切换模式



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Sequential Quadratic Programming(SQP)





对于一个带等式约束的优化问题

$$\min f(x)$$

st
$$c(x) = 0$$

构造拉格朗日函数 $\mathcal{L}(x,\lambda) = f(x) - \lambda^T c(x)$

$$A(x)^T = \left[
abla c_1(x),
abla c_2(x), \ldots,
abla c_m(x)
ight]$$

一阶KKT条件

$$F(x,\lambda) = \left[egin{array}{c}
abla f(x) - A(x)^T \lambda \ c(x) \end{array}
ight] = 0$$

对上式求Jacobian矩阵

$$F'(x,\lambda) = egin{bmatrix}
abla^2_{xx} \mathcal{L}(x,\lambda) & -A(x)^T \ A(x) & 0 \end{bmatrix}$$

更新的步长

$$\left[egin{array}{c} x_{k+1} \ \lambda_{k+1} \end{array}
ight] = \left[egin{array}{c} x_k \ \lambda_k \end{array}
ight] + \left[egin{array}{c} p_k \ p_\lambda \end{array}
ight]$$

求解Newton-KKT方程

$$egin{bmatrix}
abla^{
abla_{xx}} \mathcal{L}_k & -A_k^T \ A_k & 0 \end{bmatrix} egin{bmatrix} p_k \ p_{\lambda} \end{bmatrix} = egin{bmatrix} -
abla_k + A_k^T \lambda_k \ -c_k \end{bmatrix}$$

带不等式约束的情形

$$egin{aligned} \min f(x) \ ext{subject to } c_i(x) = 0, \qquad i \in \mathcal{E}, \ c_i(x) \geq 0, \qquad i \in \mathcal{I}. \end{aligned}$$

对约束做一阶泰勒展开

$$egin{aligned} \min_{p} & f_{k} +
abla f_{k}^{T} p + rac{1}{2} p^{T}
abla_{xx}^{2} \mathcal{L}_{k} p \ \end{aligned}$$
 subject to $egin{aligned}
abla c_{i}(x_{k})^{T} p + c_{i}\left(x_{k}
ight) &= 0, \quad i \in \mathcal{E}, \
abla c_{i}(x_{k})^{T} p + c_{i}\left(x_{k}
ight) &\geq 0, \quad i \in \mathcal{I}. \end{aligned}$

对于最小二乘问题

$$\min_{\lambda}\left\|
abla_{x}\mathcal{L}\left(x_{k},\lambda
ight)
ight\|_{2}^{2}=\left\|
abla f_{k}-A_{k}^{T}\lambda
ight\|_{2}^{2}$$

解析解为

$$\hat{\lambda}_{k+1} = \left(A_k A_k^T
ight)^{-1} A_k
abla f_k$$

Line search

每次迭代除了计算更新方向,还要对最优步长做一维搜索。

信赖域(Trust Region) SQP方法

优化问题可以写为

$$egin{aligned} \min_{p} f_k +
abla f_k^T p + rac{1}{2} p^T
abla_{xx}^2 \mathcal{L}_k p \ & ext{subject to }
abla c_i(x_k)^T p + c_i\left(x_k
ight) = 0, \quad i \in \mathcal{E} \ &
abla c_i(x_k)^T p + c_i\left(x_k
ight) \geq 0, \quad i \in \mathcal{I} \ & \|p\| \leq \Delta_k \end{aligned}$$

Byrd-Omojokun Trust-Region SQP Method

```
选择合适的常数\epsilon > 0, \eta, \gamma \in (0,1);
选择初始解x_0,初始信赖域\Delta_0 > 0;
for k=0,1,2,\cdots
       计算f_k, c_k, 
abla f_k, A_k;
       计算\hat{\lambda}_k;
       \left\| \left\| 
abla f_k - A_k^T \hat{\lambda}_k 
ight\|_{\infty} < \epsilon 	ext{ and } \left\| c_k 
ight\|_{\infty} < \epsilon :
               \mathbf{return}\; x_k
        计算
abla_{xx}^2 \mathcal{L}_k或使用拟牛顿法近似;
        计算\rho_k = \operatorname{ared}_k/\operatorname{pred}_k;
       if 
ho_k > \eta :
                x_{k+1} = x_k + p_k;
        else:
```

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