HW2

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1. Recall the situation from the first problem set, in which $X_i \sim Poisson(\lambda)$ for $i = 1, 2, 3$ with $X_1 = 1, X_2 = 0$ and $X_3 = 3$.

(a) Find (analytically) the posterior distribution if the prior distribution on λ is a gamma distribution with a shape parameter of 2 and a rate parameter of 10, using the fact that the gamma distribution is the conjugate prior to the Poisson distribution.

From Jackman, the posterior density is $Gamma(a^*, b^*)$ where $a^* = a + S$ and $b^* = b + n$.

$$S = \sum_{1}^{3} X_i = 1 + 0 + 3 = 4$$

Thus the posterior density is:

$$Gamma(2 + 4, 10 + 3) = Gamma(6, 13)$$

(b) Use the previous two results to compute the posterior probability that $\lambda > 1$ under this prior.

Here, I draw 100,000 sample from the posterior gamma distribution and calculate the probability that the draw is greater than 1:

```
### Set seed for reproducibility
set.seed(1111)

sum(rgamma(100000,
           shape = 6,
           rate = 13) > 1) / 100000
```

```
## [1] 0.01106
```

Thus, the probability that $\lambda > 1$ is .01. This result is comparable to the first HW.