E & E Homework #4

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① Bottle in a Bottle 2

a) How many molecules of gas does the large bottle contain?

$PV = Nk_BT$ where N is number of molecules

N is a unitless quantity, so our answer's units should cancel, leaving us with just a quantity.

Given in the problem is $P = 106\ Pa$, $T = 300K$, $V = 0.001\ m^3$.
Plugging in, we get

$$(106\,Pa)(0.001\,m^3) = N\left(1.38\times10^{-23}\,\frac{J}{K}\right)(300K)$$

$$N = \frac{\left(106\left(\frac{N}{m^2}\right)\right)(0.001\,m^3)}{\left(1.38\times10^{-23}(N\cdot m)\left(\frac{1}{K}\right)\right)(300K)} \rightarrow N = \frac{(0.106\ N\cdot m)}{(4.14\times10^{-21}\ N\cdot m)}$$

$\boxed{N = 2.56\times10^{19}}$ ← No units, checks out.

① cont. Bottle in a Bottle 2

b) Compute $\int \frac{dQ}{T}$ and find ΔS.

Since the work done in the system is $\emptyset$, we can say that

$$\Delta U = Q + W \rightarrow \Delta U = Q + (\emptyset) \rightarrow \Delta U = Q$$

$$Q = \frac{3}{2} N k_B T \qquad (1)$$

Plugging in initial values, we get.

$$Q_i = \frac{3}{2}(2.50 \times 10^{19})(1.38 \times 10^{-23}\ J/K)(300K) \rightarrow Q_i = 0.159\ J$$

So, $$S_i = \frac{Q_i}{T_i} \rightarrow S_i = \frac{(0.159J)}{(300K)} \rightarrow S_i = 5.3 \times 10^{-4}$$

Now we need to find these values of the system in the final state. Since the bottles are insulated T_i will be equal to T_f, and since the system is sealed, N_i will also be equal to N_f

Using equation (1) we can solve.

① Bottle in a Bottle 2 cont.

$$Q_f = \frac{3}{2} N k_B T \rightarrow Q_f = \frac{3}{2}(2.56\times10^{19})(1.38\times10^{-23} \tfrac{J}{\cancel{K}})(300\cancel{K})$$

$$Q_f = 0.159 \text{ J}$$

So, $S_f = \dfrac{Q_f}{T_f} \rightarrow S_f = \dfrac{(0.159\text{ J})}{(300\text{ K})} \rightarrow S_i = 5.3\times10^{-4}$

Now plugging into our general formula,

$$\Delta S = \int \frac{dQ}{T} \rightarrow \frac{Q_f}{T_f} - \frac{Q_i}{T_i} \rightarrow \boxed{\Delta S = 0}$$

we get that entropy is 0. Also, here's the pressure in the big bottle because I solved for it, realized I didn't need it, and couldn't bring myself to erase it.

$$PV = N k_B T \rightarrow (0.01 \text{ m}^3) P = (2.56\times10^{19})(1.38\times10^{-23} \tfrac{J}{\cancel{K}})(300\cancel{K})$$

$$P = \frac{(0.106 \text{ J})}{(0.01 \text{ m}^3)} \rightarrow P = \frac{0.106 \text{ N}\cdot\cancel{\text{m}}}{0.01 \text{ m}^3} \rightarrow P = 10.6 \text{ Pa}$$

(1) Bottle in a Bottle 2 Cont.

c) Okay, sense making time. Does it make sense to get $\emptyset$ entropy? Yes, because our system is reversible. According to the second Law, reversible processes will leave the entropy of the system unchanged. Therefore, $\emptyset$ is an acceptable answer for the ΔS of our system.

② Free Expansion

$$U = U(T,N) \quad ; \quad PV = Nk_BT$$

a) The change in entropy of the gas is positive. I know this because the system is free expansion and free expansion is irreversible. According to the second law an irreversible process has a $\Delta S > 0$. Therefore, our system must have non-zero positive entropy.

Let our system be an isothermal process with the same initial conditions, we can then say that

$$\Delta U = W + Q \text{ , } \Delta U = 0 \rightarrow W = -Q$$

Thus, $\quad W = -P\,dV \rightarrow -Q = -P\,dV \rightarrow Q = P\,dV$

Then, $\Delta S = \int \frac{(P\,dV)}{T} \rightarrow P = \frac{Nk_BT}{V} \rightarrow \Delta S = \int_{V_0}^{2V_0} \frac{\left(\frac{Nk_B\cancel{T}}{V}\right)dV}{\cancel{T}}$

Positive

$$\Delta S = Nk_B \int_{V_0}^{2V_0} \left(\frac{1}{V}\right) dV \rightarrow \Delta S = Nk_B(\ln(2V_0 - V_0)) \rightarrow \boxed{\Delta S = Nk_B \ln(V_0)}$$

Since ΔS is a state function, we know this will be equal to the change in entropy we are looking for.

(2) Free Expansion cont.

b) Since we are given that our system is adiabatic, we know that $Q = 0$. we also know that no work is done (given). Therefore we can say that

$$\Delta U = Q + W \rightarrow \Delta U = (0) + (0) \rightarrow \Delta U = 0$$

we know that $U = \frac{3}{2} N k_B T$ for an ideal gas, so

$$(0) = \frac{3}{2} N k_B T \rightarrow T = \frac{0}{\left(\frac{3}{2} N k_B T\right)} \rightarrow \boxed{T = 0}$$

3 Melting Ice Lab Questions

a) Okay, so there's No work done on the system because it's isolated ($W_{tot}=0$). There's also no change in heat because the system is insulated ($Q_{tot}=0$). So using the First Law, our equation becomes

$$\Delta U = Q + W.$$

$$\Delta U_{tot} = Q_{tot} + W_{tot} \rightarrow \Delta U = (0)+(0) \rightarrow \Delta U = 0$$

which shows that the total energy of the system does not change.

Since $Q_{tot}=0$, we know that $Q_{ice} + Q_{water} = 0$ because $Q_{tot} = Q_{ice} + Q_{water}$. From our previous homework we can recognize that

$$Q_{ice} + Q_{water} = 0 \rightarrow (-mL_f) + (mC\Delta T) = 0$$

As, ice is changing phase and water is changing temp

Thus, $Q_{ice} = -mL_f$ and $Q_{water} = mC\Delta T$

③ Melting Ice Lab Questions Cont.

Now, using the values we are given in the problem, we can plug in and solve for ΔS_{water}

$$C = 4.18\,\frac{J}{gK},\quad T_{water} = 40^\circ C,\quad mass = 205g$$

$$T_f = (273.15K) = 0^\circ C \qquad T_i = (312.15\,K) = 40^\circ C$$

$$\Delta S_{water} = \int \frac{dQ_{water}}{T_f} \rightarrow \Delta S_{water} = \int_{T_i}^{T_f} \frac{(mC\,dT)}{T} = \int_{313.15}^{273.15} \frac{mC}{T}\,dT$$

$$\Delta S_{water} = (205\cancel{g})(4.18\,\frac{J}{\cancel{g}\cdot K})\ln\left(\frac{273.15}{313.15}\right) \rightarrow \boxed{\Delta S_{water} = -117.1\ \frac{J}{K}}$$

b) To solve for the change in entropy of ice we can just plug our expression for Q_{ice} ($m_{elt} L_f$) into our ΔS expression and solve. However, we don't know what m_{melt} is, so we need to find that first. we can do this by writing an equation for the energy of the melting process.

$$\Delta U_{melt} = U_{Ice \rightarrow water} - U_{water(40^\circ) \rightarrow water(0^\circ)} \rightarrow U_{melt} = U_{Ice} + U_{water}$$

③ Melting Ice Lab Questions Cont.

From the First law of Thermo we know that

$$U_{Ice} = Q_{ice} + W \quad \text{and} \quad U_{water} = Q_{water} + W$$

Though, the work done in the system is Ø, so

$$U_{Ice} = Q_{ice} \quad \text{and} \quad U_{water} = Q_{water}$$

Plugging into ΔU_{melt}, we get

$$\Delta U_{melt} = (m_{ice} L_f) - (m_{water} C \Delta T)$$

$$\Delta U_{melt} = \left[(131\cancel{g})(333 \tfrac{J}{\cancel{g}})\right] - \left[(205\cancel{g})(4.18 \tfrac{J}{\cancel{g \cdot k}})(40\cancel{k})\right]$$

$$\Delta U_{melt} = (43623\ J) - (34276\ J)$$

$$\Delta U_{melt} = 9347\ J$$

This number is the energy required to melt the remaining ice after the system cools down.

(3) Melting Ice Lab Questions Cont.

This means that 43623 J - 9347 J = 34276 J is the amount of energy used in melting the ice. So, to find out how much ice is left we can plug this into $U_{ice} = m_{melt} L_f$ and solve for mass.

$$U_{ice} = m_{melt} L_f \rightarrow (34276\ J) = m_{melt}(333\ J/g) \rightarrow m_{melt} = 102.93g$$

Now, with this known, we can plug everything into our change in entropy expression and solve.

Latent Heat = $333\ J/g$, $m_{melt} = 102.93g$

$$\Delta S_{ice} = \int \frac{dQ_{ice}}{T_f} \rightarrow \Delta S_{ice} = \int \frac{(m_{melt} L_f)}{T_f} \rightarrow \Delta S_{ice} = \frac{(102.93\ g)(333\ \frac{J}{g})}{(273.15K)}$$

$$\boxed{\Delta S_{ICE} = 125.48\ J/K}$$

③ Melting Ice Lab Questions Cont.

c) Moreover, we can check this by checking the equation $\Delta S_{tot} = \Delta S_{ICE} + \Delta S_{water}$ with the answers we got from parts a and b.

$$\Delta S_{ICE} = 125.48\ J/K \quad , \quad \Delta S_{water} = -117.1\ \frac{J}{K}$$

$$\Delta S_{tot} = \Delta S_{ice} + \Delta S_{water}$$

$$\Delta S = (125.48\ J/K) + (-117.1\ \frac{J}{K})$$

$$\boxed{\Delta S_{tot} = 8.4\ J/K}$$

4 PDM Elevator

1 Big ball = 12 small ball

b = mass of small ball

	Left Ball	Right Ball	L_x	R_x	F_L	F_R	W_L	W_R	
1	0	0	38	38	12b(g)	0	[12b(g)]38	0	
2	1	8	38	40.2	13b(g)	8b(g)	[13b(g)]38	8b(g)(40.2)	
3	2	11	38	41	14b(g)	11b(g)	[14b(g)]38	11b(g)(41)	
4	3	13	38	41.4	15b(g)	13b(g)	[15b(g)]38	13b(g)(41.4)	
5	4	15	38	42	16b(g)	15b(g)	[16b(g)]38	15b(g)(42)	
Cat is now in the elevator									
6	4	17	37.5	42.5	16b(g)	17b(g)	[16b(g)]37.5	17b(g)(42.5)	
7	4	19	36.7	43.5	16b(g)	19b(g)	[16b(g)]36.7	19b(g)(43.5)	
8	4	21	36.4	44	16b(g)	21b(g)	[16b(g)]36.4	21b(g)(44)	
9	4	23	35.6	45	16b(g)	23b(g)	[16b(g)]35.6	23b(g)(45)	
10	4	25	34.7	46.5	16b(g)	25b(g)	[16b(g)]34.7	25b(g)(46.5)	
cat is at the top of the elevator									ΔW_{tot}
11	3	23	34.7	45.5	15b(g)	23b(g)	[15b(g)]34.7	23b(g)(45.5)	
12	2	21	34.7	45	14b(g)	21b(g)	[14b(g)]34.7	21b(g)(45)	
13	1	19	34.7	44.7	13b(g)	19b(g)	[13b(g)]34.7	19b(g)(44.7)	
14	0	17	34.7	44.5	12b(g)	17b(g)	[12b(g)]34.7	17b(g)(44.5)	

cat is out at top of elevator

④ PDM Elevator Cont.

15	0	15	35.6	43.5	12b(g)	15b(g)	[12b(g)]35.6	15b(g)(43.5)	
16	0	13	36.3	42.5	12b(g)	13b(g)	[12b(g)]36.3	13b(g)(42.5)	
17	0	11	37	41.7	12b(g)	11b(g)	[12b(g)] 37	11b(g)(41.7)	
18	0	9	37.3	41	12b(g)	9b(g)	[12b(g)]37.3	9b(g)(41)	
19	0	7	37.5	40.2	12b(g)	7b(g)	[12b(g)]37.5	7b(g)(40.2)	
20	0	5	37.7	39.5	12b(g)	5b(g)	[12b(g)]37.7	5b(g)(39.5)	
21	0	3	37.9	39	12b(g)	3b(g)	[12b(g)]37.9	3b(g)(39)	
22	0	0	38	38	12b(g)	0	[12b(g)] 38	0	

The elevator has returned

b) To calculate the work done on each elevator side, I just calculated $F = mg$, since no other force is acting on the system except gravity. Though, since we don't know the mass of the balls, everything is multiplied by "b" which is representing the mass of a small ball. Then, to calculate work, since we know that $W = F \cdot d$, I simply multiplied all the forces by the distances travelled. I didn't mark it down (lack of space), but all the forces and work should be negative since the force is pointing (physically) downward.

$\Delta W_{2tot_L} = -4bg(38)$ $\Delta Wtot_R = 15bg(42)$

4) PDM Elevator Cont.

c) So, To find the total energy we can just sum up all the work done at each part of the process. So,

Cat Loading: $\sum W_L = [12b(g)]38 + [13b(g)]38 + [14b(g)]38 + [15b(g)]38 + [16b(g)]38$

$\sum W_L = [70b(g)]38 = 2660b(g) = \Delta E_L \ (kg \cdot \frac{m^2}{s^2})$

$\sum W_R = 0 + 8b(g)(40.2) + 11b(g)(41) + 13b(g)(41.4) + 15b(g)(42)$

$\sum W_R = (321.6)b(g) + (451)b(g) + (538.2)b(g) + (630)b(g)$

$\sum W_R = 1940.8b(g) = \Delta E_R \ (kg \cdot \frac{m^2}{s^2})$

Top Cat: $\sum W_L = [16b(g)]37.5 + [16b(g)]36.7 + [16b(g)]36.4 + [16b(g)]35.6 + [16b(g)]34.7$

$\sum W_L = [16b(g)](180.9) = 2894.4b(g) = \Delta E_L \ (kg \cdot \frac{m^2}{s^2})$

$\sum W_R = 17b(g)(42.5) + 19b(g)(43.5) + 21b(g)(44) + 23b(g)(45) + 25b(g)(46.5)$

$\sum W_R = 722.5b(g) + 826.5b(g) + 924b(g) + 1035b(g) + 1162.5b(g)$

$\sum W_R = 4670.5\, b(g) = \Delta E_R \ (kg \cdot \frac{m^2}{s^2})$

Cat Unload: $\sum W_L = [15b(g)]34.7 + [14b(g)]34.7 + [13b(g)]34.7 + [12b(g)]34.7$

$\sum W_L = b(g)[520.5 + 485.8 + 451.1 + 416.4]$

$\sum W_L = (1873.8)b(g) = \Delta E_L \ (kg \cdot \frac{m^2}{s^2})$

4 PDM Elevator Cont.

$$\sum W_R = 23b(g)(45.5) + 21b(g)(45) + 19b(g)(44.7) + 17b(g)(44.5)$$

$$\sum W_R = b(g)[1046.5 + 945 + 849.3 + 756.5]$$

$$\sum W_R = 2655.4\, b(g) = \Delta E_R \quad (kg \cdot \tfrac{m^2}{s^2})$$

No Cat:

$$\sum W_L = [12b(g)]35.6 + [12b(g)]36.3 + [12b(g)]37 + [12b(g)]37.3 + [12b(g)]37.5 + [12b(g)]37.7 + [12b(g)]37.9 + [12b(g)]38$$

$$\sum W_L = [12b(g)](297.3) = 3567.6\, b(g) = \Delta E_L \; (J)$$

$$\sum W_R = 15b(g)(43.5) + 13b(g)(42.5) + 11b(g)(41.7) + 9b(g)(41) + 7b(g)(40.2) + 5b(g)(39.5) + 3b(g)(39) + 0$$

$$\sum W_R = b(g)[625.5 + 552.5 + 458.7 + 369 + 281.4 + 197.5 + 117]$$

$$\sum W_R = 2601.6\, b(g) = \Delta E_R \; (J)$$

The total energy of the system is going to be mostly potential energy. This is because the internal energy of our balls is pretty low, so all the energy of the system is coming from the adding/subtracting of mass to the elevator.

4) PDM Elevator Cont.)

d) There is a lot of error in the system. The ball masses have error, the ruler we are measuring has error, the friction of the system causes error, etc. How we can account for this is error bars and assigning uncertainty to the known sources of error. i.e. the ruler has 5% error and we add error bars to subsequent graphs containing measurements related to it and allow for error in calculation.