The AEGON Algebra
A Finite Semantic Algebra of Failure Classes for Deterministic System Interpretation

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Abstract

Modern distributed systems exhibit unbounded scale and complexity, yet their observable behaviors remain structurally constrained. This paper introduces the AEGON algebra, a finite semantic algebra over system failure modes, demonstrating that system behavior admits a total deterministic classification independent of system size. We formalize a finite ontology of failure classes, define a partial order over these classes, and show how policy actions can be compiled from semantic classifications. The AEGON algebra provides a foundation for deterministic system interpretation, policy generation, and operational reasoning without reliance on probabilistic or heuristic models.

Contents

1 Structural Generators of the AEGON Algebra

Definition 1.1 (Failure Class). A failure class is a semantic atom representing a violated structural invariant of a system.

Definition 1.2 (Failure-Class Ontology). Let $F = \{F_0, F_1, \ldots, F_{13}\}$ be a finite, closed ontology of failure classes, where F_0 denotes the neutral (no-failure) class.

Definition 1.3 (Observation Space). Let O denote the space of *normalized* system observations (e.g., structured signals, derived invariants, or canonical payloads), where normalization enforces representation invariance with respect to irrelevant operational variation. Elements of O are assumed to be sufficient for semantic classification.

Remark 1.1. This paper covers (i) the AEGON cloud application (deterministic classifier), and (ii) the AEGON compiler/language (policy compilation). These are distinct artifacts unified by the same semantic algebra.

2 Classification as Functorial Interpretation

Definition 2.1 (Deterministic Classifier). A deterministic classifier is a total function

$$C : O \to F$$

such that for identical $o \in O$, repeated evaluation yields identical output in F.

Definition 2.2 (Failure-Class Category). Define a category $\mathcal{F}$ whose objects are the failure classes in F and whose morphisms represent semantic escalation relations (defined formally in §4).

Definition 2.3 (Functorial View of Classification). When O is regarded as a category $\mathcal{O}$ of observations (with structure-preserving maps between observations), classification can be viewed as a functor

$$C : \mathcal{O} \to \mathcal{F}.$$

Corollary 2.1 (Repeatability). For all $o \in O$, $C(o)$ is stable under re-evaluation: $C(o) = C(o)$ under all evaluation contexts.

Proof. Determinism is a definitional property of C.

3 Finite Semantic Closure of System Behavior

Theorem 3.1 (Finite Semantic Closure). System behavior (as represented in O) admits a total deterministic classification into the finite ontology F, independent of system size.

Lemma 3.2 (Semantic Coverage Principle). If the failure-class ontology F is finite and closed with respect to admissible observations in O, then the classifier $C : O \to F$ induces a complete semantic interpretation of system state for the purposes of policy generation.

Proof. By totality of C and closure of F, every admissible observation maps to exactly one policy-relevant semantic regime.

Proof. Since $C : O \to F$ is total and F is finite, every observation maps to exactly one class in a finite codomain.

Corollary 3.1 (Bounded Interpretive Complexity). Even when operational state space is unbounded, interpretive state space is bounded by $|F|$.

Proof. The classifier's image is a subset of F.

4 The Failure-Class Lattice and Thin Category Structure

Definition 4.1 (Failure-Class Partial Order). Let $\leq$ be a partial order on F such that $F_i \leq F_j$ means class F_j semantically dominates or escalates F_i.

Semantic Interpretation. The partial order $\leq$ does not encode temporal ordering, causality, or probabilistic likelihood. Rather, $F_i \leq F_j$ expresses that the semantic regime F_j strictly dominates or subsumes F_i with respect to system interpretation and policy relevance.

Definition 4.2 (Join-Semilattice (Policy-Relevant)). Assume $(F, \leq)$ admits binary joins $\vee$ (least upper bounds) whenever policy composition must reconcile multiple triggered regimes.

Lemma 4.1 (Thinness of the Failure-Class Category). The category $\mathcal{F}$ induced by $(F, \leq)$ is thin: for any $F_i, F_j \in F$ there exists at most one morphism $F_i \to F_j$.

Proof. In a poset-category, $\mathrm{Hom}(F_i, F_j)$ is either empty or a singleton depending on whether $F_i \leq F_j$.

Remark 4.1. Thinness is the categorical formalization of no competing interpretations: for any two failure classes, there exists at most one admissible semantic escalation path. This eliminates ambiguity in regime dominance and ensures that reconciliation is canonical rather than heuristic.

5 Policy Compilation Semantics

Definition 5.1 (Policy Action Set). Let P be the set of policy actions (e.g., restrict rollout, freeze control-plane, page on-call, isolate dependency, etc.).

Definition 5.2 (Policy Monoid). A policy monoid is a triple $(P, \oplus, e)$ where $\oplus$ is associative policy composition and e is the identity (no-op) policy.

Proof. Associativity enables stable bundling; e ensures explicit "no action" is representable.

Definition 5.3 (Policy Compiler). A policy compiler is a deterministic map

$$\Pi : F \to P$$

that assigns a canonical policy bundle to each failure class.

Theorem 5.1 (Determinism of Policy Compilation). The composed function $\Pi \circ C : O \to P$ is deterministic.

Proof. Composition of deterministic functions is deterministic.

6 Formal Semantic Examples

6.1 Example 6.1: Single-Regime Classification and Compilation

Let $o \in O$ be an observation whose invariant-violations correspond to a unique regime F_k (e.g., Control Plane Saturation). Then:

$$C(o) = F_k, \qquad (\Pi \circ C)(o) = \Pi(F_k).$$

Interpretation: the system resolves to one stable semantic regime; compilation emits one canonical policy bundle.

6.2 Example 6.2: Multi-Regime Join and Canonical Reconciliation

Let $o \in O$ trigger evidence consistent with two regimes F_a and F_b. If $(F, \leq)$ admits join:

$$C(o) = F_a \vee F_b.$$

Compiled policy is:

$$\Pi(C(o)) = \Pi(F_a \vee F_b).$$

Interpretation: join selects a unique least-dominating semantic regime (no ambiguity), and compilation produces a single resolved policy.

6.3 Example 6.3: Explicit Neutral Outcome (No-Action as a First-Class Result)

If o violates no encoded invariants, define:

$$C(o) = F_0, \qquad \Pi(F_0) = e.$$

Interpretation: non-action is not absence of meaning; it is the meaning of semantic stability.

7 Minimal AEGON Policy DSL and JSON Bundle Target

Remark 7.1. This section is intentionally minimal: it demonstrates the "language $\rightarrow$ policy bundle" idea without over-engineering the compiler.

7.1 Five-Construct Minimal Policy DSL (Sketch)

We define a minimal DSL with at most five constructs:

- C1: `ON <FailureClass>` (trigger)

- C2: `DO <Action>` (emit action)

- C3: `WITH <Key=Value>` (attach parameters)

- C4: `TIER <Name>` (capability gate)

- C5: `END` (close block)

7.2 DSL Example Compiling to JSON Policy Bundle

```
TIER pro
ON CONTROL_PLANE_SATURATION
DO freeze_control_plane
WITH window_minutes=30
DO page_oncall
WITH severity=high
END
```

Compiled JSON Bundle (Target):

```
{
  "tier": "pro",
  "on": "CONTROL_PLANE_SATURATION",
  "actions": [
    { "name": "freeze_control_plane",
      "params": { "window_minutes": 30 } },
    { "name": "page_oncall",
      "params": { "severity": "high" } }
  ]
}
```

8 Interpretive Remarks on Classification Totality and Semantic Neutrality

Remark 8.1 (Poset-as-Category View). Treating $(F, \leq)$ as a thin category formalizes semantic dominance structurally rather than procedurally. No probabilistic scoring, confidence weighting, or human arbitration is required to determine escalation.

Remark 8.2 (Policy Neutrality). The policy monoid admits a neutral element e representing no action. Neutral outcomes are first-class semantic results, indicating system stability rather than classifier failure.

9 Conclusion

The AEGON Algebra expresses a conceptual result: despite unbounded system scale, observable behavior admits a finite deterministic semantic regime. By equipping failure classes with a partial order (and joins when needed) and compiling semantic regimes into monoidal policy bundles, AEGON supports stable interpretation and action generation without heuristics or learning.

References

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