neos Guide

Broyden’s Method

See Also: [Unconstrained Optimization (/guide/types/#unconstrained)] > [Nonlinear Optimization (/guide/types/nonlin/)]

Recall that a potential shortcoming of Newton’s method for nonlinear equations is that the derivatives required for the Jacobian may not be available or may be difficult to calculate. *Secant methods*, also known as *quasi-Newton* methods, do not require the calculation of the Jacobian; they construct an approximation to the matrix, which is updated at each iteration, so that it behaves similarly to the true Jacobian along the step. *Broyden’;s method* is the most successful secant-method for solving systems of nonlinear equations.

Let B_k be the Jacobian approximation at iteration k and let $s_k = x_{k+1} - x_k$. Then, the updated Jacobian approximation B_{k+1} must satisfy the *secant equation*

$$B_{k+1}\, s_k = f(x_{k+1}) - f(x_k). \quad (1)$$

Given an initial matrix B_0 (often a finite-difference approximation to the Jacobian matrix), Broyden's method generates subsequent matrices by the update formula

实际应用中，y_k是使用y_k - y_(k-1)替代

$$B_{k+1} = B_k + \frac{(y_k - B_k s_k) s_k^T}{\|s_k\|_2^2}, \quad (2)$$

where $y_k = f(x_{k+1}) - f(x_k)$.

The remarkable feature of Broyden's method is that it is able to generate a reasonable approximation to the Jacobian matrix with no additional evaluations of the function. This feature is partially explained by noting that, because of equation (1), the updated B_{k+1} mimics the behavior of the true Jacobian along the line joining x_k to x_{k+1}.

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