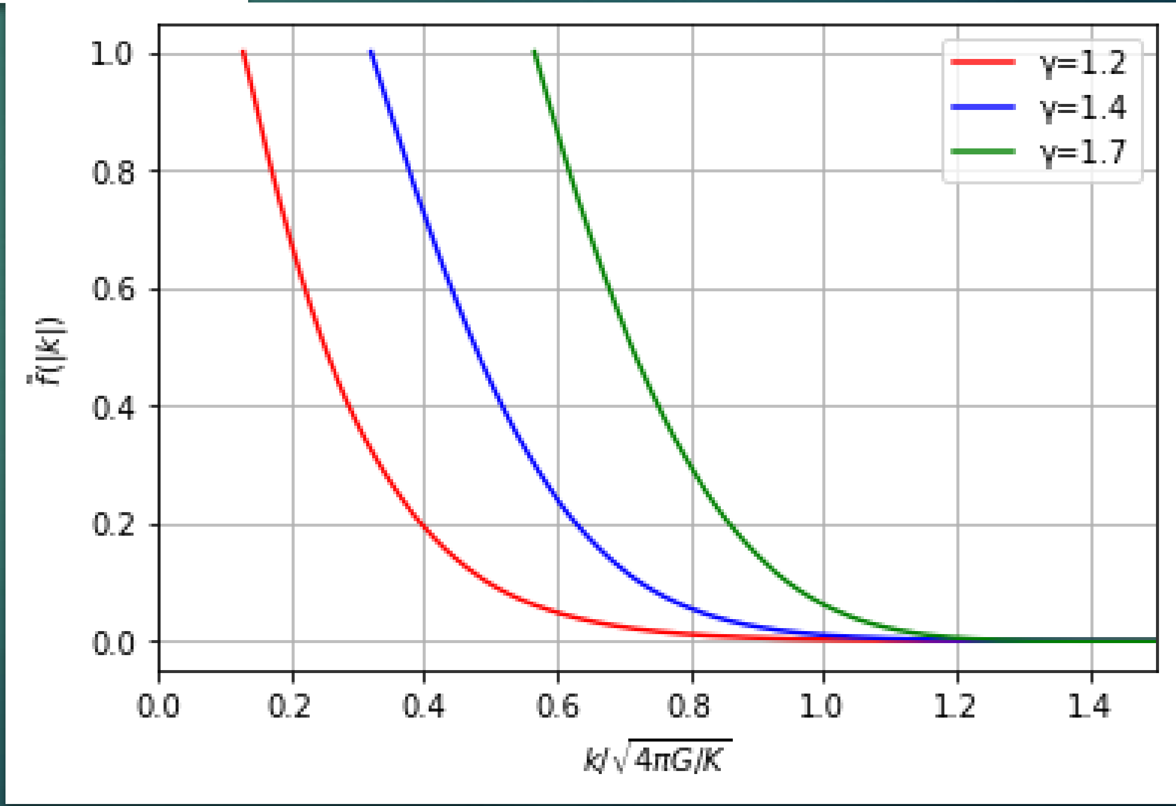




- Line 183 – Line 266: by dividing the Lane Emden equation to 2 different differential equations we can solve the system $\begin{cases} \frac{d\theta}{d\xi} = -\frac{\varphi}{\xi^2} \\ \frac{d\varphi}{d\xi} = \theta^{1/(\gamma-1)}\xi^2 \end{cases}$ for different gammas.
- We are using we use "odeint" from scipy to solve the system
- Then we get rid of the NaN values from the arithmetic approach.
- We procced to interpolate the Lane Emden with "interp1d" from scipy and then calculate the $\tilde{f}(|k|)$.
- Finally, we plot the $\tilde{f}(|k|)\ - k/\sqrt{4\pi G/K}$ for different γ.

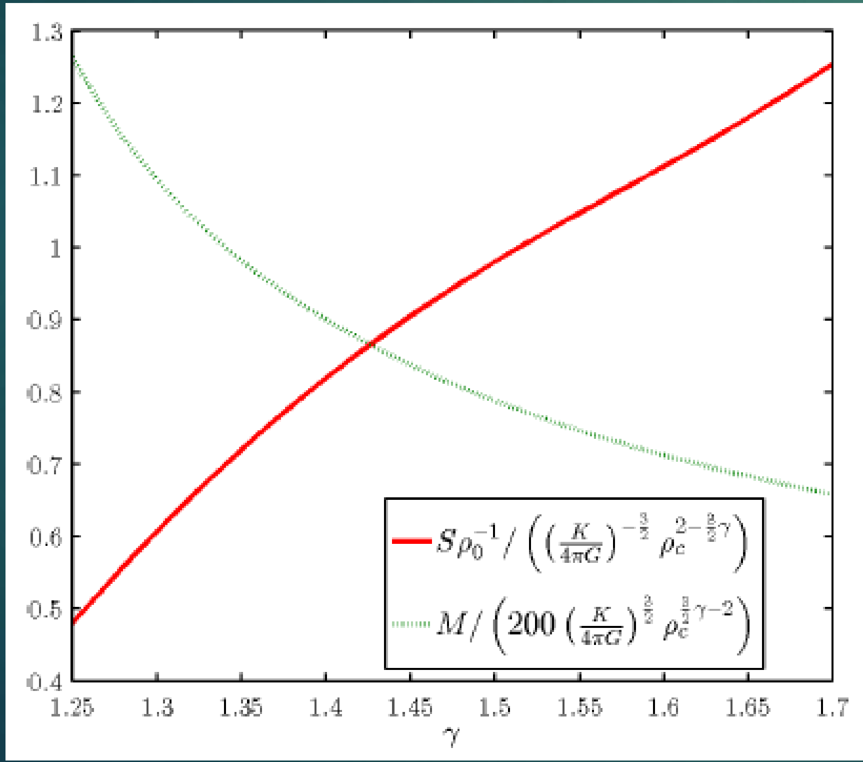

Lane-Emden Solutions
θ(ξ)
ξ
γ=1.2
γ=1.4
γ=1.7

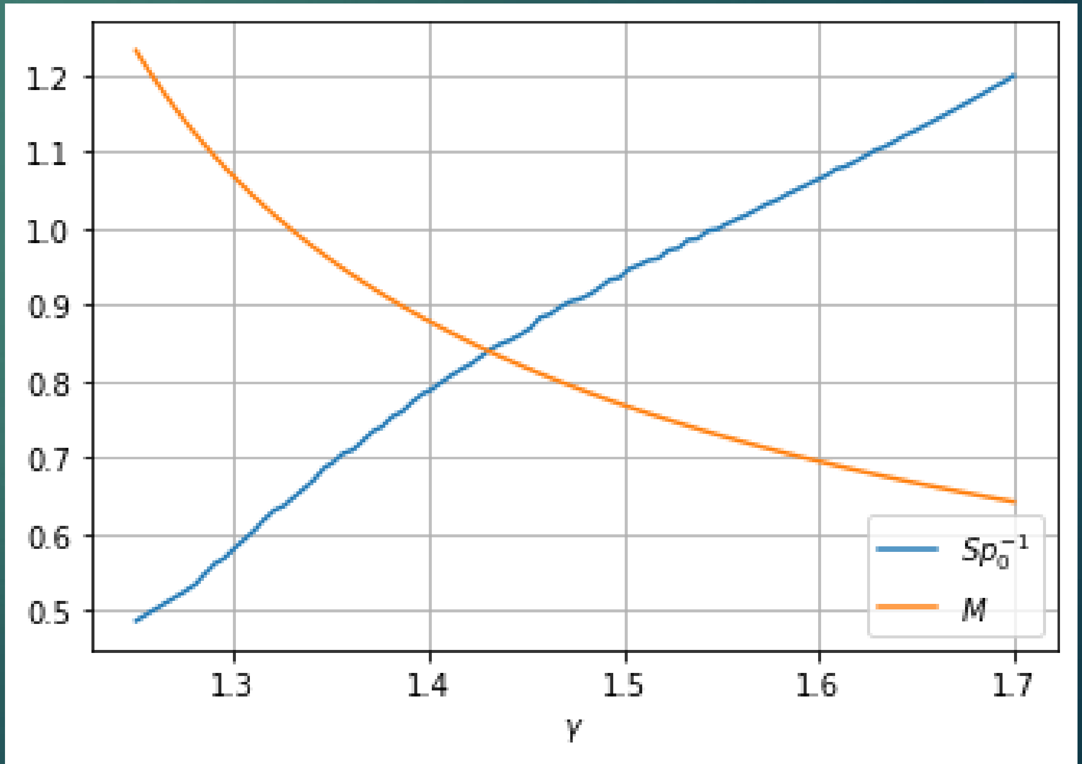

$\tilde{f}(|\mathbf{k}|)$
$k/\sqrt{4\pi G/K}$
$\gamma = 1.2$
$\gamma = 1.4$
$\gamma = 1.7$


$\tilde{f}(|k|)$
$k/\sqrt{4\pi G/K}$
γ=1.2
γ=1.4
γ=1.7

2^{nd} CODE AND FIGURES

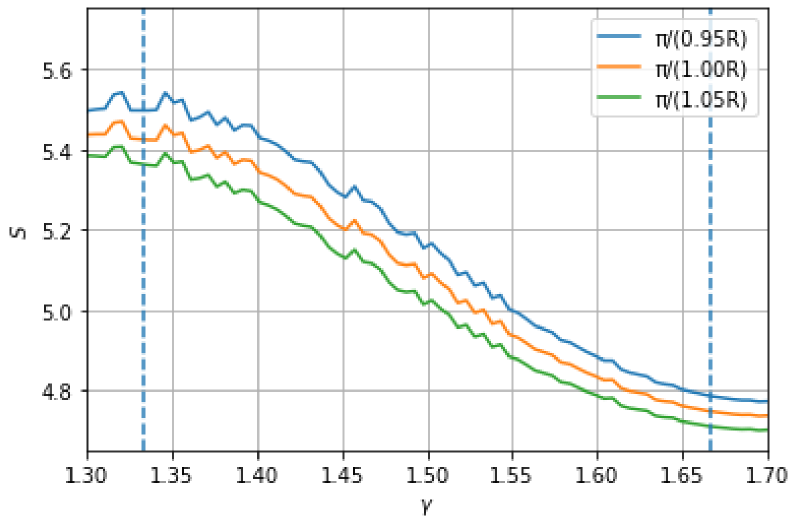
- Line 1- Line 7 : libraries used
- Line 10 - 75 : Again, we use "odeint" to solve the Lane Emden and then we proceed to Mass (M) and Configurational Entropy (S) for different γ. Interpolation is used to calculate $\tilde{f}(|k|)$ and "quad" from Scipy is used to arithmetically calculate the integrals.

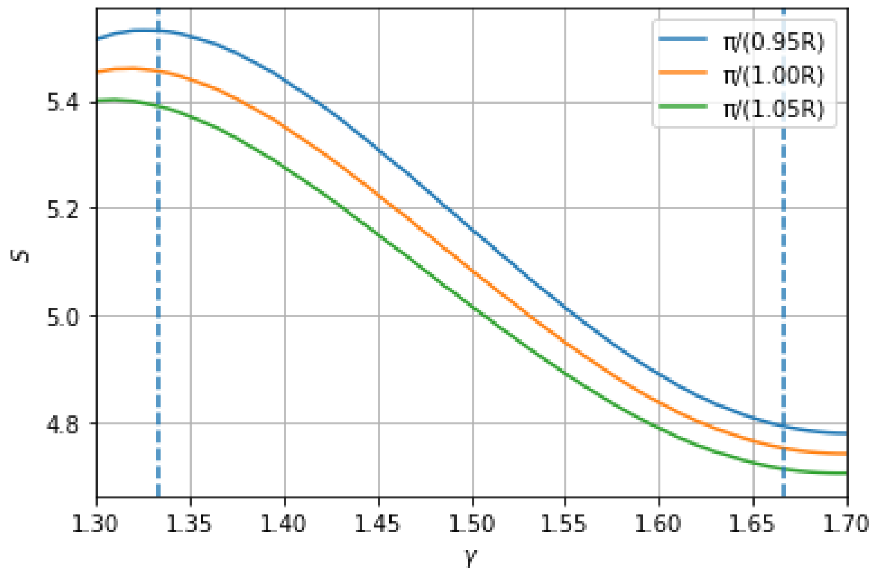


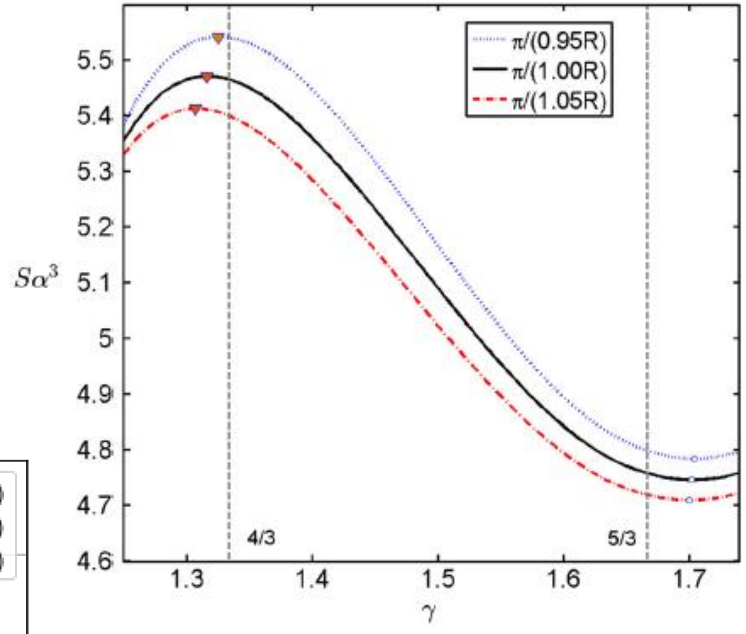


3^{rd} & 4^{th} CODE AND FIGURES

- The polytropic entropy for every γ was calculated using 2 ways.
- The main difference between the two was that the 1^{st} used "odeint" to calculate the solution of the Lane-Emden equation, in contrast to the 2^{nd} one that used "solve_ivp" from Scipy module.
- The first one appear to have fluctuation that are being lowered signific ally when the γ rises. The second one appears to be smooth, without any fluctuations and closer to the diagram of the paper.







THANK YOU FOR YOUR TIME!