Semantic Regime Theory:
From Metric Spaces to Action-Invariant State Spaces

Adrian Diamond

February 7, 2026

Abstract

We introduce *Semantic Regime Theory*, a structural framework for transforming metric or signal-driven spaces into action-invariant semantic state spaces. The theory formalizes semantic filtering, regime collapse, and ontology minimization as mathematically valid operations, independent of prediction. We define a categorical functor from metric spaces to semantic regimes, and establish invariance theorems.

Contents

1 Introduction

Most computational systems operate over metric or signal-based spaces: distances, similarities, scores, probabilities, or continuous measurements. Such spaces are often high-dimensional, noisy, and operationally unstable. Yet decisions, policies, and actions occur not in metric spaces but in *state spaces* whose elements represent semantically meaningful regimes.

This paper studies the existence and properties of semantic state spaces arising from action invariance over metric domains. The resulting objects, which we call *semantic regimes*, discard unnecessary metric detail while preserving operational meaning.

The motivating observation is simple: in many systems, action is invariant across large regions of metric space. Semantic Regime Theory studies this invariance as a first-class mathematical object.

We emphasize that the goal of this framework is not prediction, estimation, or optimization, but *semantic stabilization* under action.

2 Metric Spaces and Semantic Motivation

Definition 2.1 (Metric Space)**.** A metric space is a pair (X, d) where X is a set and $d : X \times X \to \mathbb{R}_{\geq 0}$ satisfies the usual axioms.

Metric spaces encode proximity but not meaning. In applied systems, distances often arise from heterogeneous signals whose magnitudes do not directly correspond to operational relevance.

Remark 2.2. Semantic Regime Theory does not assume that the metric is meaningful in isolation. The metric is treated as raw structure from which semantic distinctions may be extracted.

3 Semantic Filters and Ontologies

Definition 3.1 (Ontology)**.** An ontology $\mathcal{O}$ over a space X is a finite or countable set of semantic predicates

$$\mathcal{O} = \{\phi_i : X \to \{0, 1\}\}.$$

4 Semantic Regimes and State Spaces

Definition 4.1 (Semantic Regime)**.** A semantic regime is a tuple

$$\mathcal{R} = (S, A, \sim)$$

where S is a set of semantic states, A is an action set, and $\sim$ is an action-invariance relation on X such that $x \sim y$ if all admissible actions yield identical effects.

Definition 4.2 (Induced State Space)**.** The induced semantic state space is the quotient

$$S = X / \sim .$$

Remark 4.3. This construction is fundamentally non-metric. Distances are irrelevant once action invariance is established.

5 Action Invariance and Regime Collapse

Definition 5.1 (Action Invariance)**.** An action $a \in A$ is invariant on a subset $U \subseteq X$ if its outcome is constant for all $x \in U$.

Theorem 5.2 (Regime Collapse Theorem)**.** *Let (X, d) be a metric space with action set A. If A is invariant on each equivalence class induced by Φ, then the semantic quotient $X/\sim$ is well-defined and stable under refinement of d.*

6 Semantic Regimes as Quotients

Lemma 6.1 (Semantic Quotient Lemma)**.** *The projection $\pi : X \to S$ is surjective and action-preserving.*

Proof. Surjectivity follows from quotient construction. Action preservation follows from invariance definition. □

7 Categorical Formulation

Definition 7.1. Let **Met** be the category of metric spaces and non-expansive maps. Let **Sem** be the category of semantic regimes and action-preserving morphisms.

Theorem 7.2. *There exists a functor*

$$F : \mathbf{Met} \to \mathbf{Sem}$$

mapping metric spaces to semantic regimes via semantic filtering and action invariance.

Proposition 7.3. *The functor F is generally non-faithful.*

Remark 7.4. This loss of information is intentional and structurally necessary.

References

[1] F. W. Lawvere, *Metric Spaces, Generalized Logic, and Closed Categories*, Reprints in Theory and Applications of Categories, 2002.

[2] S. Mac Lane, *Categories for the Working Mathematician*, Springer, 1971.