Dirichlet boundary condition

14 languages

In the mathematical study of differential equations, the **Dirichlet** (or **first-type**) **boundary condition** is a type of boundary condition, named after Peter Gustav Lejeune Dirichlet (1805–1859).[1] When imposed on an ordinary or a partial differential equation, it specifies the values that a solution needs to take along the boundary of the domain.

In finite element method (FEM) analysis, *essential* or Dirichlet boundary condition is defined by weighted-integral form of a differential equation.[2] The dependent unknown *u in the same form as the weight function w* appearing in the boundary expression is termed a *primary variable*, and its specification constitutes the *essential* or Dirichlet boundary condition.

The question of finding solutions to such equations is known as the Dirichlet problem. In applied sciences, a Dirichlet boundary condition may also be referred to as a **fixed boundary condition**.

Examples [edit]

ODE [edit]

For an ordinary differential equation, for instance,

$$y'' + y = 0,$$

the Dirichlet boundary conditions on the interval $[a,b]$ take the form

$$y(a) = \alpha, \quad y(b) = \beta,$$

where α and β are given numbers.

PDE [edit]

For a partial differential equation, for example,

$$\nabla^2 y + y = 0,$$

where ∇^2 denotes the Laplace operator, the Dirichlet boundary conditions on a domain $\Omega \subset \mathbf{R}^n$ take the form

$$y(x) = f(x) \quad \forall x \in \partial\Omega,$$

where f is a known function defined on the boundary $\partial\Omega$.

Applications [edit]

For example, the following would be considered Dirichlet boundary conditions:

- In mechanical engineering and civil engineering (beam theory), where one end of a beam is held at a fixed position in space.
- In heat transfer, where a surface is held at a fixed temperature.
- In electrostatics, where a node of a circuit is held at a fixed voltage.
- In fluid dynamics, the no-slip condition for viscous fluids states that at a solid boundary, the fluid will have zero velocity relative to the boundary.

Other boundary conditions [edit]

Many other boundary conditions are possible, including the Cauchy boundary condition and the mixed boundary condition. The latter is a combination of the Dirichlet and Neumann conditions.

See also [edit]

- Neumann boundary condition
- Robin boundary condition
- Boundary conditions in fluid dynamics

References [edit]

1. ^ Cheng, A.; Cheng, D. T. (2005). "Heritage and early history of the boundary element method". *Engineering Analysis with Boundary Elements*. **29** (3): 268–302. doi:10.1016/j.enganabound.2004.12.001.
2. ^ Reddy, J. N. (2009). "Second order differential equations in one dimension: Finite element models". *An Introduction to the Finite Element Method* (3rd ed.). Boston: McGraw-Hill. p. 110. ISBN 978-0-07-126761-8.

Category: Boundary conditions