Oregon State University
Department of Physics

Physics 431: Electrodynamics Capstone
Fall 2020

Assignment Two

Due: Wednesday, October 7, 2020 (11:59 p.m.) **Points Possible:** 50

Written questions and problems are to be neatly written up as described in the syllabus and turned in via Gradescope. Remember that for Gradescope submissions *each problem must be written on a separate page*, with the problem number clearly indicated. I (or the TA) may choose only a subset of problems to grade carefully each assignment – we won't tell you in advance which those might be! – and rely on you to study the solutions to make sure you fully understand the remainder. Problems will be worth 10 points each unless otherwise stated.

As described in detail in the syllabus, any resources you consult (faculty, friends, books, papers, web sites, etc.) must be cited on a problem-by-problem basis. Individual web page urls, book chapter, section and page numbers etc. should be included; simply listing e.g. the site or the name of the book is not enough.

In most all problems, a good sketch or plot will be your friend. Any additional computer files (e.g. *Mathematica* or python code) should be *both* uploaded to *Canvas* **and** submitted with the rest of your Gradescope submission.

Reading:
Griffiths §3.1-3

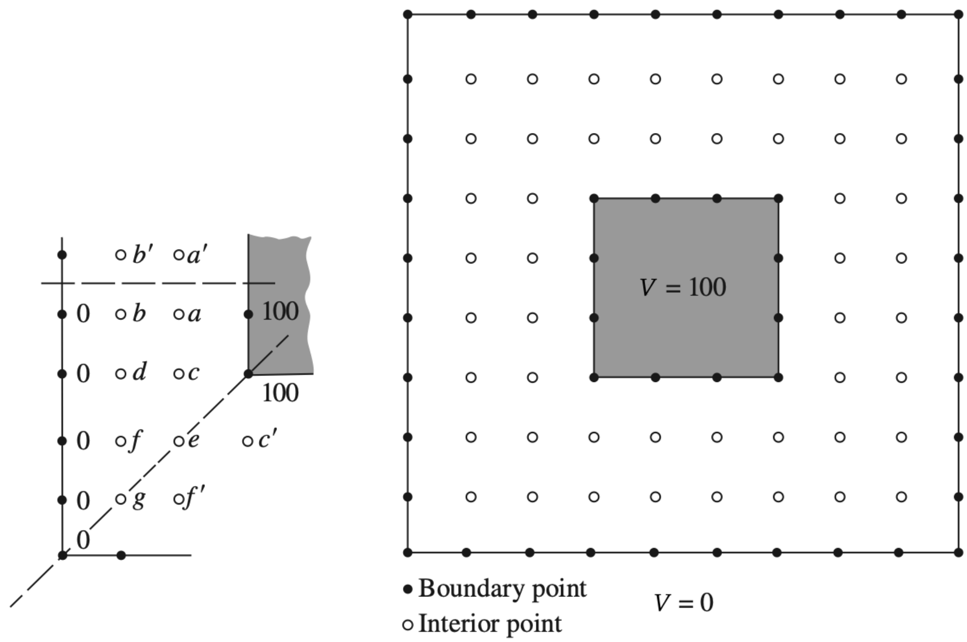
Griffiths **Problems:**
Chapter 2 Problems: 43
Chapter 3 Problems: 13, 15 *Extra Credit:* 7, 16

Notes:
3.13, 3.15: Plot your results for $V(x, y)$, the profile of the potential on any $z =$ constant slice, using *Mathematica* or your preferred tool. (I'm looking for something along the lines of Figs. 3.18 and 3.21.) For your plots, if you need to choose $a = 1$, $b = 2$, and $V_0 = 3$. Google for help with the plotting as much as you please.

Other Problems:

1. (**10 points total**) Consider a parallel plate capacitor consisting of 2 circular plates 3.0 cm in diameter with a plate separation of 0.5 mm. Calculate the capacitance. If the voltage on the capacitor is 10.0 V, find both the electrostatic pressure on the capacitor plates, and the net electrostatic force on each plate.

2. (**10 points total**) ***METHOD OF RELAXATION I*** Before we tackle the method of relaxation on a computer, let's work through a simpler example by hand (with a calculator, that is.)



In the figure there are two square equipotential boundaries, one inside the other. The problem is to find an approximate numerical solution to Laplace's equation for the potential $V(x, y)$ in the region between these conductors for the given boundary conditions, $V = 0$ V on the outer boundary and $V = 100$ V on the inner. Normally one does this on a computer, but to get a feel for what's happening let's begin by doing it by hand. To that end we'll make the array rather coarse, to keep the labor within reasonable bounds. You should take advantage of the symmetry of the configuration: only seven different interior values actually need to be computed! (These are labeled *a*-*g* in the figure.) You could start with any values at the interior points, but time will be saved with a little judicious guesswork. We know the correct values must lie between 0 and 100 (*why?*), and we expect that points closer to the inner boundary will have higher values than those closer to the outer boundary. Some reasonable starting values are $a = 50$, $b = 25$, $c = 50$, $d = 25$, $e = 50$, $f = 25$, and $g = 25$. With the given boundary conditions and these starting values, go over these seven interior lattice points in some systematic manner, replacing the value at each interior point by the average of its four neighbors. (At each step reset $a' = a$, $b' = b$, $c' = c$, and $f' = f$.) Repeat until all changes resulting from a sweep over the array are acceptably small. "Acceptably small" as a rule depends on how accurate one needs the answer to be. For this exercise, let us agree that it will be time to quit when no change larger in absolute magnitude than one unit occurs in the course of the sweep. Enter your final values on the array, and sketch the approximate course that the two equipotentials $V = 25$ V and $V = 50$ V would have in the actual continuous $V(x, y)$.

(*Aside:* The relaxation of the grid values toward an eventually unchanging distribution is closely related to the physical phenomenon of diffusion. If you start with much too high a value at one point, it will "spread" to its nearest neighbors, then to its next nearest neighbors, and so on, until the bump is smoothed out.)

It won't be graded, but I strongly suggest working out how to implement what you just did by hand on a computer, using *Mathematica* or python, as preparation for the next problem, and making sure your results agree.

3. (***Extra Credit:*** **10 points total**) Consider any solution of Laplace's equation $\phi(x, y, z)$, so $\nabla^2\phi = 0$. Expand ϕ in a Taylor series about *each* of six points surrounding the point (x_0, y_0, z_0): $(x_0 + \delta, y_0, z_0)$, $(x_0, y_0 + \delta, z_0)$, $(x_0, y_0, z_0 + \delta)$, $(x_0 - \delta, y_0, z_0)$, $(x_0, y_0 - \delta, z_0)$, and $(x_0, y_0, z_0 - \delta)$. Show explicitly that the average of the values of ϕ at these six points is equal to the value of ϕ at the center, namely, $\phi(x_0, y_0, z_0)$, through terms of order δ^3.

4. (**30 points total**) **THIS PROBLEM IS DUE IN *TWO* WEEKS. I AM PUTTING IT IN YOUR HANDS NOW SO YOU HAVE PLENTY OF TIME TO WORK ON IT. DON'T HAND IN ANYTHING THIS WEEK; THIS WILL APPEAR AGAIN ON NEXT WEEK'S ASSIGNMENT.** Consider a square metal pipe with one edge running along the z-axis. The sides, at $x = 0$, $x = a$, $y = 0$, and $y = a$, are insulated from each other. (Be sure to draw a picture of this so you are clear on the geometry.) The sides are maintained at the following potentials:

   $\mathbf{y = 0}$**:** $V(x, 0) = 0$

   $\mathbf{y = a}$**:** $V(x, a) = V_0$

   $\mathbf{x = 0}$**:** $V(0, y) = V_0$

   $\mathbf{x = a}$**:** $V(a, y) = V_0 \cdot \frac{y}{a}$,

   where $V_0 = 4$ V. Since there is no z-dependence, this problem is effectively two-dimensional.

   (a) (20 points total) Find the potential $V(x, y)$ inside the pipe numerically using the *method of relaxation.* Feel free to use *Mathematica*, *Maple*, *Matlab*, python, or *Excel.* For the numerical solution, set $a = 1$ and use a rectangular grid of 20 points in both the x- and y-directions, for a total of 400 grid points. (Please don't be shy about asking for help!) Plot your result. *Upload your code to Canvas when finished.*

   (b) (10 points total) Solve the same problem using separation of variables. (Give your answer in terms of V_0 and a.) Plot your result using the values of V_0 and a given above and compare with your numerical solution.