Oregon State University
Department of Physics

Assignment Three

Due: Wednesday, October 14, 2020 (11:59 p.m.) **Points Possible:** 90

Written questions and problems are to be neatly written up as described in the syllabus and turned in via Gradescope. Remember that for Gradescope submissions *each problem must be written on a separate page*, with the problem number clearly indicated. I (or the TA) may choose only a subset of problems to grade carefully each assignment – we won't tell you in advance which those might be! – and rely on you to study the solutions to make sure you fully understand the remainder. Problems will be worth 10 points each unless otherwise stated.

As described in detail in the syllabus, any resources you consult (faculty, friends, books, papers, web sites, etc.) must be cited on a problem-by-problem basis. Individual web page urls, book chapter, section and page numbers etc. should be included; simply listing e.g. the site or the name of the book is not enough.

In most all problems, a good sketch or plot will be your friend. Any additional computer files (e.g. *Mathematica* or python code) should be *both* uploaded to *Canvas* **and** appended to your solution and submitted with the rest of your Gradescope submission. (Remember, though, code is not a substitute for a problem solution. You are attaching your code for reference only.)

Reading:
Griffiths Chapter 3

***Griffiths* Problems:**
Chapter 3 Problems: 28, 30, 33, 35

Notes:
3.28, 3.30: For 5 points *Extra Credit* (each), plot an interesting equipotential for both of these configurations. (Plots in 2D are fine, though 3D would be more fun.) How does this give you an idea of the corresponding electric field? Google for help with the plotting as much as you please.

3.35: Plot the electric field. Google for help with the plotting as much as you please.

Other Problems:

1. (**20 points total**) In this problem we'll explore similarities and differences between a so-called "pure dipole", a charge distribution for which the *only* non-zero multipole moment is the dipole moment, and a "physical dipole", two equal-but-opposite charges q separated by a distance d, which therefore has a dipole moment of magnitude $p = qd$.

    (a) (**10 points total**) Make separate plots of the electric fields due to both a pure dipole and a physical dipole (note you've already done the latter) in two dimensions. (Make sure the dipole moments p of the two confgurations are *equal*.) Then put them together on the *same plot*. (It'll probably be easier to compare them if you plot field lines rather than vectors.) Briefly discuss the similarities and differences between the two fields. Be sure to explore what happens as you adjust the separation d between the charges in the physical dipole – but don't forget to keep the dipole moment p fixed!

    (b) (**10 points total**) If the electric field of a physical dipole is not identical to that of a "pure" dipole, a physical dipole must have some non-zero higher multipole moments. Find the first one that is not zero, and try to figure out why it isn't zero. What can you say about the values of any higher moments? (Orient the dipole along the z-axis and put the origin at the middle of the dipole.)

2. (**30 points total**) Consider a square metal pipe with one edge running along the z-axis. The sides, at $x = 0$, $x = a$, $y = 0$, and $y = a$, are insulated from each other. (Be sure to draw a picture of this so you are clear on the geometry.) The sides are maintained at the following potentials:

    $\mathbf{y = 0:}$ $V(x, 0) = 0$

    $\mathbf{y = a:}$ $V(x, a) = V_0$

    $\mathbf{x = 0:}$ $V(0, y) = V_0$

    $\mathbf{x = a:}$ $V(a, y) = V_0 \cdot \frac{y}{a}$,

    where $V_0 = 4$ V. Since there is no z-dependence, this problem is effectively two-dimensional.

    (a) (20 points total) Find the potential $V(x, y)$ inside the pipe numerically using the *method of relaxation*. Feel free to use *Mathematica*, *Maple*, *Matlab*, python, or *Excel*. For the numerical solution, set $a = 1$ and use a rectangular grid of 20 points in both the x- and y-directions, for a total of 400 grid points. (Please don't be shy about asking for help!) Plot your result. *Upload your code to Canvas when finished.*

    (b) (10 points total) Solve the same problem using separation of variables. (Give your answer in terms of V_0 and a.) Plot your result using the values of V_0 and a given above and compare with your numerical solution.

3. (***Extra Credit:*** **20 points total**) Consider a charged rod of length L. The total charge on the rod is Q. (Put the center of the rod at the origin and align it along the z-axis. Use spherical coordinates throughout.)

    (a) (10 points) Suppose the (linear) density of charge λ on the rod is uniform. Find the first three terms in the multipole expansion of the potential. Plot an interesting equipotential.

    (b) (10 points) Suppose the density of charge λ instead is 0 at one end of the rod and increases linearly from there. Find an expression for $\lambda(s)$, where s is the distance along the rod from the 0-charge end. Find the first three terms in the multipole expansion of the potential. Plot an interesting equipotential.