E&M Assignment Eight

Blake Evans

Chapter 9 P.9)

a) Traveling in -x-direction, polarized in z-direction

From the book, equation 9.51 and 9.52 says

$$\vec{E}(\vec{r},t) = E_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \delta)\,\hat{n}, \quad \vec{B}(\vec{r},t) = \frac{E_0}{c}\cos(\vec{k}\cdot\vec{r} - \omega t + \delta)(\hat{k}\times\hat{n})$$

In the problem statement, we are given

$$\boxed{k = -\omega/c\,\hat{x}\ ;\ \hat{n} = \hat{z}}$$

From our knowledge, we know $\hat{r} = x\hat{x} + y\hat{y} + z\hat{z}$. Thus we can calculate $\hat{k}\cdot\hat{r}$ and $\hat{k}\times\hat{n}$.

$$\hat{k}\cdot\hat{r} = (-\omega/c\,\hat{x})\cdot(x\hat{x} + y\hat{y} + z\hat{z}) = -\omega/c\,x$$

$$\hat{k}\times\hat{n} = (-\omega/c\,\hat{x})\times(\hat{z}) = \hat{y}$$

Now, we can plug this into our equation

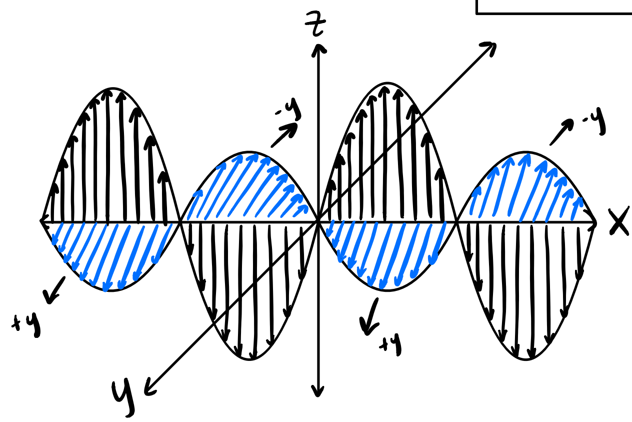
cos is even, $\cos(-\theta) = \cos\theta$

$$\vec{E}(x,t) = E_0\cos(-\omega/c\,x + \omega t)\,\hat{z} \rightarrow \boxed{\vec{E}(x,t) = E_0\cos(\omega/c\,x - \omega t)\,\hat{z}}$$

Chapter 9 P.9 cont.)

$$\vec{B}(x,t) = \frac{E_0}{c}\cos\left(-\omega/c\, x - \omega t\right)(\hat{y}) \rightarrow \boxed{\vec{B}(x,t) = \frac{E_0}{c}\cos\left(\omega/c\, x - \omega t\right)\hat{y}}$$

cos is even, $\cos(-\theta) = \cos\theta$



b) Traveling in the direction of (1,1,1) from (0,0,0)

Right off the bat, we know that our vector should look like $\hat{r} = \hat{x} + \hat{y} + \hat{z}$. Though, this is not normalized and we need it to be. So,

$$|\hat{r}| = (N\vec{r})\cdot(N\vec{r}) \longrightarrow 1 = N^2(\vec{r}\cdot\vec{r}) \quad \text{where} \quad \hat{r} = \hat{x} + \hat{y} + \hat{z}$$

$$\frac{1}{N^2} = (1\cdot1)(\hat{x}\cdot\hat{x}) + (1\cdot1)(\hat{y}\cdot\hat{y}) + (1\cdot1)(\hat{z}\cdot\hat{z}) \rightarrow \frac{1}{N^2} = 1+1+1 \longrightarrow N = \sqrt{\frac{1}{3}}$$

Thus, we can say our normalized vector $(\hat{k})$ is

$$\hat{r} = N\hat{x} \longrightarrow \vec{k} = \omega/c\,\hat{r} \longrightarrow \boxed{\vec{k} = \omega/c \cdot \frac{1}{\sqrt{3}}(\hat{x}+\hat{y}+\hat{z})}$$

chapter 9 P.9 cont.)

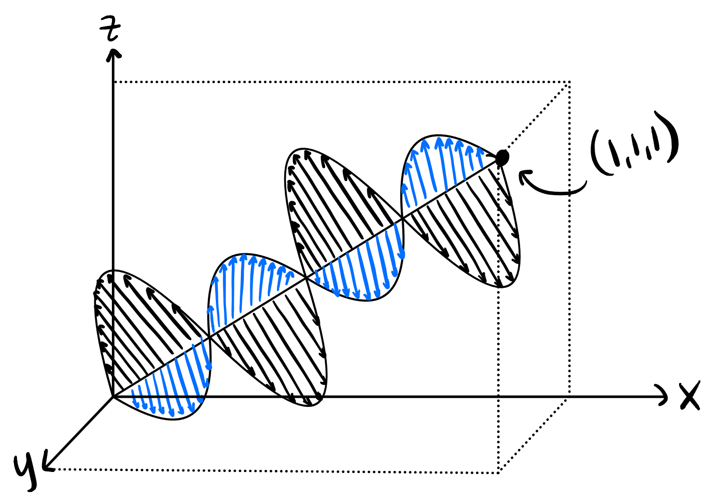
now, for n, we know n is parallel to the xz plane, so $\vec{y}=0$ and $\hat{n}=a\hat{x}+b\hat{z}$. If we dot this with $\hat{r}$, we should get 0 and be able to solve for a and b.

$$\hat{n}\cdot\hat{r}=0 \rightarrow (a\hat{x}+b\hat{y})\cdot(\hat{x}+\hat{y}+\hat{z})=0 \rightarrow a+b=0 \rightarrow a=-b$$

Letting $a=1$, b will be -1 and $\hat{n}=(1,0,-1)$.

normalize $\hat{n}=(1,0,-1) \longrightarrow |n|^2=(1)+(1) \longrightarrow \boxed{\hat{n}=\frac{1}{\sqrt{2}}(\hat{x}-\hat{z})}$

Our new wave will look the same as before but pointed towards (1,1,1).



Chapter 9 P.9 cont.)

Thus our electric and magnetic fields are

$$\vec{E} = E_0 \cos(\vec{k}\cdot\vec{r} - \omega t)\,\hat{n} \longrightarrow \boxed{\vec{E} = E_0 \cos\left[\frac{\omega}{\sqrt{3}}(x+y+z) - \omega t\right]\left(\frac{1}{\sqrt{2}}(\hat{x}-\hat{z})\right)}$$

$$\vec{B} = \frac{E_0}{c}\cos(\vec{k}\cdot\hat{r} - \omega t)(\hat{k}\times\hat{n}) \longleftarrow \text{we need } \hat{k}\times\hat{n}$$

$$\hat{k}\times\hat{n} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix} = \left.\begin{aligned} &\hat{x}(-1-0) \\ &-\hat{y}(-1-1) \\ &+\hat{z}(0-1) \end{aligned}\right\} \quad \frac{-x+2\hat{y}-\hat{z}}{\sqrt{6}}$$

$$\boxed{\vec{B} = \frac{E}{c}\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right]\left(\frac{1}{\sqrt{6}}(-\hat{x}+2\hat{y}-\hat{z})\right)}$$

Chapter 9 P.10)

From the problem statement, we are given

$$\text{Intensity} = 1300 \, W/m^2 \qquad P = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{I}{c}$$

Plugging Intensity in, we get

$$P = \frac{I}{c} \rightarrow P = \frac{(1300 \, W/m^2)}{(3.0 \times 10^8 \, m/s)} \rightarrow \boxed{P = 4.3 \times 10^{-6} \, N/m^2}$$

Next, for a perfect reflector system, the pressure is doubled.

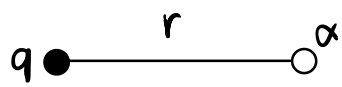
$$P_{reflect} = 2 P_{incident} = 8.6 \times 10^{-6} \, N/m^2$$

And lastly, to find out the fraction of atmospheric pressure, we just divide by standard pressure.

$$1 \, atm = 1.01 \times 10^5 \, kg/m \cdot s^2 \rightarrow \frac{(8.6 \times 10^{-6} \, N/m^2)}{1.01 \times 10^5 \, kg/m \cdot s^2}$$

$$\boxed{8.3 \times 10^{-11} \, atm}$$

Chapter 4 P.4)



We know that the electric field of the point charge q is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Thus, the induced dipole moment of the atom will be $p = \alpha E$

$$p = \alpha E \rightarrow p = \alpha\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\right) \rightarrow p = \frac{\alpha}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

The electric field of a perfect dipole is given by equation 3.103 in the book as

$$E_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

because of how the charge is placed in our system,

Chapter 4 P.4 cont.)

our θ will just be $\theta = \pi$ and our $\sin\theta$ term will just go to $\emptyset$. So, plugging in p and knowing this, we can say

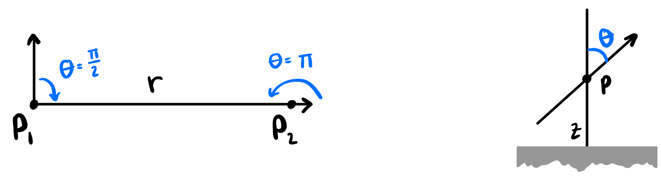
$$\vec{F} = q\left(\frac{p}{4\pi\varepsilon_0 r^2}\left(2\cos(\pi)\hat{r} + \sin(\pi)\hat{\theta}\right)\right)$$

$$\vec{F} = \frac{q}{4\pi\varepsilon_0 r^2}\left(\frac{\alpha}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\right)\left(2(1)+0\right)$$

$$\boxed{\vec{F} = \left(\frac{q}{4\pi\varepsilon_0 r^2}\right)^2 \frac{2\alpha}{r}}$$

The force is attractive

Chapter 4 P.5)



From equation 3.104 and 4.4 in the book, we know that

$$\vec{N} = \vec{p} \times \vec{E} \quad \text{and} \quad E_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

From general physics, we know that the torque acting on two things is the cross product of those things. So, to find the torque on p_2 from p we can plug in $\theta_1 = \pi/2$ and say

$$\vec{N}_{2\to1} = \vec{p}_2 \times \vec{E}(p_1)$$

$$\vec{N}_{2\to1} = \vec{p}_2 \times \left(\frac{p_1}{4\pi\varepsilon_0 r^3}\left(2\cos\left(\tfrac{\pi}{2}\right)\hat{r} + \sin\left(\tfrac{\pi}{2}\right)\hat{\theta}\right)\right)$$

$$\vec{N}_{2\to1} = \vec{p}_2 \times \left(\frac{p_1}{4\pi\varepsilon_0 r^3}\left(0 + (1)\hat{\theta}\right)\right)$$

Chapter 4 P.5 Cont.)

Plug in $\theta_1 = \frac{\pi}{2}$

$$\vec{N}_{2\to1} = |P_2| \sin\theta \left| \frac{P_1}{4\pi\varepsilon_0 r^3} \hat{\theta} \right| \longrightarrow \vec{N}_{2\to1} = \frac{P_2 \cdot P_1}{4\pi\varepsilon_0 r^3} \sin(\pi/2)$$

$$\boxed{\text{Torque on } P_2 : \vec{N}_{2\to1} = \frac{P_1 \cdot P_2}{4\pi\varepsilon_0 r^3}}$$ into the page

Now, we can complete this process again for the torque on P_1 from P_2, using $\theta_2 = \pi$.

$$N_{1\to2} = \vec{P}_1 \times \vec{E}(P_2)$$

$$\vec{N}_{1\to2} = \vec{P}_1 \times \left(\frac{P_2}{4\pi\varepsilon_0 r^3} \left(2\cos(\pi)\hat{r} + \sin(\pi)\hat{\theta} \right) \right)$$

$$\vec{N}_{1\to2} = \vec{P}_1 \times \left(\frac{P_2}{4\pi\varepsilon_0 r^3} \left(2(-1)\hat{r} + (0) \right) \right)$$

Plug in $\theta_1 = \frac{\pi}{2}$

$$\vec{N}_{1\to2} = |P_1| \sin\theta \left| \frac{2P_2}{4\pi\varepsilon_0 r^3} \hat{r} \right| \longrightarrow \vec{N}_{2\to1} = \frac{2 P_1 \cdot P_2}{4\pi\varepsilon_0 r^3} \sin(\pi/2)$$

$$\boxed{\text{Torque on } P_1 : \vec{N}_{1\to2} = \frac{2 P_1 \cdot P_2}{4\pi\varepsilon_0 r^3}}$$ into the page

chapter 4 P.10)

a) $\vec{P}(\hat{r}) = k\hat{r}$, Calculate bound changes σ_b and ρ_b

From equation 4.11 in the book, we know that

$\sigma_b = \vec{P} \cdot \hat{n}$ where $\hat{n}$ is the normal unit vector

Since our surface is a sphere, $\hat{n} = \hat{r}$ and $r = R$.
So, we can say the following

$$\sigma_b = \vec{P}(r) \cdot \hat{r} = (kr) \cdot \hat{r} = k(R\hat{r}) \cdot \hat{r} \rightarrow \boxed{\sigma_b = kR}$$

To find the volume density, we can use the formula
given in equation 4.12 : $\rho_b = -\vec{\nabla} \cdot \vec{P}$

$$\rho_b = -\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2 P_r\right) + \frac{1}{r\sin\theta}\frac{d}{d\theta}\left(\sin\theta\, P_\theta\right) + \frac{1}{r\sin\theta}\frac{d}{d\phi}(P_\phi)\right]$$

$$\rho_b = -\left[\frac{1}{r^2}\frac{d}{dr}\left[r^2(kr)\right] + (0) + (0)\right] \rightarrow \rho_b = -\frac{1}{r^2}\frac{d}{dr}(kr^3)$$

$$\rho_b = -\frac{3k\cancel{r^2}}{\cancel{r^2}} \rightarrow \boxed{\rho_b = -3k}$$

chapter 4 P.10 cont.)

b) Find the field inside and outside the sphere.

Since we want Electric field, let's use Gauss' Law. We have charge density σ_b which we know to be [Charge]/[Area]. So we can multiply by $A = (4\pi R^3)$ to get charge (Q).

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_0} \rightarrow \oint \vec{E} \cdot d\vec{a} = \frac{kR(4\pi R^2)}{\varepsilon_0}$$

$$E(4\pi (R)^2) = \frac{k4\pi R^3}{\varepsilon_0} \rightarrow E = \frac{kR\cancel{(4\pi R^2)}}{\varepsilon_0 \cancel{(4\pi R^2)}} \rightarrow E_\sigma = \frac{kR}{\varepsilon_0}\hat{r}$$

Now let's repeat this process with $\rho_b = -3k$. But, Since this is a charge-volume we need to use $V = \frac{4}{3}\pi r^2$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_0} \rightarrow \oint \vec{E} \cdot d\vec{a} = \frac{-3k(\frac{4}{3}\pi r^3)}{\varepsilon_0}$$

Let r=R

$$E_\rho(4\pi R^2) = \frac{-k4\pi (R)^3}{\varepsilon_0} \rightarrow E_\rho = \frac{-kR\cancel{(4\pi R^2)}}{\varepsilon_0 \cancel{(4\pi R^2)}} \rightarrow E_\rho = \frac{-kR}{\varepsilon_0}\hat{r}$$

Chapter 9 P.10 Cont.)

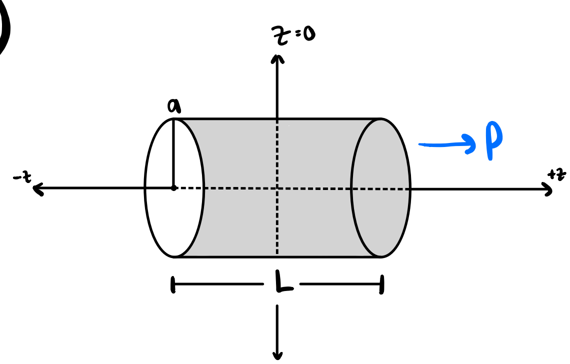
Now, all we need to do to find the total outside electric field is to sum up our solutions.

$$E_{tot} = \Sigma E \rightarrow E_{tot} = E_\sigma + E_\rho \rightarrow E_{tot} = \left(\frac{kR}{\varepsilon_0}\right) + \left(-\frac{kR}{\varepsilon_0}\right)$$

Thus, our total outside electric field is $\boxed{\vec{E}_{out} = 0}$

Our inside electric field only depends on the enclosed field E_ρ, so $\boxed{\vec{E}_{in} = -\frac{kr}{\varepsilon_0}\hat{r}}$

chapter 9 P.11)



The bound charge for the system will be $\sigma_b = P \cdot \hat{n}$. We can let $\vec{P} = P\hat{z}$ and $\hat{n}$ be outward from the z-axis in the $\hat{r}$ direction (the sides of the cylinder).

<u>Sides:</u>

$\sigma_b = (P\hat{z})\cdot(\hat{r}) \rightarrow \hat{z}$ and $\hat{r}$ are perpendicular $\longrightarrow \sigma_b = 0$

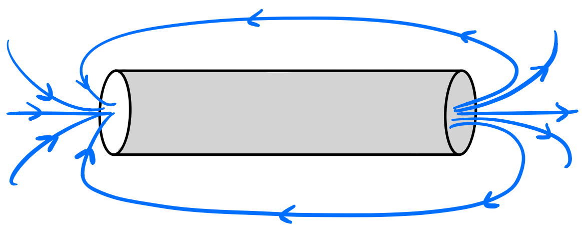
<u>Top:</u>

$\sigma_b = (P\hat{z})\cdot(\hat{z}) = P \qquad (\hat{z}\cdot\hat{z} = 1)$

<u>Bottom:</u>

$\sigma_b = (P\hat{z})\cdot(-\hat{z}) = -P$

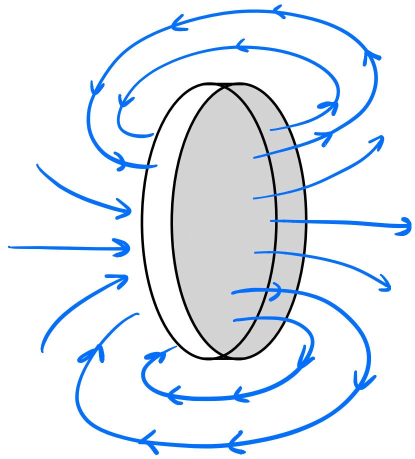
(i) $L \gg a$



The ends appear like point charges while the system acts like a dipole.

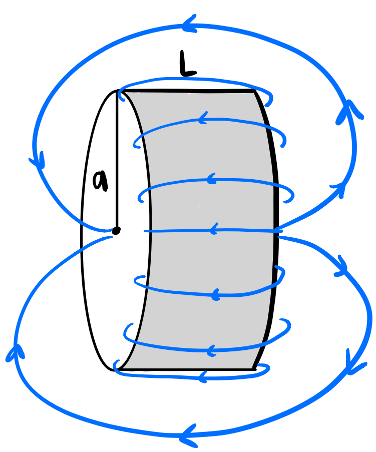
Chapter 4 P.11 cont.)

(ii) $L \ll a$



The system acts like two parallel plates - like a parallel plate capacitor

(iii) $L \approx a$



I don't really know how to describe this one.

Chapter 4 P.12)

Calculate the potential of a uniformly polarized sphere.

Equation 4.9 gives us an equation for potential

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{P(\vec{r}')\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau' \longrightarrow V(\vec{r}) = \frac{P(\vec{r}')}{4\pi\varepsilon_0}\int_V \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

Now, we know that the equation for the electric field is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho\hat{r}}{r^2}\,d\tau$$

From this, we can recognize that our potential expression houses our $\vec{E}$ field inside it.

$$V(\vec{r}) = \frac{P(\vec{r}')}{\rho}\left(\frac{1}{4\pi\varepsilon_0}\int\frac{\rho\hat{r}}{r^2}\,d\tau\right) \longrightarrow V(\vec{r}) = P(\vec{r}')\left(\frac{E(\vec{r})}{\rho}\right)$$

Chapter 4 P.12 cont.)

Plugging in evaluated E, we get

$$V(\vec{r}) = \vec{P}\left(\frac{1}{\rho}\right)\cdot\left(\frac{1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi R^3\right)\frac{\rho}{r^2}\hat{r}\right) \quad \text{for } r > R$$

$$V(\vec{r}) = \vec{P}\left(\frac{1}{\rho}\right)\cdot\left(\frac{1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi R^3\right)\frac{\rho}{R^3}\hat{r}\right) \quad \text{for } r < R$$

Simplifying, we get

$$V(\vec{r}) = \frac{R^3(\vec{P}\cdot\hat{r})}{3\varepsilon_0 r^2} \quad \text{for } r > R$$

$$V(\vec{r}) = \frac{\vec{P}\cdot\hat{r}}{3\varepsilon_0} \quad \text{for } r < R$$

Complete the dot product to get

$$\boxed{V(\vec{r}) = \frac{R^3(P\cos\theta)}{3\varepsilon_0 r^2} \text{ for } r > R \;;\; V(\vec{r}) = \frac{P\cos\theta\, r}{3\varepsilon_0} \text{ for } r < R}$$

Other Problem #1)

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \quad ; \quad \vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$$

a) $\nabla\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right) = i\vec{k}\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

LHS: $\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$$\frac{\partial}{\partial x}\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{x} + \frac{\partial}{\partial y}\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{y} + \frac{\partial}{\partial z}\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{z}$$

$$\left(ik_x\,e^{(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{x} + \left(ik_y\,e^{(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{y} + \left(ik_z\,e^{(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{z}$$

$$i\left(k_x\hat{x} + k_y\hat{y} + k_z\hat{z}\right)e^{(\vec{k}\cdot\vec{r}-\omega t)} \longrightarrow i(\vec{k})\,e^{(\vec{k}\cdot\vec{r}-\omega t)}$$

b) $\nabla\cdot\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}\right) = i\vec{k}\,\hat{n}\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

LHS: $\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}\right)$

$$\frac{\partial}{\partial x}\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}\right)\hat{x} + \frac{\partial}{\partial y}\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}\right)\hat{y} + \frac{\partial}{\partial z}\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}\right)\hat{z}$$

$$\left(ik_x\,e^{(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}\right)\hat{x} + \left(ik_y\,e^{(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}\right)\hat{y} + \left(ik_z\,e^{(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}\right)\hat{z}$$

$$i\left(k_x\hat{x} + k_y\hat{y} + k_z\hat{z}\right)e^{(\vec{k}\cdot\vec{r}-\omega t)}\hat{n} \longrightarrow i(\vec{k})\,\hat{n}\,e^{(\vec{k}\cdot\vec{r}-\omega t)}$$

Other Problem #1 Cont.)

c) $\nabla \times (e^{i(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}) = i\vec{k}\times\hat{n}e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

Cross product rules

LHS: $e^{i(\vec{k}\cdot\vec{r}-\omega t)}(\vec{\nabla}\times\hat{n}) \rightarrow (\nabla\cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)})\times\hat{n}$

We can compute $\nabla\cdot(e^{i(\vec{k}\cdot\vec{r}-\omega t)})$

$$\left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)$$

$$\frac{\partial}{\partial x}\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{x}+\frac{\partial}{\partial y}\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{y}+\frac{\partial}{\partial z}\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{z}$$

$$\left(ik_x e^{(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{x}+\left(ik_y e^{(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{y}+\left(ik_z e^{(\vec{k}\cdot\vec{r}-\omega t)}\right)\hat{z}$$

$$i(k_x\hat{x}+k_y\hat{y}+k_z\hat{z})e^{(\vec{k}\cdot\vec{r}-\omega t)} \rightarrow i(\vec{k})e^{(\vec{k}\cdot\vec{r}-\omega t)}$$

Now we plug this back into our equation

$$(\nabla\cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)})\times\hat{n} \longrightarrow \left(i\vec{k}e^{(\vec{k}\cdot\vec{r}-\omega t)}\right)\times\hat{n}$$

Pull out non-vectors, $i\vec{k}\times\hat{n}e^{(\vec{k}\cdot\vec{r}-\omega t)}$ ✓

Other Problem #2)

Maxwell's equations in a vacuum read

$$\text{(i)}\ \nabla\cdot E = 0 \qquad \text{(iii)}\ \nabla\times E = -\frac{\partial B}{\partial t}$$

$$\text{(ii)}\ \nabla\cdot B = 0 \qquad \text{(iv)}\ \nabla\times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t^2}$$

(Given by equation 9.40 in Griffiths)

So, Ampere's Law and Faraday's law tell us that the changing $\vec{E}$ field induces a $\vec{B}$ field, and a changing $\vec{B}$ field induces an $\vec{E}$ field. Therefore, we would expect them both to change continually for a moving electromagnetic wave. And, In our video we see the $\vec{B}$ and $\vec{E}$ field change continually and synchronously as the wave moves. Further, curl gives you a perpendicular vector, and we can see that $\vec{E}\perp\vec{B}$.