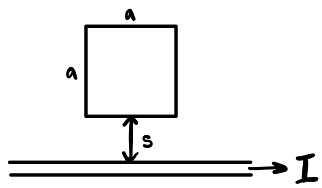
E & M Homework #5

Blake Evans

Chapter 7 P.8)



a) To find the flux through the loop, we can use the equation

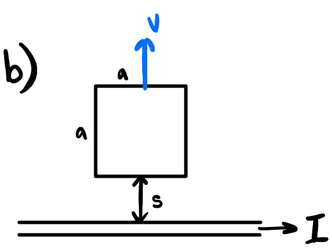
$\Phi_B = \int \vec{B} \cdot d\vec{a}$ where $\vec{B} = \frac{\mu_0}{2\pi} \frac{I}{(s)} \hat{\phi}$ and $da = a\,ds$

Plugging this all in, we can find that

$$\Phi_B = \int_a^{a+s} \left(\frac{\mu_0}{2\pi} \frac{I}{(s)}\right)(a\,ds) \rightarrow \Phi_B = \frac{\mu_0}{2\pi} I \int_a^{a+s} \frac{1}{s} (a\,ds) \longrightarrow \Phi_B = \left(\frac{\mu_0 I a}{2\pi}\right)\left(\ln(s)\right)\Big|_s^{a+s}$$

and finally $\boxed{\Phi_B = \frac{\mu_0 I a}{2\pi}\left(\ln(a+s) - \ln(s)\right)}$

b)



The equation that know for the emf of a system is

$$emf = -\frac{\Delta \Phi_B}{\Delta t}$$

Plugging in the flux from before, we can see that

Chapter 7 P.8 Cont.)

$$\text{emf} = -\frac{\left(\frac{\mu_0 I a}{2\pi}\left(\ln(s+a) - \ln(a)\right)\right)}{(dt)} \rightarrow \text{emf} = -\frac{\mu_0 I a}{2\pi}\left[\frac{d}{dt}\left(\ln(s+a)\right) - \frac{d}{dt}\left(\ln(a)\right)\right]$$

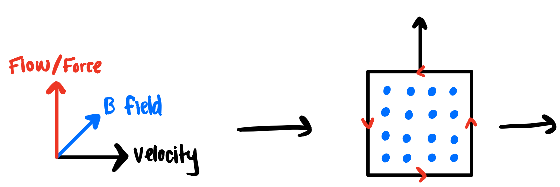
as time evolves, we know that Δs will change with Δt giving us ds/dt which we can recognise as velocity. So,

$$\text{emf} = -\frac{\mu_0 I a}{2\pi}\left[\left(\frac{ds}{dt}\right)\left(\frac{1}{s+a}\right) - \left(\frac{ds}{dt}\right)\left(\frac{1}{s}\right)\right]$$

$$\text{emf} = -\frac{\mu_0 I a}{2\pi}\left[\frac{v}{s+a} - \frac{v}{s}\right] \rightarrow \text{emf} = -\frac{\mu_0 I a v}{2\pi}\left[\frac{s}{s(s+a)} - \frac{(s+a)}{s(s+a)}\right]$$

$$\text{emf} = -\frac{\mu_0 I a v}{2\pi(s(s+a))}\left[s - s + a\right] \rightarrow \text{emf} = -\frac{\mu_0 I a v\,(a)}{2\pi s(s+a)} \rightarrow \boxed{\text{emf} = -\frac{\mu_0 I a^2 v}{2\pi s(s+a)}}$$

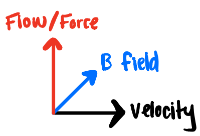
Now, using the Right hand rule we can find which way the current in the loop is flowing.



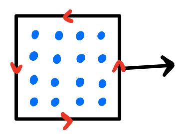
Counter-Clockwise

Chapter 7 P.8 cont.)

c) Since the loop is only traveling right, Δs isn't changing, and therefore the Emf also will not change. We also know that, If we pull the Loop to the right instead of up, the right hand rule will be



$\rightarrow$



$\rightarrow$

$$\boxed{\begin{array}{c}\text{Emf} = 0 \\ \text{Counter-Clockwise}\end{array}}$$

Chapter 7 P.10)

Since Emf depends on magnetic flux, Let's start by finding the flux (Φ_B).

$$\Phi_B = \oint \vec{B} \cdot d\vec{a} \longrightarrow \Phi_B = |\vec{B}| \cos\theta \, |\vec{A}| \quad \text{where } A = (a)(a) = a^2$$

Thus, $\Phi_B = Ba^2 \cos\theta$

Since the loop has an angular velocity of ω, we know that $\theta = \omega t$. We can plug that in.

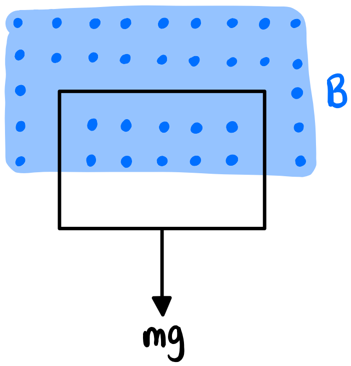
$$\Phi_B = Ba^2 \cos(\omega t)$$

Now all we need to do is differentiate Φ_B with respect to t to solve for the Emf.

$$\text{Emf} = \frac{\Delta\Phi}{dt} \longrightarrow \text{Emf} = \frac{d}{dt}\left(Ba^2 \cos(\omega t)\right)$$

$$\boxed{\text{Emf} = -Ba^2(\omega \sin(\omega t))}$$

Chapter 7 P.11 [Extra Credit])



Due to the motion of the Loop through the magnetic field, we can express the Emf as

$$\mathcal{E}mf = B\ell v$$ where v = velocity of loop

we also can express Emf as $\mathcal{E}mf = IR$. Equating the two together, we can get that

$$\mathcal{E}mf = B\ell v = IR \rightarrow I = \frac{B\ell v}{R}$$

Now, Since the upward magnetic force for the system is written as $F = BI\ell$, we can plug in our current (I).

$$F = BI\ell \rightarrow F = B\left(\frac{B\ell v}{R}\right)\ell \rightarrow F = \frac{B^2\ell^2 v}{R}$$

This is the magnetic force on the loop. To get the total force, we need to add this with the force of gravity on the ring.

Chapter 7 P.11 [Extra Credit] cont.)

$$F_{net} = F_{mag} - F_g \rightarrow (ma) = \left(\frac{B^2\ell^2 v}{R}\right) - (mg)$$

To find v as a function of time (t) we can rewrite "a" as dv/dt and differentiate.

$$\frac{dv}{dt} = \frac{B^2\ell^2 v}{mR} - g \rightarrow dv\left(\frac{1}{g - \frac{B^2\ell^2 v}{mR}}\right) = -(1)dt \rightarrow \int\left(\frac{1}{g - \frac{B^2\ell^2 v}{mR}}\right)dv = -\int dt$$

$$\left(\frac{1}{\frac{B^2\ell^2 v}{mR}}\right)\ln\left(g - \frac{B^2\ell^2 v}{mR}\right) = -t \rightarrow \ln\left(g - \frac{B^2\ell^2 v}{mR}\right) = -\left(\frac{B^2\ell^2 v}{mR}\right)t \rightarrow g - \frac{B^2\ell^2 v}{mR} = e^{-\left(\frac{B^2\ell^2 v}{mR}t\right)}$$

$$\boxed{v(t) = \left(\frac{B^2\ell^2}{mR}\right)g\left(1 - e^{-\left(\frac{B^2\ell^2 v}{mR}\right)t}\right)}$$

Though, the problem asks for terminal velocity. And at terminal velocity we know (by definition) that we aren't accelerating anymore $(a=0)$, so our expression becomes (at terminal velocity)

$$(m(0)) = \left(\frac{B^2\ell^2 v_t}{R}\right) - (mg) \rightarrow mg = \frac{B^2\ell^2 v_t}{R} \rightarrow \boxed{v_t = \frac{mgR}{B^2\ell^2}}$$

Chapter 7 P.11 [Extra Credit] cont.)

So, to find how long it takes to reach 90% terminal velocity we can use the equation $v/v_t = 0.9$, plug in and simplify.

$$0.9 = \frac{v}{v_t} \rightarrow 0.9 = \frac{\left[\cancel{\left(\frac{B^2\ell^2}{mR}\right)g}\left(1 - e^{-\left(\frac{B^2\ell^2 v}{MR}\right)t}\right)\right]}{\cancel{\left(\frac{B^2\ell^2}{MR}\right)\left(\frac{g}{1}\right)}} \rightarrow 0.9 = \left(1 - e^{-\left(\frac{B^2\ell^2}{MR}\right)t}\right)$$

$$-0.1 = -e^{-\left(\frac{B^2\ell^2}{MR}\right)t} \rightarrow \ln(0.1) = -\left(\frac{B^2\ell^2}{MR}\right)t \rightarrow \left(\frac{mR}{B^2\ell^2}\right)\ln\left(\left(\frac{1}{10}\right)^{(-1)}\right) = t_{90\%}$$

$$\boxed{t_{90\%} = \left(\frac{mR}{B^2\ell^2}\right)\ln(10)}$$

If the loop were cut then it is not a closed loop and no interference with the field is made and the loop falls as if it weren't there at all. It would fall at mg.

Chapter 7 P.13 [Extra Credit])

From the book and class, we know that we can find the magnetic field with the equation $\Phi = \int B \cdot dA$. Though, the sides of the loop here lie on a coordinate plane so we can use $A = xy$ and $dA = dx\,dy$. Further, since we are given the field, we can plug that in as well.

$$\Phi = \int B \cdot ds \rightarrow \Phi = \int B \cdot (dx\,dy) \rightarrow \Phi = \iint (ky^3t^2)\,dx\,dy$$

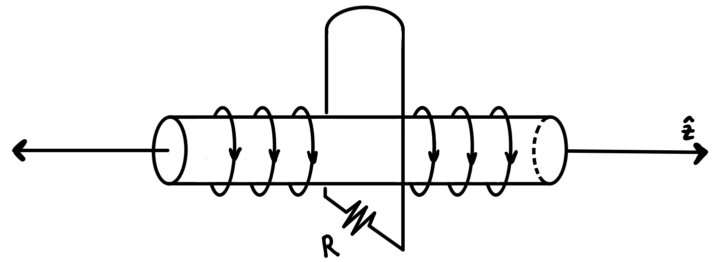
$$\Phi = (kt^2)\iint y^3\,dx\,dy \rightarrow \Phi = (kt^2)(1)\int y^3\,dy \rightarrow \Phi = kt^2\left(\frac{y^5}{4}\right)$$

Now all we need to do is plug this into our Emf equation and solve.

$$\text{Emf} = \frac{\Delta Q}{\Delta t} \rightarrow \text{Emf} = \frac{d}{dt}\left(\frac{kt^2y^5}{4}\right) \rightarrow \boxed{\text{Emf} = -\frac{kty^5}{2}}$$

Chapter 7 P.17)

radius a , n turns



a) The solenoid is increasing at a rate $\frac{dI}{dt} = k$, and we know from class that the field inside it is $B = \mu_0 n I$ in the $\hat{z}$-direction. Plugging this into our expression for magnetic flux Φ_B, we get.

$$\Phi_B = \vec{B} \cdot \vec{A} \rightarrow \Phi_B = (\mu_0 n I)(\pi (a)^2) \quad \text{where } A = \pi r^2 = \pi (a)^2$$

$$\text{meaning that } \Phi_B = \mu_0 n I \pi a^2$$

From class, we know that Emf is a potential voltage, and as such can sometimes replace ΔV in our equations. If we let emf be equal to $\mathcal{E}mf = I_{loop} R$, we can just use our flux (Φ_B) to solve for Emf and then I_{loop}.

$$\mathcal{E}mf = -\frac{\Delta \Phi_B}{\Delta t} \rightarrow -\frac{(\mu_0 n \pi a^2) I}{dt} \rightarrow -(\mu_0 n \pi a^2)\frac{dI}{dt} \rightarrow \mathcal{E}mf = -\mu_0 n \pi a^2 (k)$$

$$\mathcal{E}mf = I_{loop} R \rightarrow (-\mu_0 n \pi a^2 k) = I_{loop} R \rightarrow \boxed{I_{loop} = -\frac{\mu_0 n \pi a^2 k}{R}}$$

Current always opposes flux, so, since B = +, $\boxed{I = - \text{ (counterclockwise)}}$

Chapter 7 P.17 Cont.)

b) Since current hasn't changed, the magnetic field (B) hasn't changed. So our flux equation is still

$$\Phi_B = \mu_0 n I \pi a^2$$

However, since the solenoid is being pulled out to the left, the magetic field becomes negative and our change in flux becomes

$$\Delta\Phi_B = \Delta\Phi_{B_i} - \Delta\Phi_{B_f} \rightarrow \Delta\Phi_B = (\mu_0 n I \pi a^2) - (-\mu_0 n I \pi a^2)$$

$$\Delta\Phi_B = 2\mu_0 n I \pi a^2$$

Now, we are looking for charge Q. We know that current can be written as $\Delta Q/\Delta t$, and that current can also be written as Emf/R (Like before).

$$I = \frac{\Delta Q}{\Delta t} = \frac{Emf}{R} \rightarrow \frac{dQ}{dt} = \left(\frac{1}{R}\right)\left(\frac{\Delta Q}{dt}\right) \rightarrow \Delta Q = \frac{1}{R}\left(2\mu_0 n I \pi a^2\right)$$

$$\boxed{\Delta Q = \frac{2\mu_0 n I \pi a^2}{R}}$$

Chapter 7 P.17 Cont.)

c) Lenz' Law states that $\mathcal{E}mf = -N \frac{\Delta \Phi_B}{\Delta t}$

we still have an expression for flux from before (ϕ_B)

$$\Phi_B = \mu_0 n I \pi a^2$$

If we plug in the current ($I(t)$) expression given to us, we get

$$I(t) = I_0 \cos(\omega t) \rightarrow \Phi_B = \mu_0 n (I_0 \cos(\omega t)) \pi a^2$$

Plugging this into our $\mathcal{E}mf$ expression, we find

$$\mathcal{E}mf = -N \frac{\Delta \Phi_B}{\Delta t} \rightarrow \mathcal{E}mf = -N \frac{d}{dt}\left(\mu_0 n (I_0 \cos(\omega t)) \pi a^2\right)$$

$$\mathcal{E}mf = -N\left(\mu_0 n I_0 \pi a^2 (-\omega \sin(\omega t))\right) \rightarrow \mathcal{E}mf = \mu_0 n N I_0 \pi a^2 \sin(\omega t)$$

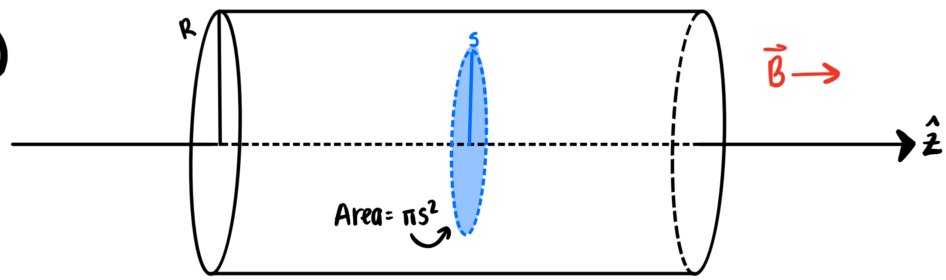
Now, to solve for $I_r(t)$ we set it equal to $\mathcal{E}mf = I_r(t) R$.

$$\left(\mu_0 n N I_0 \pi a^2 \sin(\omega t)\right) = I_r(t) \cdot R \rightarrow \boxed{I_r(t) = \left(\frac{1}{R}\right)\left(\mu_0 n N I_0 \pi a^2 \sin(\omega t)\right)}$$

Therefore, from Lenz's Law we can derive $I_r(t)$.

Other Problem #1)

a)



using the units from the problem, we will let "s" be the radius from the center of the solenoid to where we are measuring the electric field.

To find the electric field in the solenoid, we can start by writing the magnetic field down from class and plugging in our expression for current we were given.

$$B = \mu_0 n I \, \hat{z} \rightarrow \vec{B} = \mu_0 n (Ct^2) \, \hat{z}$$

we can then plug this into our formula for flux.

$$\Phi_B = \oint B \cdot d\vec{a} \rightarrow |B| \cos\theta |A| \rightarrow (\mu_0 n C t^2)(\pi s^2) \cos(0) \rightarrow \Phi_B = \mu_0 n C t^2 (\pi s^2)$$

Finally, to find the electric field, we can use the generalized form of Faraday's Law.

Other Problem #1 cont.)

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt} \rightarrow E(2\pi s) = -\frac{d}{dt}\left(\mu_0 n c t^2 \pi s^2\right)$$

$$E(\cancel{2}\pi s) = -\left(\mu_0 n c(\cancel{2}t)\pi \cancel{s}^2\right) \rightarrow \boxed{E_{in} = -\mu_0 n c t s}\,\hat{\phi} \text{ for } s < R$$

Now, for the electric field outside the solenoid, we know that the magnetic field (B) will just be Ø. Therefore, the only field that contributes to the flux is inside the solenoid. So, using the field (B) before, we find that.

$$\oint \vec{E}_{out}\cdot d\vec{a} = -\frac{d\Phi}{dt} \rightarrow E(2\pi s) = -\frac{d}{dt}\left(\mu_0 n c t^2 \pi R^2\right)$$

$$E_{out}(\cancel{2}\pi s) = -\left(\mu_0 n c(\cancel{2}t)\pi R\right) \longrightarrow \boxed{E_{out} = -\frac{\mu_0 n c t R^2}{s}}\,\hat{\phi}$$

b) We can check to see if this satisfies Fariday's Law by just plugging it in. using the formula from the book, we see (First E_{in})

$$\vec{\nabla}\times\vec{E} = \left[\frac{1}{s}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial E_s}{\partial z} - \frac{\partial E_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sE_\phi) - \frac{\partial E_s}{\partial \phi}\right]\hat{z}$$

In our case, we only need to worry about the $\hat{z}$ term.

Other Problem #1 Cont.)

So, our expression becomes

$$\nabla \times E_{in} = \frac{1}{s}\left[\frac{\partial}{\partial s}(s(-\rho_0 ncts)) - 0\right]\hat{z}$$

$$= \frac{1}{s}\left[\frac{\partial}{\partial s}(-\rho_0 ncts^2)\right]\hat{z}$$

$$= \frac{1}{s}\left[2(-\rho_0 ncts)\right]\hat{z}$$

$$\boxed{\nabla \times E_{in} = -2\rho_0 nct\ \hat{z}}$$

which is the same answer we got before, and hence verified. Moreover, we can do the same thing to the outside E-field (E_{out}).

$$\nabla \times E_{out} = \frac{1}{s}\left[\frac{\partial}{\partial s}\left(\cancel{s}\left(-\frac{\rho_0 nctR^2}{\cancel{s}}\right)\right) - 0\right]\hat{z}$$

Other Problem #1 Cont.)

$$\nabla \times E_{out} = \frac{1}{s}\left[(0)\right] \rightarrow \boxed{\nabla \times E_{out} = 0 \ \hat{z}}$$

Here also, we get the same answer from before in the problem. That we recieved no contributions from outside the solenoid.