Exam 1 – Take Home
EE 324/Phys 324
SPRING 2016

- **Due date:** Wednesday, March 2.
- Open book/notes.
- This exam is intended to be an independent effort your part; you are therefore not permitted to discuss it with **anyone**! If you need clarification on a problem you may email me, or see me in person.
- Upon completion of the exam please sign the Honor code below.

EE Honor Code
"No aid received, given or observed"

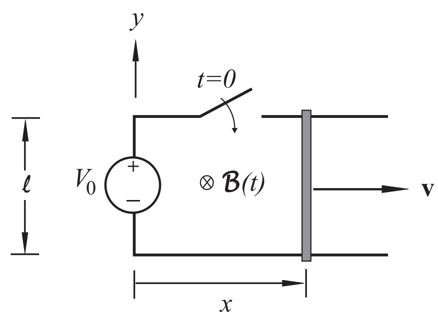
Signature: ____________________________________

1. A bar with total resistance R, makes electrical contact with a system perfectly conducting rails, as shown in the figure. At $t = 0$ the switch is closed so that the DC voltage source (with magnitude V_0) results in current flow through the closed loop. This, in turn, gives rise to a net force on the bar, given by,

$$\mathbf{F} = \mathbf{I}\ell \times \mathbf{B},$$

which causes the bar to accelerate to the right. For a constant magnetic field $\boldsymbol{\mathcal{B}} = -\hat{z}B_0$ throughout, and assuming frictionless rails, determine the velocity of the bar as $t \to \infty$.

Your solution should be in terms of the variables V_0, B_0 and ℓ.



2. The electric field intensity of a linearly polarized uniform plane wave propagating in the $+z$–direction in seawater ($\epsilon_r = 80$, $\mu_r = 1$, $\sigma = 4$ S/m) at $z = 0$ is,

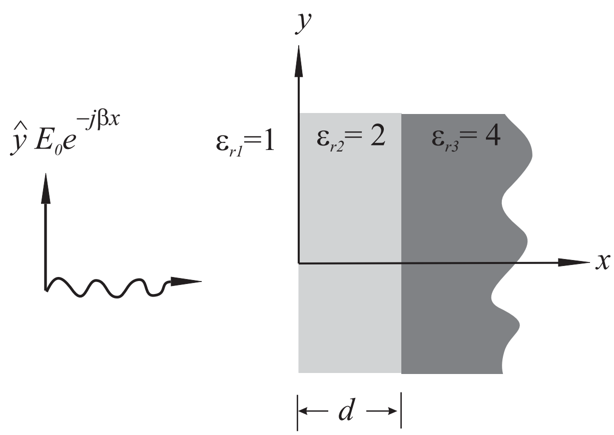
$$\boldsymbol{\mathcal{E}} = \hat{x}10\sin\left(2\pi \times 10^6 t - \pi/8\right).$$

(a) Determine the attenuation constant α, the phase constant β, the intrinsic impedance η, the phase velocity, the wavelength.

(b) Find the distance the wave must propagate for the magnitude of $\mathbf{E}$ to be reduced to 0.01 (V/m).

(c) Write expressions for $\boldsymbol{\mathcal{E}}(z,t)$ and $\boldsymbol{\mathcal{H}}(z,t)$.

3. A uniform plane-wave in free-space (i.e. of the form $\mathbf{E}^i = \hat{y}E_0 e^{-jk_0 z}$) is incident on a lossless layered dielectric slab. The first layer has thickness d and permittivity ϵ_2, with the second layer infinite in extent and having a permittivity ϵ_3, as shown in the sketch below. Assume $\mu = \mu_0$ throughout.



(a) Write an expression for the reflection coefficient at $x = 0$.

(b) If $d = .25\lambda$, what is the net reflection coefficient (magnitude and angle) at $x = 0$.

(c) Use the Smith chart to verify your result from part (b).

4. Derive the general expressions for the attenuation and phase constants $k = \beta - j\alpha$ for conducting medium,

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2},$$

and

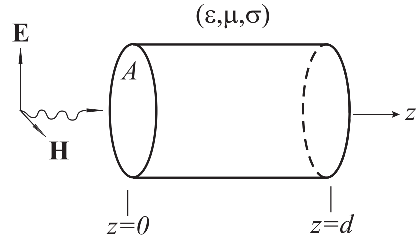
$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2}.$$

Obtain expressions for α and β in the limit as $\omega \to \infty$ and $\omega \to 0$.

5. A plane wave in a lossy medium (i.e. with material properties ϵ, μ, σ) propagates in the $+z$-direction such at its electric field is given by,

$$\mathbf{E}(z) = \hat{x}E_0 e^{-\alpha z}e^{-j\beta z},$$

with the propagation constant $k = \beta - j\alpha$. Determine the total real power absorbed within a cylinder of cross section A, and length d along the z-axis, as shown in the sketch below.



Do this by,

(a) evaluating the Poynting vector,

$$\mathbf{P} = \frac{1}{2}\Re\mathrm{e}[\mathbf{E} \times \mathbf{H}^*].$$

and determining the difference in real power flow across the two interfaces.

(b) Show that this is exactly the power absorbed resulting from the conductivity, σ, given by

$$\frac{1}{2}\iiint_V \sigma |\mathbf{E}|^2 dv,$$

where V is the volume of the cylinder and $|\mathbf{E}|^2 \equiv \mathbf{E} \cdot \mathbf{E}^*$.

Hint: You will need to make use of results from the previous problem.