Adaptively Exploiting d-Separators with Causal Bandits

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We have no guarantees that observing Z_t will help us...but we would like to exploit it when we can.

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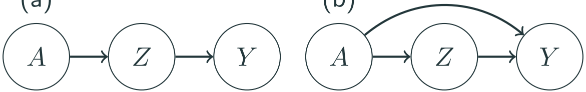

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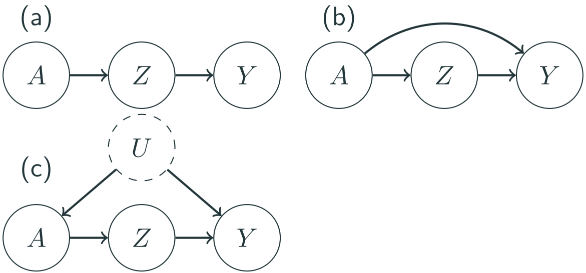

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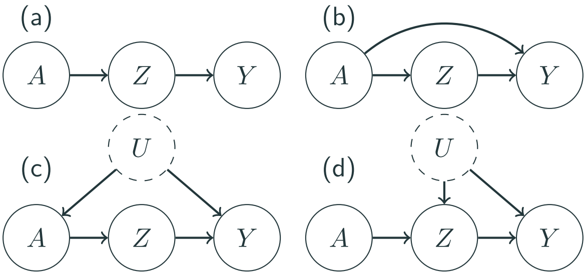

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Proposition

If Z satisfies the front-door criterion with respect to (A, Y) on $\mathcal{G}$ then Z d-separates Y from A on $\mathcal{G}_{\overline{A}}$.

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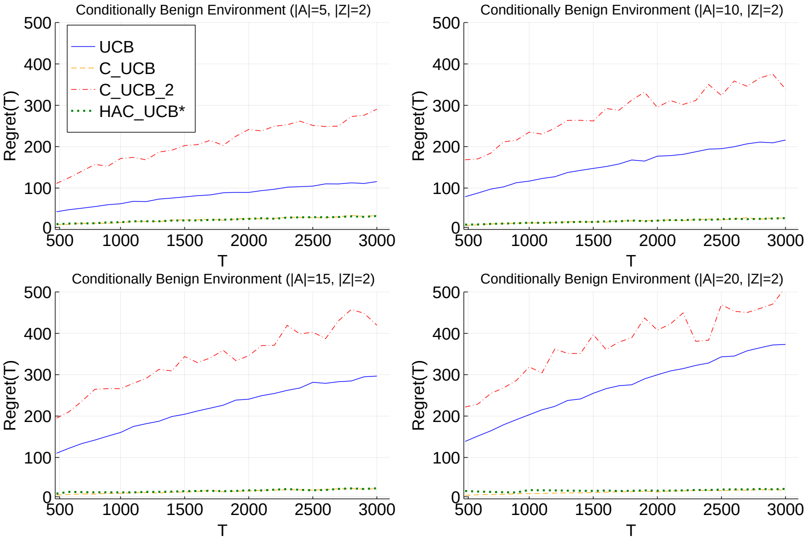

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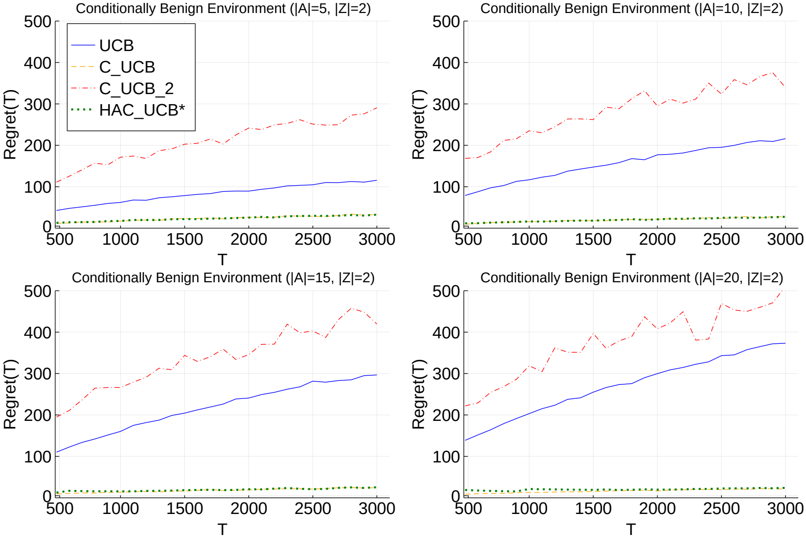

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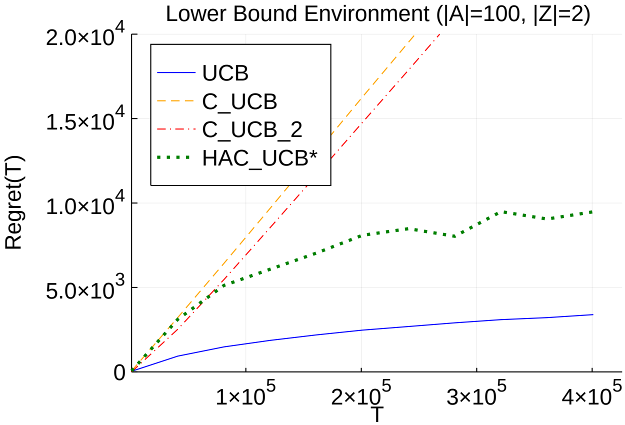

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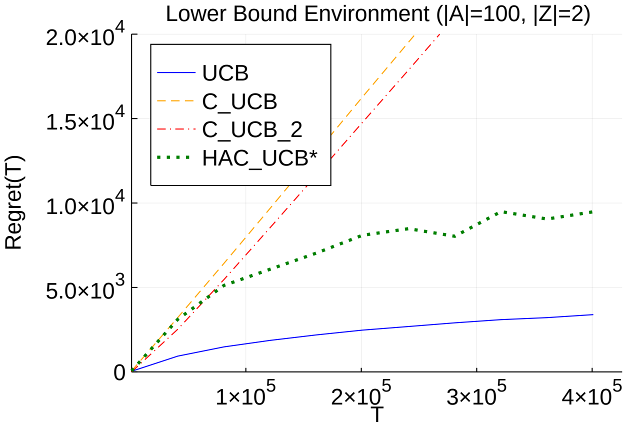

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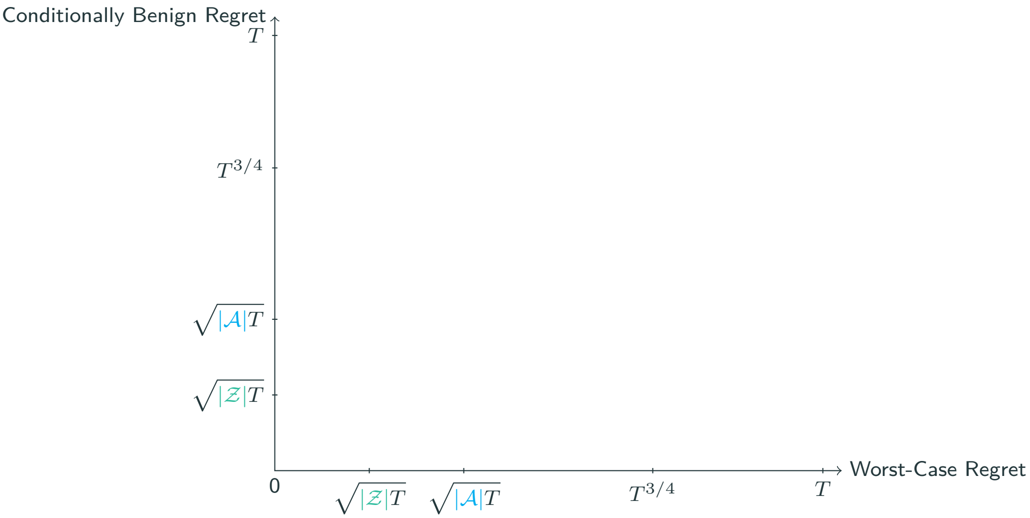

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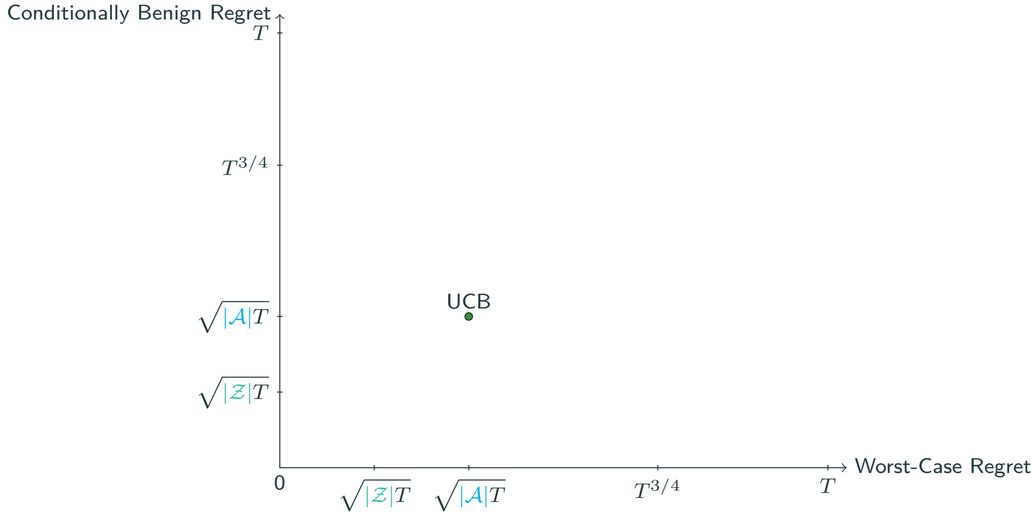

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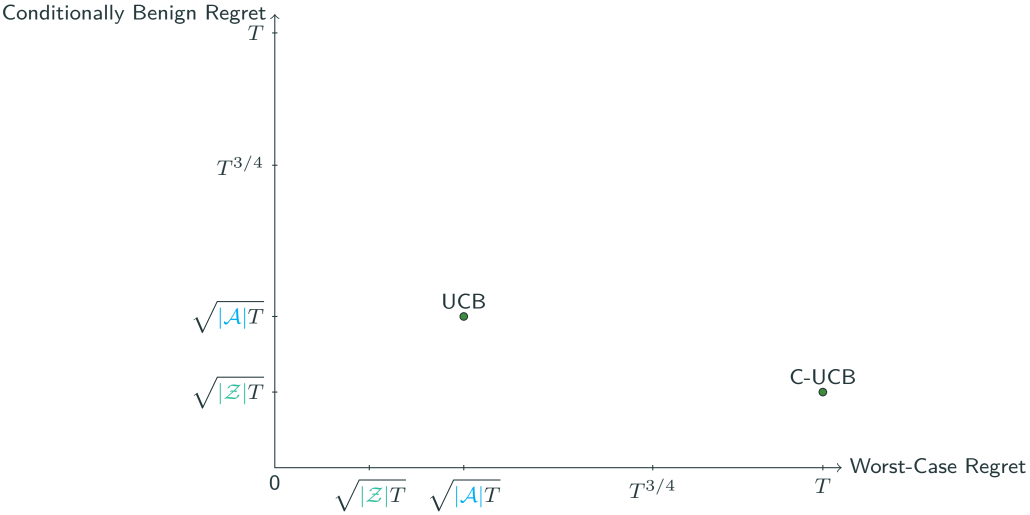

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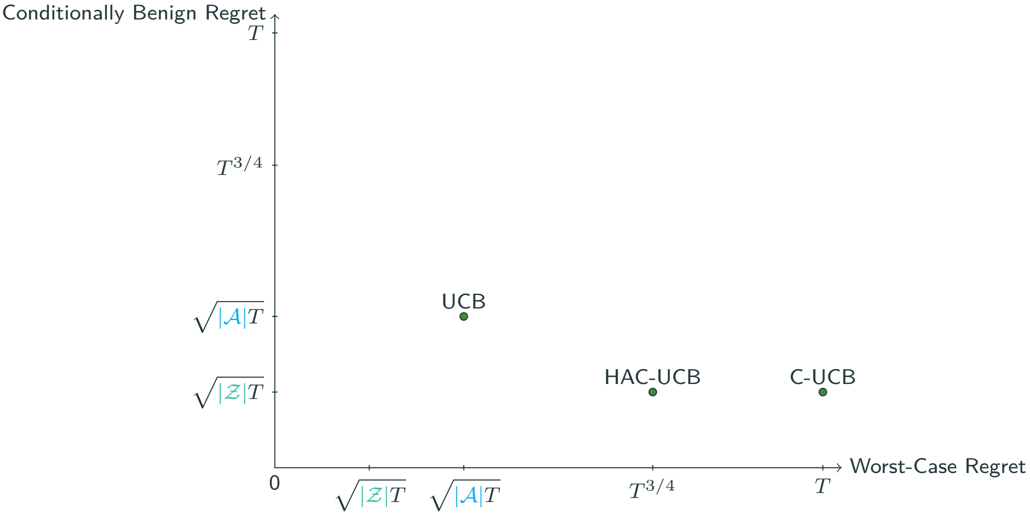

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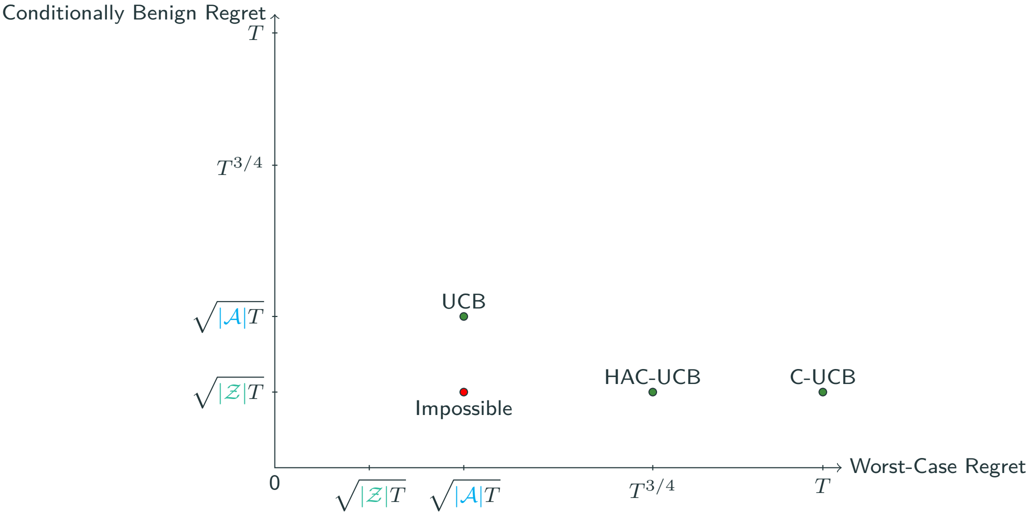

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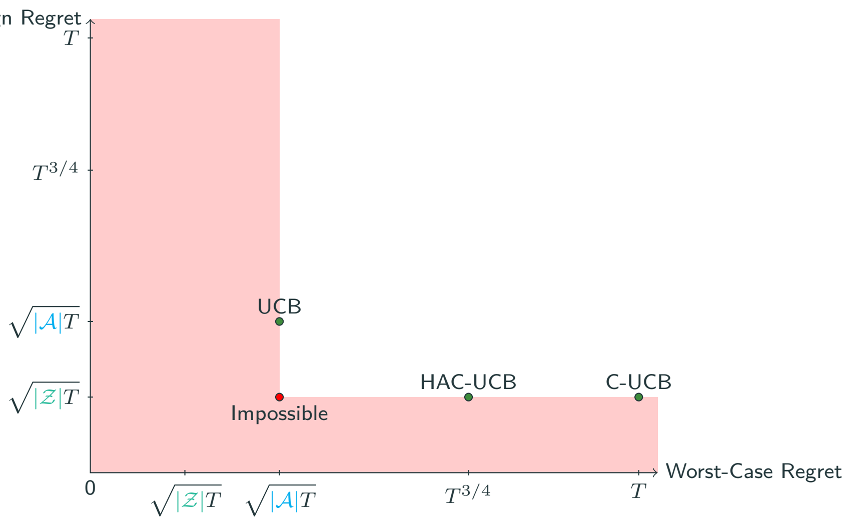

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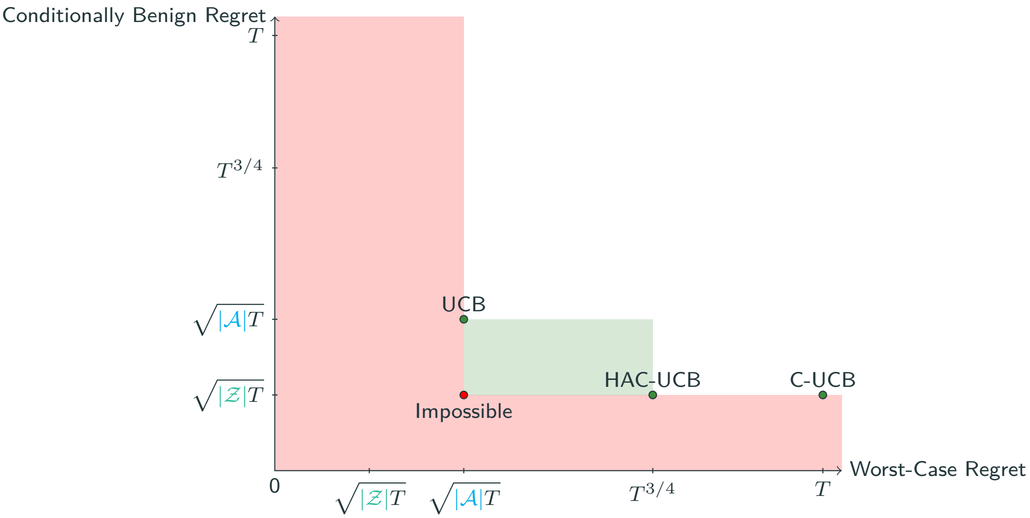

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