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Tutorials for Finite Element Methods

Lagrange Elements

Element Encylopedia » Lagrange Elements

The Lagrange element, or nodal elment, was first defined with use of Lagrange interpolation polynomials in Courant's introduction of the finite elment method [1].

Element definition

The approximating basis functions are determined by the Kronecker delta function.

The element can extended the polynomial definition to arbitrary order and requires only an affine transformation. (1) The requirement of nodes is determined by the necessary order, see examples below for more details.

Examples

One dimension

One dimensional basis functions are the same as the Lagrange polynomials. For each segment there will be nodes where is the order of the polynomial space. The basis functions will be:

Simplices

(2)

For simplices, each edge and face must have the correct number of nodes for the space, and the additional nodes place anywhere within the cell. Nodes on are typically put on a lattice over the element. A common way to tabulate the necessary basis functions is using a vandermonde matrix.

For example using a 2nd order triangular element represented functions will be of the form:

which will require 6 nodes (or degrees of freedom). Since each edge needs 3 nodes, all the node will be on the   (3) edges. For the reference element we have the following nodes:

Vertices	(0,0)	(1,0)	(0,1)
Edge-node	(.5,0)	(.5,.5)	(0,.5)

Thus the vandermonde system will be:

where is 1 in the th component and 0 otherwise. Solving for produces the following basis functions: (4)

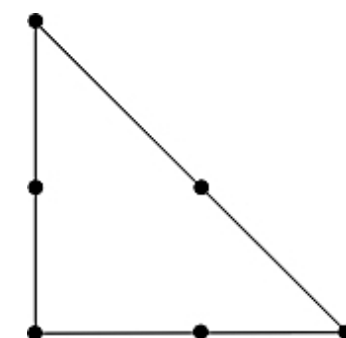|		
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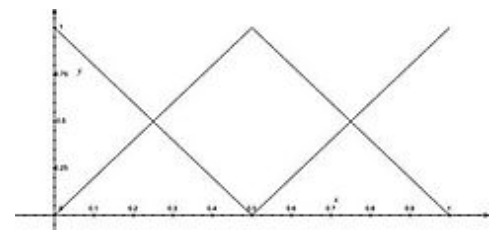
1D Lagrange elements order 1
The first element with nodes at 0 and .5. A second with nodes at .5 and 1.0
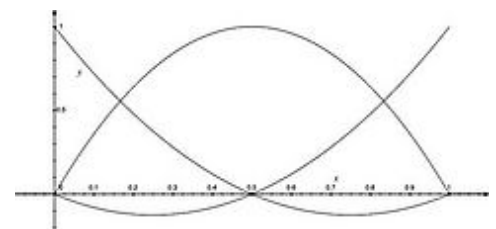
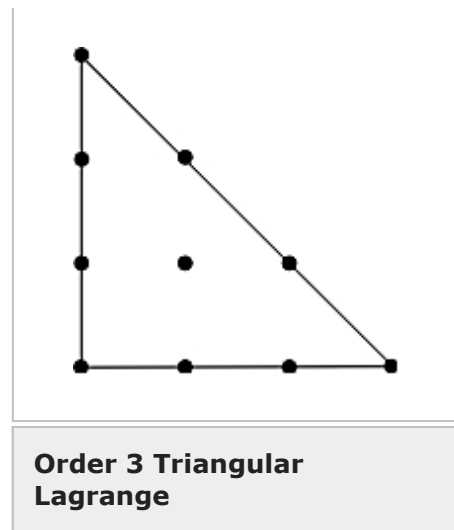**1D Lagrange elements order 2**
with nodes at 0, .5, and 1.0

Order 2 Triangular Lagrange

1	
2	
3	
4	
5	

A similar process may be applied for tetrahedra. The basis polynomials go as:

Order 3 Triangular Lagrange

	Triangles		Tetrahedra	
order	polynomial	number of nodes	polynomial	number of nodes
0		1		1
1		1 + 2		1 + 3
2		1 + 2 + 3		1 + 3 + 5
k	...	1 + 2 + ... +	...	1 + 3 + ... +

Quadralateral and hexahedrals

Basis functions for quadralaterals and hexahedrals can be formed the same way as the simplex counterpart. For lower order function spaces the number of nodes required on each edge is more than required in total, for quadralateral of order 1 there are 4 nodes defined by the edges but only 3 needed to define the space. To resolve this additional higher order terms are added, for order 1 the term is added.

Bibliography

1. Courant, R. L. (1943). "Variational Methods for the Solution of Problems of Equilibrium and Vibration". Bulletin of the American Mathematical Society 49: 1-23.