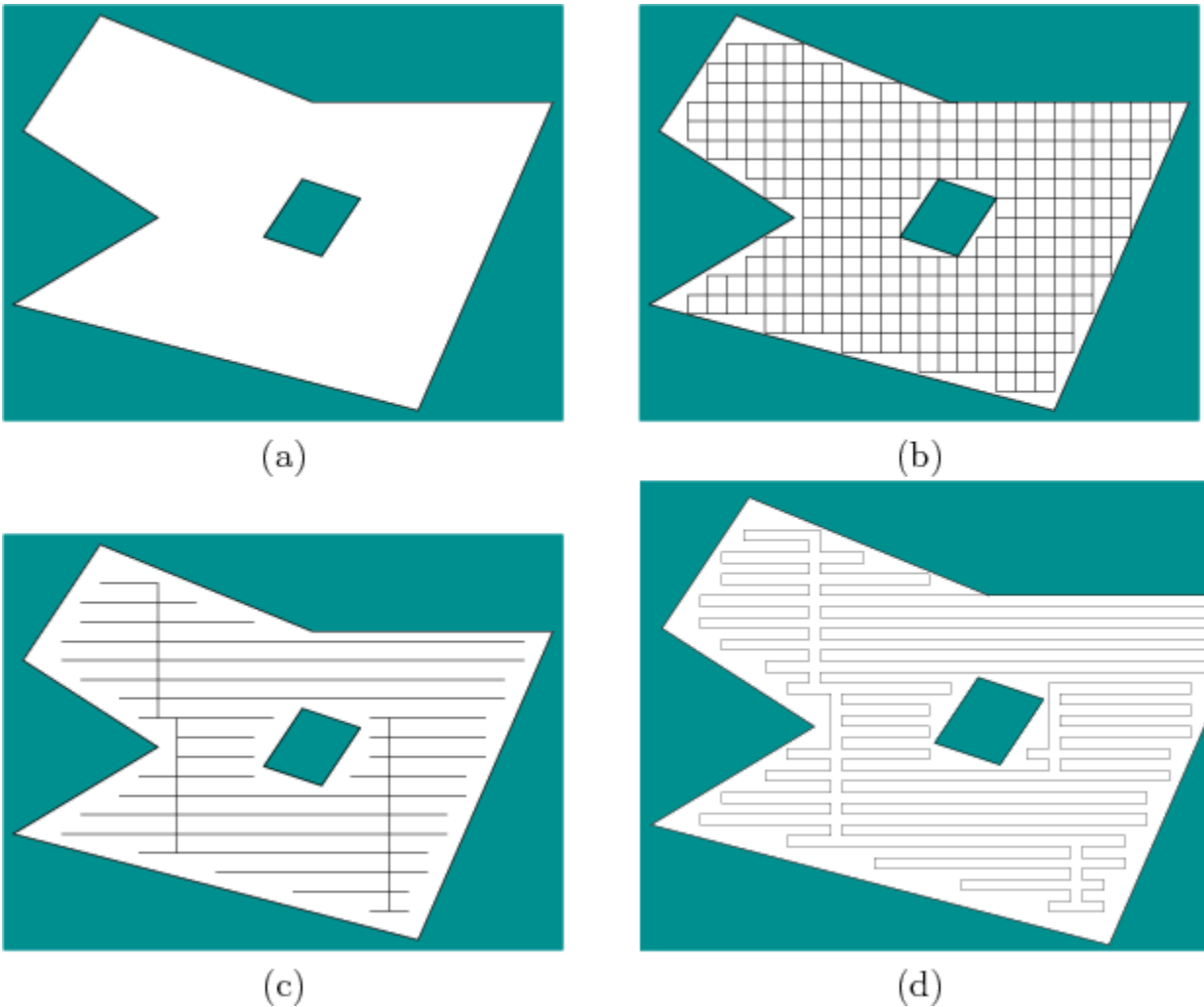**Spanning tree covering**

Figure 7.37: (a) An example used for spanning tree covering. (b) The first step is to tile the interior with squares. (c) The spanning tree of a roadmap formed from grid adjacencies. (d) The resulting coverage path.

An interesting approximate method was developed by Gabriely and Rimon; it places a tiling of squares inside of $\mathcal{C}_{free}$ and computes the spanning tree of the resulting connectivity graph [372,373]. Suppose again that $\mathcal{C}_{free}$ is polygonal. Consider the example shown in Figure [7.37](#)a. The first step is to tile the interior of $\mathcal{C}_{free}$ with squares, as shown in Figure [7.37](#)b. Each square should be of width ϵ, for some constant $\epsilon > 0$. Next, construct a roadmap $\mathcal{G}$ by placing a vertex in the center of each square and by defining an edge that connects the centers of each pair of adjacent cubes. The next step is to compute a *spanning tree* of $\mathcal{G}$. This is a connected subgraph that has no cycles and touches every vertex of $\mathcal{G}$; it can be computed in $O(n)$ time, if $\mathcal{G}$ has n edges [683]. There are many possible spanning trees, and a criterion can be defined and optimized to induce preferences. One possible spanning tree is shown Figure [7.37](#)c.
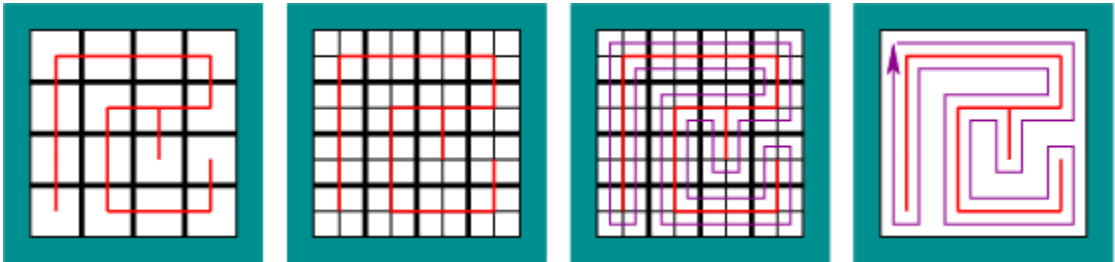

Figure 7.38: A circular path is made by doubling the resolution and following the perimeter of the spanning tree.

Once the spanning tree is made, the robot path is obtained by starting at a point near the spanning tree and following along its perimeter. This path can be precisely specified as shown in Figure [7.38](#). Double the resolution of the tiling, and form the corresponding roadmap. Part of the roadmap corresponds to the spanning tree, but also included is a loop path that surrounds the spanning tree. This path visits the centers of the new squares. The resulting path for the example of Figure [7.37](#)a is shown in Figure [7.37](#)d. In general, the method yields an optimal route, once the approximation is given. A bound on the uncovered area due to approximation is given in [372]. Versions of the method that do not require an initial map are also given in [372,373]; this involves reasoning about information spaces, which are covered in Chapter [11](#).

Steven M LaValle 2020-08-14