

CS 170 Homework 13 (Optional)

1 Convexity Potpourri

For each of the following statements, state whether it is true or false. Justify your answers.

- (a) The complement of a convex set $\mathcal{S} \in \mathbb{R}^d$ is always a convex set.
- (b) The complement of a convex set $\mathcal{S} \in \mathbb{R}^d$ is never a convex set.
- (c) The sum of two convex functions is a convex function.
- (d) The difference of two convex functions is a convex function.
- (e) The set of points (x, y) such that $x^2 + y^2 \geq 10$ is a convex set.

2 Fair Allocation

Consider the following problem:

FAIR ALLOCATION

There are N dollars to be allocated among n employees. Based on work experience, employee i is entitled at least ℓ_i dollars.

An allocation is defined to be fair if no subset of $n/10$ employees or fewer receives more than half the dollars.

Find a fair allocation if possible; otherwise, report that none exists.

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- (a) Write a linear program for the above problem (with possibly exponentially many constraints).
 - (b) Describe a polynomial-time algorithm implementing the separation oracle¹ for the linear program from part (a).

3 Setting Tolls

Consider the following problem:

SETTING TOLLS

The city council has decided to impose tolls on some its most popular roads to discourage people from driving, aiming to reduce traffic and emissions. In particular, the goal is to discourage all driving from location s to location t in the city. The formal description of the problem is as follows.

Input: An undirected unweighted graph $G = (V, E)$, a pair of nodes s and t , and a positive real number B .

Output: A set of edge weights $w : E \rightarrow \mathbb{R}^+$ such that:

1. For every $s - t$ path P in the graph G , the total weight on the path is at least 1
2. The total weight of all edges in the graph is at most B

(weights w_e are the “tolls” on the edges)

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- (a) Write a linear program for the above problem (with possibly exponentially many constraints).

¹The separation oracle a linear program determines a constraint that is violated by a point x , if there exists one.

- (b) Describe a polynomial-time algorithm implementing the separation oracle for the linear program.