Blake Evans

PH 431 Assignment One

P. 29) Poisson's → $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

Potential → $V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')}{\mathscr{r}}\, d\tau'$

$$\nabla^2 \left(\frac{1}{4\pi\varepsilon_0}\right)\int \frac{\rho(r')}{\mathscr{r}}\, d\tau' \rightarrow \frac{1}{4\pi\varepsilon_0}\int \nabla^2\left(\frac{1}{\mathscr{r}}\right)\rho(r')\, d\tau' = -\frac{\rho}{\varepsilon_0}$$

we know that $\nabla^2\left(\frac{1}{\mathscr{r}}\right) = -4\pi\,\delta^3(\mathscr{r})$ from 1.102

$$\frac{1}{4\pi\varepsilon_0}\int\left(-4\pi\,\delta^3(\mathscr{r})\right)\rho(r')\, d\tau' = -\frac{\rho}{\varepsilon_0} \rightarrow \frac{1}{\varepsilon_0}\int\delta^3(\mathscr{r})\,\rho(r')\, d\tau' = -\frac{\rho}{\varepsilon_0}$$

Now, we know $\mathscr{r} = r - r'$ and that $\int \delta(x-a) f(x)\, dx = -f(a)$

If we apply these to our equation we get

$$\int\delta^3(r-r')\,\rho(r')\, d\tau' = -\rho(r) \rightarrow \boxed{-\rho(r) = -\rho(r)}$$ checks out.

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P.30) Eq. 2.33 → $E_{above} - E_{below} = \frac{\sigma}{\varepsilon_0}\hat{n}$

a) Ex. 2.5

The field due to an infinite plane is

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

we choose $\hat{n}$ to be the normal vector to the top of the plane, giving us

$$E_{above} = \frac{\sigma}{2\varepsilon_0}\hat{n}, \quad E_{below} = -\frac{\sigma}{2\varepsilon_0}\hat{n}$$

Plugging in, we get

$$E_{above} - E_{below} = \left(\frac{\sigma}{2\varepsilon_0}\hat{n}\right) - \left(-\frac{\sigma}{2\varepsilon_0}\hat{n}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\hat{n} + \frac{\sigma}{2\varepsilon_0}\hat{n} = \boxed{\frac{\sigma}{\varepsilon_0}\hat{n}}$$

Ex. 2.6 Two infinite parallel plates with equally opposing charge have field

$$E = \frac{\sigma}{\varepsilon_0}\hat{n} \text{ (Inside)}$$

This means that we have

$$E_{above} = \frac{\sigma}{\varepsilon_0}\hat{n}, \quad E_{below} = 0$$

Plugging in, we get

$$\Delta E = \frac{\sigma}{\varepsilon_0}\hat{n} - 0 = \boxed{\frac{\sigma}{\varepsilon_0}\hat{n}}$$

Problem 2.11 The electric field of a sphere of charge (with charge density σ) is

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}\hat{r} \quad \text{(where } r = R\text{)}$$

outside, the field is zero so we only have E_{above}

$$\Delta E = \frac{\sigma R^2}{\varepsilon_0 (R)^2}\hat{r} - 0 = \boxed{\frac{\sigma}{\varepsilon_0}\hat{r}}$$

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P 30) Eq. 2.33 $\rightarrow E_{above} - E_{below} = \frac{\sigma}{\varepsilon_0}\hat{n}$

b) The electric field inside the tube is zero, because the charge inside it is zero. The electric field outside the tube is

$$E_{out}(2\pi r \ell) = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma(2\pi R \ell)}{\varepsilon_0}$$

$$E_{out} = \frac{\sigma R}{\varepsilon_0 r}$$

we can sub R in for r and simplify

$$E_{out} = \frac{\sigma R}{\varepsilon_0 (R)}\hat{r} \rightarrow E_{out} = \frac{\sigma}{\varepsilon_0}\hat{r}$$

$$\Delta E = E_{out} - E_{in} = \left(\frac{\sigma}{\varepsilon_0}\hat{r}\right) - 0 = \boxed{\frac{\sigma}{\varepsilon_0}\hat{r}}$$

c) From Ex. 2.8 the potential is given as

$$V_{out} = \frac{R^2\sigma}{\varepsilon_0 z} \; ; \; V_{in} = \frac{R\sigma}{\varepsilon_0}$$

If we substitute R for z we get

$$V_{out} = \frac{(R^2)\sigma}{\varepsilon_0 (R)} = \frac{R\sigma}{\varepsilon_0}$$

Thus the inside and outside potentials are equal and consistent with 2.34.

If we differentiate both and sub $z \rightarrow R$ we get.

$$\frac{\partial V_{out}}{\partial z} = -\frac{R^2\sigma}{\varepsilon_0 z^2} \xrightarrow{z\rightarrow R} -\frac{\sigma}{\varepsilon_0} \; ; \; \frac{\partial V_{in}}{\partial z} = \varnothing \xrightarrow{z\rightarrow R} \varnothing$$

$$\frac{\partial V_{out}}{\partial z} - \frac{\partial V_{in}}{\partial z} = -\frac{\sigma}{\varepsilon_0} - 0 = \boxed{-\frac{\sigma}{\varepsilon_0}}$$ which is consistent with 2.36.

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P.38)

a) Find σ @ a, b, R

At R, $\sigma = \frac{q}{4\pi R^2}$

At a, surface charge density is $\sigma = -\frac{q}{4\pi(a)^2}$ (because charge at a is $-q$)

At b, $\sigma = +\frac{q}{4\pi(b)^2}$ (because charge at b is $+q$)

b) $$E(r) = \begin{cases} 0 & r<R \\ \frac{q}{4\pi\varepsilon_0 r^2} & R<r<a \\ 0 & a<r<b \\ \frac{q}{4\pi\varepsilon_0 r^2} & b<r \end{cases}$$

$$V(\text{center}) = -\int_{\infty}^{0} E(r)\,dr$$

$$= -\int_a^b \frac{q}{4\pi\varepsilon_0 r^2}\,dr - \int_b^a 0\,dr - \int_a^R \frac{q}{4\pi\varepsilon_0 r^2}\,dr + 0$$

$$= \boxed{\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} + \frac{1}{R} - \frac{1}{a}\right) = V(\text{center})}$$

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P.38)

c) At R, σ becomes

$$\sigma_R = \frac{q}{4\pi R^2}$$

At a, σ becomes

$$\sigma_a = -\frac{q}{4\pi a^2}$$

At b, σ becomes

$$\sigma_b = 0$$

Now,
$$E(r) = \begin{cases} 0 & r<R \\ \dfrac{q}{4\pi\varepsilon_0 r^2} & R<r<a \\ 0 & a<r<b \\ 0 & r<b \end{cases}$$

Making $V(\text{center}) = -\int_{\infty}^{0} E(r)\,dr$

$$= \int_a^R \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \boxed{\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{a}\right)}$$

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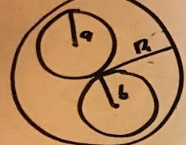
P.39)



a) Surface area of a:

$$A = 4\pi a^2$$

Surface density of a:

$$\sigma_a = -\frac{q_a}{A} = \boxed{-\frac{q_a}{4\pi a^2}}$$

Surface area of b:

$$B = 4\pi b^2$$

Surface density of b:

$$\sigma_b = -\frac{q_b}{B} = \boxed{-\frac{q_b}{4\pi b^2}}$$

The surface area of the sphere w/ radius R is

$$R = 4\pi R^2$$

The total charge on the sphere should be

$$q_{induced} = q_a + q_b$$

Therefore, charge density is

$$\sigma_R = \frac{(q_a+q_b)}{R}$$

$$\boxed{\sigma_R = \frac{q_a+q_b}{4\pi R^2}}$$

b) using the charge from before,

$$q_{ind} = q_a + q_b$$

we can plug into the electric field expression to get

$$E = \frac{q_{ind}}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$\boxed{E = \frac{q_a+q_b}{4\pi\varepsilon_0 r^2}\hat{r}} \text{ outside}$$

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P.53)

a) Poisson's Equation – $\nabla^2 V = \frac{\rho}{\varepsilon_0}$

Since V & ρ only vary along x-axis

$$\boxed{\frac{\partial^2 V_x}{\partial x^2} = -\frac{\rho}{\varepsilon_0}}$$

b) Using conservation of energy, we know potential difference = change in kinetic energy. So,

$$V_x e = \frac{1}{2} m v_x^2 \rightarrow \boxed{v_x = \sqrt{\frac{2eV_x}{m}}}$$

c) Volume Charge Density – $\rho \frac{dq}{V}$

$$dq = \rho V \rightarrow dq = \rho(A\,dx)$$

Rate of Flow of Charge – $I = \frac{dq}{dt}$

$$I = A\rho \frac{dx}{dt} \rightarrow \boxed{I = A\rho v_x}$$

d) Using our previous answers, we can say

$$\frac{\partial^2 V_x}{\partial x^2} = -\frac{\rho}{\varepsilon_0} = -\left(\frac{I}{Av_x}\right)\frac{1}{\varepsilon_0} = \frac{-I}{A\varepsilon_0}\left(\sqrt{\frac{2eV_x}{m}}\right)$$

If we let $C = -\frac{I}{\varepsilon_0 A}\sqrt{\frac{m}{2e}}$, the we get

$$\boxed{\frac{\partial^2 V}{\partial x^2} = C V_x^{-1/2}}$$

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P. 3.2) Earnshaw's Theorem:

A charged particle cannot be held in a stable equilibrium by electrostatic forces alone.

For Laplace's equations to be true there must be no extrema; on the cube, since there are charges on the verticies, the center would be a local minimum and thus does not comply with Laplace.

P 3.3) From other classes, we know

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Here we are given that V only depends on R.

So, $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)$

we can then set this equal to Ø and solve

$$\emptyset = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) \rightarrow \emptyset = \frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) \rightarrow r^2\frac{\partial V}{\partial r} = C$$

Integrating both sides gives us $\boxed{V = -\frac{C}{r} + B}$

Now, $\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial^2 z}$

Doing similar steps as before we get

$$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) = \emptyset \rightarrow \frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) = \emptyset \rightarrow s\frac{\partial V}{\partial s} = C \rightarrow \partial V = \frac{C}{s}\partial s$$

Integrating both sides, we get $\boxed{V = C\ln(s) + B}$

```python
import numpy as np
import matplotlib.pyplot as plt

x,y = np.meshgrid(np.linspace(-5,5,20),np.linspace(-5,5,20))

#Dipole Electric Field
Ex = (x + 1)/((x+1)**2 + y**2) - (x - 1)/((x-1)**2 + y**2)
Ey = y/((x+1)**2 + y**2) - y/((x-1)**2 + y**2)

plt.figure()
plt.streamplot(x,y,Ex,Ey)
plt.title('Dipole Electric Field')

plt.figure()
plt.quiver(x,y,Ex,Ey,scale=50)
plt.title('Dipole')

# Non-Zero Divergence
u = x/np.sqrt(x**2 + y**2)
v = y/np.sqrt(x**2 + y**2)

plt.figure()
plt.title("Non-Zero Divergence")
plt.quiver(x,y,u,v)

# Non-Zero Curl
w = -y**2/np.sqrt(x**2 + y**2)
z = x/np.sqrt(x**2 + y**2)

plt.figure()
plt.title("Non-Zero Curl")
plt.quiver(x,y,w,z)

# For Fun
a = np.sin(y);
b = np.cos(x);

plt.figure()
plt.title("For Fun Plot")
plt.quiver(x,y,a,b)
plt.show()
```

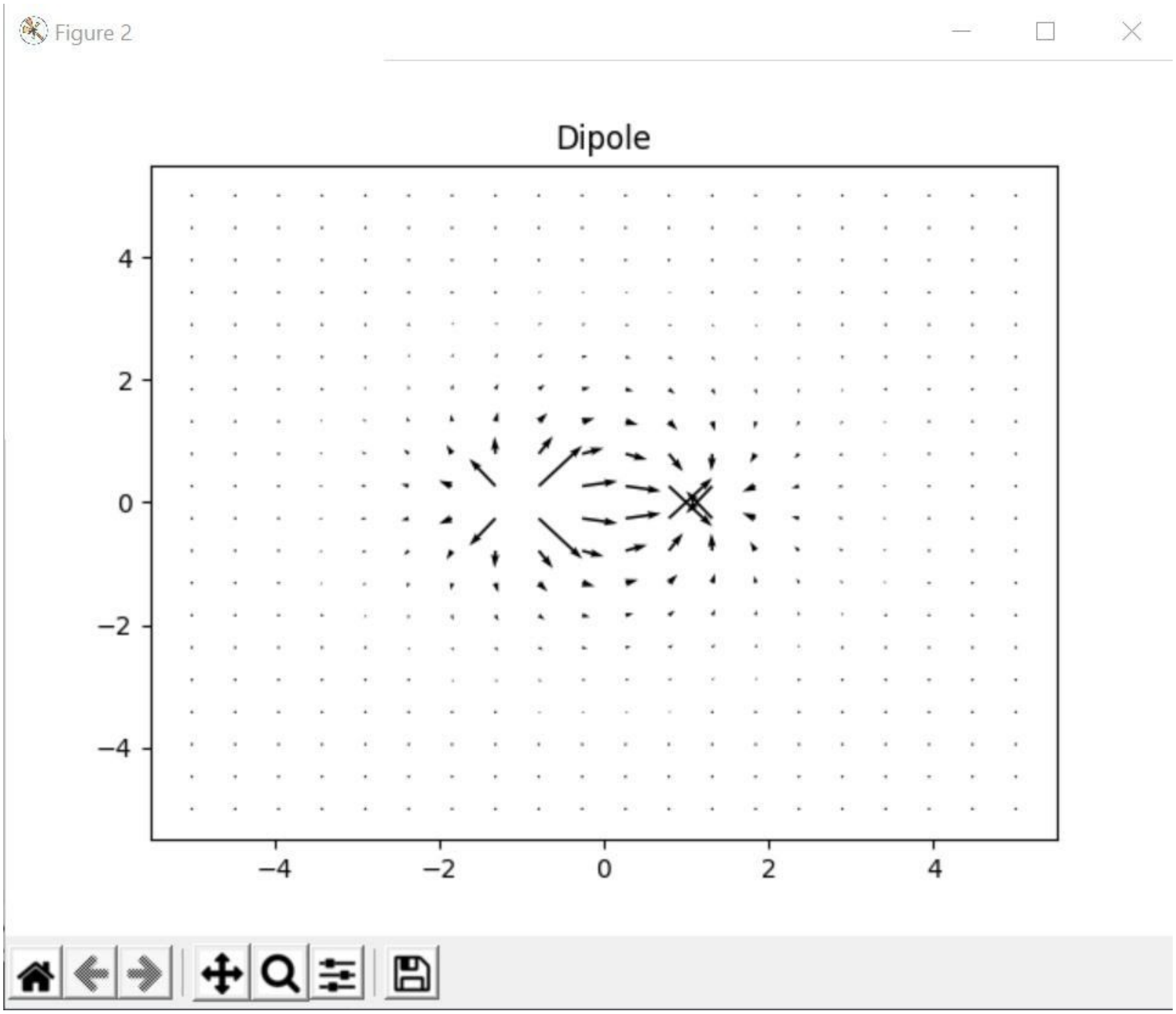
Figure 2
Dipole
4
2
0
−2
−4
−4
−2
0
2
4

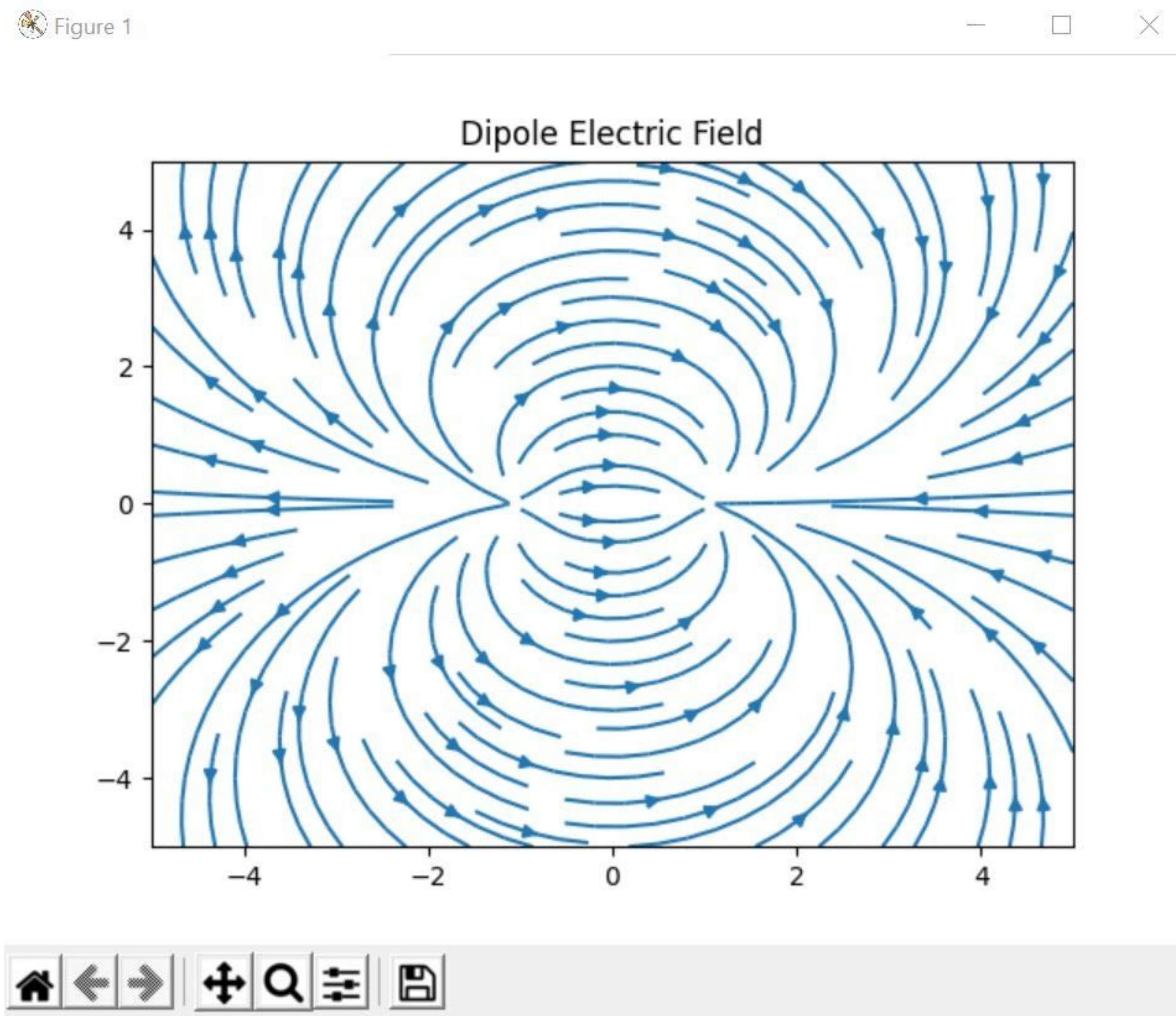
Figure 1
Dipole Electric Field
4
2
0
−2
−4
−4
−2
0
2
4

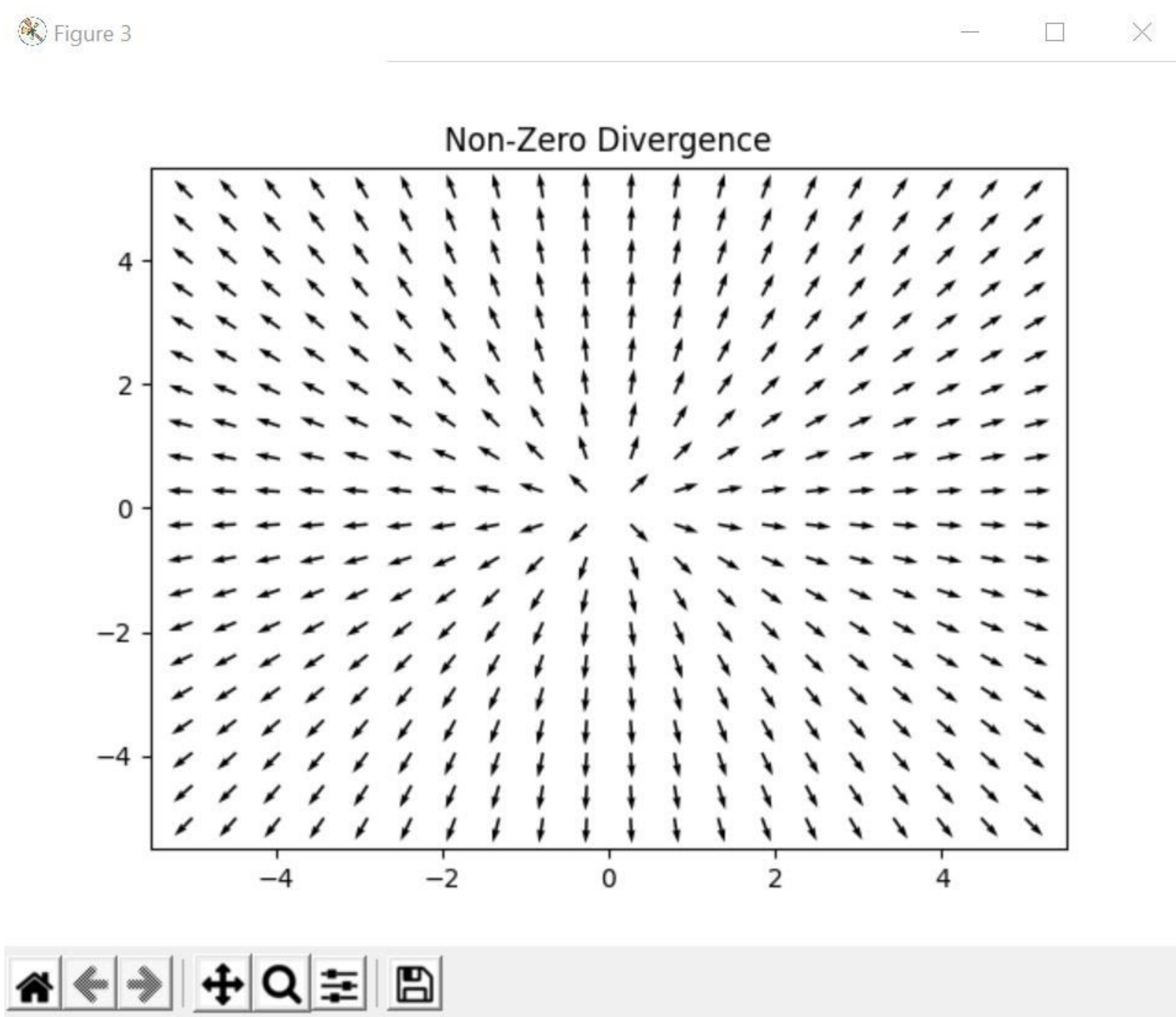
Figure 3
Non-Zero Divergence
4
2
0
−2
−4
−4
−2
0
2
4

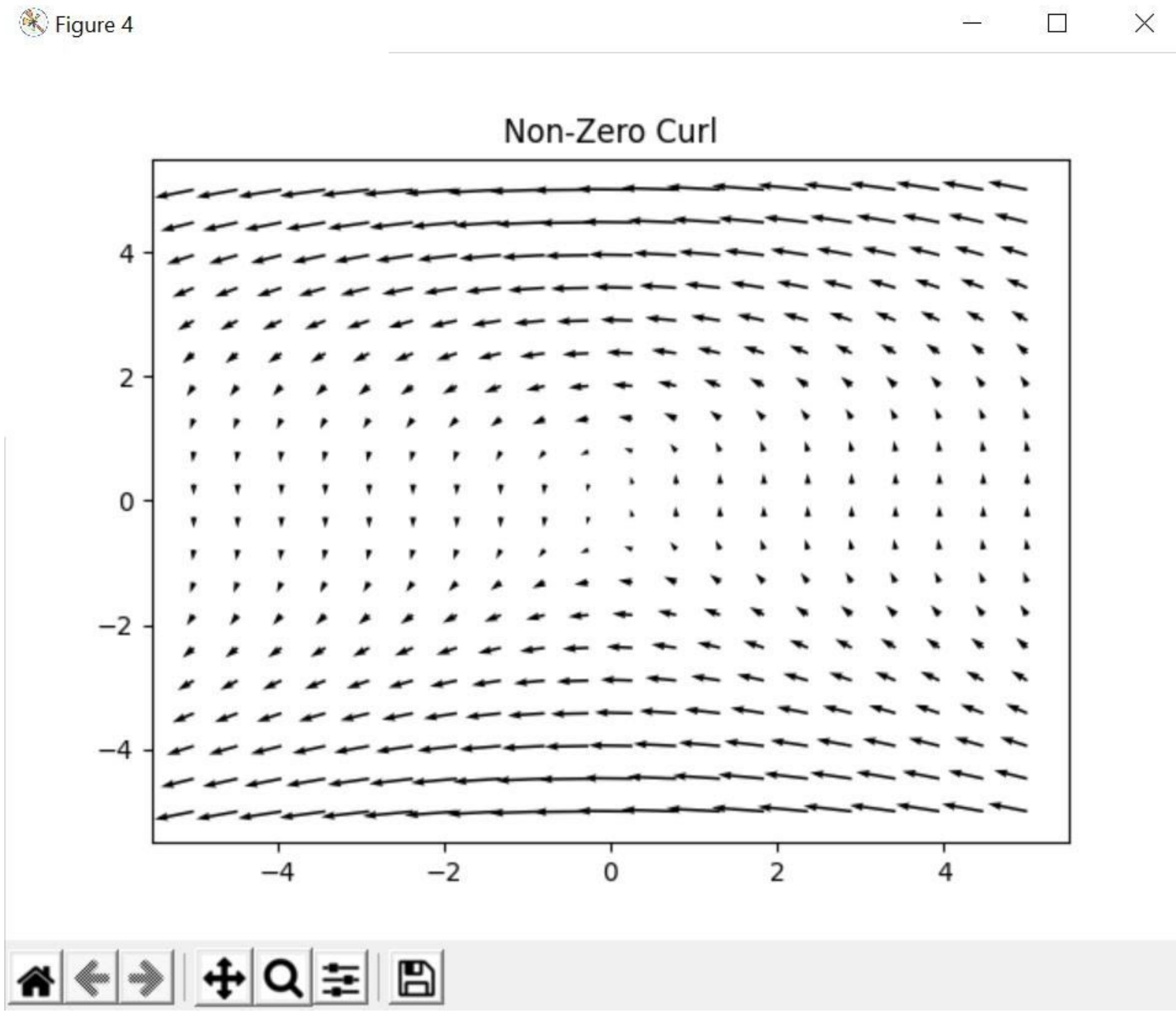
Figure 4
Non-Zero Curl
4
2
0
−2
−4
−4
−2
0
2
4

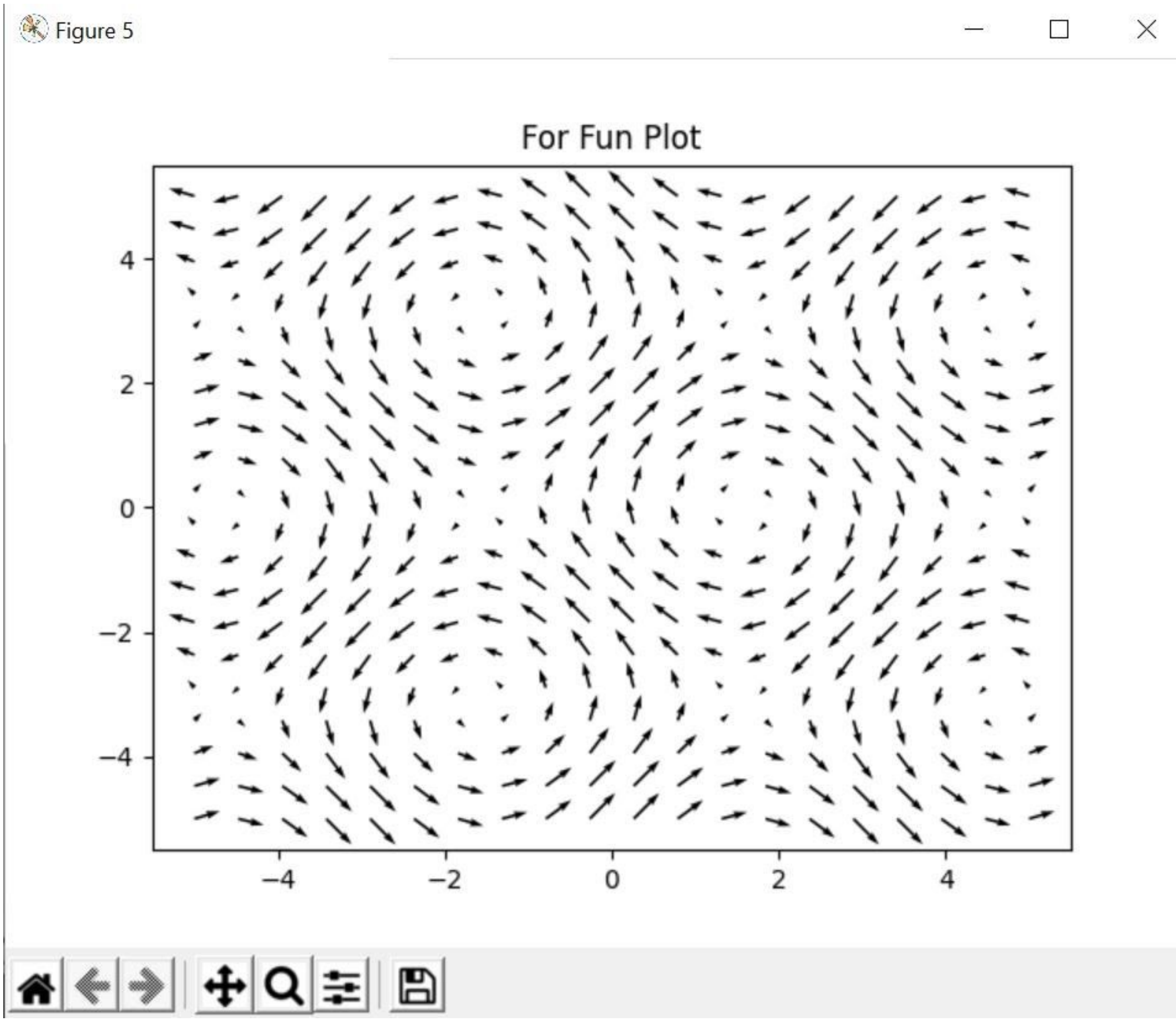
Figure 5
For Fun Plot
4
2
0
−2
−4
−4
−2
0
2
4