1. Demand：求解三维空间无碰撞的存续联通路径
2. Project Address：<https://github.com/bladesaber/MAPF_Pipeline>
3. Algorithm Structure:

算法组成：

1. 求解三维空间存续无碰撞路径
   1. 参考方案1: CBS[10] + A-star + CAT(Conflict Avoidance Table)[12]
   2. 参考方案2: Prioritized Planning[19, 20] + A-star

初步评价：方案1的求解路径为理论最短路径，但求解空间大，需要计算资源大，求解时间长。在方案初步结果确认后，可考虑添加Operator Decomposition[6,7,8]与Bypass[17]方法进行优化。方案2的求解路径为次优路径，但无法保证一定有求解结果。

1. 所得路径进行局部平滑
   1. 参考方案1：Hyprid A-star
   2. 参考方案2：Bezier Smoothing（以路径点为控制点）

初步评价：后续需要结合流体方面的函数设计

1. 管道效果可视化
   1. 参考方案1：基于PyVista库实现（参考https://docs.pyvista.org/）
   2. 参考方案2（候选）：基于Mayavi库实现
2. Plan：

2023-03-10：

1. 计划2023-04-01前完成算法组成（1）中两个方案（基于C++）。不确认在三维空间上A-star会不会由于搜索空间过大的问题导致崩溃，因此初期Grid空间设计为（50x50x50）先做尝试。
2. 之后需要一个粗糙的可视化代码（基于Python）。
3. 需要部分模拟应用场景的参数，对比以上两种算法求解该部分场景的Metric差异，以及对比人工设计与算法求解路径的差异（忽略局部路径平滑），已确认下一步是否继续进行。

2023-04-03：

确认新的一个版本的一些特点：

1. Conflict检测在连续空间进行：Ball Tree或KD Tree
2. 采用类似Hybrid A-star的算法结构：
   1. 改用Greedy search可能更合适，不确定
   2. State近似采用（X， Y， Z， alpha， beta）
   3. 采用Any Angel方法，线与球体最短距离进行测障
   4. 启发式或者限制条件都是为了减少搜索节点，我目前不太确认好的方法，所以我打算使用粗糙的限制，例如不允许搜索方向与目标方向接近反向的角度，或者搜索空间限制。我需要考虑角度的启发式（二维曲线的三维离散化，相切平面线，不一定非要准确解析解，可以用模糊的解，但会造成高估启发式）
   5. 我认为解析解是否需要的
3. 采用Any Angel的方法进行修改
4. 采用约束优化的方法进行最后的曲线平滑Paper Reference:

Survey：

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2. Multi-Agent Path Finding – An Overview , Roni Stern, 2022
3. Multi-Agent Pathfinding: Definitions, Variants, and Benchmarks, Roni Stern
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MAPF：

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4. M-star: Wagner, G., Choset, H.: Subdimensional expansion for multi-robot path planning. Artificial Intelligence 219, 1–24 (2015) 这里有很多参考文章(Why M-star is complete and optimal ?)
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2. MA-CBS: Sharon, G., Stern, R., Felner, A., Sturtevant, N.R.: Conflict-based search for optimal multi-agent pathfinding. Artificial Intelligence 219, 40–66 (2015)
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9. Disjoint Splitting for Multi-Agent Path Finding with Conflict-Based Search, Jiaoyang Li, ICAPS 2019)

Prioritized Based Method:

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   1. ECBS + focal A-star
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10. J. Petereit, T. Emter, C. W. Frey, T. Kopfstedt and A. Beutel, "Application of Hybrid A\* to an Autonomous Mobile Robot for Path Planning in Unstructured Outdoor Environments, " ROBOTIK 2012; 7th German Conference on Robotics, Munich, Germany, 2012, pp. 1-6. (sequence goals hybrid A-star + potential field)
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4. Geometric A-Star Algorithm: An Improved A-Star Algorithm for AGV Path Planning in a Port Environment. GANG TANG. School of Logistics Engineering, Shanghai Maritime University, Shanghai 201306, China. 2021.
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   1. Andreychuk, A., Yakovlev, K.: Two techniques that enhance the performance of multi-robot prioritized path planning. In: International Conference on Autonomous Agents and Multi Agent Systems (AAMAS). pp. 2177–2179 (2018)
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   3. HCBS + RTC: Improved Heuristics for Multi-Agent Path Finding with Conflict-Based Search

记录：

2023-03-06：

这段时间可以完成low-level的搜索方法与high-level规划方法的确定就很好了，第二个比较麻烦的方面就是要找到合适的参考代码并进行移植。对于传统的搜索方法的改进主要体现在：（1）启发式（2）限制branch数目（3）independent detection（4）创建window移动窗体（4）Operator decomposition （5）Conflict Avoid Table

2023-03-08：

与多智体路径规划不同的是，管道规划中行走过的路径不允许再被占据，所以Prioritized Planning 才会比较占主导，不过CBS应该仍然是最优结果的求解器。但我估计在三维上使用CBS会比在二维上开销大得多。

1. M-star似乎不适合于管道路径问题，因为路径不允许二次占据

2023-03-09:

总结来说，与常规的多智慧体路径规划相对比，主要是agent merge的操作失效了，（1） Operator Decomposition（单纯作为branch发散的问题方案使用）（2）ByPass方法 （3）启发式 这部分仍然是有效的。我的计划：

1. 完成两个求解器：
   1. 基于CBS + Search 的求解器 (CBS+A\*, ECBS)
   2. 基于 Prioritized Planning 的求解器 (PBS)
2. 我需要对比一下人工设计与算法设计的纯路径的差异
3. 路径平滑还是一个大问题，我预估这是一个Motion Planning Problem，我可能需要参考一下Hybrid A\* 或 基于simulation 的方法（例如Model Predictive Control，RRT\*）

2023-03-10:

A\*-star的变种应该是有用的：A-star（wiki）

2023-03-22：

目前完成了简单的Space-Time Astar与CBS的基础。在确认了方向无误后，先考虑完成一个完整的应用，先在小的测试环境上完成对（1）不同尺寸（2）平滑路径（3）合并与交集，三个问题的处理。

我认为我之前参考multi-agent path finding是有一定的偏差的，因为agent path finding的冲突需要考虑运动时间，但管道不需要考虑运动时间，因此可能Any-Angel Path finding与CBS混合是一个更合适的选择。

2023-03-23：

早期的两个思路是：（1）先用Astar搜索最优路径，然后做路径短接后处理，再在转角位使用特定的平滑方法。（2）直接在非完整的连续空间进行搜索

目前的三个问题主要是：（1）管道尺寸不一致，（2）平滑可能产生干涉或密集区域无法平滑

1. 关于LA-MAPF的问题，我认为如果使用disjoint splitting的方法则约束为一整个球体，否则用常规方式，则使用MC-CBS的方法，用交集约束

2023-03-24：

需求比我预想中多，而且由于我对目前的情况理解太少，似乎很多的需求缺乏启发式，它们的衍生是非线性的，那意味着无法在一开始的时候就知悉。这样迭代式的开发可能比想象中重要。

这个项目使用连续搜索比离散搜索更合适。MAPF-LNS有迭代形式的思想，我需要参考一下。

那些流体参数可能可以塞进路径的梯度优化里面。

可以通过全局梯度下降来平滑曲线。

2023-03-25：

既然找不到参考，要不直接看RRT的路径规划文章

或者先弱化一下graph

2023-03-27：

我介意的问题在于：

1. 特殊的约束是设计前知道，设计中知道，还是设计后知道
2. 特殊的约束是设计中一直保持，还是设计中存在
3. 约束是不是软约束，即是同一约束不同选择有不同值
4. 进口与出口一定要与截面正交（希望如此）

我没太多的把握，第1，这是个NP-Hard难题，这意味着必须有人的启发式，否则无法解决问题。第2，这几乎一定是个迭代求解问题，并且需要约束启发式的选择可能。第3，既然是个迭代问题，应该尽量使求解快速。

角度的两条限制式：

杜宾启发式在三维上完备，但怎么确定最短？（这些曲线圆滑限制太大，我不想用），我希望可以直接使用梯度优化确定最优曲线。

我需要方法来bound住路径

2023-03-28:

1. CBS + Reactive Path finding（例如CBS + Voronoi 势能场，可能陷入局部点无法逃逸）

CBS + Reactive Path finding + hybrid method可能是个好选择，其实我主要执着于A\*的branch问题上，我希望有替代的方式

杜宾曲线是不是转弯半径越小则结果路径越小呢？

2023-03-29:

我确认思路为以下两种

1. CBS+角度离散限制的Hybrid-Astar+梯度曲线平滑（可能不使用Dubins的启发式），一些很特别的问题需要注意：
   1. 障碍物的形状不再是cube，这就造成了基于Grid的启发式会失效
   2. 最小外接圆形成的CSC联通域一定大于其他外接圆形成的CSC联通域
2. CBS+Reactive方法（当障碍超出一定阈值使用A\*作为启发式）+Bezier曲线平滑，一些很特别的问题需要注意：
   1. CBS对单个节点的解决方案非常的敏感，如果单个节点的解决方案不是最优就会造成无效探索
   2. Grid启发式失效，必须想方法妥善解决U型局部最优点的问题
   3. 必须添加回溯的方法
3. 仿造ICTS，采用CBS+A-star+smoothing（当smoothing产生conflict则不接受当前grid得最短路径，继续搜索知道超出最优路径thresold后，搜索失败）（X）不可行
4. CBS+Sampling/APF方法，只要可以保证Sample/APF方法可以有一个bound就行了

在目前的CBS+A-star基础上直接做梯度优化

2023-03-30：

如果在无约束下的path1<=path2，那平滑后smooth path1 <= smooth path2 是否仍然成立？？我觉得不是，而且单条曲线的平滑曲线不止一条

如果我知道最小曲率，就可以根据grid的大小，退化为折线A-star问题

Hybrid A-star是不完备的

2023-04-01：

针对采样与搜索的方法，目前结论是不存在在连续空间中找到最优路线的可能，当需要求解的空间本身非常苛刻时，不存在找到结果的可能。唯一有点可能的是人工势场的方法。我需要Hybrid A-star+维诺图（或者不是维诺图而是势场）+ theta\* 思路，用深度优先可能比A-star好。PHI\*