Assignment #6 Blake Evans

Problem #1)

Hamiltonian of electron is $H = H_0 - g L \cdot S$

(1) Find energy eigenstates

(2) Find the number of degenerate states for each energy level.

(3) If at $t=0$ an electron is at 2p state with spin in the positive z direction, Calculate $\langle L_z(t) \rangle = \langle \psi(t) | L_z | \psi(t) \rangle$

(1) $H = H_0 - g L \cdot S \qquad \vec{J} = \vec{L} + \vec{S}$

$$\vec{J} \cdot \vec{J} = (\vec{L} + \vec{S}) \cdot (\vec{L} + \vec{S}) = (L^2 + S^2 + 2 L^2 \cdot S^2)$$

$$\vec{L} \cdot \vec{S} = (J^2 - L^2 - S^2)\, ^1\!/_2$$

$$\langle \vec{L} \cdot \vec{S} \rangle = [J(J+1) - L(L+1) - S(S+1)]\, ^{\hbar^2}\!/_2$$

$$\langle H \rangle = (H_0 - g L \cdot S)$$

$$\langle H \rangle = \frac{E_0}{n^2} - \frac{g\hbar^2}{2}[J(J+1) - L(L+1) - S(S+1)]$$

From this we can see that our hamiltonian has a dependency on variables: n, J, L (J is dependant on m because Bo said so). And we can say our eigenstate is

$$\boxed{|n,J,L,m\rangle}$$

(2) Looking at our eigenstate, our Hamiltonian has a dependancy on all the variables but m. So, for every m value, we have a different eigenvalue and thus will have $m = 2j+1$ degenerate states.

$$\boxed{2j+1 \text{ degenerate States}}$$

(3) $\psi(0) = |2,1,1\rangle \longrightarrow |n=2, \ell=1, m_\ell=1, \uparrow\rangle$

We can re-write this as $|n, J, \ell, J_z = m_\ell + m_s\rangle$ where $m_\ell = 1$, $m_s = 1/2 \rightarrow J_z = 3/2$

Our state is then $|\psi(0)\rangle = \sum_{J,J_z} C_{J,J_z} |2, J, 1, 3/2\rangle$

we know $\ell = 1$ and $s = 1/2$, and $|\ell - s| \leq J \leq |\ell + s|$. meaning $J = 1/2, 3/2$, which makes our state

$$|\psi(0)\rangle = C_{\frac{1}{2},\frac{3}{2}}|2,\tfrac{1}{2},1,3/2\rangle + C_{\frac{3}{2},\frac{3}{2}}|2,\tfrac{3}{2},1,3/2\rangle$$

Now, looking at the table, we see

Table 11.3 Clebsch-Gordan Coefficients for $j_1 = 1$ and $j_2 = \frac{1}{2}$

$j_1 = 1$, $j_2 = \frac{1}{2}$	j	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	m	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
m_1	m_2						
1	$\frac{1}{2}$	1	0	0	0	0	0
1	$-\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	0	0	$\sqrt{\frac{2}{3}}$	0
0	$\frac{1}{2}$	0	$\sqrt{\frac{2}{3}}$	0	0	$-\frac{1}{\sqrt{3}}$	0
0	$-\frac{1}{2}$	0	0	$\sqrt{\frac{2}{3}}$	0	0	$\frac{1}{\sqrt{3}}$
-1	$\frac{1}{2}$	0	0	$\frac{1}{\sqrt{3}}$	0	0	$-\sqrt{\frac{2}{3}}$
-1	$-\frac{1}{2}$	0	0	0	1	0	0

and we get

$$|\psi(0)\rangle = (0) + (1)|2,\tfrac{3}{2},1,3/2\rangle$$

which makes time evolving easy, because we can just say that

$$|\psi(t)\rangle = e^{-\frac{iEt}{\hbar}}|2,3/2,1,3/2\rangle$$

Now, we can just use the formula $L_z|\psi\rangle = m_\ell \hbar|\psi\rangle$ to solve for $\langle L_z(t)\rangle$

$$\langle L_z(t)\rangle = \langle\psi(t)|L_z|\psi(t)\rangle$$

$$= \cancelto{1}{\left(e^{\frac{iE_z t}{\hbar}}\right)\left(e^{\frac{-iE_z t}{\hbar}}\right)}\langle 2,\tfrac{3}{2},1,\tfrac{3}{2}|L_z|2,\tfrac{3}{2},1,\tfrac{3}{2}\rangle$$

$$= (1)\langle 2,3/2,1,3/2|L_z|2,3/2,1,3/2\rangle$$

$$= \langle 2,3/2,1,3/2|(1)\hbar|2,3/2,1,3/2\rangle$$

$$= \hbar\,\cancelto{1}{\langle 2,3/2,1,3/2|2,3/2,1,3/2\rangle}$$

$$\boxed{\langle L_z(t)\rangle = \hbar\,(1)}$$