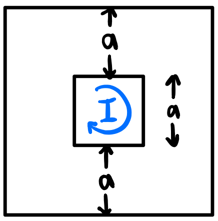
E & M Assignment Six

Blake Evans

Chapter 7 P.23) Find the emf in the loop



Current in the loop is increasing by $dI/dt = k$ where k is a constant.

We need to solve the emf of the system, $\text{emf} = -d\Phi/dt$. Though, we don't know Φ. From the book we know that $\Phi_w = MI$, so we can plug in magnetic field (B) and solve since we know current.

$$\Phi_w = MI = \int_a^{2a} B \cdot da \rightarrow MI = \int_a^{2a} \frac{\mu_0 I}{2\pi s} \cdot (a\, ds)$$

$$MI = \frac{\mu_0 I a}{2\pi} \int_0^{2a} \frac{1}{s}\, ds \longrightarrow MI = \frac{\mu_0 I a}{2\pi} \left[\ln(s)\right]_a^{2a}$$

$$MI = \frac{\mu_0 I a}{2\pi} \ln(2)$$

Now, Φ_w is for a singular wire. And, since our system has 2 wires, we need to double it. Thus,

$$\Phi = 2\Phi_w \rightarrow \Phi = 2\left(\frac{\mu_0 I a}{2\pi} \ln(2)\right) \rightarrow \Phi = \frac{\mu_0 I a}{\pi} \ln(2)$$

Chapter 7 P.23 Cont.)

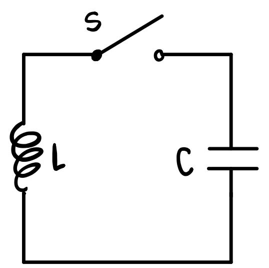
Plugging this into an expression we have for Emf, we get

$$\mathcal{E}mf = -\frac{d\Phi}{dt} \rightarrow \mathcal{E}mf = -\frac{d}{dt}\left(\frac{\mu_0 I a}{\pi}\ln(2)\right) \rightarrow \mathcal{E}mf = -\frac{\mu_0 a}{\pi}\ln(2)\frac{dI}{dt}$$

And since $\frac{dI}{dt} = k$ we can say $\boxed{\mathcal{E}mf = -\frac{\mu_0 a(k)\ln(2)}{\pi}}$

The net flux here is inward and increasing. From Lenz's Law, induced Emf always opposes flux. Thus, the Emf and magnetic field is outward, and the current has to flow Counterclockwise in the big loop.

Chapter 7 P.27)



Find the current in the circuit as a function of time.

The emf across an inductor is $\text{emf} = -L\frac{dI}{dt}$
And we know $I = dQ/dt$, thus we get

$$\text{emf} = -L\left(\frac{dI}{dt}\right) \rightarrow \text{emf} = -L\left[\frac{d}{dt}\left(\frac{dQ}{dt}\right)\right] \rightarrow \text{emf} = -L\left(\frac{d^2Q}{dt^2}\right)$$

Since emf is a potential difference, we can say that $\text{emf} = \Delta V = Q/C$ where Q is capacitor charge, and also

$$\text{emf} = Q/C = -L\left(\frac{d^2Q}{dt^2}\right) \rightarrow \frac{d^2Q}{dt^2} + \left(\frac{1}{CL}\right)Q = 0$$

$(1/CL)$ Looks like angular velocity $(\omega = 1/\sqrt{LC})$,
So we can say

$$\frac{d^2Q}{dt^2} + (\omega)^2 Q = 0$$ ← which is a differential equation

Chapter 7 P.27 Cont.)

we know that the solution to this differential equation will follow the form

$$Q(t) = A\cos(\omega t) + B\sin(\omega t)$$

at $t=0$, $Q(0) = A\cos\left(\frac{1}{\sqrt{LC}}(0)\right) + B\sin\left(\frac{1}{\sqrt{LC}}(0)\right) \rightarrow Q(0) = A$

Since Q is also equal to CV, we can say $A = CV$. Further, $dQ/dt = I$ so we can differentiate and find current.

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left(CV\cos(\omega t) + B\sin(\omega t)\right)$$

$$I(t) = -CV\omega\sin(\omega t) + B\omega\cos(\omega t)$$

$$I(0) = -CV\omega\sin(\omega(0)) + B\omega\cos(\omega(0))$$

$$0 = -(0) + B\omega(1) \rightarrow B = 0$$

Chapter 7 P.27 Cont.)

Thus, plugging everything in, we get

$$\boxed{I(t) = -CV\left(\frac{1}{\sqrt{LC}}\right)\sin\left(\frac{t}{\sqrt{LC}}\right)}$$

If we add a resistor all we need to do to account for it is to add an extra term (IR) into our Emf equation.

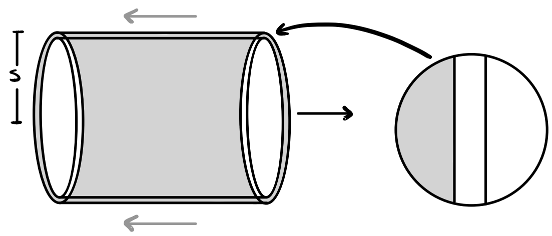
$$\text{Emf} = -L\left(\frac{d^2Q}{dt^2}\right) = \frac{Q}{C} + IR \rightarrow \frac{d^2Q}{dt^2} + \left(\frac{R}{L}\right)\frac{dQ}{dt} + \frac{Q}{LC} = 0$$

Since $Q = CV$, we say $\frac{d^2(CV)}{dt^2} + \left(\frac{R}{L}\right)\frac{d(CV)}{dt} + \frac{(CV)}{LC} = 0$

$$C\left(\frac{d^2V}{dt^2} + \left(\frac{R}{L}\right)\frac{dV}{dt} + \frac{V}{LC}\right) = 0 \rightarrow \frac{d^2V}{dt^2} + \left(\frac{R}{L}\right)\frac{dV}{dt} + \frac{V}{LC} = 0$$

Then we would just solve as normal like before. The only thing that has changed is the addition of the IR term.

Chapter 7 P.30)



Find Self Inductance per unit length.

Let's start off by finding the magnetic field from Ampere's Law.

$$\int B \cdot dl = \mu_0 I_{enc} \rightarrow B(2\pi s) = \mu_0 I_{enc} \rightarrow B = \frac{\mu_0 I_{enc}}{2\pi s}$$

Though, for our system, we have effectively two wires and our enclosed current will be dependant on the ratios between the distance that separates them. Let (l) be the distance to the outermost layer of wire.

$$B = \frac{\mu_0 I_{enc}}{2\pi s} \rightarrow B = \frac{\mu_0 I\left(\frac{s^2}{l^2}\right)}{2\pi s} \rightarrow B = \frac{\mu_0 I s}{2\pi l^2}$$

we can now plug this into the book Eq. 7.35 for work.

$$W = \frac{1}{2\mu_0}\int B^2 d\tau \rightarrow W = \frac{1}{2\mu_0}\int_0^{l}\int_0^{2\pi}\int_0^{r}\left(\frac{\mu_0 I s}{2\pi l^2}\right)^2 s \, dr\, d\phi\, ds$$

Chapter 7 P.30 cont.)

$$W = \frac{\mu_0 I^2}{6\pi^2 \ell^4} \int_0^{\ell} s^2 (2\pi s) r \, ds \rightarrow W = \frac{\mu_0 I^2}{4\pi \ell^4} \left(\frac{s^4 r}{4}\right)\Bigg|_0^{\ell}$$

$$W = \frac{\mu_0 I^2 \cancel{(\ell)^4} r}{16\pi \cancel{\ell^4}} \rightarrow W = \frac{\mu_0 I^2 r}{16\pi}$$

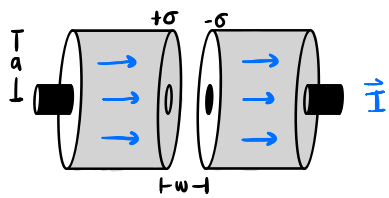
Looking at equation 7.30 from the book, we also know this is equal to $W = \frac{1}{2} L I^2$ and can use it to solve for L.

$$W = \frac{\mu_0 I^2 r}{16\pi} = \left(\frac{1}{2}\right) L I^2 \rightarrow L = \frac{\mu_0 r}{8\pi}$$

Further, we know self inductance to be $\mathcal{L} = L/r$. Using this we can arrive at an answer.

$$\mathcal{L} = \frac{L}{r} \rightarrow \mathcal{L} = \left(\frac{1}{\cancel{r}}\right)\left(\frac{\mu_0 \cancel{r}}{8\pi}\right) \rightarrow \boxed{\mathcal{L} = \frac{\mu_0}{8\pi}}$$

Chapter 7 P. 34)



Find the field in the gap of width $w \ll a$

The current density is $J = I/A$ for the wire. Since the radius of the wire is a, we know the face area will be $A = \pi r^2$ and our current density becomes $J = I/(\pi a^2)$. Given this, we can draw an Amperian loop with an arbitrary radius $\ell < a$. where $I_{enc} = J(\pi \ell^2)$ ←area of loop

$$\int B \cdot d\ell = B(2\pi \ell) = \mu_0 I_{enc} \longrightarrow B(2\pi\ell) = \mu_0 \left(\frac{I}{\pi a^2}\right)(\pi \ell^2)$$

$$B = \frac{\mu_0 I \ell^2}{a^2}\left(\frac{1}{2\pi \ell}\right) \rightarrow B = \frac{\mu_0 I \ell}{2\pi a^2} \rightarrow \boxed{B = \frac{\mu_0 I \ell}{2\pi a^2}\hat{\varphi}}$$

Chapter 7 P.32 [Extra Credit])

a) The hint says to use magnetic dipole and since i'm a lemming, let's write it out.

Eq. 5.88 $$B_{dip}(r) = \nabla \times A = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

$$B_{dip}(r) = \frac{\mu_0}{4\pi}\left(\frac{I}{r^3}\right)[3(m\cdot\hat{r})\hat{r} - m]$$

Okay, cool. This is the magnetic field, and we can put this in terms of flux (ϕ). We also know that $\phi = LI$ where L is inductance. So, because I guess it's a thing we can do, we set $L = M$ and find flux (ϕ) to solve our expression. ($a_2 = A$ for clarity)

$$\phi_B = B\cdot A \rightarrow \phi_B = \left(\frac{\mu_0}{4\pi}\left(\frac{1}{r^3}\right)[3(m\cdot\hat{r})\hat{r} - m]\right)(A)$$

$$(MI) = \frac{\mu_0}{4\pi}\left(\frac{I}{r^3}\right)\left[3(a\cdot\hat{r})A\hat{r} - aA\right]$$

$$\boxed{M = \frac{\mu_0}{4\pi}\left(\frac{1}{r^3}\right)\left[3(a\cdot\hat{r})A\hat{r} - aA\right]}$$

Chapter 7 P.32 [Extra Credit] cont.)

b) So, I read the word Emf and I know what that is. We can write Emf as $\text{Emf} = d\Phi/dt$ and $\Phi = ML$ is Something we used in part a – feels like this is the right way to go.

$$\text{Emf} = \frac{d\Phi}{dt} \rightarrow \text{Emf} = \frac{d}{dt}(MI_2) \rightarrow \text{Emf} = M\frac{d(I_2)}{dt}$$

If we multiply both sides by "I_1" it Sure does look a lot like the equation above 7.30. So let's use that.

$$I_1(\text{Emf}) = M(I_1)\frac{dI_2}{dt} = \frac{dW}{dt} \rightarrow W = MI_1I_2$$

Now plug it all in

$$W = \left(\frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(a\cdot\hat{r})(A\cdot\hat{r}) - aA\right]\right)I_1 \cdot I_2$$

Since $m = Ia$, we can say $\boxed{W = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3m_1\hat{r}(m_2\hat{r}) - m_1m_2\right]}$

Chapter 7 P.32 [Extra Credit] cont.)

Equation 6.35 is $U = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[m_1 m_2 - 3(m_1 \hat{r})(m_2 \hat{r}) \right]$

This is exactly opposite to our answer and thus $U = -W$.

Chapter 7 P.35 [Extra Credit])

a) we know that the equation for an electric field in a capacitor is

$$E = \frac{\sigma(t)}{\varepsilon_0}\hat{z} \quad \text{where } z \text{ is } \perp \text{ to the plates}$$

Since $\sigma(t)$ is charge density, we can rewrite it as charge over area ($^{Q(t)}/_{A}$) where $A = \pi a^2$ (the capacitor does not have a specified radius, so we'll call it a where $a \gg s$).

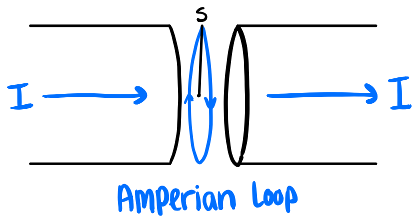
$$E = \frac{\sigma(t)}{\varepsilon_0}\hat{z} \rightarrow E = \frac{\left(\frac{Q(t)}{\pi a^2}\right)}{\varepsilon_0}\hat{z} \rightarrow E = \frac{Q(t)}{(\pi a^2)\varepsilon_0}\hat{z}$$

Now, we just rewrite $Q(t)$ as It ($Q = It$), and bam!

$$E = \frac{(It)}{\pi a^2 \varepsilon_0}\hat{z} \rightarrow \boxed{E = \frac{It}{\pi a^2 \varepsilon_0}\hat{z}} \quad a \gg s$$

Chapter 7 P.35 [Extra Credit] cont.)

b)



First things first. We need to find the current density in the amperian loop $J = \frac{I_{enc}}{A}$

using the area of a circle, we can say

$$J = \frac{I_{enc}}{A} \rightarrow J = \frac{I_{enc}}{(\pi s^2)}$$

we also know that current density takes the form

$$J = \varepsilon_0 \frac{dE}{dt} = \frac{I_{enc}}{(\pi s^2)} \rightarrow I_{enc} = \frac{dE}{dt}(\pi s^2)$$

well, we need an expression for dE/dt, and it so happens that we solved for $E(t)$ in part a. Let's differentiate and substitute.

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{It}{\pi a^2 \varepsilon_0}\right) \rightarrow \frac{dE}{dt} = \frac{I}{\pi a^2 \varepsilon_0} \Rightarrow I_{enc} = \cancel{\varepsilon_0}\left(\frac{I}{\cancel{\pi} a^2 \cancel{\varepsilon_0}}\right)(\pi s^2)$$

Thus the current through the loop is $I_{enc} = I \frac{s^2}{a^2}$

Chapter 7 P.35 [Extra Credit] cont.)

Now that we have I_{enc}, we can substitute it into Ampere's Law.

$$\int B \cdot dl = \mu_0 I_{enc} \rightarrow B(2\pi s) = \mu_0 \left(I \frac{s^2}{a^2}\right)$$

$$B = \frac{\mu_0 I s^2}{(2\pi s) a^2} \rightarrow \boxed{B = \frac{\mu_0 I s}{2\pi a^2} \hat{\phi}}$$

c) Okay, so now the surface current flows radially outward over the left plate. If we let $I(s)$ be the total current crossing over a circular surface radius s, then we know I_{enc} will now be $I_{enc} = [I_{tot} - I(s)]$. And, since (from before) we know that $I_{enc} = I_{tot} \frac{s^2}{a^2}$ we can set them equal and solve for $I(s)$.

$$I_{enc} = [I_{tot} - I(s)] = I_{tot} \frac{s^2}{a^2} \rightarrow I(s) = I_{tot} - I_{tot} \frac{s^2}{a^2}$$

$$I(s) = I_{tot}\left(1 - \frac{s^2}{a^2}\right) \Longrightarrow I_{enc} = I_{tot} - \left[I_{tot}\left(1 - \frac{s^2}{a^2}\right)\right]$$

$$I_{enc} = \cancel{I_{tot}} - \cancel{I_{tot}} - I_{tot} \frac{s^2}{a^2} \rightarrow I_{enc} = I_{tot} \frac{s^2}{a^2}$$

Chapter 7 P.35 [Extra Credit] cont.)

Then, since we know I_{enc} we can plug into Ampere's Law and Solve.

$$\int B \cdot dl = \mu_0 I_{enc} \rightarrow B(2\pi s) = \mu_0 \left(I_{tot} \frac{s^2}{a^2} \right)$$

$$B = \frac{\mu_0 I_{tot} s^2}{(2\pi s) a^2} \rightarrow \boxed{B = \frac{\mu_0 I_{tot} s}{2\pi a^2} \hat{\phi}}$$

Chapter 7 P.36 [Extra Credit])

a) we are given that $E(s,t) = \frac{\mu_0 I_0 w}{2\pi} \sin(wt) \ln\left(\frac{a}{s}\right)\hat{z}$

we can plug this into our current density equation $J_d = \varepsilon_0 \, dE/dt$ for E and solve.

$$J_d = \varepsilon_0 \frac{dE}{dt} \rightarrow J_d = \varepsilon_0 \frac{d}{dt}\left(\frac{\mu_0 I_0 w}{2\pi} \sin(wt) \ln\left(\frac{a}{s}\right)\hat{z}\right)$$

$$J_d = \varepsilon_0 \left(\frac{\mu_0 I_0 w^2}{2\pi} \cos(wt) \ln\left(\frac{a}{s}\right)\hat{z}\right)$$

we know that current can be written $I = I_0 \cos(wt)$. so,

$$\boxed{J_d = \frac{\varepsilon_0 \mu_0 w^2}{2\pi}(I) \ln\left(\frac{a}{s}\right)\hat{z}}$$

b) Integrate it. $I_d = \int J_d \cdot da$

$$I_d = \int \frac{\varepsilon_0 \mu_0 w^2}{2\pi}(I) \ln\left(\frac{a}{s}\right) \cdot da \rightarrow I_d = \frac{\varepsilon_0 \mu_0 w^2}{2\pi} I \int_0^a \int_0^{2\pi s} \ln\left(\frac{a}{s}\right) d\ell \, ds$$

Chapter 7 P.36 [Extra Credit] cont.)

$$I_d = \frac{\varepsilon_0 \mu_0 w^2 I}{\cancel{2\pi}} \int_0^a (\ln(a) - \ln(s))(\cancel{2\pi} s)\, ds$$

$\longrightarrow$ Wolfram Alpha $\longrightarrow$

$$I_d = \varepsilon_0 \mu_0 w^2 I \left[-\frac{s^2 \ln(s)}{2} + \frac{\ln(a) s^2}{2} + \frac{s^2}{4} \right]_0^a$$

$$I_d = \varepsilon_0 \mu_0 w^2 I \left[\cancel{-\frac{(a^2)\ln(a)}{2}} + \cancel{\frac{(a^2)\ln(a)}{2}} + \frac{(a)^4}{4} \right]$$

$$I_d = \varepsilon_0 \mu_0 w^2 I \left[\frac{a^4}{4} \right] \rightarrow \boxed{I_d = \frac{\varepsilon_0 \mu_0 w^2 I a^4}{4}}$$

c) To compare the two, let's divide I_d by I.

$$\frac{I_d}{I} = \left(\frac{\varepsilon_0 \mu_0 w^2 I a^4}{4} \right) \frac{1}{I} \rightarrow \frac{I_d}{I} = \frac{\varepsilon_0 \mu_0 w^2 a^4}{4}$$

we are given that radius $a = 1\,\text{mm}$ $(1 \times 10^{3}\,\text{m})$, $\frac{I_d}{I} = 1\%$ $\left(\frac{1}{100}\right)$, $\varepsilon_0 \mu_0 = 1/c^2$. So we can plug these in.

Chapter 7 P.36 [Extra Credit] cont.)

$$\left(\frac{1}{100}\right)=\left(\frac{1}{c^2}\right)\frac{w^2(1\times10^{-3})^2}{4} \rightarrow \frac{4}{100}=\left(\frac{1}{3\times10^8\,\frac{m}{s}}\right)^2 w^2(10^{-3}\,m)^2$$

$$\frac{4}{100}=w^2(1.11\times10^{-23}) \rightarrow w^2=\frac{4(1.11\times10^{23}\,\frac{1}{s^2})}{100}$$

$$w^2=4.44\times10^{21}\,\frac{1}{s^2} \longrightarrow \boxed{w=6.66\times10^{10}\,\frac{rad}{s}}$$

Also, with the equation $v=\frac{w}{2\pi}$ we see that this equals

$$v=\frac{w}{2\pi} \rightarrow v=\frac{(6.66\times10^{10}\,\frac{rad}{s})}{2\pi} \rightarrow \boxed{v=1.06\times10^{10}\,Hz}$$

chapter 7 P.55 [Extra Credit])

we know that the emf associated with self inductance is emf = $B\ell v$ where

$$emf = B\ell v = -L\frac{dI}{dt} \rightarrow -\frac{B\ell v}{L} = \frac{dI}{dt}$$

Similarly, we know that $F_M = I\ell(B)$ where F_M is also equal to ma ($F=ma$).

$$F_M = I\ell(B) = ma = m\frac{dv}{dt} \longrightarrow BI\ell = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{BI\ell}{m}$$

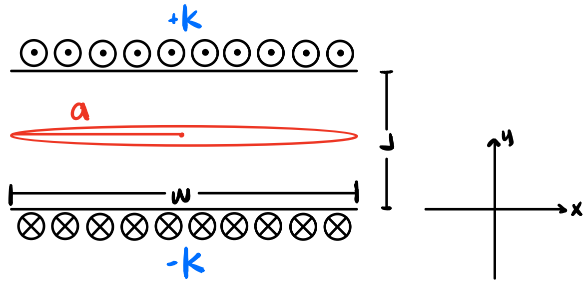
we can differentiate both sides $\left(\frac{d}{dt}\right)$ and plug in the expression from before, to get

$$\frac{d^2v}{dt^2} = \frac{B\ell}{m}\frac{dI}{dt} \rightarrow \frac{d^2v}{dt^2} = \frac{BI\ell}{m}\left(\frac{B\ell v}{-L}\right) \rightarrow \frac{d^2v}{dt^2} = -\frac{B^2\ell^2Iv}{mL}$$

we know $\omega^2 = \frac{B^2\ell^2}{mL} \rightarrow \omega = \frac{B\ell}{\sqrt{mL}}$. so, $\boxed{\frac{d^2v}{dt} = -\omega^2 v}$

Other Problem #1)

a)



According to Eq. 7.30 in the book, the energy stored in a magnetic field is $W = \frac{1}{2}LI^2$. We can start to solve by finding the current density and working from there. Let the radius of our loop be "a" where $a < w$.

$$J = \frac{I_{enc}}{A} \rightarrow J = \frac{I_{enc}}{(\pi a^2)}$$

we also know that current density takes the form

$$J = \varepsilon_0 \frac{dE}{t} = \frac{I_{enc}}{\pi a^2} \rightarrow I_{enc} = \frac{dE}{dt}(\pi a^2)$$

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{It}{\pi a^2 \varepsilon_0}\right) \rightarrow \frac{dE}{dt} = \frac{I}{\pi a^2 \varepsilon_0} \Rightarrow I_{enc} = \cancel{\varepsilon_0}\left(\frac{I4}{\cancel{\pi} a^2 \cancel{\varepsilon_0}}\right)(\cancel{\pi} w^2)$$

Other Problem #1 Cont.)

Thus the current through the loop is $I_{enc} = I\frac{w^2}{a^2}$
Now that we have I_{enc}, we can substitute it into Ampere's Law.

$$\int B \cdot dl = \mu_0 I_{enc} \rightarrow B(2\pi w) = \mu_0 \left(I\frac{w^2}{a^2}\right)$$

$$B = \frac{\mu_0 I w^2}{(2\pi w)a^2} \rightarrow B = \frac{\mu_0 I w}{2\pi a^2}\hat{\phi}$$

we know an expression for emf that puts it in terms of L and Φ. So we can plug in what we found and solve.

$$\text{emf} = \frac{d\Phi}{dt} = -L\frac{dI}{dt} \rightarrow -\frac{Blv}{L} = \frac{dI}{dt}$$

$$\frac{d(B \cdot A)}{dt} = -L\frac{dI}{dt} \rightarrow \left(\frac{\mu_0 I w}{2\pi a^2}\right)(\pi a^2) = -LI$$

If we divide both sides by two, the right hand side looks identical to $W = \frac{1}{2}LI$ from the book.

Other Problem #1 Cont.)

$$\left(\frac{1}{2}\right)\frac{\mu_0 I w}{2} = -LI\left(\frac{1}{2}\right) \rightarrow \frac{W}{I} = -\frac{LI}{2} = \frac{\mu_0 I w}{4}$$

Current is given to us in the problem as $I = \vec{K}$, so we can plug that in. Also, the problem asks for Energy "per unit length," so we need to divide our expression by l.

$$W = \frac{\mu_0 (\vec{K})^2 w}{4} \rightarrow \frac{W}{l} = \frac{\mu_0 \vec{K}^2 w}{4w} \rightarrow \boxed{\frac{W}{l} = \frac{\mu_0 \vec{K}^2}{4}\frac{w}{l}}$$

where l is an arbitrary length variable out of the page.

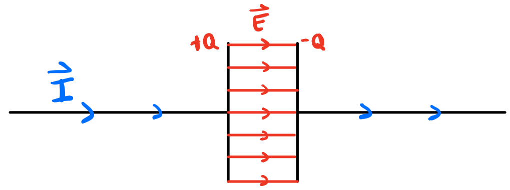
b) Before, we had the equation

$$\left(\frac{\mu_0 I w}{2\pi a^2}\right)(\pi a^2) = -LI \rightarrow \frac{\mu_0 I}{2} = -LI$$

Simplifying, we get $\boxed{L = -\frac{\mu_0}{2}}$

Other Problem #2)

a) Field in a Capacitor



Calculate $\vec{B}$ induced by $\vec{E}$.

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{I} = \frac{Q}{\varepsilon_0 A}\hat{I}$$

$$\int B \cdot dl = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\frac{\partial E}{\partial t} = \frac{d}{dt}\left(\frac{Q(t)}{\varepsilon_0 A}\right) = \frac{dQ(t)}{dt}\left(\frac{1}{\varepsilon_0 A}\right) = \frac{I}{\varepsilon_0 A} \quad ; \quad J = \varepsilon_0\left(\frac{\partial E}{\partial t}\right)$$

Thus,
$$\int B \cdot dl = \mu_0 (I_{enc}) + \mu_0 \int \left(\frac{I}{A}\right) da$$

$$\int B \cdot dl = \mu_0 I_{enc} + \frac{\mu_0 I (\pi s^2)}{A}$$

Since $I_{enc} = 0$ and $\hat{I} = \left(\frac{\hat{\varphi}}{A}\right)$ $\Big\}$ $B(s) = 0 + \frac{\mu_0 I (\pi s^2)}{(2\pi s)}\left(\frac{\hat{\varphi}}{A}\right) \rightarrow B(s) = \frac{\mu_0 I(s)}{2A}\hat{\varphi}$

Now let's write out our given values. $s = 0.5\,m$

Capacitor $V = 174\,kV$; $w = 50\,Hz$; $d = 1.5\,m \rightarrow r = 0.75\,m$

Solenoid $N = 813$; diameter $= 4.0\,cm \rightarrow r = 2\,cm$; $\mu_0 \varepsilon_0 = \frac{1}{c^2}$

Other Problem #2)

To simplify things, let's pre assemble some variables

A = Capacitor plate area $= \pi r^2 = \pi(0.75\text{ m})^2 = 0.563\pi \text{ m}^2 = 1.767 \text{ m}^2$

$$V = \frac{Q}{A\varepsilon_0} \rightarrow Q = V(A\varepsilon_0) \rightarrow \frac{dQ}{dt} = I = \frac{d}{dt}(VA\varepsilon_0) \rightarrow I = \frac{dV}{dt}A\varepsilon_0$$

Now we can plug everything in.

$$B(s) = \frac{\mu_0 I(s)}{2A}\hat{\varphi} \rightarrow B = \frac{\mu_0\left(\frac{dV}{dt}\cancel{A}\varepsilon_0\right)(0.5\text{ m})}{2\cancel{A}} \rightarrow B = \frac{(\mu_0\varepsilon_0)\frac{dV}{dt}(0.5\text{ m})}{2}$$

$$B = \left(\frac{dV}{dt}\right)\frac{\left(\frac{1}{2}\right)}{c^2 2} \rightarrow B = \left(\frac{1}{4c^2}\right)\frac{dV}{dt} \rightarrow B = \left(\frac{1}{4}\right)\left(\frac{V_0\omega\cos(\omega t)}{c^2}\right)$$

$$\vec{B} = \frac{(174\times10^3\text{ V})(50\text{ Hz})\cos(50\text{ Hz}\cdot t)}{4\left(3\times10^8\ \frac{m}{s}\right)^2}\hat{\phi}$$

$$\boxed{\vec{B} = (2.42\times10^{-11}\text{ T})\cos(t\cdot 50\text{ Hz})\,\hat{\phi}}$$

6) Now, we need $\mathcal{E}mf = -M\frac{dI}{dt}$

we can find the mutual inductance from $\Phi = MI$, where $\Phi = B \cdot A$.

$$B(s) = \frac{\mu_0 I(s)}{2A}\hat{\varphi} \rightarrow \Phi = \left(\frac{\mu_0 I(s)}{2(\pi r_{cap}^2)}\right)\left(\pi (r_{sol})^2\right) \text{ where } r_{sol} = 2\text{ cm}$$

$$\Phi = \frac{\mu_0 I(s)(r_{sol})^2}{2 r_{cap}^2} = MI \rightarrow M = \frac{\mu_0 s (r_{sol})^2}{2(r_{cap})^2}\left(\frac{T \cdot m^2}{A}\right)$$

$$\mathcal{E}mf = -\left(\frac{\mu_0 s (r_{sol})^2}{2(r_{cap})^2}\right)\frac{dI}{dt} \rightarrow \mathcal{E}mf = \left(\frac{\mu_0 s (r_{sol})^2}{2(r_{cap})^2}\right)\left(V_0 w \cos(wt)\right)$$

$$\mathcal{E}mf = \frac{\left(4\pi \times 10^{-7}\frac{(T \cdot m)}{A}\right)(0.5\text{ m})\left(2 \times 10^{-2}\text{ m}\right)^2}{2(0.75\text{ m})^2}\left[(175 \times 10^3 V)(50 Hz)\cos(t \cdot 50 Hz)\right]$$

$$\boxed{\mathcal{E}mf = 1.955 \times 10^{-3}\cos(t \cdot 50 Hz)\ \frac{T \cdot m^2 \cdot V}{A \cdot s}} \text{ or } H\left(\frac{V}{s}\right) \text{ or } \frac{V^2}{A}$$

$$\left(\frac{T \cdot m^2}{A}\right)\frac{V}{s} = H\left(\frac{V}{s}\right) \rightarrow \frac{H}{s}(V) = \frac{\left(\frac{V \cdot s}{A}\right)}{s}(V) \rightarrow \frac{(V)}{A}(V) = \frac{V^2}{A}$$

Theo Rex CPA
Cold Blooded on
Your Taxes!