E & M Assignment Nine

Blake Evans

Chapter 4 P.13)

Show that the field outside the cylinder is $\Big\}$ $E(r) = \dfrac{a^2}{2\varepsilon_0 s^2}\left[2(\vec{p}\cdot\hat{s})\hat{s} - p\right]$

Let's use Gauss's Law

$$\int \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} \longrightarrow E\underbrace{(2\pi s \cancel{\ell})}_{\text{cylinder Area}} = \frac{\rho\overbrace{(\pi s^{\cancel{2}} \cancel{\ell})}^{\text{cylinder volume}}}{\varepsilon_0} \longrightarrow E = \frac{\rho s}{2\varepsilon_0}$$

Since the cylinder is uniformly charged, we can let it be two equal cylinders of opposite charge where $\vec{E} = \vec{E}_+ + \vec{E}_-$ (let $s_+ = r$, $s_- = r'$)

$\vec{\mathscr{r}} = r - r'$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \left(\frac{\rho\vec{s}_+}{2\varepsilon_0}\right) + \left(-\frac{\rho\vec{s}_-}{2\varepsilon_0}\right) = \frac{\rho}{2\varepsilon_0}(\vec{r} - \vec{r}') = \frac{\rho}{2\varepsilon_0}(-\vec{\mathscr{r}})$$

S_+ and S_- are just halve the length of the cylinder. So let's set that equal to $-\vec{\mathscr{r}}$ pointing from the negative axis to positive and say.

$$\vec{S}_+ - \vec{S}_- = -\mathscr{r} \longrightarrow \vec{E} = \frac{\rho(-\vec{\mathscr{r}})}{2\varepsilon_0} \longrightarrow \vec{E} = -\frac{\rho\vec{\mathscr{r}}}{2\varepsilon_0} \longrightarrow \vec{E} = -\frac{(\vec{p})}{2\varepsilon_0}$$

where $\vec{p} = \rho\cdot\vec{\mathscr{r}}$. so $\boxed{E_{inside} = -\dfrac{p}{2\varepsilon_0}}$

Chapter 4 P.13 Cont.)

Now, for the outside let's do the same thing with Gauss's law.

$$\int \vec{E}\cdot d\vec{a} = \frac{(\rho\cdot\pi a^2 l)}{\varepsilon_0} \longrightarrow \vec{E}\cdot(2\pi s \cancel{l}) = \frac{\rho}{\varepsilon_0}\pi a^2 \cancel{l} \longrightarrow E = \frac{\rho a^2}{2\varepsilon_0}\frac{\hat{s}}{s}$$

we can split it again like before

$$E = E_+ + E_- \longrightarrow \left(\frac{\rho a^2}{2\varepsilon_0}\frac{\hat{s}_+}{s_+}\right) + \left(-\frac{\rho a^2}{2\varepsilon_0}\frac{\hat{s}_-}{s_-}\right) \longrightarrow E = \frac{\rho a^2}{2\varepsilon_0}\left(\frac{\hat{s}_+}{s_+} - \frac{\hat{s}_-}{s_-}\right)$$

Now, I don't know why, but if you make the arbitrary substitution $s_\pm = s \mp \vec{d}/2$ things work out. So, let's make like a mathematician and make assumptions.

Since $\left(\frac{\hat{s}_+}{s_+} - \frac{\hat{s}_-}{s_-}\right) = \frac{\hat{s}_\pm}{s_\pm^2}$ we can say

$$s_\pm = s \mp \frac{\vec{d}}{2} \Rightarrow \text{divide it by } s_\pm^2 \longrightarrow \frac{\left(\vec{s} \pm \frac{\vec{d}}{2}\right)}{\left(s^2 + \frac{\vec{d}^2}{4} \mp (\vec{s}\cdot\vec{d})\right)}$$

$$\frac{s_\pm}{s_\pm^2} = \frac{1}{s^2}\left(\frac{\vec{s} \mp \frac{\vec{d}}{2}}{1 \pm \frac{\vec{s}\cdot\vec{d}}{s^2}}\right) \longrightarrow \left(\frac{\hat{s}_+}{s_+} - \frac{\hat{s}_-}{s_-}\right) = \frac{1}{s^2}\left(\vec{s} \mp \frac{d}{s}\right)\left(1 \pm \frac{\vec{s}\cdot\vec{d}}{s^2}\right)$$

Chapter 4 P.13 Cont.)

Let's split our expression up.

$$\frac{\hat{S}_+}{S_+} - \frac{\hat{S}_-}{S_-} = \frac{1}{S^2}\left[\left(\vec{S} + \vec{S}\frac{(\vec{S}\cdot\vec{r})}{S^2} - \frac{r}{2}\right) - \left(\vec{S} - \vec{S}\frac{(\vec{S}\cdot\vec{r})}{S^2} + \frac{\vec{S}}{2}\right)\right]$$

$$\frac{\hat{S}_+}{S_+} - \frac{\hat{S}_-}{S_-} = \frac{1}{S^2}\left[2\left(\frac{\vec{S}(\vec{S}\cdot\vec{r})}{S^2}\right) - \vec{r}\right]$$

Let's Plug this back in

$$E = \frac{\rho a^2}{2\varepsilon_0}\left[\frac{1}{S^2}\left(2\frac{\vec{S}\cdot(\vec{S}\cdot\vec{r})}{S^2}\right) - \vec{r}\right]$$

$$E = \frac{a^2}{2\varepsilon_0}\frac{1}{S^2}\left(\frac{2\rho(\vec{S}\cdot\vec{r})\vec{S}}{S^2} - \rho\vec{r}\right) \longleftarrow$$ Since $\rho\vec{r} = P$ we can say

$$E = \frac{a^2}{2\varepsilon_0}\frac{1}{S^2}\left(\frac{2(\vec{S}\cdot\vec{P})\vec{S}}{S^2} - \vec{P}\right)$$

and finally, $$\boxed{E = \frac{a^2}{2\varepsilon_0}\frac{1}{S^2}\left(2(\vec{S}\cdot\vec{P})\vec{S} - \vec{P}\right)}$$

Chapter 4 P.15)

a) $P(r) = \frac{k}{r}\hat{r}$; $\oint D \cdot da = Q_{f\,enc}$ ← Eq. 2.13

Locate all the bound charge using Gauss's Law.

So, to use Gauss's Law we first need to find Qenc for the sphere. We know the equation for volume charge density to be $\rho_b = -\nabla \cdot \vec{P}$ where $\vec{P}$ is potential. We know the potential of a dielectric sphere to be $P(r) = k/r\,\hat{r}$ (Given), so

$$\rho_b = \nabla \cdot \vec{P}(r) = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\left(\frac{k}{r}\right)\right)\right] \rightarrow \rho_b = \frac{\partial}{\partial r}\left(\frac{k}{r}\right) \rightarrow \rho_b = -\frac{k}{r^2}$$

So, from the figure, we know a few things. First, that in the area $r < a$ there's no charge that is enclosed, meaning $Q_{enc\; r<a} = 0$

for $r<a$

Therefore, $\int \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} \rightarrow E(2\pi r^2) = \frac{(0)}{\varepsilon_0} \rightarrow \boxed{E = 0}$

Chapter 4 P.15 Cont.)

Second, the area $a<r<b$ will just be the surface density of the sphere $(r=a)$ plus the charge between a and b, since charge before a is Ø. So, we can say that surface charge density is

$$\sigma_b = \vec{P}\cdot\hat{n} \rightarrow \sigma_b = \left(\frac{k}{r}\hat{r}\right)\cdot(\hat{r}) \longrightarrow \sigma_b = \frac{k}{r}(\hat{r}\cdot\hat{r}) \longrightarrow \sigma_b = \frac{k}{r}$$

Following this, we can say

$$q_{enc} = q_{bound} + q_{surface} \longrightarrow q_{enc} = \left[\int_a^r \rho_b(V)\,dr\right] + \left[\sigma_b(A)\right]$$

$$q_{surface} = \left(\frac{k}{r}\right)(4\pi r^2) \longrightarrow q_{surface} = 4\pi k r \leftarrow r=a$$

$$q_{bound} = \int_a^r \left(-\frac{k}{r^2}\right)(4\pi r^2)\,dr \rightarrow q_{bound} = -4\pi k(r-a)$$

Thus, $q_{enc} = (-4\pi k a) + \left[4\pi k(r-a)\right] \rightarrow q_{enc} = -4\pi k r$

Therefore, $\int \vec{E}\cdot d\vec{a} = \frac{(-4\pi k r)}{\varepsilon_0} \rightarrow E(4\pi r^2) = -\frac{4\pi r}{\varepsilon_0}k$

$$\boxed{E = -\frac{k}{r\varepsilon_0}\cdot\hat{r}}$$ for region $a<r<b$

Chapter 4 P.15 cont.)

Third, for $r>b$ the q_{enc} will be the surface charge density at $r=b$ plus what we had for $a<r<b$. So, we can say

$$\sigma_b = \vec{P}\cdot\hat{n} \rightarrow \sigma_b = \left(\frac{k}{r}\hat{r}\right)\cdot(\hat{r}) \longrightarrow \sigma_b = \frac{k}{r}(\hat{r}\cdot\hat{r}) \longrightarrow \sigma_b = \frac{k}{(b)}$$

$$q_{out} = \sigma_b(A) \longrightarrow q_{out} = \left(\frac{k}{b}\right)(4\pi r^2) \rightarrow q_{out} = 4\pi k r$$

So, when $r>a$ we can say

$$q_{enc} = q_{bound} + q_{surface} + q_{out}$$

$$q_{enc} = (-4\pi k r) + q_{out} \longrightarrow q_{enc} = -4\pi k r + 4\pi k r$$

Thus, we can see that $q_{enc} = \emptyset$

for $r>b$

$$\int E\cdot da = \frac{q_{enc}}{\varepsilon_0} \longrightarrow E(4\pi r^2) = \frac{0}{\varepsilon_0} \longrightarrow \boxed{E=0}$$

Chapter 4 P.15 cont.)

b) Equation 4.23 states $D = \varepsilon_0 E + P$

Since D only accounts for free charge, and our system contains no free charge, we can let $D = 0$ and solve for E.

$$D = \varepsilon_0 E + P \rightarrow (0) = \varepsilon_0 E + P \rightarrow E = -\frac{P}{\varepsilon_0}$$

We are given that $P = \frac{k}{r}\hat{r}$

$$E = -\frac{\left(\frac{k}{r}\hat{r}\right)}{\varepsilon} \rightarrow \boxed{E = -\frac{k}{\varepsilon_0 r}\hat{r}}$$ for area $a < r < b$

chapter 4 P.16)

a) Find the field at the center in terms E_0, P.
Find the displacement at the center in terms D_0, P.

From example 4.3 in the book, we know that $E_{center} = -\frac{1}{3\varepsilon_0}P$. From the given, we know that $E_{shell} = E_0$. So, let's subtract E_{center} from E_{shell} to get E_{tot}.

$$E_{tot} = E_{shell} - E_{center} \rightarrow E_{tot} = (E_0) - \left(-\frac{P}{3\varepsilon_0}\right)$$

$$E_{tot} = E_0 + \frac{P}{3\varepsilon_0}$$

Now, from equation 4.24 we know that

$$D = \varepsilon_0 E \longrightarrow D = \varepsilon_0 (E_{tot}) \longrightarrow D = \varepsilon_0 \left(E_0 + \frac{P}{3\varepsilon_0}\right)$$

$$D = \varepsilon_0 E_0 + \frac{1}{3}P \longleftarrow \text{Plug in } \varepsilon_0 E_0 = D_0 - P$$

$$D = (D_0 - P) + \frac{1}{3}P \rightarrow \boxed{D = D_0 - \frac{2}{3}P}$$

Chapter 4 P.16 Cont.)

b) Since a needle is thin and long, we can just let $E_{tot} = E_0$. So, using what we had before, we can say

$$D = \varepsilon_0 E \longrightarrow D = \varepsilon_0 (E_{tot}) \longrightarrow D = \varepsilon_0 (E_0)$$

We can Plug in $\varepsilon_0 E_0 = D_0 - P$

$$D = (D_0 - P) \rightarrow \boxed{D = D_0 - P}$$

c) For the waifer case, our $E_{center} = -\frac{P}{\varepsilon_0}$. Using this, our E is $E = E_0 - \left(-\frac{P}{\varepsilon_0}\right) \longrightarrow E = E_0 + P/\varepsilon_0$. Thus, plugging into our Maxwell equation we get.

$$D = \varepsilon_0 \left(E_0 + P/\varepsilon_0\right) \rightarrow D = \varepsilon_0 E_0 + P$$

We can Plug in $\varepsilon_0 E_0 = D_0 - P$

$$D = (D_0 - P) + P \rightarrow \boxed{D = D_0}$$

chapter 4 P.18) ;)

a) Okay, we are looking for the D. To do this we can use the Gauss's Law D form.

$$\int \vec{D} \cdot d\vec{a} = Q_{fenc} \rightarrow D\cancel{(A)} = \sigma\cancel{(A)} \rightarrow \boxed{D = \sigma}$$

b) Now we need to find E. Since this is a parallel plate capacitor we know the field will be

$$E = \frac{\sigma}{2\varepsilon} \longrightarrow \text{Since there are two} \rightarrow E_1 = \frac{\sigma}{2\varepsilon_1} \; ; \; E_2 = \frac{\sigma}{2\varepsilon_2}$$

Since ε_1 and ε_2 is permittivity for slabs 1 and 2, we can set $\varepsilon_1 = k_1\varepsilon_0$ and $\varepsilon_2 = k_2\varepsilon_0$. So,

$$E_1 = \frac{\sigma}{2(k_1\varepsilon_0)} \; ; \; E_2 = \frac{\sigma}{2(k_2\varepsilon_0)}$$

Since k in this context is the dielectric constant we can plug in the values given (slab 1 → $k_1 = 2$, slab 2 → $k_2 = 1.5$)

Chapter 4 P.18 Cont.)

$$\boxed{E_1 = \frac{\sigma}{2\varepsilon_0} \; ; \; E_2 = \frac{2\sigma}{3\varepsilon_0}}$$

c) Now we need to find Polarization P for both slabs. From the book we have the expression $P = \varepsilon_0 \chi_e E$. Knowing that $E = \frac{\sigma}{\varepsilon}$ and $\varepsilon = \varepsilon_0 \varepsilon_r$ we can say

$$P = \varepsilon_0 \chi_e E \rightarrow P = \varepsilon_0 \chi \left(\frac{\sigma}{\varepsilon}\right) \rightarrow P = \frac{\cancel{\varepsilon_0} \chi \sigma}{(\cancel{\varepsilon_0} \varepsilon_r)} \rightarrow P = \frac{\chi_e}{\varepsilon_r} \sigma$$

using $\chi_e = \varepsilon_r - 1$ we can write

$$P = \frac{(\varepsilon_r - 1)}{\varepsilon_r} \sigma \rightarrow P = \left(\frac{\varepsilon_r}{\varepsilon_r} - \frac{1}{\varepsilon_r}\right) \sigma \rightarrow P = \sigma\left(1 - \frac{1}{\varepsilon_r}\right)$$

For ε_1 we get $P = \sigma\left(1 - \frac{1}{(2)}\right) \longrightarrow \boxed{P = \frac{\sigma}{2}}$

For ε_2 we get $P = \sigma\left(1 - \frac{1}{(\frac{3}{2})}\right) \rightarrow \boxed{P = \frac{\sigma}{3}}$

chapter 4 P.18 cont.)

d) This time 'round, we need potential between the plates. The expression for potential of a capacitor is $V = E \cdot d$ (d is separation distance). Then, the potential between Slab 1 and Slab 2 can be shown as

$$V = E_1 a + E_2 a$$ where a is $\frac{1}{2}$ separation.

We can sub in our values from before for E and get

$$V = \left(\frac{\sigma}{2\varepsilon_0}\right)a + \left(\frac{2\sigma}{3\varepsilon_0}\right)a \longrightarrow V = \frac{\sigma}{\varepsilon_0}\left(\frac{1}{2} + \frac{2}{3}\right)a$$

meaning our total potential will be $\boxed{V = \frac{7\sigma a}{6\varepsilon_0}}$

e) Find the location and amount of bound charge. Since it's a capacitor ρ_b will be Ø and all charge will be surface charge σ_b.

Chapter 4 P.18 cont.)

$\sigma_b = \vec{P} \cdot \hat{n}$ where $\hat{n}$ points up and $\vec{P}$ points down.

Slab 1: $\sigma_b = \pm\left(\frac{\sigma}{2}\hat{n}\right)\cdot\hat{n} \rightarrow \sigma_b = \pm\frac{\sigma}{2}(1)$

Slab 2: $\sigma_b = \pm\left(\frac{\sigma}{3}\hat{n}\right)\cdot\hat{n} \longrightarrow \sigma_b = \pm\frac{\sigma}{3}(1)$

Thus, $\sum \sigma_b = \cancel{\frac{\sigma}{2}} + \left(\cancel{-\frac{\sigma}{2}}\right) + \cancel{\frac{\sigma}{3}} + \left(\cancel{-\frac{\sigma}{3}}\right) \rightarrow \sum \sigma_b = 0$

$\boxed{\text{Therefore, there is no bound charge and it's located nowhere.}}$

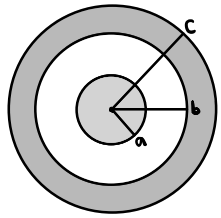
f) Confirm (b) with bound/free charge.

Slab 1: total above charge — $\sigma - \left(\frac{\sigma}{2}\right) = \frac{\sigma}{2}$
total below charge $-\left(\frac{\sigma}{2}\right) - \cancel{\left(\frac{\sigma}{3}\right)} + \cancel{\left(\frac{\sigma}{3}\right)} - \sigma = -\frac{\sigma}{2}$

Slab 2: total above charge — $\sigma - \left(\frac{\sigma}{2}\right) + \left(\frac{\sigma}{2}\right) - \left(\frac{\sigma}{3}\right) = \frac{2\sigma}{3}$
total below charge — $\left(\frac{\sigma}{3}\right) - \sigma = -\frac{2\sigma}{3}$

Thus $E_1 = \frac{\left(\frac{\sigma}{2}\right)}{\varepsilon_0} \longrightarrow \boxed{E_1 = \frac{\sigma}{2\varepsilon_0}}$ ✓ & $E_2 = \frac{\left(\frac{2\sigma}{3}\right)}{\varepsilon_0} \longrightarrow \boxed{E_2 = \frac{2\sigma}{3\varepsilon_0}}$ ✓

Chapter 4 P.21 [Extra Credit])



Find the capacitance per length.

Let's start by using the D form of Gauss's Law to find Q_{fenc}

$$\oint \vec{D}\cdot d\vec{a} = Q_{fenc} \longrightarrow D(2\pi s \ell) = Q_{fenc} \rightarrow D = \frac{Q_{fenc}}{2\pi s \ell}$$

From the book, we know that $E = D/\varepsilon_0$ when $P=0$. So we can get an expression for E.

$$E = \frac{D}{\varepsilon_0} \longrightarrow E = \frac{\left(\frac{Q_{fenc}}{2\pi s \ell}\right)}{\varepsilon_0} \rightarrow E = \frac{Q_{fenc}}{2\pi s \ell \varepsilon_0} \quad \text{where } a<s<b$$

Since $0 \rightarrow a$ is a conductor and $c \rightarrow$ out is a conductor, we need to do this process once more from $b<r<c$.

$$\oint \vec{D}\cdot d\vec{a} = Q_{fenc} \longrightarrow D(2\pi s \ell) = Q_{fenc} \rightarrow D = \frac{Q_{fenc}}{2\pi r \ell}$$

$$E = \frac{D}{\varepsilon} \longrightarrow E = \frac{\left(\frac{Q_{fenc}}{2\pi r \ell}\right)}{\varepsilon} \rightarrow E = \frac{Q_{fenc}}{2\pi r \ell \varepsilon} \quad \text{where } b<r<c$$

Chapter 4 P.21 [Extra Credit] cont.)

Now, we know that $V = -\int E \cdot d\ell$, and we need V to plug into $C/\ell = \left(\frac{Q}{V}\right)\frac{1}{\ell}$.

$$V = \int_a^b \left(\frac{Q_{fenc}}{2\pi s \ell \varepsilon_0}\right)\frac{ds}{s} + \int_b^c \left(\frac{Q_{fenc}}{2\pi r \ell \varepsilon}\right)\frac{dr}{r}$$

$$V = \frac{Q_{fenc}}{2\pi \varepsilon_0 \ell}\left[\ln\left(\frac{b}{a}\right) + \frac{\varepsilon_0}{\varepsilon}\ln\left(\frac{c}{b}\right)\right]$$

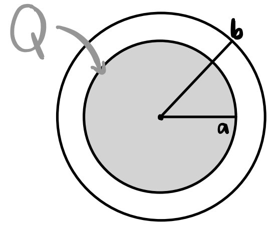
Plug V into $Q/V\ell$

$$\frac{\text{capacitance}}{\text{length}} = \frac{C}{\ell} = \left(\frac{Q}{V}\right)\frac{1}{\ell} = \frac{\cancel{Q_{fenc}}\left(\frac{1}{\cancel{\ell}}\right)}{\left(\frac{\cancel{Q_{fenc}}}{2\pi\varepsilon_0\cancel{\ell}}\left[\ln\left(\frac{b}{a}\right) + \frac{\varepsilon_0}{\varepsilon}\ln\left(\frac{c}{b}\right)\right]\right)}$$

$\varepsilon = \varepsilon_0 \varepsilon_r$

$$\boxed{\frac{\text{capacitance}}{\text{length}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right) + \left(\frac{1}{\varepsilon_r}\right)\ln\left(\frac{c}{b}\right)}}$$

chapter 4 P.26)



Find energy of setup in terms χ_e.

Equation 4.29 says $W = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}\, d\tau$

Example 4.5 is very similar to our problem, so let's use some conclusions from that problem. we can say

$$\mathbf{D} = \begin{cases} 0 & (r<a) \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r} & (r>a) \end{cases} \qquad \mathbf{E} = \begin{cases} 0 & (r<a) \\ \dfrac{Q}{4\pi\varepsilon r^2}\hat{r} & (a<r<b) \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r} & (r>b) \end{cases}$$

using this with eq. 4.29, we can say

$$W = \frac{1}{2}\int \left(\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}\right)\cdot\left(0 + \frac{Q}{4\pi\varepsilon r^2}\hat{r} + \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}\right) 4\pi r^2 dr$$

$$W = \frac{1}{2}\left(\frac{Q}{4\pi}\right)^2 (4\pi)\left[\int_a^b \frac{1}{\cancel{r^2}}\left(\frac{1}{\varepsilon r^2}\right)\cancel{r^2}\, dr + \int_b^\infty \frac{1}{\cancel{r^2}}\left(\frac{1}{\varepsilon_0 r^2}\right)\cancel{r^2}\, dr\right]$$

$$W = \frac{Q^2}{8\pi}\left[\frac{1}{\varepsilon}\left(-\frac{1}{r}\right)\Bigg|_a^b + \frac{1}{\varepsilon_0}\left(\frac{-1}{r}\right)\Bigg|_b^\infty\right] \rightarrow W = \frac{Q^2}{8\pi}\left[\frac{1}{\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{1}{\varepsilon_0}\left(\frac{1}{b}\right)\right]$$

Chapter 4 P.26 cont.)

Now, from equation 4.33 we know that $\varepsilon = \varepsilon_0(1+\chi_e)$. We happen to want our equation in terms of χ_e. Let's plug it in

$$W = \frac{Q^2}{8\pi}\left[\frac{1}{\varepsilon_0(1+\chi_e)}\left(\frac{1}{a}-\frac{1}{b}\right)+\frac{1}{\varepsilon_0}\left(\frac{1}{b}\right)\right]$$

$$W = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{(1+\chi_e)}\left(\frac{1}{a}-\frac{1}{b}\right)+\frac{1}{b}\right]$$

$$W = \frac{Q^2}{8\pi\varepsilon_0(1+\chi_e)}\left[\left(\frac{1}{a}-\frac{1}{b}\right)+\frac{(1+\chi_e)}{b}\right]$$

$$\boxed{W = \frac{Q^2}{8\pi\varepsilon_0(1+\chi_e)}\left[\frac{1}{a}+\frac{\chi_e}{b}\right]}$$

Other Problem #1)

Find the surface charge density of a cup of water.

Surface density can be found with $\sigma = n \cdot p$ which is the number of molecules per volume (n) times the dipole moment (p).

All we need to do is find n.

googled #'s

$$n = \frac{\rho_w}{M_{molar}}(N_A) \longrightarrow n = \frac{(1000\,\cancel{kg}/m^3)}{(0.018\,\cancel{kg}/\cancel{mol})}\left(6.02\times10^{23}\,\frac{molecules}{\cancel{mol}}\right)$$

$$n = 3.35\times10^{28}\ molecules/m^3$$

Thus, our surface charge density will be

$$\sigma = \left(3.35\times10^{28}\,\frac{molecules}{\cancel{m^3}}\right)\left(6.2\times10^{-30}\,C\cdot\cancel{m}\right)$$

from Hyperphysics

$$\boxed{\sigma = 20.54\times10^{-2}\ C/m^2}$$

Other Problem #2)

Estimate the capacitance of the jar.

We are given that the jar is 1 liter, and we need to know surface area. We can say

$$V = 1 \text{ Liter} = 1\times10^{-3}\,m^3 = \pi r^2 h \longrightarrow h = \frac{1\times10^{-3}}{\pi r^2}$$

$$S.A = 2\pi \cancel{r}\left(\frac{1\times10^{-3}}{\pi r^{\cancel{2}}}\right) + 2\pi r^2 \rightarrow S.A = \frac{2(1\times10^{-3})}{r} + 2\pi r^2$$

Now Let's use the D form of Gauss's Law to find Q_{fenc}

Sides: $\oint \vec{D}\cdot d\vec{a} = Q_{fenc} \longrightarrow D_{sides}(2\pi r h) = Q_{fenc}$

$$D_{sides} = \frac{Q_{fenc}}{2\pi r h} \xrightarrow{\text{use volume}} D_{sides} = \frac{Q_{fenc}}{\frac{2}{r}(1\times10^{-3})}$$

Other Problem #2 cont.)

Bottom: $\oint \vec{D}\cdot d\vec{a} = Q_{fenc} \longrightarrow D_{bottom}(2\pi r^2) = Q_{fenc}$

$$D_{bottom} = \frac{Q_{fenc}}{2\pi r^2} \xrightarrow{\text{use volume}} D_{bottom} = \frac{Q_{fenc}}{2\pi r^2}$$

From the book, we know that $E = D/\varepsilon_0$ when $P=0$. So we can plug in our side and bottom expressions for E and solve.

Sides: $E = \dfrac{D}{\varepsilon_0} \longrightarrow E = \left(\dfrac{Q_{fenc}}{\frac{2(1\times10^{-3})}{r}}\right)\dfrac{1}{\varepsilon_0} \longrightarrow E = \dfrac{Q_{fenc}}{\varepsilon_0\left(\frac{2(1\times10^{-3})}{r}\right)}$

Bottom: $E = \dfrac{D}{\varepsilon_0} \longrightarrow E = \left(\dfrac{Q_{fenc}}{2\pi r^2}\right)\dfrac{1}{\varepsilon_0} \longrightarrow E = \dfrac{Q_{fenc}}{2\pi r^2 \varepsilon_0}$

Now, we know that $V = -\int E \cdot dl$, and we need V to plug into $C/l = \left(\frac{Q}{V}\right)\frac{1}{l}$ (where $l = r$).

$$V = -\int E\cdot dr \longrightarrow V = \int_0^d \left(\frac{Q_{fenc}}{\varepsilon_0\left(\frac{2}{r}(1\times10^{-3})\right)} + \frac{Q_{fenc}}{\varepsilon_0(2\pi r^2)}\right)\frac{dr}{r}$$

Other Problem #2 cont.)

$$V = \frac{Q_{fenc}}{\varepsilon_0}\left[\frac{r}{2(1\times10^{-3})} + \left(-\frac{1}{4\pi r^2}\right)\right]_0^d \rightarrow V = \frac{Q_{fenc}}{\varepsilon_0}\left(\frac{\cancel{(2\times10^{-3})}}{\cancel{(2\times10^{-3})}} - \frac{1}{4\pi(2\times10^{-3})^2}\right)$$

$$V = \frac{Q_{fenc}}{\varepsilon_0}\left(1 - \frac{1}{4\pi(2\times10^{-3})^2}\right) \rightarrow V = \frac{Q_{fenc}}{\varepsilon_0}(-1.99\times10^4)$$

Plug v into Q/V

$$\text{capacitance} = C = \left(\frac{Q}{V}\right) = \frac{\cancel{Q_{fenc}}}{\left(\frac{\cancel{Q_{fenc}}}{\pi\varepsilon_0}(-1.99\times10^4)\right)}$$

$$C = \frac{\pi\varepsilon_0}{(-1.99\times10^4)} \rightarrow C = \frac{\pi(8.85\times10^{-12})}{(-1.99\times10^4)} \rightarrow C = -1.397\times10^{-15}F$$

Last we just multiply this number by ε_r which here is equal to $\varepsilon_r = 4$.

$$C = -1.397\times10^{-15}F\,(4) \rightarrow \boxed{C = -5.59\times10^{-15}F}$$