PHP 2517 Homework #1

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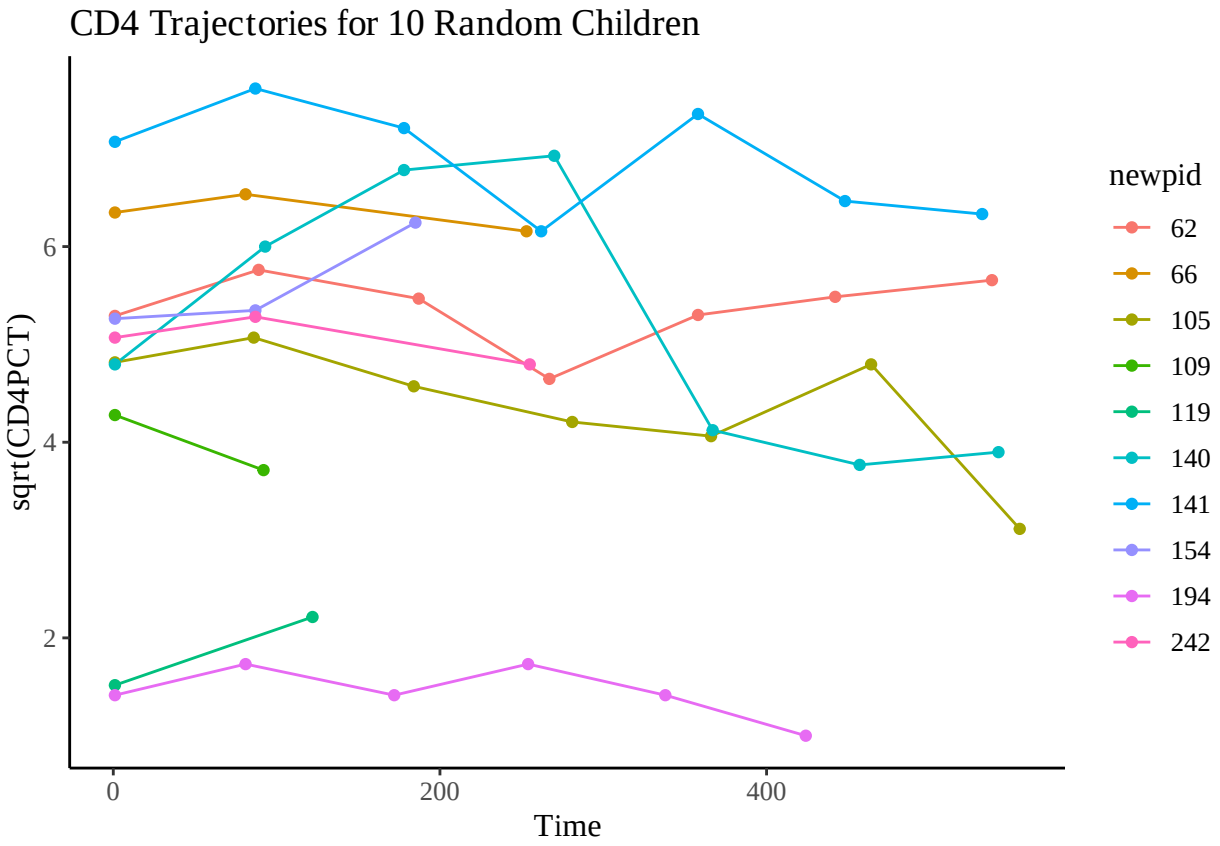
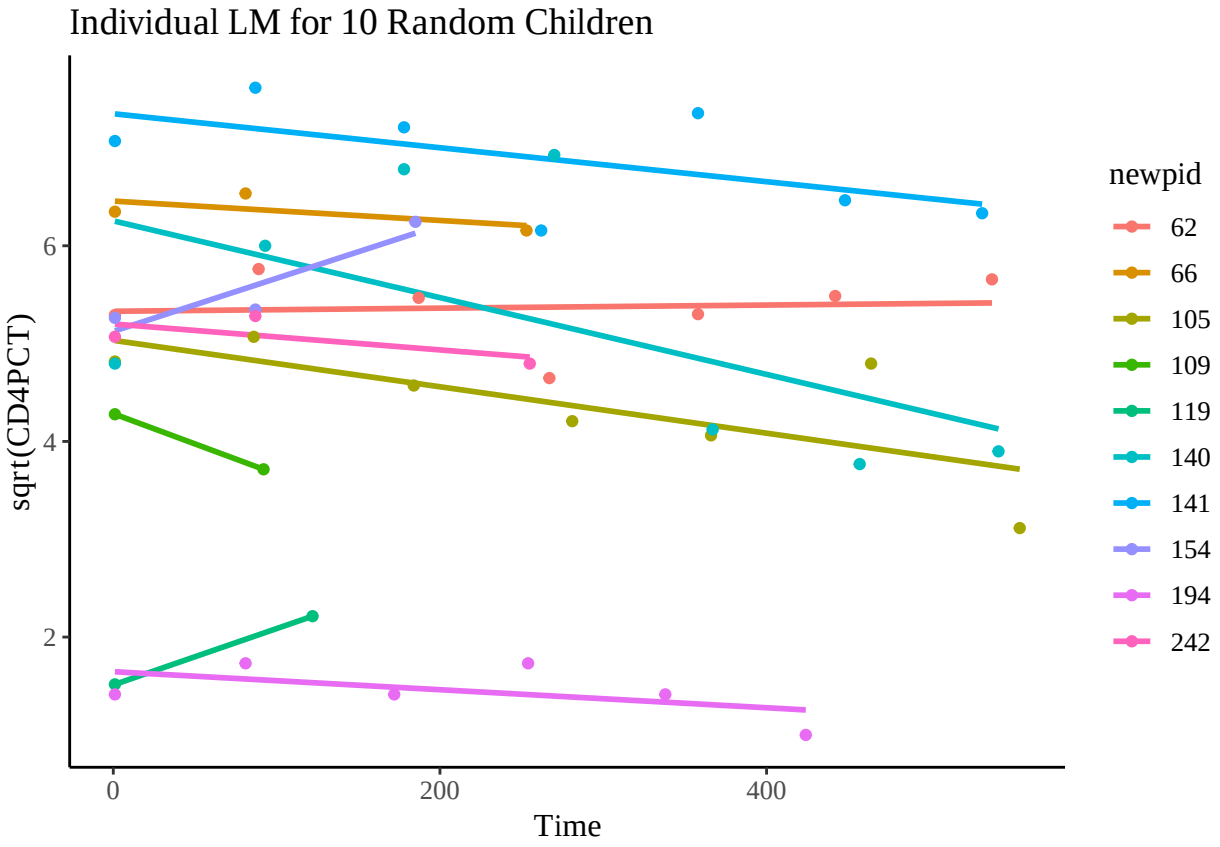
Q1 GH Chapter 11: Exercise 4

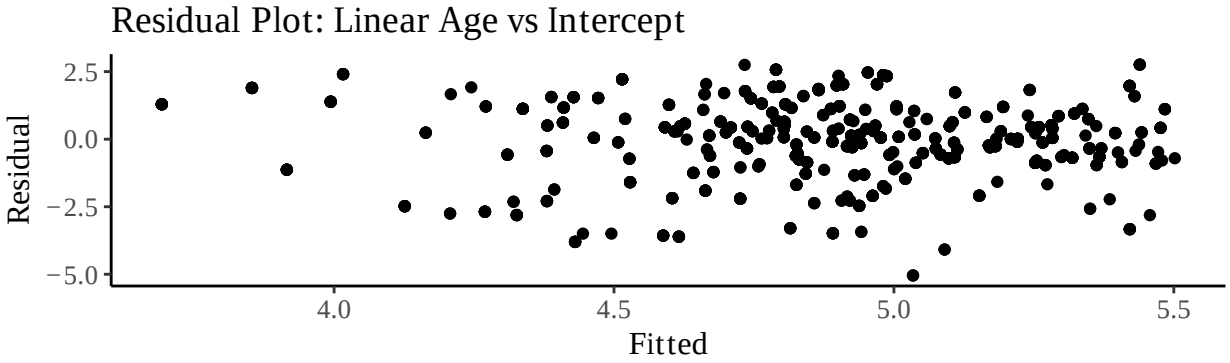
a.) Graph the outcome (the CD4 percentage, on the square root scale) for 10 children as a function of time.

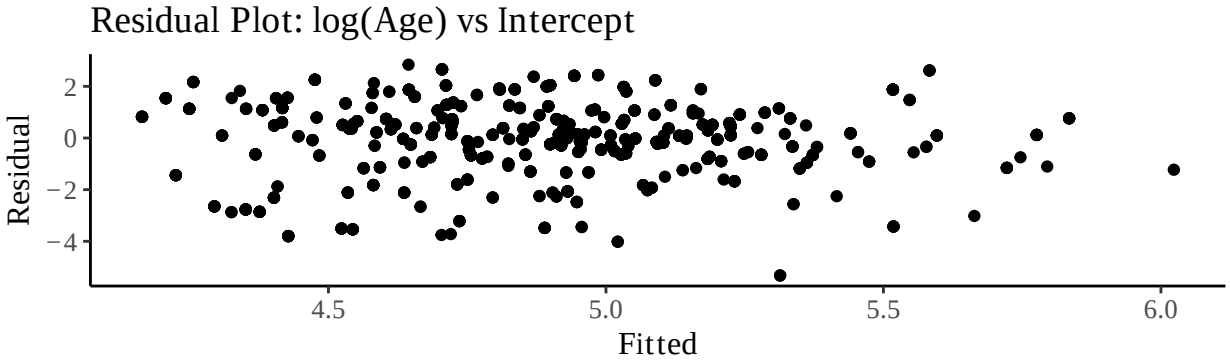


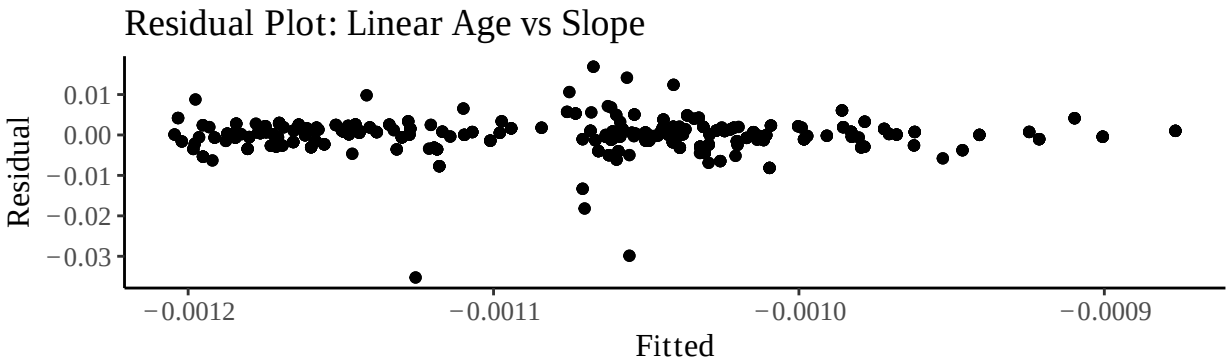
b.) Each child's data has a time course that can be summarized by a linear fit. Estimate these lines and plot them for 10 children.



c.) Set up a model for the children's slopes and intercepts as a function of the treatment and age at baseline. Estimate this model using the two-step procedure–first estimate the intercept and slope separately for each child, then fit the between-child models using the point estimates from the first step.







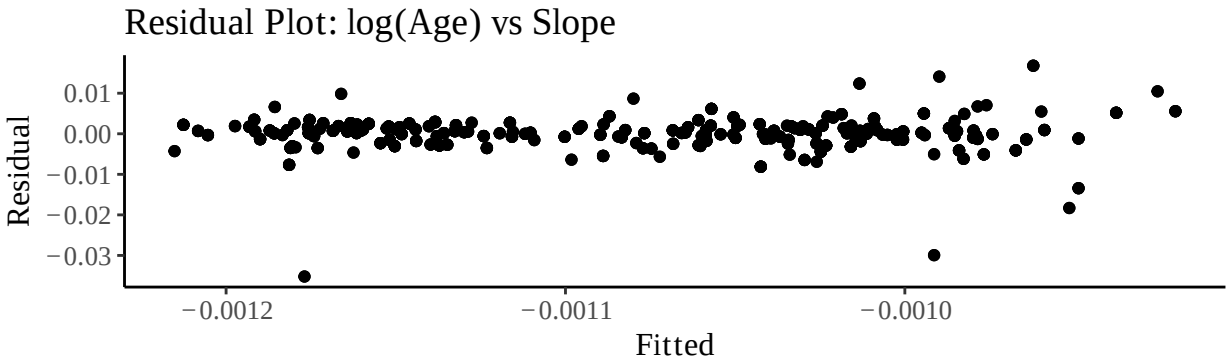


Table 1: Regression on Intercept and Slope

	Dependent variable:	
	beta0	beta1
	(1)	(2)
Treatment = 2	0.451***	−0.0001
	(0.092)	(0.0003)
log(baseage)	−0.349***	−0.0001
	(0.065)	(0.0002)
Constant	5.014***	−0.001***
	(0.089)	(0.0002)
Observations	960	936
R^2	0.052	0.0004
Adjusted R^2	0.050	−0.002
Residual Std. Error	1.428 (df = 957)	0.004 (df = 933)
F Statistic	26.155*** (df = 2; 957)	0.170 (df = 2; 933)
Note:		*p<0.1; **p<0.05; ***p<0.01
	Individuals with only one observation do not have a slope estimate.	

Q2 GH Chapter 12: Exercise 2

a.) Write a model predicting CD4 percentage as a function of time with varying intercepts across children. Fit using lmer() and interpret the coefficient for time.

Level 1 decribes the within child variability:

$$Level1 : \sqrt{CD4PCT_{ij}} \sim \mathcal{N}(\alpha_j + \beta time_{ij}, \sigma^2)$$

Level 2 decribes the between children variability:

$$Level2 : \alpha_j \sim \mathcal{N}(\mu, \tau^2)$$

Here is the regression result:

```
rand.int.a = lmer(sqrt(CD4PCT) ~ time + (1 | newpid), data = cd4)
texreg(rand.int.a, caption.above = TRUE,
       caption = "Random Intercept Model",
       custom.model.names = "sqrt(CD4PCT)",
       float.pos = 'H',
       digits = 3)
```

Table 2: Random Intercept Model

	sqrt(CD4PCT)
(Intercept)	4.796***
	(0.103)
time	−0.001***
	(0.000)
AIC	2825.035
BIC	2844.503
Log Likelihood	-1408.518
Num. obs.	960
Num. groups: newpid	221
Var: newpid (Intercept)	1.981
Var: Residual	0.592

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

The coefficient on time, β, is -.001. This means that for each additional day, we expect the square root of CD4 percentage to decrease by .001.

b.) Extend the model in (a) to include child-level predictors (that is, group-level predictors) for treatment and age at baseline. Fit using lmer() and interpret the coefficients on time, treatment, and age at baseline.

Again, level 1 describes the within child variability:

$$Level1 : CD4PCT_{ij} \sim \mathcal{N}(\alpha_j + \beta time_{ij}, \sigma^2)$$

Level 2 describes the between children variability (now includes group level predictors):

$$Level2 : \alpha_j \sim \mathcal{N}(\mu + \gamma_0 Treat + \gamma_1 log(baseage), \tau^2)$$

Here is the code we used to fit the model:

```
rand.int.b = lmer(sqrt(CD4PCT) ~ time + treatmnt + log(baseage) + (1|newpid), data = cd4)

texreg(rand.int.b, caption.above = TRUE,
       caption = "Random Intercept Model with Child Level Predictors",
       custom.model.names = "sqrt(CD4PCT)",
       float.pos = 'H',
       digits = 3)
```

Table 3: Random Intercept Model with Child Level Predictors

	sqrt(CD4PCT)
(Intercept)	4.888***
	(0.181)
time	−0.001***
	(0.000)
treatmnt2	0.321
	(0.196)
log(baseage)	−0.263*
	(0.128)
AIC	2825.901
BIC	2855.103
Log Likelihood	-1406.951
Num. obs.	960
Num. groups: newpid	221
Var: newpid (Intercept)	1.926
Var: Residual	0.593

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

- The coefficient on time (β) is -.001. This means that we expect the square root of CD4% to decrease by .001 for each additional day on average, all else equal.
- The coefficeint on treatmnt2 (γ_0) is .32. This means that we expect the square root of CD4% to be .32 higher for the children in treatment group 2 than the children in treatment group 1 on average, all else equal.
- The coefficient on log(baseage) (γ_1) is -.263. This means that for a 1% increase in baseage, we expect the square root of CD4% to decrease by $(.263 * log(1.01) = .0026)$.

c.) Investigate the change in partial pooling from (a) to (b) both graphically and numerically.

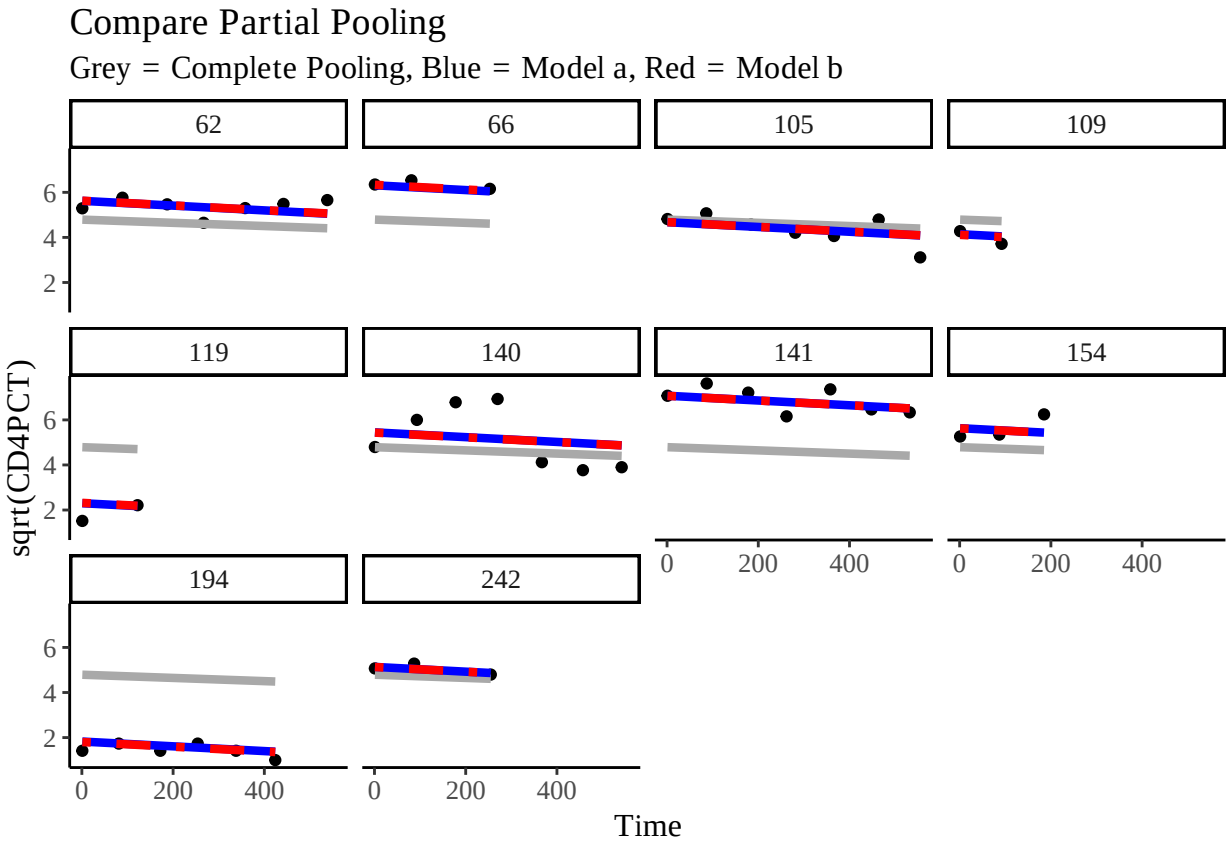
Since treatment and age at baseline are group level predictors, we expect that they may help explain some of the between group variation and leave the within group variation unchanged. First, we numerically compare the differences in the estimated within group variance (σ^2) and between group variance (τ^2) between model (a) and model (b):

Table 4: Sources of Variation

	sigma	tau
Model A	0.77	1.981
Model B	0.77	1.926

We see that the within child standard error (σ) is basically unchanged between model (a) and model (b). We see that there is a slight reduction in the between children standard errors (τ), from 1.981 to 1.926. Thus, the group level predictors, treatment and age at baseline, explain a small amount of the variation between children.

We also look at the difference in partial pooling graphically:



Graphically, we see that there is not much difference between the red and blue lines. Thus, there is not much difference in the partial pooling. This confirms what we observed from the small change in τ seen in Table 4.

Q3 GH Chapter 12: Exercise 5

Using the radon data, include county sample size as a group-level predictor and write the varying-intercept model. Fit this model using lmer().

Level 1 describes the within county variation:

$$Level1 : log(radon)_{ij} \sim \mathcal{N}(\alpha_j, \sigma^2)$$

Level 2 describes the between county variation:

$$Level2 : \alpha_j \sim \mathcal{N}(\mu + \gamma countysamples_j, \tau^2)$$

Here is the code and a table of the regression results:

Table 5: Random Intercept Model with County Level Predictor

	log(radon)
(Intercept)	1.329***
	(0.060)
county.samples	−0.005**
	(0.002)
AIC	2951.235
BIC	2971.555
Log Likelihood	-1471.618
Num. obs.	1188
Num. groups: county	96
Var: county (Intercept)	0.127
Var: Residual	0.639

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Bonus

a.)

$$var(\bar{Y}) = var(1/N * \sum_{j=1}^{J} \sum_{i=1}^{n} Y_{ij})$$

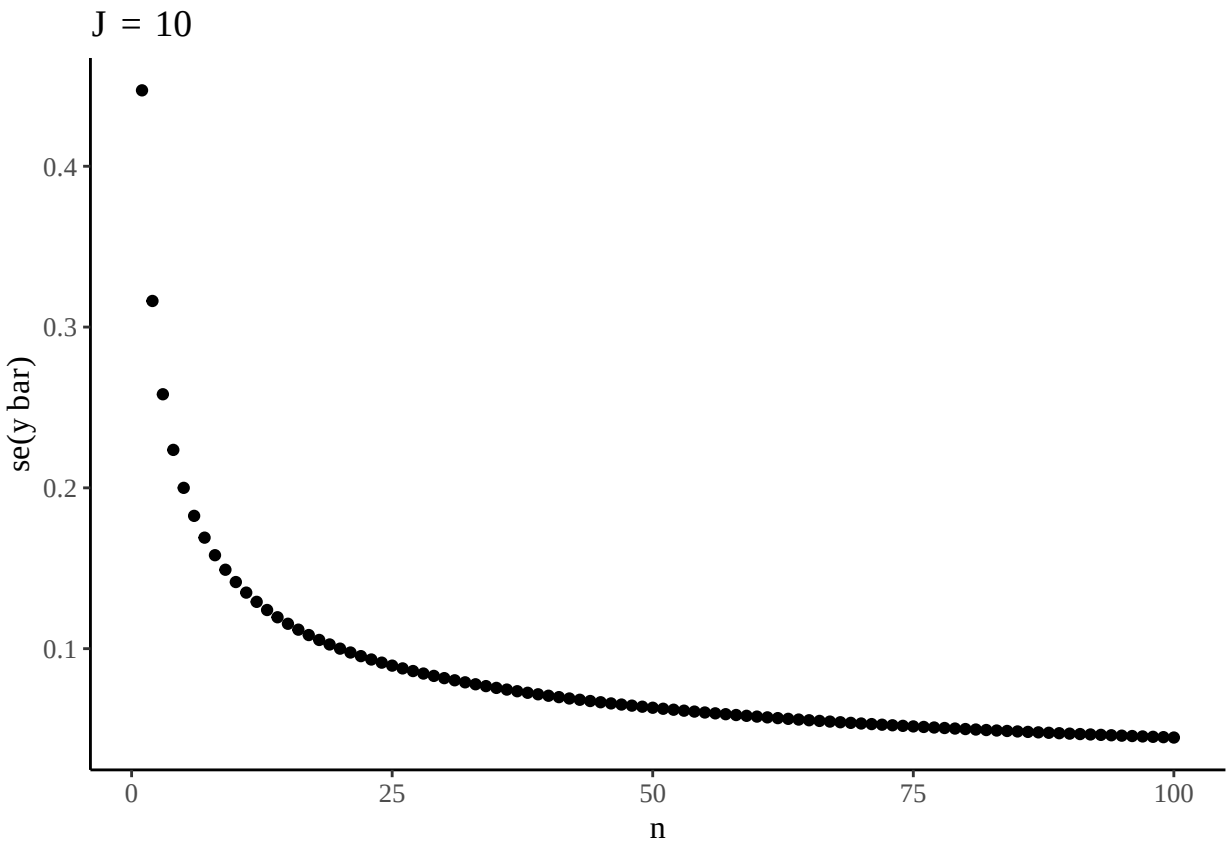
$$= 1/N^2 * \sum_{j=1}^{J} \sum_{i=1}^{n} var(Y_{ij})$$

$$= 1/N^2 * N(\sigma^2 + \tau^2)$$

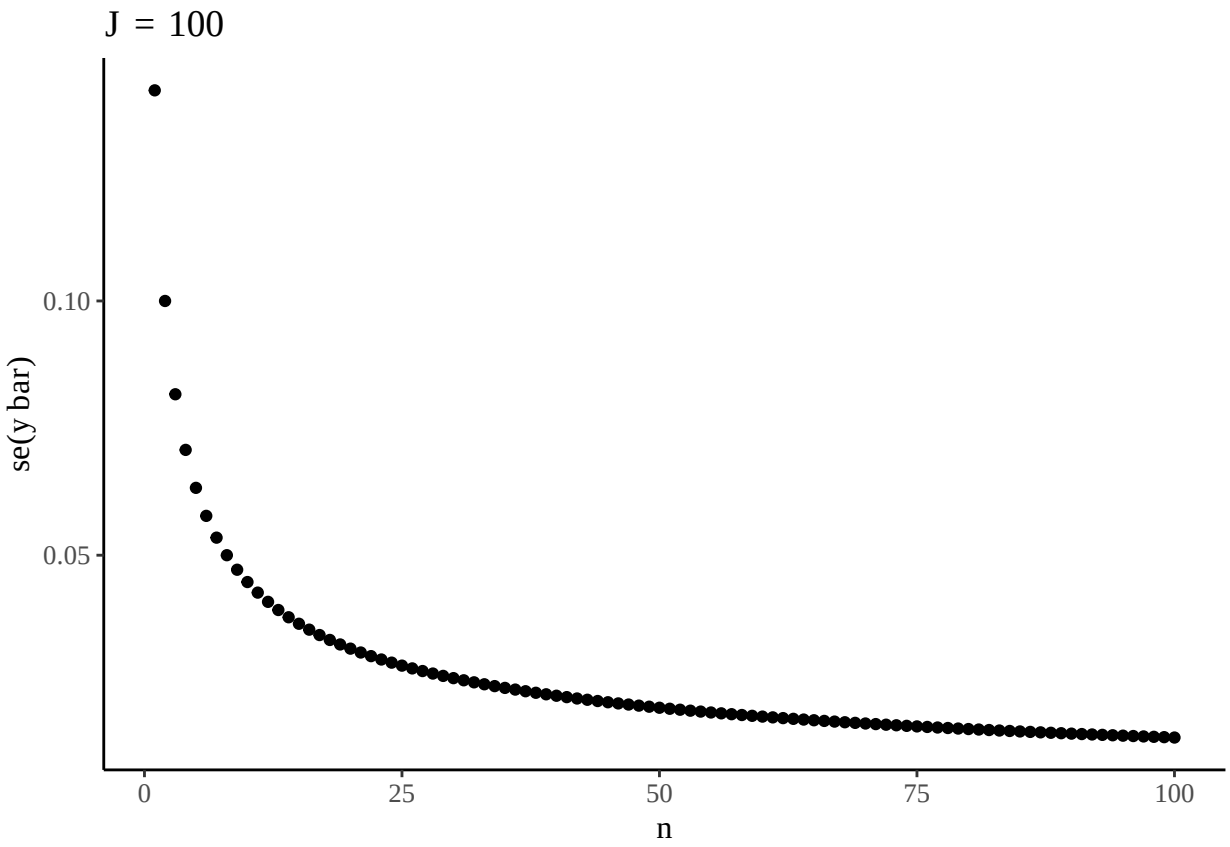
$$= \frac{(\sigma^2 + \tau^2)}{nJ}$$

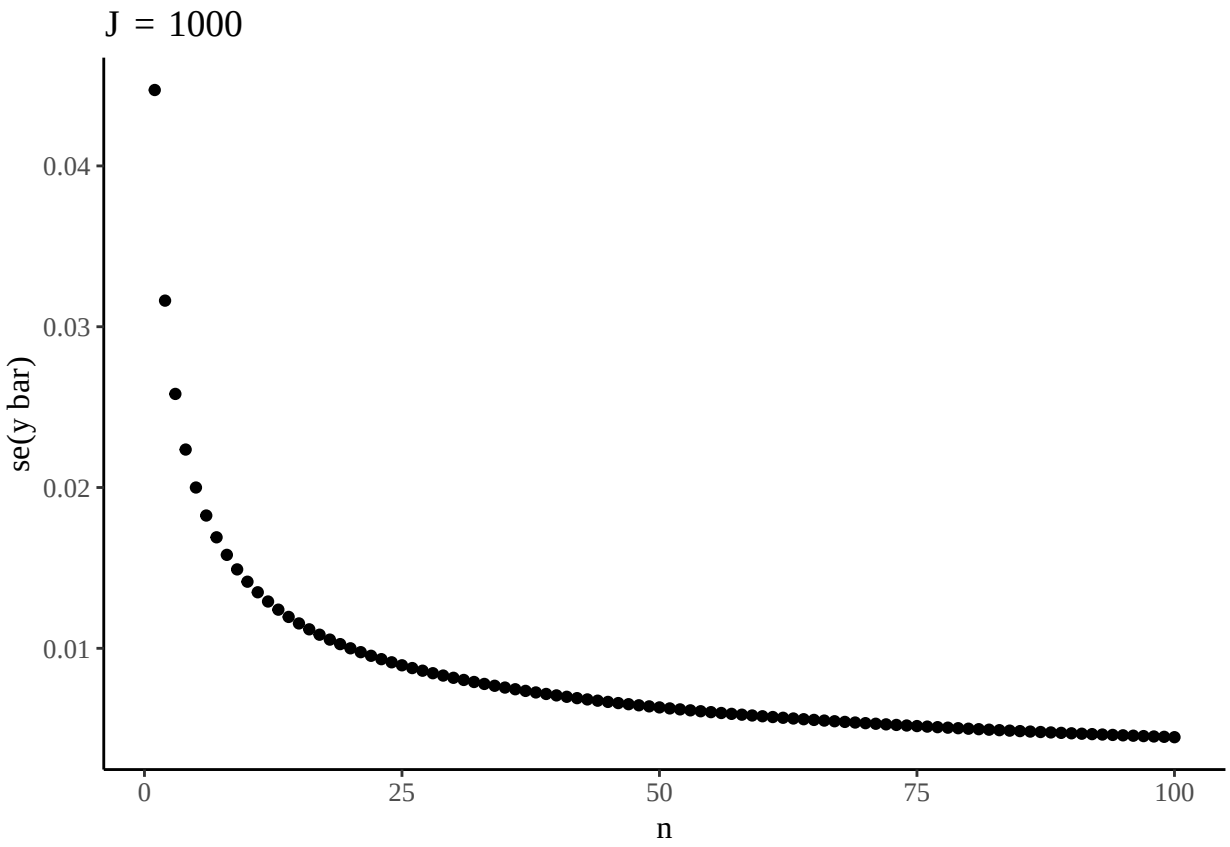
$$s.e.(\bar{Y}) = \sqrt{\frac{(\sigma^2 + \tau^2)}{nJ}}$$

b.)



c.)





d.)

We see that the the standard error of the sample mean decreases at a decreasing rate as the number of observations within each cluster (n) increases. The same is true of the number of clusters (J).