PHYSICS 77/88
Capstone Project: Capstone Bridge Assistant

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I. Abstract

The project is centered around providing assistance to Bridge players, with a focus on enhancing their experience in three key phases: before, during, and after gameplay. Our goal is to aid Bridge enthusiasts in improving their skills and comprehension of the game.

In the pre-game phase, the project presents a customizable dealing program. During gameplay, the project performs suit distribution probability analysis. a thorough examination of the game is conducted through Butler calculation and ranking.

II. Introduction

The game of Bridge, having ancient roots dating back to the 16th century, is both the ultimate trick-taking card game and a mind sport game like chess.

Bridge is played with a pack of 52 cards, consisting of 13 cards in each of four suits. The suits rank downward in the order spades, hearts, diamonds, clubs. The cards of each suit rank downward in the order Ace (A), King (K), Queen (Q), Jack (J), 10 (sometimes T for alignment), 9, 8, 7, 6, 5, 4, 3, 2. Four players play at each table, on which one direction is designated as North and other compass directions assume the normal relationship to North. North (N) plays with South (S) and East (E) with West (W). Each player receives thirteen cards which should be sorted into suits.

In the first part, we present a customizable dealing program, assisting players before gameplay. By leveraging Python programming, we have created functions to handle the tasks of shuffling and dealing cards. In addition to these fundamental features, we also provide advanced options that allow players to customize their hands with specific suit length(s) according to their practice needs, thus enhancing their card-playing skills.

The game of Bridge is divided into two phases, bidding and play. The focus of the second part of the project is on the play phase. During gameplay, one of the players, called **dummy**, spreads his entire hand face up, neatly arranged into suits for all players to see. Four cards played, one from each player in clockwise rotation, constitute a **trick**. The first card played to each trick is called the **lead**. The remaining three players must then follow suit if able. Generally, a trick is won by the player who played the highest card of the suit led. Therefore, having an understanding of the probable length or distribution of high cards in the unseen hands is crucial when devising a strategy.

In the second part, we furnish functions for assessing the likelihood of particular length distributions, as well as the likelihood of encountering specific length distributions, during gameplay. We have created functions to evaluate the probability of where certain valuable cards might be located, as well as the distribution of cards held by each opponent in a given suit. These functions can consider any additional information available. By utilizing it during training sessions, players can acquire the skills necessary to enhance their strategic approach and attain higher scores in tournaments.

After gameplay, for each table and each board, one pair of partners will be assigned a score determined by the collective performance of the four players involved. Coaches may need to evaluate how well players performed. Meanwhile, tournament organizers might need to create rankings for players. To avoid the influence of card quality on scores, duplicate bridge competitions utilize multiple tables with identical card distributions, comparing the scores of partners sitting in the same direction for ranking purposes.

In the third part, we assess players' performance in an actual tournament through the **Butler** and **Victory Point (VP)** methods. These approaches provide valuable insights into player performance and ranking, contributing to a deeper understanding of their capabilities. Coupled with the conventional **Match Point (MP)** method, we provide players and coaches with a thorough system for assessing their performance, as well as some ranking methods for tournament organizers.

III. Methods

A. Part 1. Before Gameplay: Customizable Dealing Program

We may assign a hand of specific length(s) of suit(s) to a player. In this case, we distribute the designated suits initially and keep track of the remaining valid cards that can still be allocated to the player's hand. All the other cards in the required suit(s) should not be valid. Finally, we distribute the other suits.

If we want to assign a hand of specific range(s) of length(s) of suit(s) to a player, we utilize the random.randint() function to obtain random length(s) of suit(s), then apply the preceding function.

We may assign a hand with specific card(s) to a player as well. Similarly, we distribute the designated card initially. Finally, we distribute the other cards.

After assigning a hand, we can assign the remaining cards randomly to the other three hands to the other players without constraints.

B. Part 2. During Gameplay: Suit Distribution Probability Analysis

We gain insight into the distribution between two particular players through sampling.

In high card sampling, we would like to know the probability of the "location" of some specific (high) card(s). We disregard the restriction imposed by a 13-card limit since its effect on the outcome is minimal. Instead, we randomly allocate a card to either the first or second player.

In high card conditional sampling, we have precise knowledge of the number of cards of a particular suit held across four hands. Consequently, we can distribute corresponding "slots" to the first and second players according to the quantity of suit cards they possess. Subsequently, a high card is randomly designated to one of these slots.

In length distribution sampling, we would like to know the probability of each kind of distribution of cards held by each opponent in a particular suit by considering the total number of cards held by himself and his partner. For instance, if South and dummy (North, for instance) have 9 spades in total, the distribution of spades held by their opponents could be 4-0, suggesting that one of their opponents holds all the remaining spades, or it could be 3-1, or 2-2. Similarly, we distribute 13 "slots" to each of players and start sampling. The unknown cards are sequentially designated to the slots.

In length distribution conditional sampling, we possess supplementary data suggesting that players hold a certain quantity of cards in suits other than the one in focus. We allocate a total of 13 minus the number of already accounted for cards in the other suits to each player as "slots" and commence the sampling process. The unidentified cards are then assigned to these slots in sequence.

C. Part 3. After Gameplay: Butler Calculation and Ranking

In bridge tournaments, IMP, VP, and MP scoring methods are widely used for ranking players. In this part, we employ these methods to assess and rank players in an actual tournament.

Each board's score is an integer multiple of 10. The sign indicates which pair receives the score: a positive sign indicates that North-South (N-S) earns the score, while a negative sign indicates that East-West (E-W) earns the score. After getting the scores of all boards on all tables, we should first compute the **datum** of each board. A datum or mean score can be computed from the set of the scores after discarding the top (max) and bottom (min) scores, and retaining the rest. Datum is then rounded to the nearest multiple of 10.

$$\textbf{datum} = \frac{\sum_{\text{table}} \text{score} - \max(\text{scores}) - \min(\text{scores})}{|\text{tables}| - 2}$$

For each pair (including those pairs whose scores were discarded from the datum calculation), the score for their board is taken and subtracted from the datum score. This score, positive or negative, is converted to **IMP**s (International Match Points) according to the IMP table (see Table I). If the difference reaches the number in the "Difference" column, they can obtain the corresponding Butler in the "IMP" column. Each of the pairs then scores that number of IMPs for the board. In general, for any table, one pair gets the positive score and the other the negative score, or both pairs get 0. For example, if the difference is -250, N-S pair will get -6 IMPs and E-W pair will get 6 IMPs for this board.

Difference (lower limit)	IMP
0	0
20	1
50	2
90	3
130	4
170	5
220	6
270	7
320	8
370	9
430	10
500	11
600	12
750	13
900	14
1100	15
1300	16
1500	17
1750	18
2000	19
2250	20
2500	21
3000	22
3500	23
4000	24

TABLE I
DIFFERENCE TO IMP

The Butler score for a pair on a board is numerically equivalent to the IMP they receive from that board. To determine a pair's final Butler, take the sum of their Butlers from all boards. A higher Butler indicates better performance by the pair.

A round comprises a set number of boards played by a pair against the same opposing pair. Generally, the number of boards played in a round is fixed during a tournament. VP is calculated once at the end of every round. To calculate the VP, we then transform the IMP scores using the specified function

$$\tau = \frac{\sqrt{5} - 1}{2}$$

$$\textbf{VP} = 10 + \text{sign}(\Delta\text{IMP}) \cdot 10 \cdot \frac{1 - \tau^{\min(3, \frac{|\Delta\text{IMP}|}{5\sqrt{\text{boards}}})}}{1 - \tau^3}$$

where M represents the total IMP score the pair get in the round and N denotes the number of boards played in a round.

To determine a pair's rank, take the sum of their vps from all rounds. A higher VP indicates better performance by the pair.

IV. Results

A. Part 1. Before Gameplay: Customizable Dealing Program

Applying basic dealing function, we can obtain a random board without any constraints:

```
                S: 8
                H: 9543
                D: AQ98532
                C: 3

S: KT953                       S: AJ64
H: KJ762        ---------      H: T
D: K            |   N   |      D: J764
C: 97           |W     E|      C: A542
                |   S   |
                ---------

                S: Q72
                H: AQ8
                D: T
                C: KQJT86
```

Fig. 1. Basic Deal

Assume we want a hand with exactly 4 Spades, 5 Hearts, 2 Diamonds and 2 Clubs. Applying exact suit length dealing function, we can obtain a random hand with the exact suit length(s) assigned to the specific player:

S: T754 H: AJ743 D: T4 C: 75

Assume we want a hand with 4-6 Hearts. Applying range suit length dealing function, we can obtain a random hand with the suit length(s) in range(s) assigned to the specific player:

S: AJ84 H: KQT4 D: KQ8 C: 97

Assume we want a hand with Ace of Spades, King of Hearts, 4 of Diamonds and 7 of Clubs. Applying specific card dealing function, we can obtain a random hand with the specific card(s) assigned to the specific player:

S: A7 H: AK9542 D: T643 C: 7

Assume we want a hand with exactly 2 Spades, 5 Hearts, no Diamond and 6 Clubs assigned to West. Applying these functions, we can obtain a random board with some constraints to the hand of a player:

```
                S: AKJ7
                H: QT84
                D: J64
                C: KJ

S: 62                          S: Q95
H: 97652        ---------      H: K3
D: -            |   N   |      D: T972
C: AT7653       |W     E|      C: Q942
                |   S   |
                ---------

                S: T843
                H: AJ
                D: AKQ853
                C: 8
```

Fig. 2. Conditional Deal

B. Part 2. During Gameplay: Suit Distribution Probability Analysis

Assume we are looking for A(ce) and Q(ueen) which may be held by either E(ast) or W(est). Applying high card distribution function, we can acquire samples that indicate the positioning of high cards. Figure 3 shows the frequency that East holds neither, A only, both A and Q and Q only:

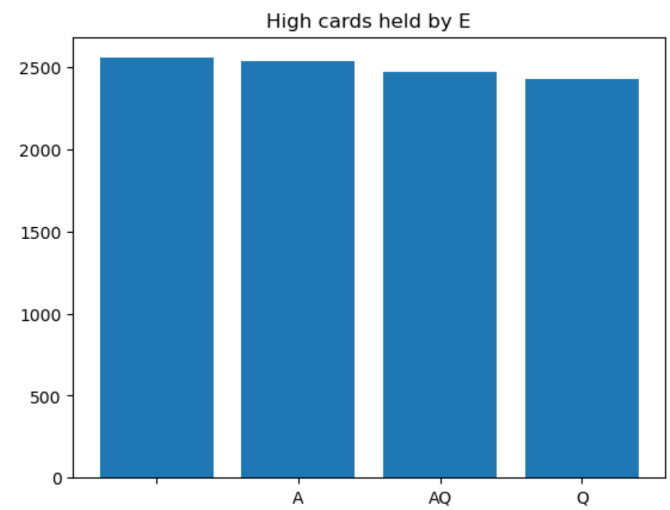

Fig. 3. High Card Distribution Sample

Assume we are looking for A(ce) and J(ack) which may be held by either E(ast) or W(est). Applying high card conditional distribution function, we can acquire samples that indicate the positioning of high cards based on the number of cards in each player's suit. Figure shows the frequency that East holds neither, A only, both A and Q and Q only:

Assume we are looking for the probability of distribution of 5 cards in a suit which may be held by either E(ast) or W(est). Applying length distribution function, we can acquire samples that indicate each player's suit length. Figure 4 and 5 shows the frequency that East and West respectively holds i and j cards, denoted by i-j:

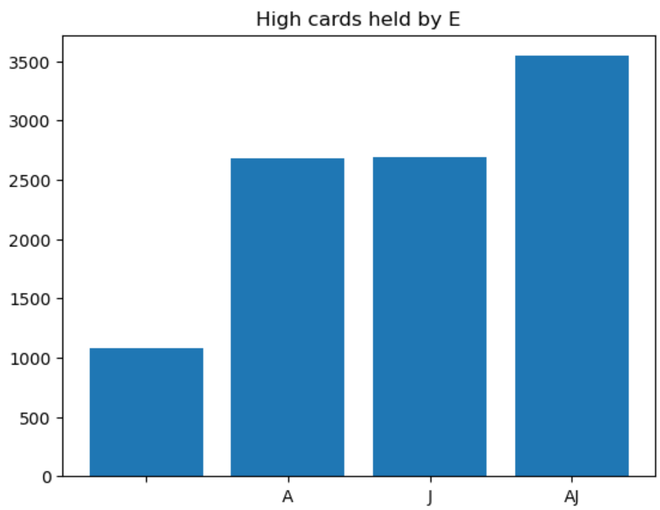

Fig. 4. High Card Conditional Distribution Sample for East

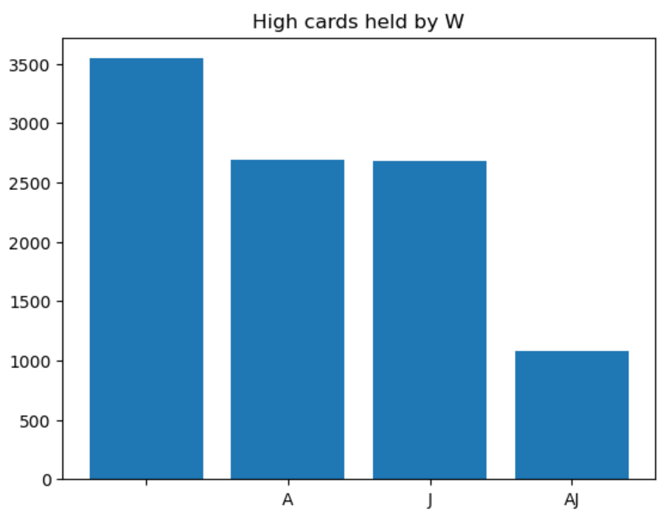

Fig. 5. High Card Conditional Distribution Sample for West

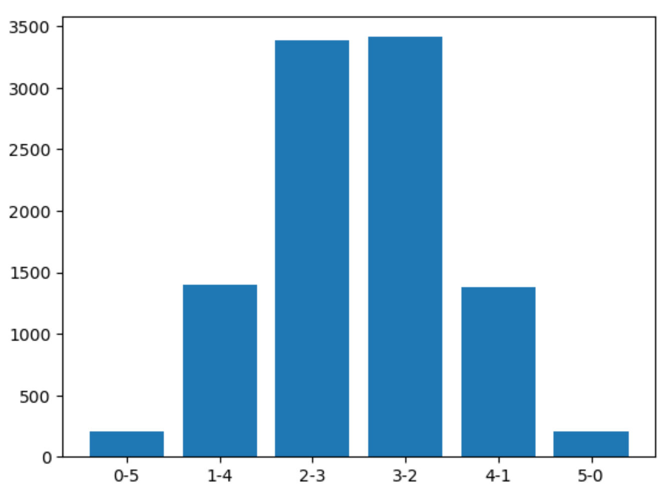

Fig. 6. Length Distribution Sample for East-West

Assume we are looking for the probability of distribution of 5 cards in a suit which may be held by either E(ast) or W(est), given East has 3 cards in other suits and West has 7. Applying length conditional distribution function, we can acquire samples that indicate each player's suit length based on the number of cards each player have in other suits. As a player has only 13 cards, it is impossible for West to have 7 cards in the suit as well as 7 in other suits. Figure 6 shows the frequency:

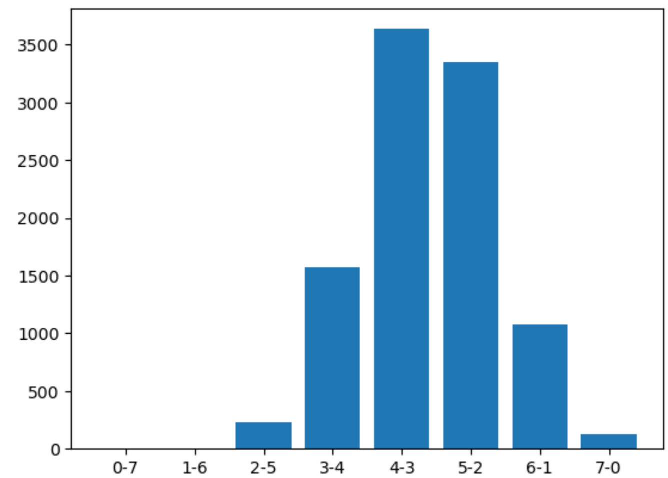

Fig. 7. Length Conditional Distribution Sample for East-West

C. Part 3. After Gameplay: Butler Calculation and Ranking

By summing the Butler scores for each pair across all boards, we can calculate the total Butler score and then rank the pairs accordingly. Refer to Table II for the results.

Rank	Pair	Butler
1	4	52
2	7	45
3	9	11
4	6	1
5	8	-2
6	12	-2
7	5	-7
8	1	-9
9	11	-15
10	2	-16
11	3	-26
12	10	-32

TABLE II
RANK BY BUTLER

By summing the VP scores for each pair across all rounds, we can calculate the total VP score and then rank the pairs accordingly. Refer to Table III for the results.

V. CONCLUSIONS

A. Part 1. Before Gameplay: Customizable Dealing Program

The project correctly shuffles and deals random boards, whether with or without any constraints to a specific hand.

Rank	Pair	VP
1	7	131.42
2	4	130.40
3	9	113.71
4	6	111.55
5	12	109.98
6	8	109.93
7	1	107.68
8	5	104.19
9	11	102.83
10	2	100.91
11	3	99.30
12	10	98.10

TABLE III
RANK BY VP

B. Part 2. During Gameplay: Suit Distribution Probability Analysis

According to the samples generated by previously defined functions, the probability of each high card in a particular direction is 0.5 when there are no constraints, and they are almost not dependent on each other. While the 13-card limit was not taken into account, it serves as a practical approximation during gameplay. Therefore, the probability of a player holding at least i specific high cards among n designated high cards is

$$\frac{1}{2^i}$$

The probability of a distribution scenario involves one player holding i cards in a specific suit, with j of those cards being among n designated high cards, while the other player holds k cards in the same suit, including the remaining $n-j$ designated high cards, is

$$\frac{C(i,j)\cdot C(k,n-j)}{C(n,j)\cdot C(i+k,n)}$$

which means the player who possesses a greater number of cards in a particular suit is more inclined to have a higher number of specific high cards within that suit.

The probability that one player holds i cards while the other holds j cards in a suit is

$$\frac{C(13,i)\cdot C(13,j)}{C(26,i+j)}$$

which means the cards are intend to be evenly distributed between the players, although it is less probable to have an absolutely even $(\frac{n}{2})$-$(\frac{n}{2})$ distribution than $(\frac{n}{2}\pm 1)$-$(\frac{n}{2}\mp 1)$ one with n cards in total. For example, 4-2 or 2-4 distribution has a higher probability (about 0.48) than 3-3 distribution (about 0.36).

The probability of one player possessing i cards while simultaneously holding k cards in different suits, while the other player holds the remaining j cards while simultaneously holding l cards in different suits, is

$$\frac{C(13-k,i)\cdot C(13-l,j)}{C(26-k-l,i+j)}$$

which means the player who possesses a greater number of cards in other suits are less likely to have a greater number of cards in the suit.

C. Part 3. After Gameplay: Butler Calculation and Ranking

The actual rank by Match Point (MP) is shown in Table IV. It can reveal the average percentage of opposing pairs they can defeat in the same direction on each board.

Rank	Pair	MP(%)
1	4	54.09
2	12	53.18
3	7	52.73
4	11	52.50
5	6	51.14
6	5	51.14
7	9	50.68
8	1	49.77
9	8	47.95
10	2	46.82
11	10	46.14
12	3	43.86

TABLE IV
RANK BY MP(%)

Comparing the results, we infer a positive correlation between the rankings of Butler and VP with the rankings of the players in the tournament from Figure 8 and 9.

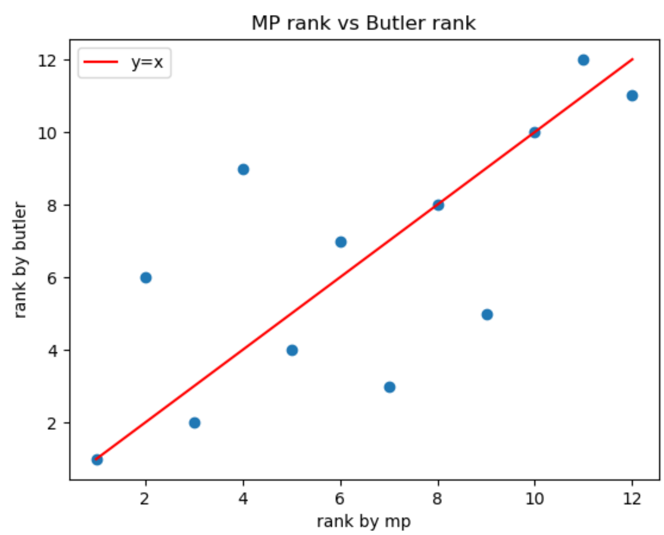

Fig. 8. Relation Between MP rank and Butler Rank

In fact, MP highlights the significance of variations in scores, wherein even a tiny score difference can mean a big change in rank. Butler, though, focuses more on snagging higher scores on certain boards, even if the gaps between scores are big. Meanwhile, VP hones in on the difference between a pair and their opponents, setting it apart from MP. So, which method to choose really depends on what parts of the game the organizer want to highlight. Tournament organizers have the flexibility to select a ranking method that precisely aligns with their particular requirements.

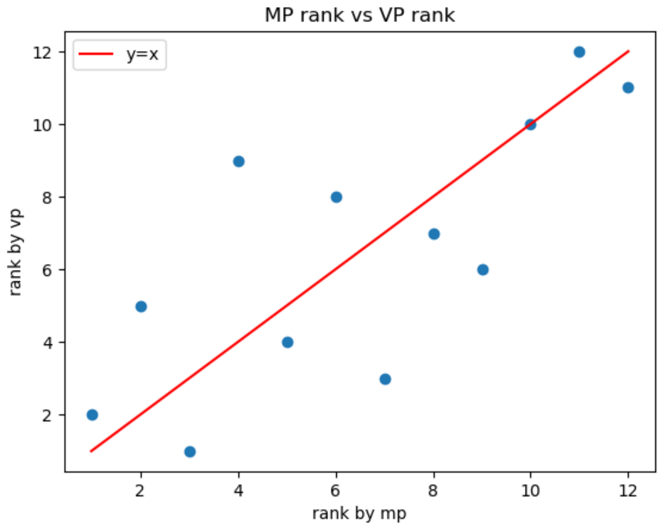

Fig. 9. Relation Between MP rank and VP Rank

VI. References

1) http://www.worldbridge.org/what-is-bridge/#about-play
2) http://www.worldbridge.org/wp-content/uploads/2016/12/2017LawsofDuplicateBridge-paginated.pdf
3) http://www.gembridge.cn/score/PairBoards?tourStart=2023-08-25&tour=23721&event=80628966-bc59-4fab-8947-61033b091aae&section=9092c949-e68d-445c-8b24-e0b0a44a938d&board=1&from=ccba