# Frequency Response

*Robert Atkinson*
*22 August 2018*

This is the abstract

## 1. Introduction

### 1.1. Administrivia

Before we begin, we load in some previously computed logic.

```
Get[NotebookDirectory[] <> "Utilities.m"]
inputDirectory = FileNameJoin[{NotebookDirectory[], "MotorPhysicsGearsInitialConditions.Output"}] <> $PathnameSeparator;
Get[inputDirectory <> "ParametersUnitsAndAssumptions.m"];
Get[inputDirectory <> "MotorModels.m"];
Get[inputDirectory <> "MotorTimeDomainFunctions.m"];
Get[inputDirectory <> "Misc.m"];
```

## 2. Frequency Response

### 2.1. Example Motor

```
aMotor = motorParameters["AM 60 A"]
(aMotorWithLoad = addMotorLoad[aMotor, flywheel[Quantity[5, "kg"], Quantity[10, "cm"]]] // siUnits) // N
```

$\langle| R \to \frac{33}{10} \mathrm{W/A^2}, L \to \frac{347}{500\,000} \mathrm{H}, N \to 60, \eta \to \frac{9}{10}, Ke \to \frac{533}{30\,000} \mathrm{kg\,m^2/(s^2\,A\,rad)}, Kt \to \frac{533}{30\,000} \mathrm{kg\,m^2/(s^2\,A\,rad)}, B \to \frac{11}{1\,080\,000} \mathrm{kg\,m^2/(s\,rad^2)},$
$J \to \frac{347}{108\,000\,000\,000} \mathrm{kg\,m^2/rad^2}, Jafter \to 0\, \mathrm{kg\,m^2/rad^2}, Bafter \to 0\, \mathrm{kg\,m^2/(s\,rad^2)}, \Delta\tau appConst \to 0\, \mathrm{kg\,m^2/(s^2\,rad)} |\rangle$

$\langle| R \to 3.3\, \mathrm{kg\,m^2/(s^3\,A^2)}, L \to 0.000694\, \mathrm{kg\,m^2/(s^2\,A^2)}, N \to 60., \eta \to 0.9, Ke \to 0.0177667\, \mathrm{kg\,m^2/(s^2\,A\,rad)},$
$Kt \to 0.0177667\, \mathrm{kg\,m^2/(s^2\,A\,rad)}, B \to 0.0000101852\, \mathrm{kg\,m^2/(s\,rad^2)}, J \to 3.21296 \times 10^{-9}\, \mathrm{kg\,m^2/rad^2},$
$Jafter \to 0.025\, \mathrm{kg\,m^2/rad^2}, Bafter \to 0.\, \mathrm{kg\,m^2/(s\,rad^2)}, \Delta\tau appConst \to 0.\, \mathrm{kg\,m^2/(s^2\,rad)} |\rangle$

```
(reflectInertia[Jafter] + J)
% /. aMotorWithLoad
% // N
```

$$J + \frac{Jafter}{\eta\, N^2}$$

$$\frac{2\,501\,041}{324\,000\,000\,000} \mathrm{kg\,m^2/rad^2}$$

$$7.71926 \times 10^{-6}\, \mathrm{kg\,m^2/rad^2}$$

```
(reflectInertia[Jafter] + J) / B
% /. aMotorWithLoad
% // N
```

$$\frac{J + \frac{\text{Jafter}}{\eta\, N^2}}{B}$$

$$\frac{2\,501\,041}{3\,300\,000}\ \text{s}$$

0.757891 s

## 2.2. Infrastructure

```
Clear[bodePlot]
bodePlot[fullModel_, title_] := Module[{model, unitlessModel, theme, gpm, bodes, nyquist, nyquist2, den},
   model = TransferFunctionModel[fullModel[s].{0, 1, 0}, s];
  unitlessModel = model /. aMotorWithLoad /. s → Quantity[s, "per second"] // siUnits // clearUnits // N;
  gpm = GainPhaseMargins[unitlessModel];
  theme = "Scientific";
  bodes = BodePlot[unitlessModel,
    ImageSize → Large,
    StabilityMargins → True,
    StabilityMarginsStyle → {Directive[Green // Darker, Thick], Directive[Green // Darker, Thick]},
    ScalingFunctions → {{"Log10", "dB"}, {"Log10", "Degree"}},
    GridLines → Automatic,
    PlotLabel → {title <> " Freq Resp (magnitude, 20Log10)", title <> " Freq Resp (phase, deg)"},
    PlotTheme → theme,
    PlotLayout → "List"];
  nyquist = NyquistPlot[unitlessModel, {ω, 10*^-5, 10^5},
    PlotLabel → title <> " Nyquist Plot",
    MaxRecursion → 10,
    AxesOrigin → {0, 0},
    ImageSize → Large,
    NyquistGridLines → Automatic,
    PlotTheme → theme];
  nyquist2 = NyquistPlot[unitlessModel,
    PlotLabel → title <> " Nyquist Plot",
    MaxRecursion → 10,
    AxesOrigin → {0, 0},
    ImageSize → Large,
    NyquistGridLines → Automatic,
    PlotTheme → theme];

  den = Collect[model[s] // Flatten // First // Denominator, s];
  ({"============================== " <> title <> " ==============================",
      "model" → model // prettyPrint,
      "poles" → (TransferFunctionPoles[model] // Flatten // prettyPrint),
      "poles" → (TransferFunctionPoles[unitlessModel] // Flatten)
     }~Join~bodes~Join~{nyquist, nyquist2}) // Column
 ]
bodePlot[motorCurrentModel, "Current"]
bodePlot[motorVelocityModel, "Velocity Before"]
bodePlot[motorVelocityAfterModel, "Velocity After"]
bodePlot[motorAccelerationModel, "Acceleration Before"]
bodePlot[motorAccelerationAfterModel, "Acceleration After"]
```

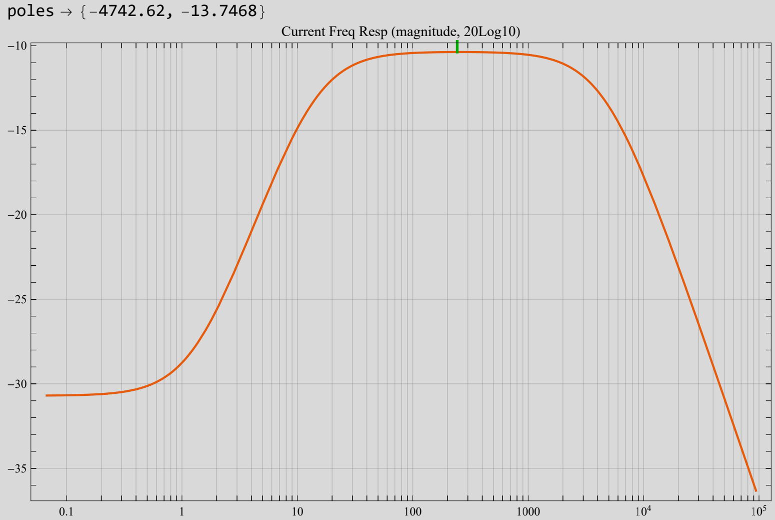
============================== Current ==============================

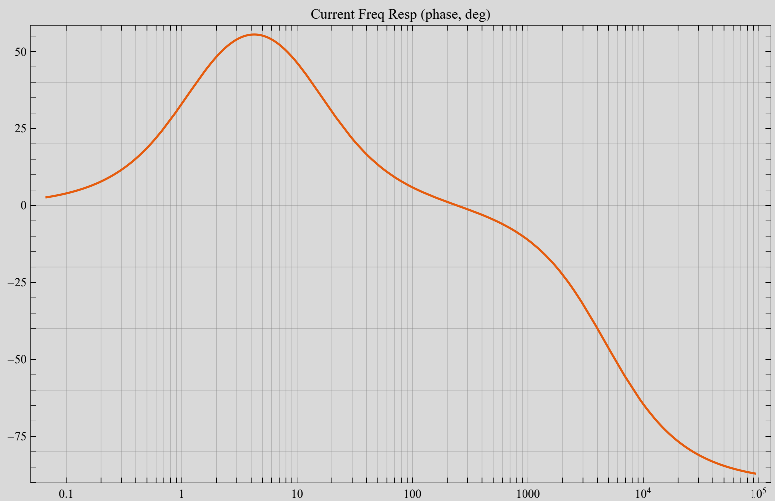
$$\text{model} \to \left(\frac{\eta\,(B+J\,s)\,N^2+\text{Bafter}+\text{Jafter}\,s}{\text{Ke Kt}\,\eta\,N^2+(R+L\,s)\left(\eta\,(B+J\,s)\,N^2+\text{Bafter}+\text{Jafter}\,s\right)}\right)^{\mathcal{T}}$$

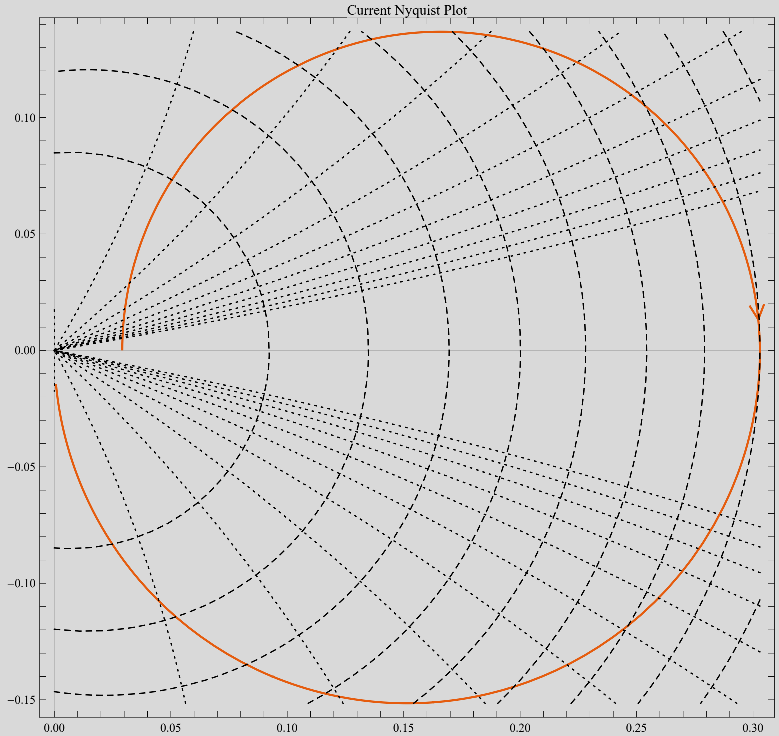
poles → $\dfrac{-B\,L\,\eta\,\mathrm{N}^2-J\,R\,\eta\,\mathrm{N}^2-\mathrm{Bafter}\,L-\mathrm{Jafter}\,R-\sqrt{\left(B\,L\,\eta\,\mathrm{N}^2+J\,R\,\eta\,\mathrm{N}^2+\mathrm{Bafter}\,L+\mathrm{Jafter}\,R\right)^2-4\left(J\,L\,\eta\,\mathrm{N}^2+\mathrm{Jafter}\,L\right)\left(\mathrm{Ke}\,\mathrm{Kt}\,\eta\,\mathrm{N}^2+B\,R\,\eta\,\mathrm{N}^2+\mathrm{Bafter}\,R\right)}}{2\left(J\,L\,\eta\,\mathrm{N}^2+\mathrm{Jafter}\,L\right)}$

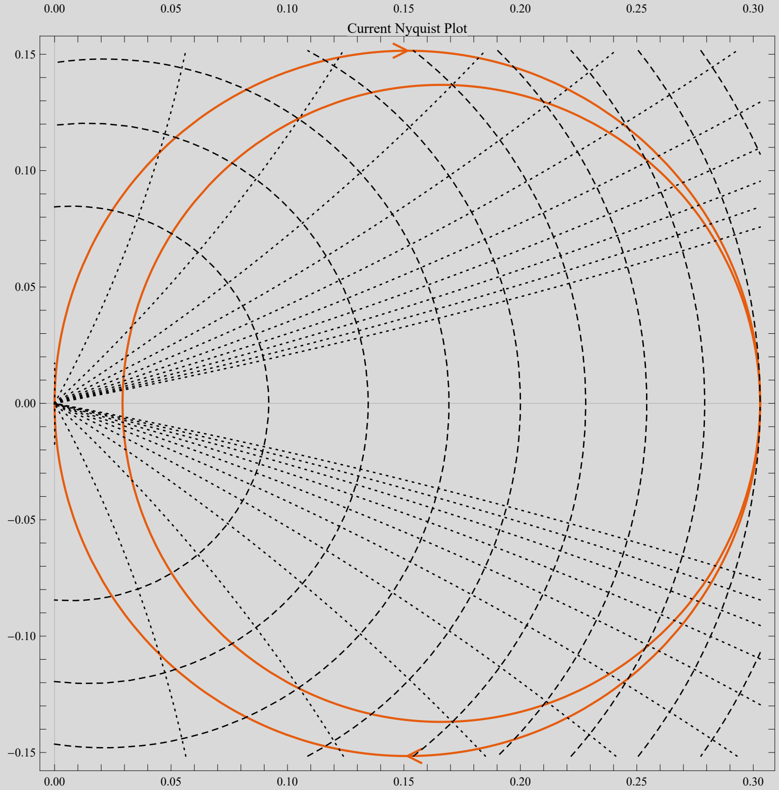
$\dfrac{-B\,L\,\eta\,\mathrm{N}^2-J\,R\,\eta\,\mathrm{N}^2-\mathrm{Bafter}\,L-\mathrm{Jafter}\,R+\sqrt{\left(B\,L\,\eta\,\mathrm{N}^2+J\,R\,\eta\,\mathrm{N}^2+\mathrm{Bafter}\,L+\mathrm{Jafter}\,R\right)^2-4\left(J\,L\,\eta\,\mathrm{N}^2+\mathrm{Jafter}\,L\right)\left(\mathrm{Ke}\,\mathrm{Kt}\,\eta\,\mathrm{N}^2+B\,R\,\eta\,\mathrm{N}^2+\mathrm{Bafter}\,R\right)}}{2\left(J\,L\,\eta\,\mathrm{N}^2+\mathrm{Jafter}\,L\right)}$

poles → {-4742.62, -13.7468}






============================ Velocity Before ============================

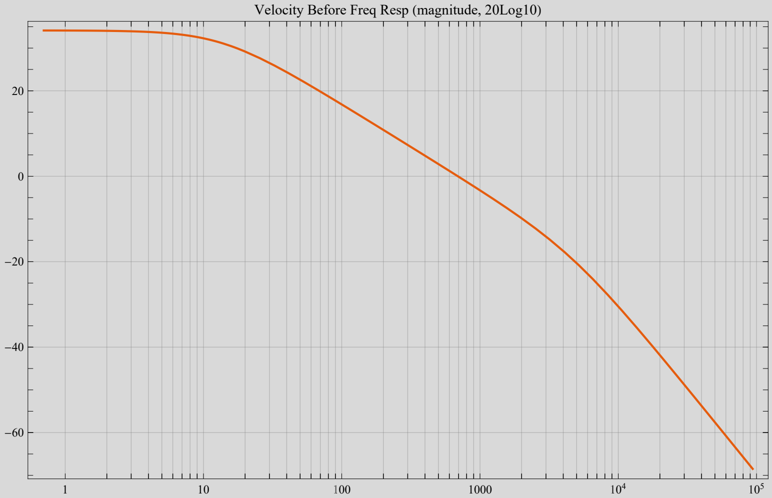
model → $\left(\frac{\text{Kt}\,\eta\,\text{N}^2}{\text{Ke Kt}\,\eta\,\text{N}^2+(R+L\,s)\,(\eta\,(B+J\,s)\,\text{N}^2+\text{Bafter}+\text{Jafter}\,s)}\right)^{\mathcal{T}}$

poles $\to \frac{-B\,L\,\eta\,N^2 - J\,R\,\eta\,N^2 - \text{Bafter}\,L - \text{Jafter}\,R - \sqrt{(B\,L\,\eta\,N^2 + J\,R\,\eta\,N^2 + \text{Bafter}\,L + \text{Jafter}\,R)^2 - 4\,(J\,L\,\eta\,N^2 + \text{Jafter}\,L)\,(\text{Ke}\,\text{Kt}\,\eta\,N^2 + B\,R\,\eta\,N^2 + \text{Bafter}\,R)}}{2\,(J\,L\,\eta\,N^2 + \text{Jafter}\,L)}$

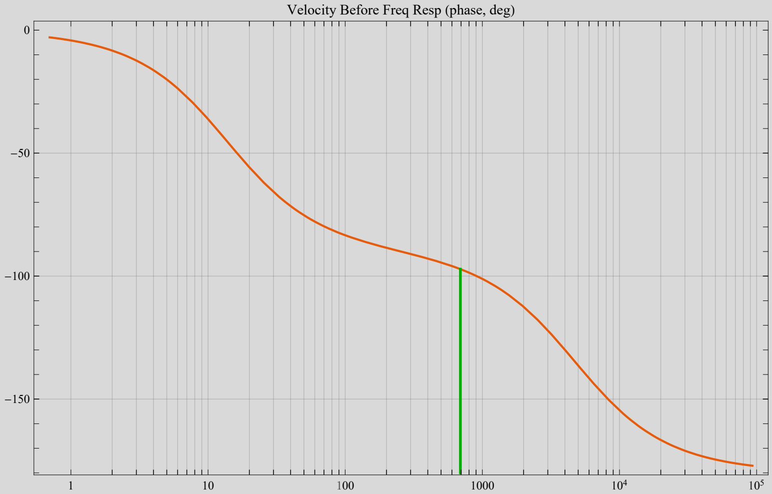
$\frac{-B\,L\,\eta\,N^2 - J\,R\,\eta\,N^2 - \text{Bafter}\,L - \text{Jafter}\,R + \sqrt{(B\,L\,\eta\,N^2 + J\,R\,\eta\,N^2 + \text{Bafter}\,L + \text{Jafter}\,R)^2 - 4\,(J\,L\,\eta\,N^2 + \text{Jafter}\,L)\,(\text{Ke}\,\text{Kt}\,\eta\,N^2 + B\,R\,\eta\,N^2 + \text{Bafter}\,R)}}{2\,(J\,L\,\eta\,N^2 + \text{Jafter}\,L)}$

poles $\to$ {-4742.62, -13.7468}

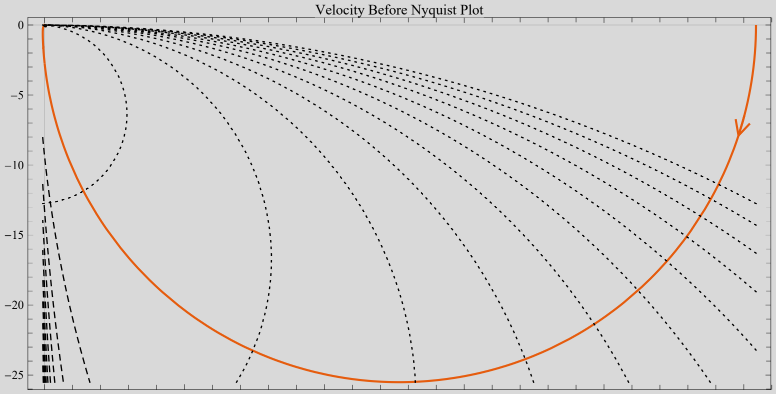
Velocity Before Freq Resp (magnitude, 20Log10)

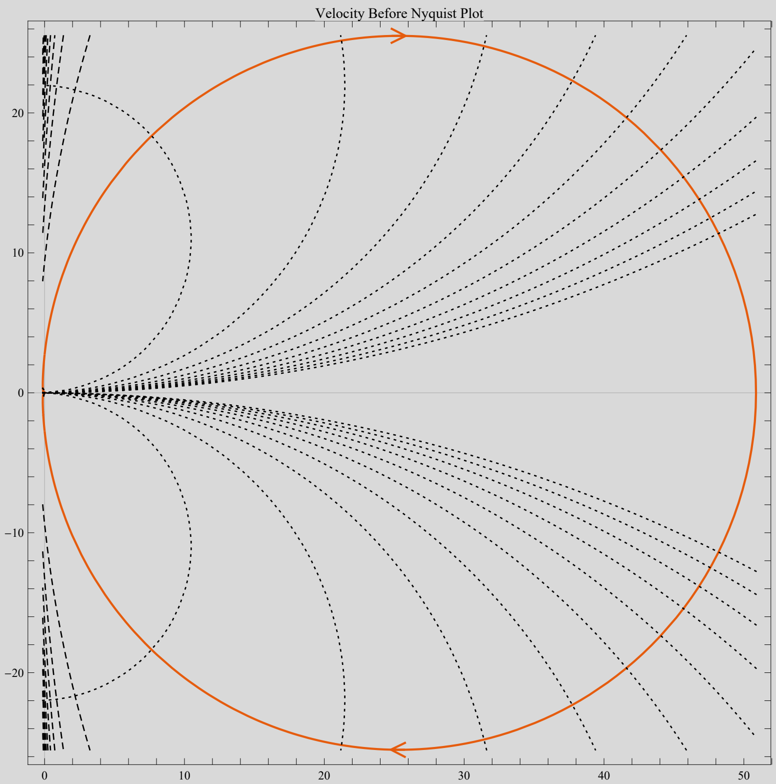


Velocity Before Freq Resp (phase, deg)



Velocity Before Nyquist Plot





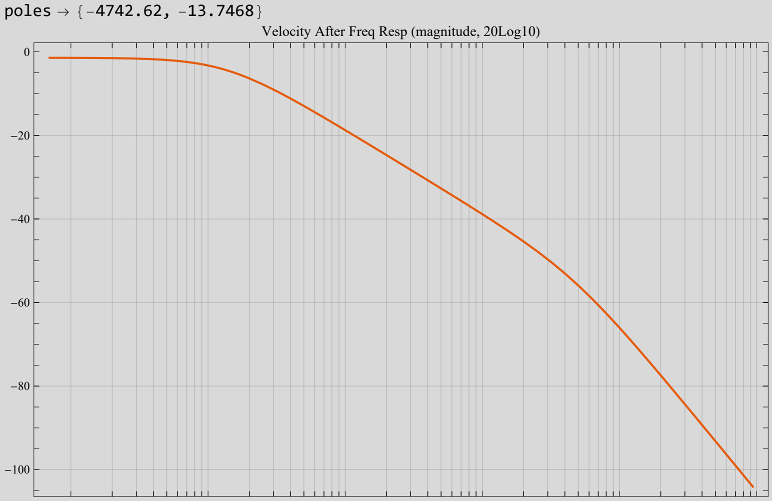

```
============================== Velocity After ==============================
```

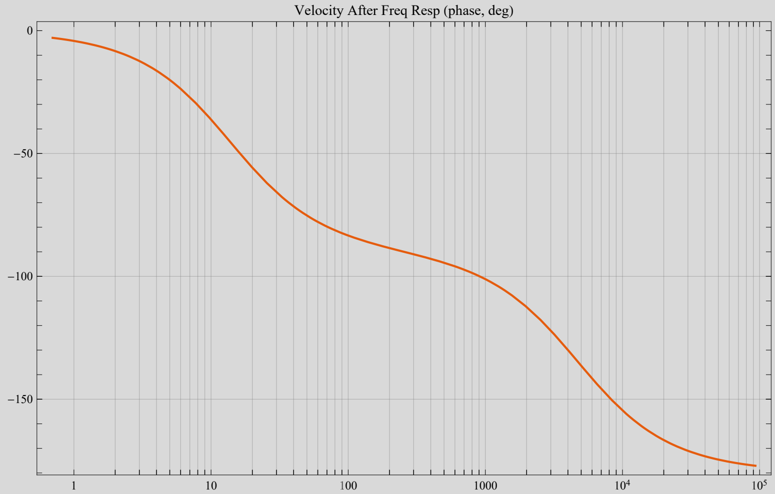
model → $\left(\dfrac{\text{Kt}\,\eta\,\text{N}}{\text{Ke}\,\text{Kt}\,\eta\,\text{N}^2+(R+L\,s)\,(\eta\,(B+J\,s)\,\text{N}^2+\text{Bafter}+\text{Jafter}\,s)}\right)^{\mathcal{T}}$

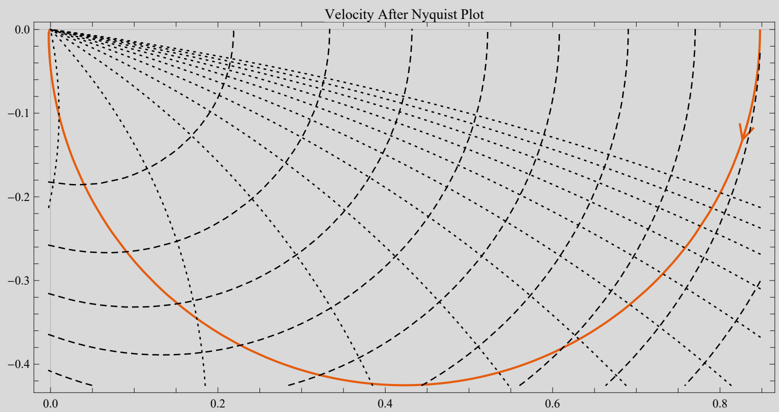
poles → $\dfrac{-B\,L\,\eta\,\text{N}^2-J\,R\,\eta\,\text{N}^2-\text{Bafter}\,L-\text{Jafter}\,R-\sqrt{(B\,L\,\eta\,\text{N}^2+J\,R\,\eta\,\text{N}^2+\text{Bafter}\,L+\text{Jafter}\,R)^2-4\,(J\,L\,\eta\,\text{N}^2+\text{Jafter}\,L)\,(\text{Ke}\,\text{Kt}\,\eta\,\text{N}^2+B\,R\,\eta\,\text{N}^2+\text{Bafter}\,R)}}{2\,(J\,L\,\eta\,\text{N}^2+\text{Jafter}\,L)}$

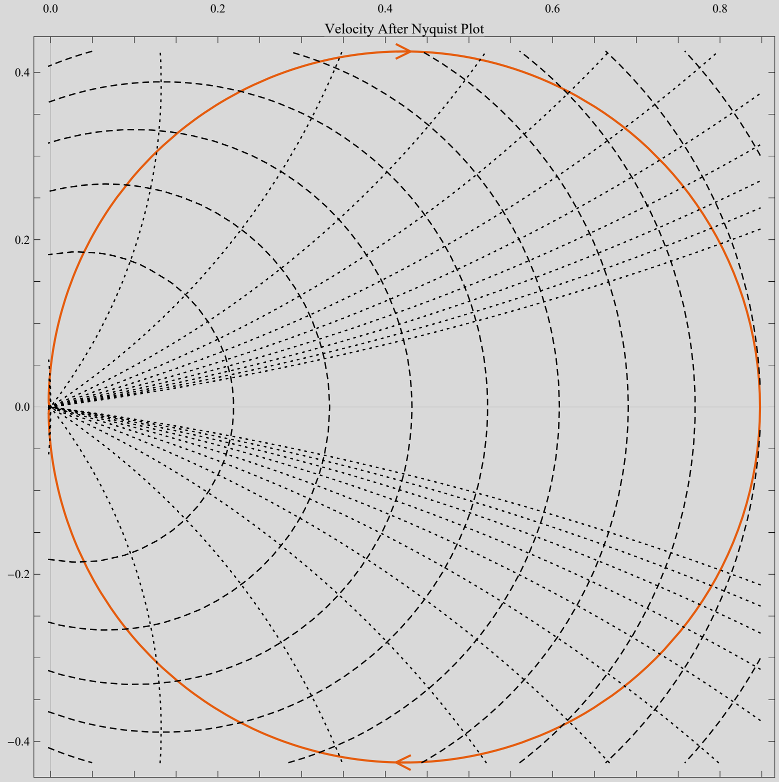
$\dfrac{-B\,L\,\eta\,\text{N}^2-J\,R\,\eta\,\text{N}^2-\text{Bafter}\,L-\text{Jafter}\,R+\sqrt{(B\,L\,\eta\,\text{N}^2+J\,R\,\eta\,\text{N}^2+\text{Bafter}\,L+\text{Jafter}\,R)^2-4\,(J\,L\,\eta\,\text{N}^2+\text{Jafter}\,L)\,(\text{Ke}\,\text{Kt}\,\eta\,\text{N}^2+B\,R\,\eta\,\text{N}^2+\text{Bafter}\,R)}}{2\,(J\,L\,\eta\,\text{N}^2+\text{Jafter}\,L)}$

poles → {-4742.62, -13.7468}




Velocity After Freq Resp (phase, deg)
0
−50
−100
−150
1
10
100
1000
$10^4$
$10^5$


Velocity After Nyquist Plot
0.0
−0.1
−0.2
−0.3
−0.4
0.0
0.2
0.4
0.6
0.8


Velocity After Nyquist Plot
0.4
0.2
0.0
−0.2
−0.4
0.0
0.2
0.4
0.6
0.8

```
============================== Acceleration Before ==============================
```

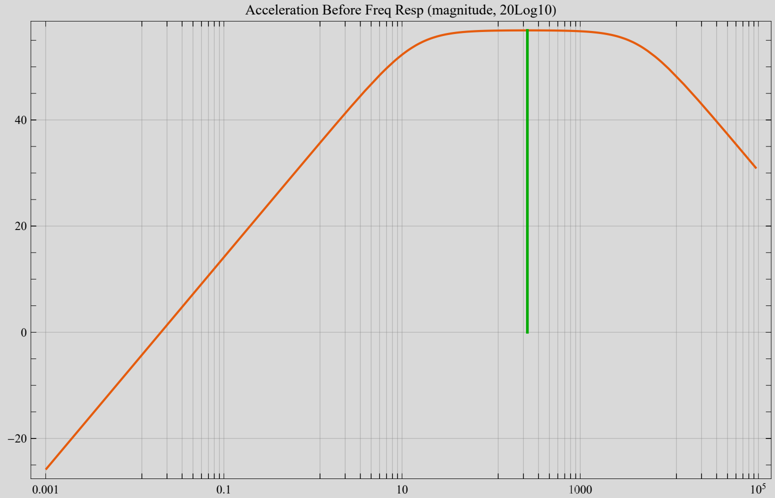
$$\text{model} \to \left(\frac{\text{Kt}\,\eta\,\text{N}^2\,s}{\text{Ke}\,\text{Kt}\,\eta\,\text{N}^2+(R+L\,s)\left(\eta\,(B+J\,s)\,\text{N}^2+\text{Bafter}+\text{Jafter}\,s\right)}\right)^{\mathcal{T}}$$

$$\text{poles} \to \frac{-B\,L\,\eta\,\text{N}^2-J\,R\,\eta\,\text{N}^2-\text{Bafter}\,L-\text{Jafter}\,R-\sqrt{\left(B\,L\,\eta\,\text{N}^2+J\,R\,\eta\,\text{N}^2+\text{Bafter}\,L+\text{Jafter}\,R\right)^2-4\left(J\,L\,\eta\,\text{N}^2+\text{Jafter}\,L\right)\left(\text{Ke}\,\text{Kt}\,\eta\,\text{N}^2+B\,R\,\eta\,\text{N}^2+\text{Bafter}\,R\right)}}{2\left(J\,L\,\eta\,\text{N}^2+\text{Jafter}\,L\right)}$$

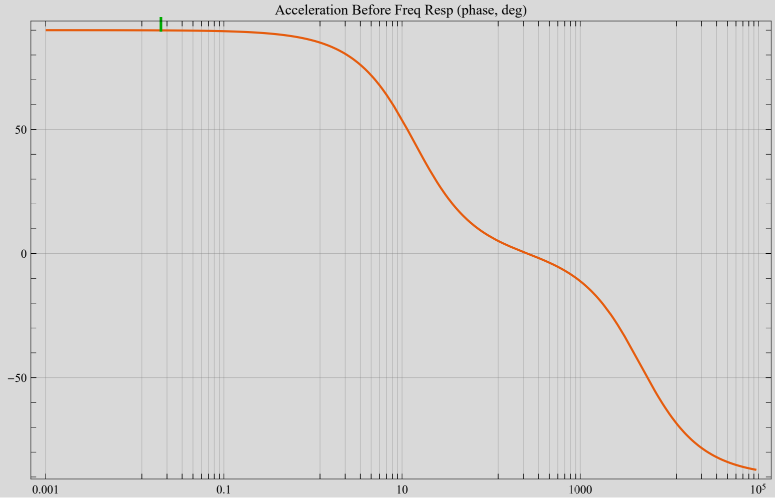
$$\frac{-B\,L\,\eta\,\text{N}^2-J\,R\,\eta\,\text{N}^2-\text{Bafter}\,L-\text{Jafter}\,R+\sqrt{\left(B\,L\,\eta\,\text{N}^2+J\,R\,\eta\,\text{N}^2+\text{Bafter}\,L+\text{Jafter}\,R\right)^2-4\left(J\,L\,\eta\,\text{N}^2+\text{Jafter}\,L\right)\left(\text{Ke}\,\text{Kt}\,\eta\,\text{N}^2+B\,R\,\eta\,\text{N}^2+\text{Bafter}\,R\right)}}{2\left(J\,L\,\eta\,\text{N}^2+\text{Jafter}\,L\right)}$$

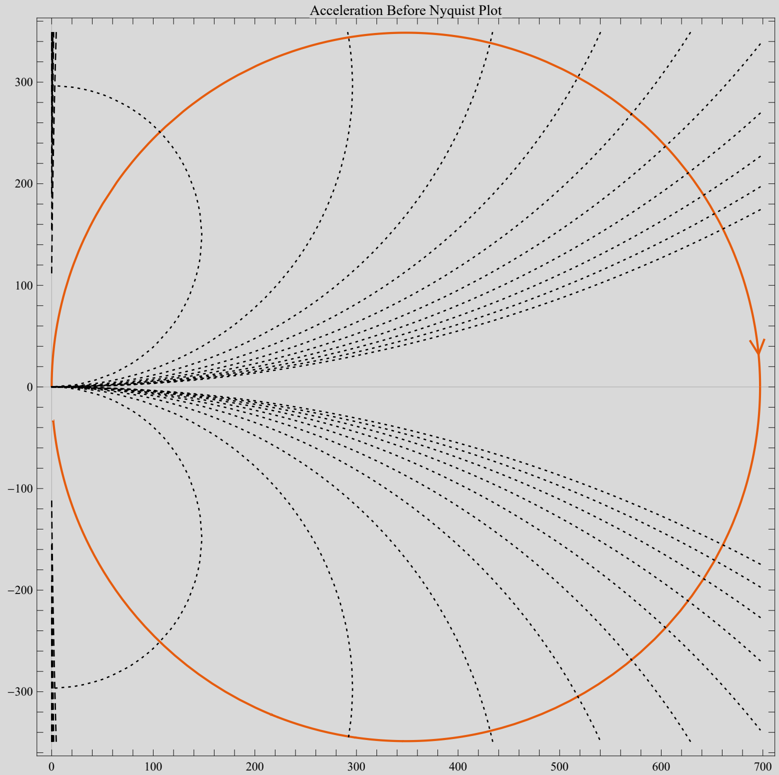
poles → {-4742.62, -13.7468}

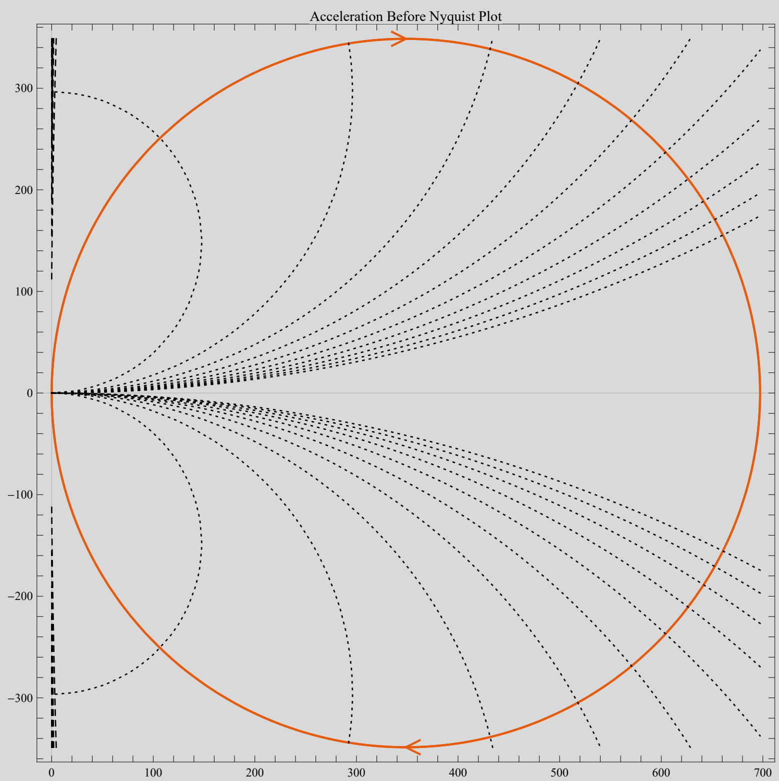
Acceleration Before Freq Resp (magnitude, 20Log10)




Acceleration Before Freq Resp (phase, deg)





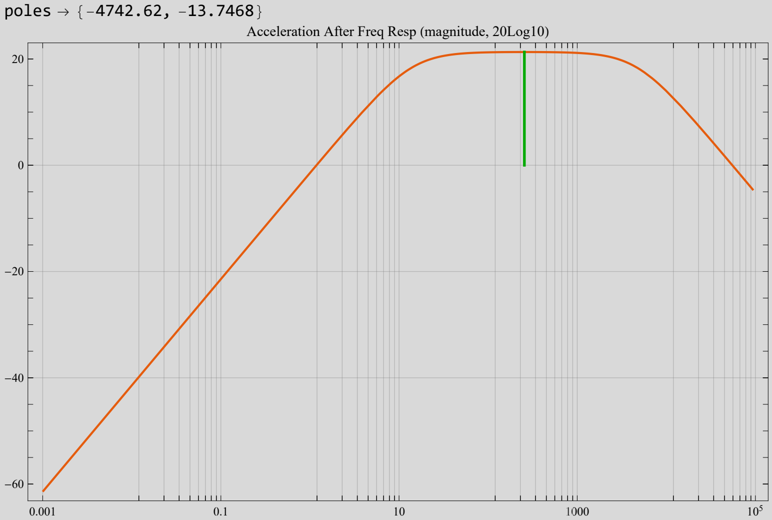

```
============================ Acceleration After ============================
```

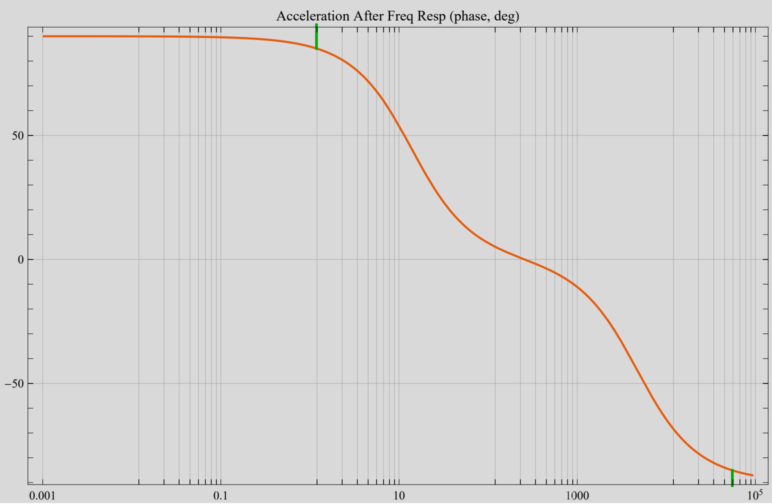
model → $\left(\frac{Kt\,\eta\,N\,s}{Ke\,Kt\,\eta\,N^2+(R+L\,s)\,(\eta\,(B+J\,s)\,N^2+Bafter+Jafter\,s)}\right)^{\mathcal{T}}$

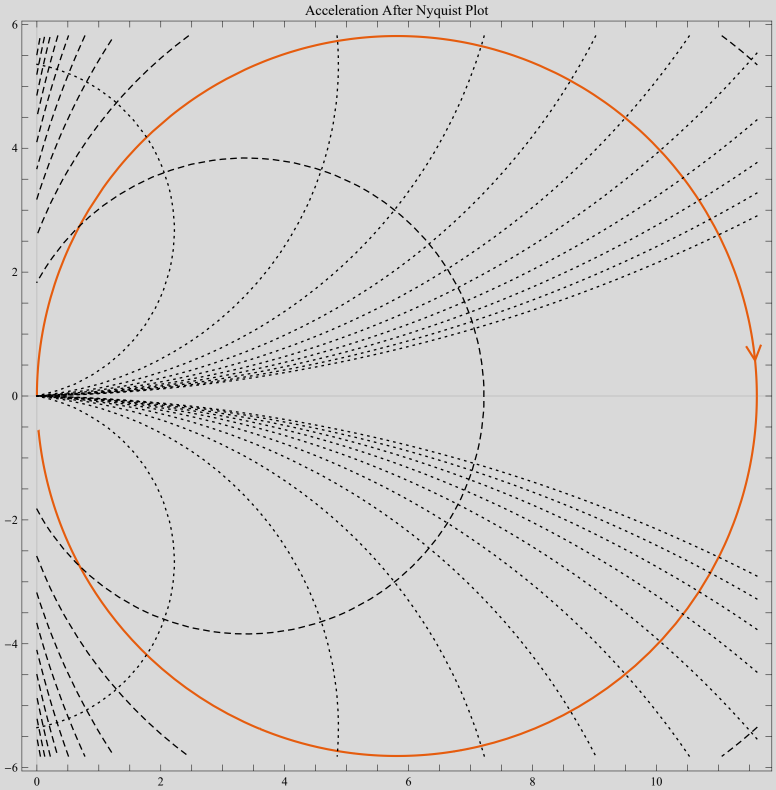
$$\text{poles} \to \frac{-B\,L\,\eta\,N^2 - J\,R\,\eta\,N^2 - \text{Bafter}\,L - \text{Jafter}\,R - \sqrt{(B\,L\,\eta\,N^2 + J\,R\,\eta\,N^2 + \text{Bafter}\,L + \text{Jafter}\,R)^2 - 4\,(J\,L\,\eta\,N^2 + \text{Jafter}\,L)\,(\text{Ke}\,\text{Kt}\,\eta\,N^2 + B\,R\,\eta\,N^2 + \text{Bafter}\,R)}}{2\,(J\,L\,\eta\,N^2 + \text{Jafter}\,L)}$$

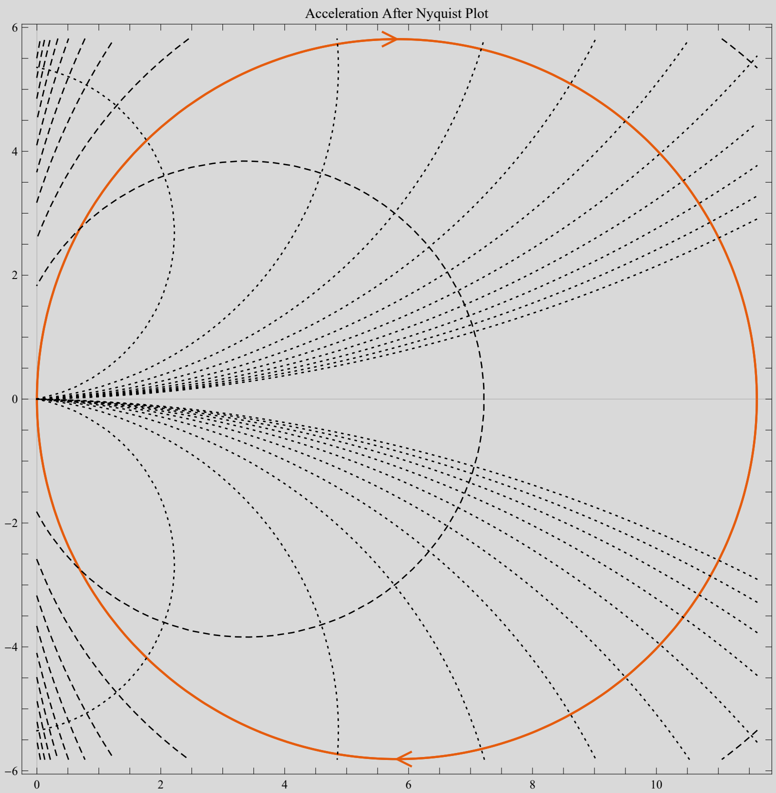
$$\frac{-B\,L\,\eta\,N^2 - J\,R\,\eta\,N^2 - \text{Bafter}\,L - \text{Jafter}\,R + \sqrt{(B\,L\,\eta\,N^2 + J\,R\,\eta\,N^2 + \text{Bafter}\,L + \text{Jafter}\,R)^2 - 4\,(J\,L\,\eta\,N^2 + \text{Jafter}\,L)\,(\text{Ke}\,\text{Kt}\,\eta\,N^2 + B\,R\,\eta\,N^2 + \text{Bafter}\,R)}}{2\,(J\,L\,\eta\,N^2 + \text{Jafter}\,L)}$$

poles → {-4742.62, -13.7468}






## 3. Revision History

2018.08.22. Initial version.