Survey Weighting

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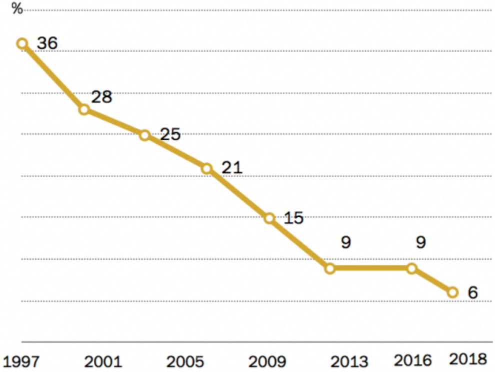
- Unfortunately, in the real world we don't get a true random sample
- Specifically, if some variable influences both $P(D_i = 1)$ and Y_i, the simple mean will be a biased estimator

Declining response rates have intensified this problem

After brief plateau, telephone survey response rates have fallen again

Response rate by year (%)



Note: Response rate is AAPOR RR3. Only landlines sampled 1997-2006. Rates are typical for surveys conducted in each year.
Source: Pew Research Center telephone surveys conducted 1997-2018.

PEW RESEARCH CENTER

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- Other times, you're just stuck with what there is (i.e. MTurk)
- The same general idea applies – want to draw conclusions about a population that's different from our sample

Weighting to the rescue

In an ideal world, we'd know the true probability that a unit was sampled from the population: $P_i = P(D_i = 1)$. Then:

$$\mathbb{E}\left(\sum_{i=1}^{N} \frac{P_i^{-1} Y_i}{\sum_{i=1}^{N} P_i^{-1}}\right) = \mathbb{E}\left(\sum_{i=1}^{N} \frac{Y_i}{N}\right)$$

Of course, in the real world, we don't actually know the weights, so instead we guess

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- Specifically, will use covariates with a known frequency in the target population to produce estimates that will be representative of the target population

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$$w_i = \frac{P(X_i \text{ in Population})}{P(X_i \text{ in Sample Frame})} P(\text{i drawn from sample frame})$$

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$$\frac{\sum_{i=1}^{N} w_i Y_i}{\sum_{i=1}^{N} w_i}$$

Multi-level Regression and Post-stratificiation (MrP)

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- There are two advantages of this:
  - Allows us to make predictions about $\widehat{\mathbb{E}\left(Y_i|X_i = x\right)}$ for empty cells
  - Bayesian methods regularize estimates for $\widehat{\mathbb{E}\left(Y_i|X_i = x\right)}$ for small cells

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- *Raking* is a procedure to produce weights will match the population marginal means, while maintaining the sample joint distribution
- There are many variations of the exact raking procedure, but they generally involve iteratively balancing the the marginal distribution of each variable

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- This level is usually chosen in an ad hoc way (e.g. mak esure no weights are greater than a 10 or a 100)
- More principled techniques exist, but they're not frequently used

Go Over Code

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- Researchers should focus weighting on variables that are likely to influence both the outcome and the sampling probability
- This is a very active area of methodological research – the number covariates available for weighting has increased while samples have become increasingly less representative
- Nonetheless, these basic principles will be sufficient for designing weights for most applied projects.