CS 5/7320 Artificial Intelligence

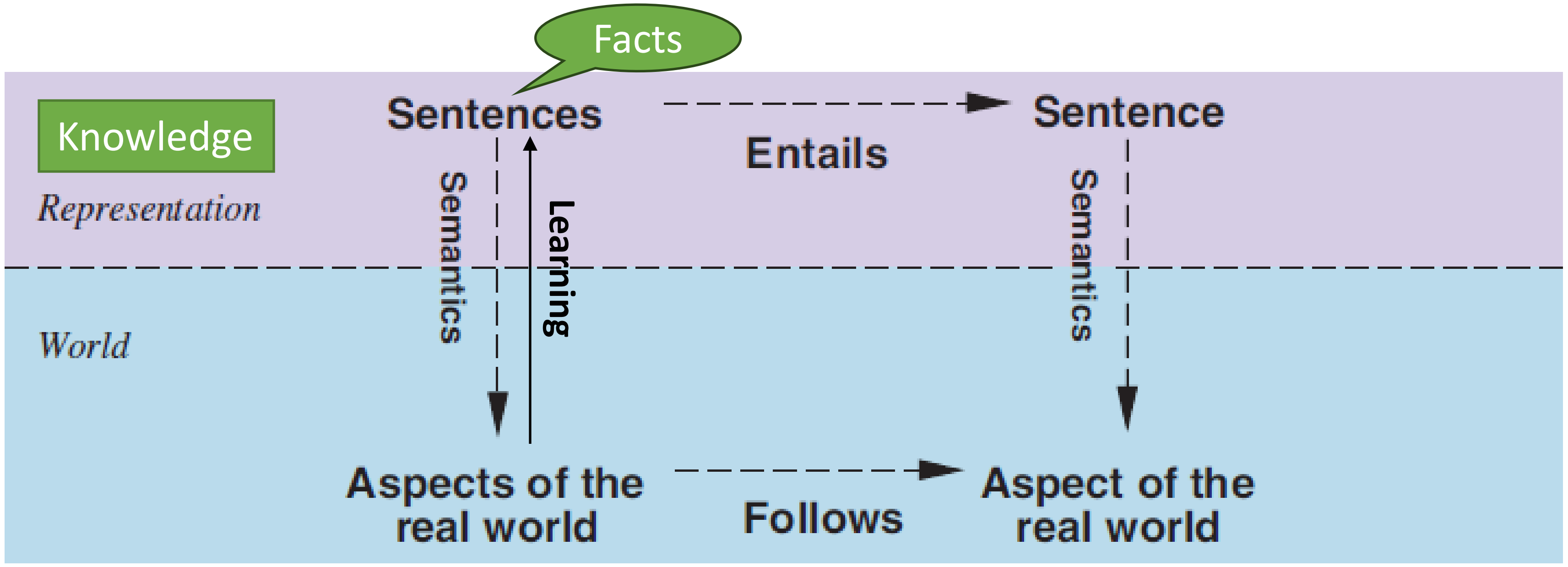
Knowledge-Based Agents
AIMA Chapters 7-9

Slides by Michael Hahsler
based on slides by Svetlana Lazepnik
with figures from the AIMA textbook

Outline

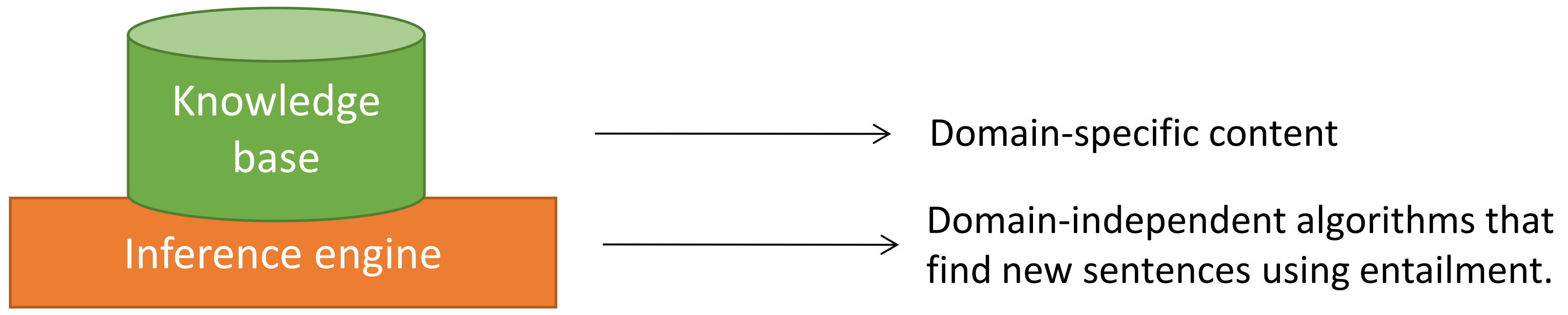
- Knowledge-Based Agents
- Logical Agents
- Large Language Models
- Probabilistic Reasoning

Reality vs. Knowledge Representation



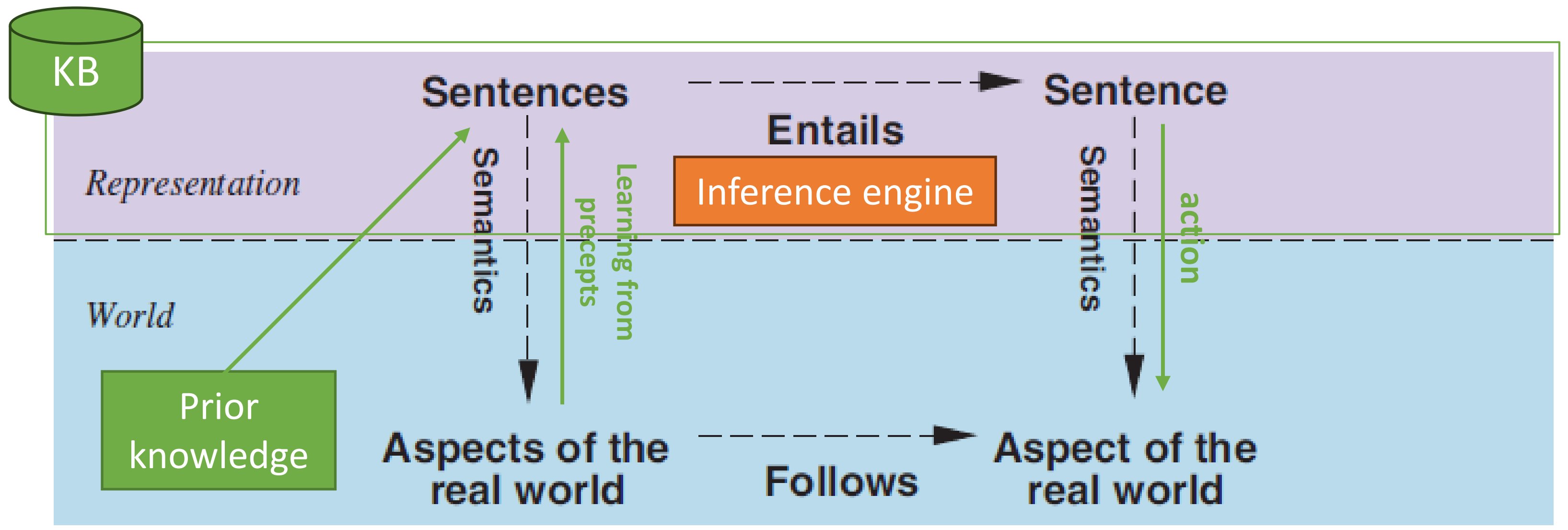
- **Facts:** Sentences we know to be true.
- **Possible worlds**: all worlds/models which are consistent with the facts we know (compare with belief state).
- **Learning** new facts reduces the number of possible worlds.
- **Entailment:** A new sentence logically follows from what we already know.

Knowledge-Based Agents


Knowledge base
Inference engine
Domain-specific content
Domain-independent algorithms that find new sentences using entailment.

- Knowledge base (KB) = **set of facts.** E.g., set of **sentences** in a **formal language** that are known to be true.
- **Declarative** approach to building an agent: Define what it needs to know in its KB.
- **Separation** between data (knowledge) and program (inference).
- Actions are based on knowledge (sentences + inferred sentences) + an **objective function**. E.g., the agent knows the effects of 5 possible actions and chooses the action with the largest utility.

Generic Knowledge-based Agent



```
function KB-AGENT(percept) returns an action
    persistent: KB, a knowledge base
                t, a counter, initially 0, indicating time

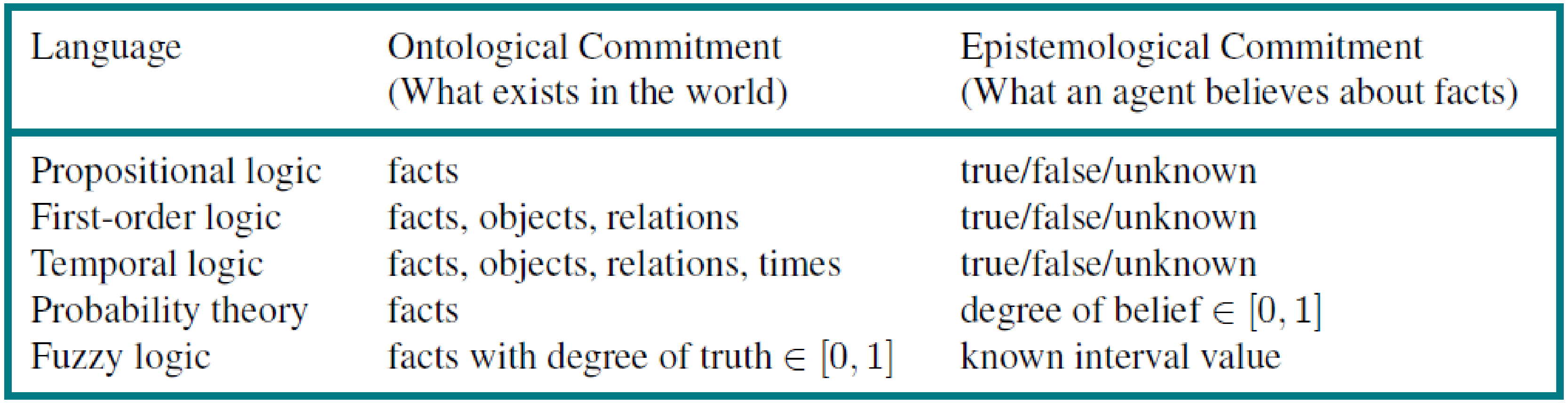
    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

Memorize percept at time t

Ask for logical action given an objective

Record action taken at time t

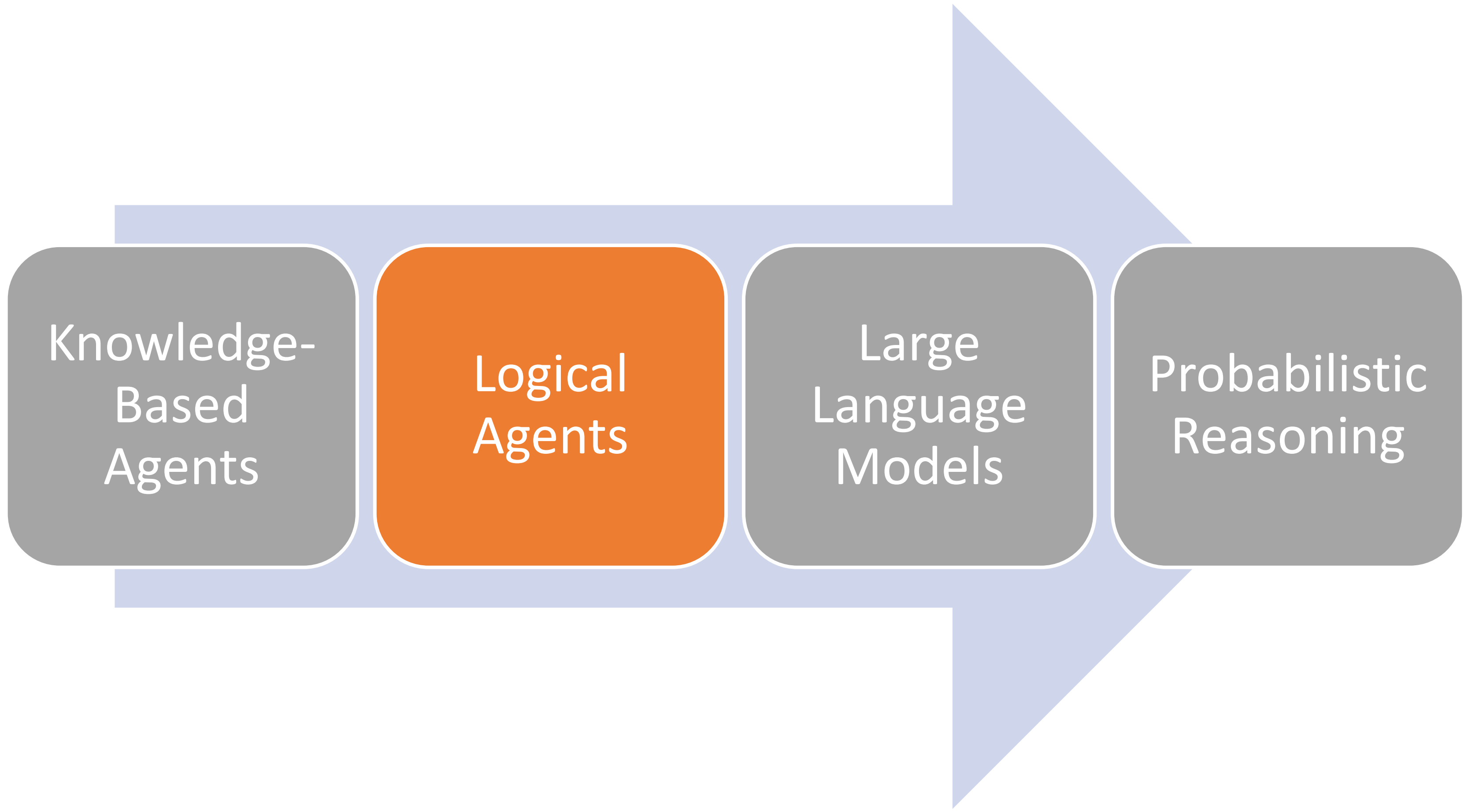
Different Languages to Represent Knowledge



Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

+ Natural Language	word patterns representing facts, objects, relations, …	???

Outline


Knowledge-Based Agents
Logical Agents
Large Language Models
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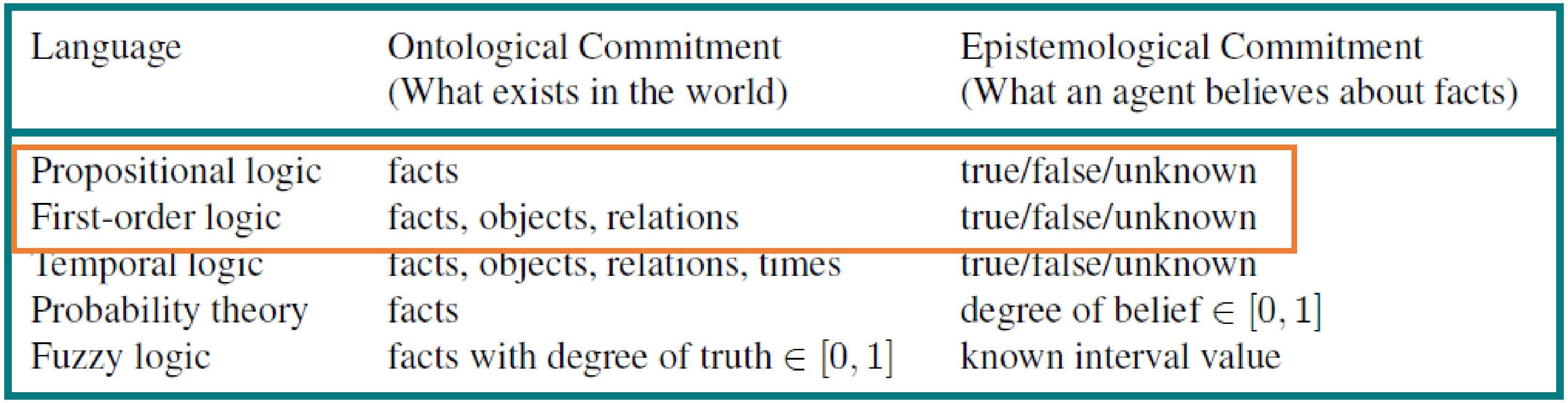
Logical Agents



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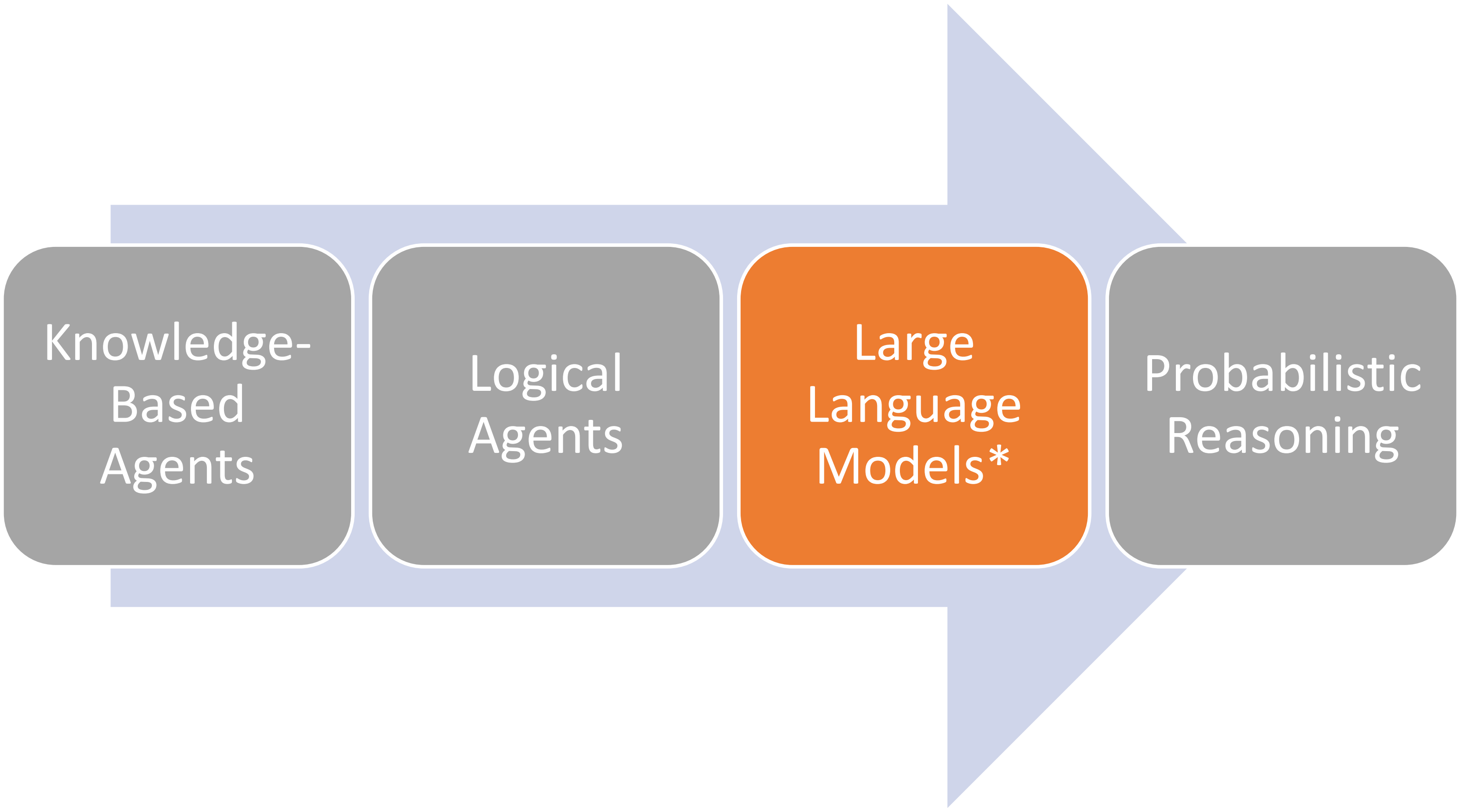
- Facts are logical sentences that are known to be true.
- Inference: Generate new sentences that are entailed by all known sentences.
- Implementation: Typically using Prolog
  - Declarative logic programing language.
  - Runs queries over the program (= the knowledge base)

Issues:

- Inference is computationally very expensive.
- Logic cannot deal with uncertainty.

Outline



*** This is not in the AIMA textbook!**

LLMs - Large Language Models

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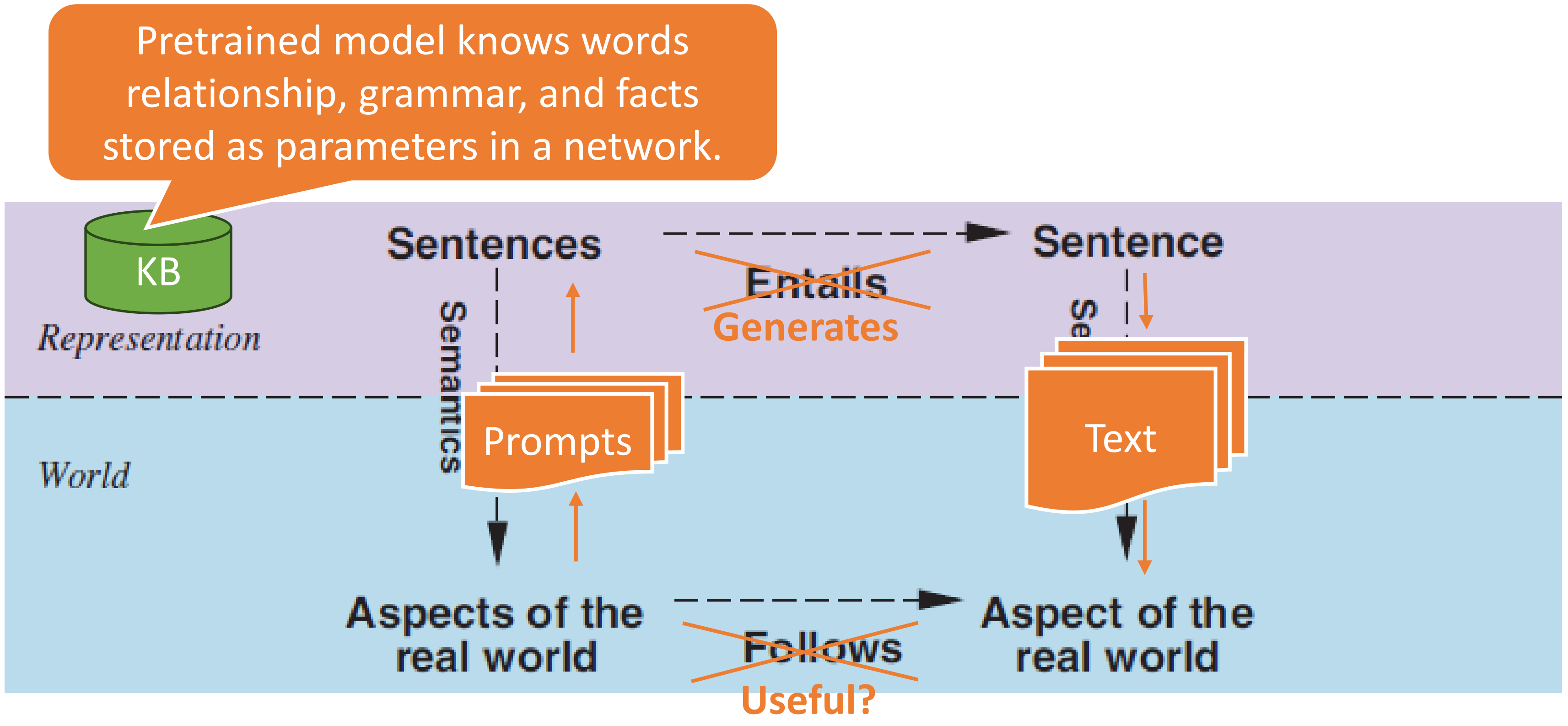
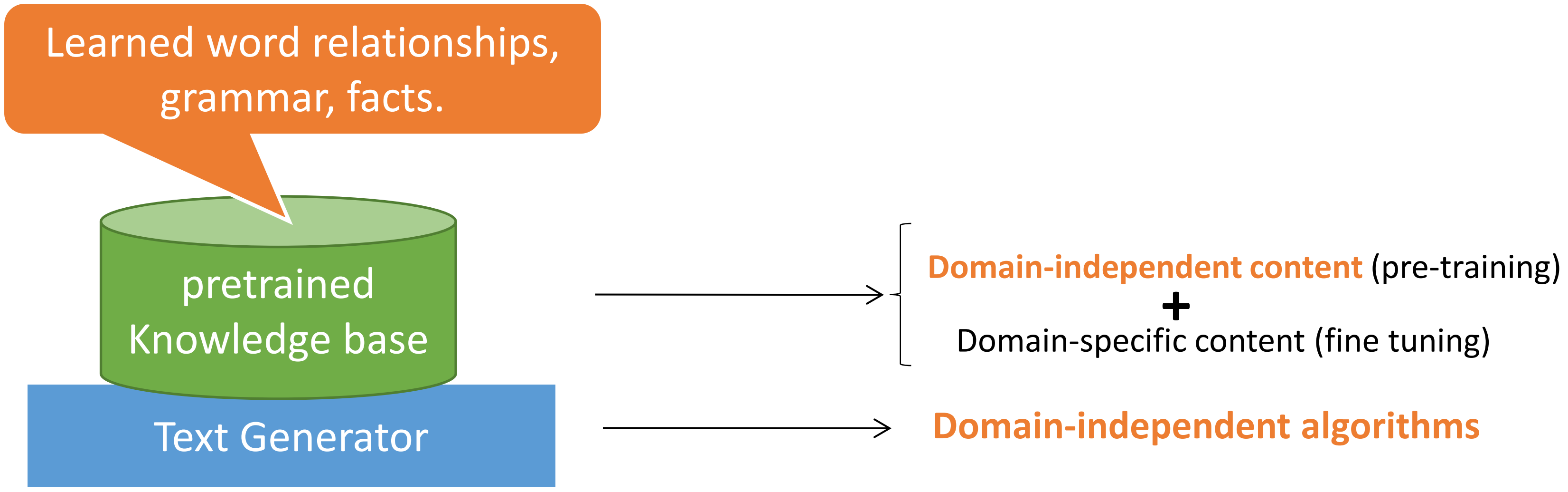
- Store knowledge as parameters in a deep neural networks.

Using Natural Language for Knowledge Representation



- The user formulates a question about the real world as a natural language prompt (a sequence of tokens).
- The LLM generates text using a model representing its knowledge base.
- The text (hopefully) is useful in the real world. The **objective function** is not clear. Maybe it is implied in the prompt?

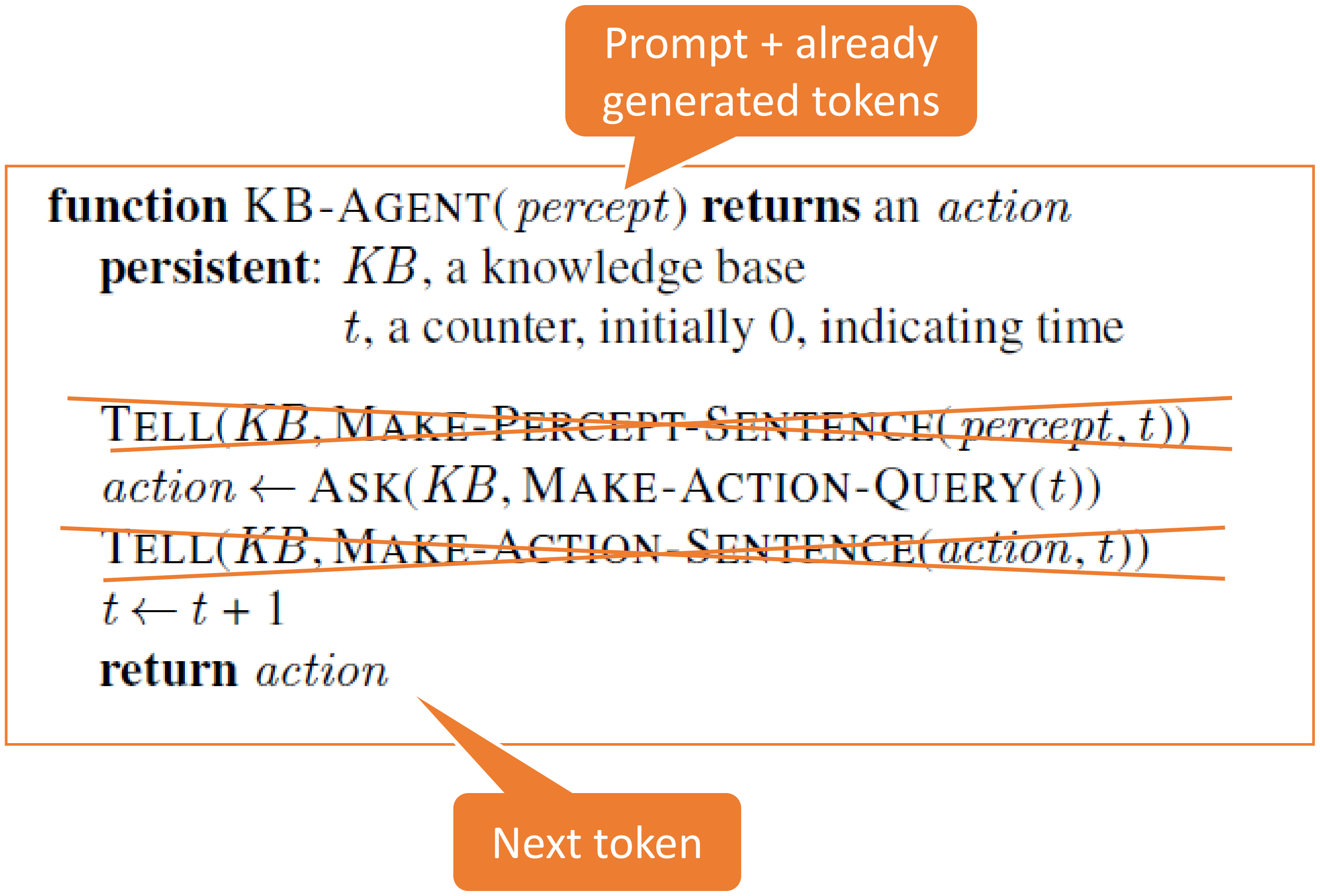
LLM as a Knowledge-Based Agents


Learned word relationships, grammar, facts.
pretrained Knowledge base
Text Generator
Domain-independent content (pre-training)
+
Domain-specific content (fine tuning)
Domain-independent algorithms

Current text generators are:

- Pretrained decoder-only transformer models (e.g., GPT stands for Generative Pre-trained Transformer). The knowledge base is not updated during interactions.
- Tokens are created autoregressively. One token is generated at a time based on all the previous tokens using the transformer attention mechanism.

LLM as a Generic Knowledge-based Agent



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    return action
```

Prompt + already generated tokens
Next token

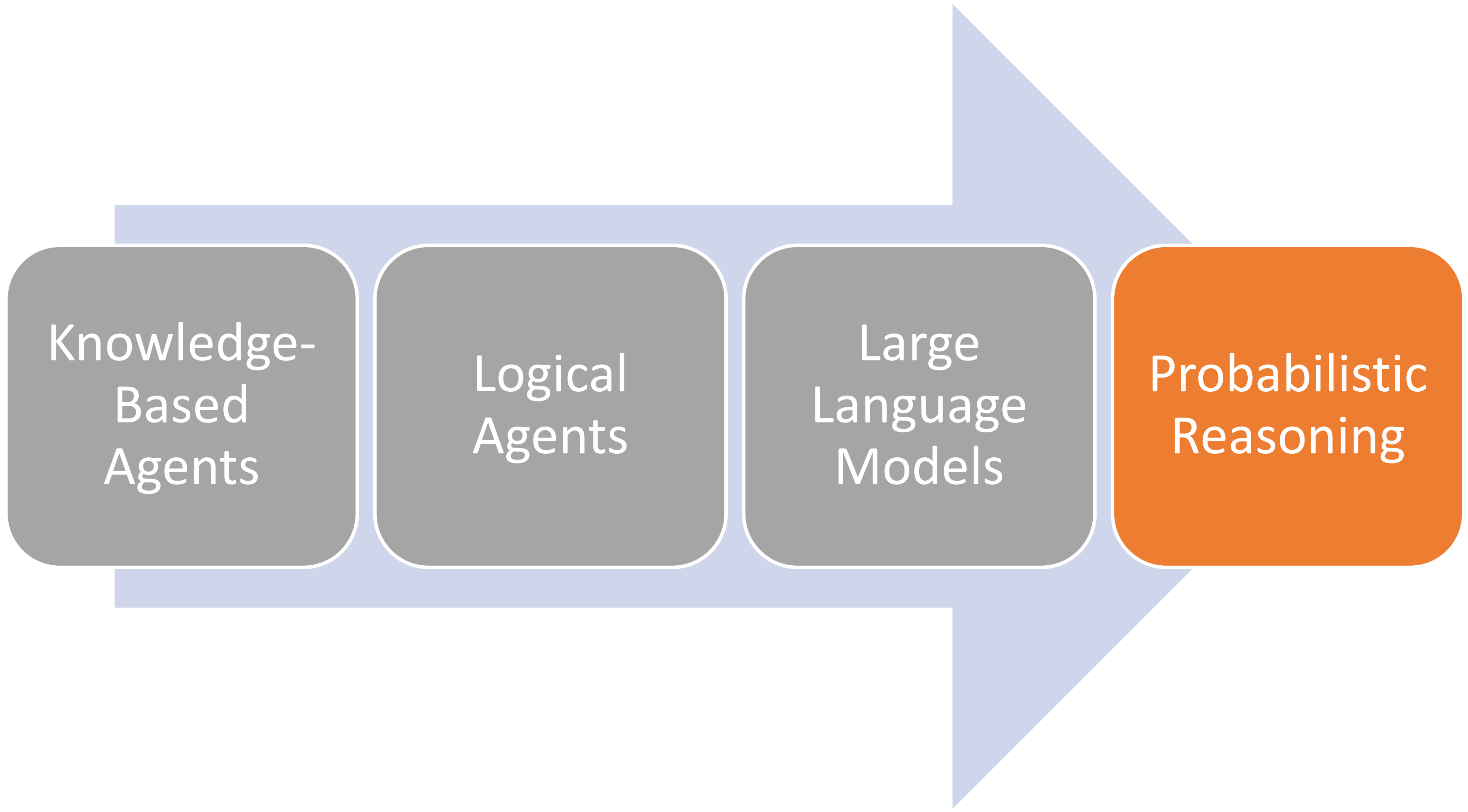
- A chatbot repeatedly calls the agent function till the agent function returns the ‘end’ token.

Many Open Questions about LLMs

- Correlation is not causation: **Can LLMs reason** to solve problems?
- Generative stochasticity leads to **hallucinations**: LLM makes up facts.
- Autoregression is an exponentially **diverging** diffusion process.
- The training data contains **biases**, nonsense and harmful content.
- **Security**: LLM can reveal sensitive information it was trained on.
- **Rights-laundering**: Copyrighted or licensed material can be in the training data.
- Leaky data makes it hard to evaluate true **reasoning performance**.

Reading: [2307.04821] Amplifying Limitations, Harms and Risks of Large Language Models (arxiv.org)

Outline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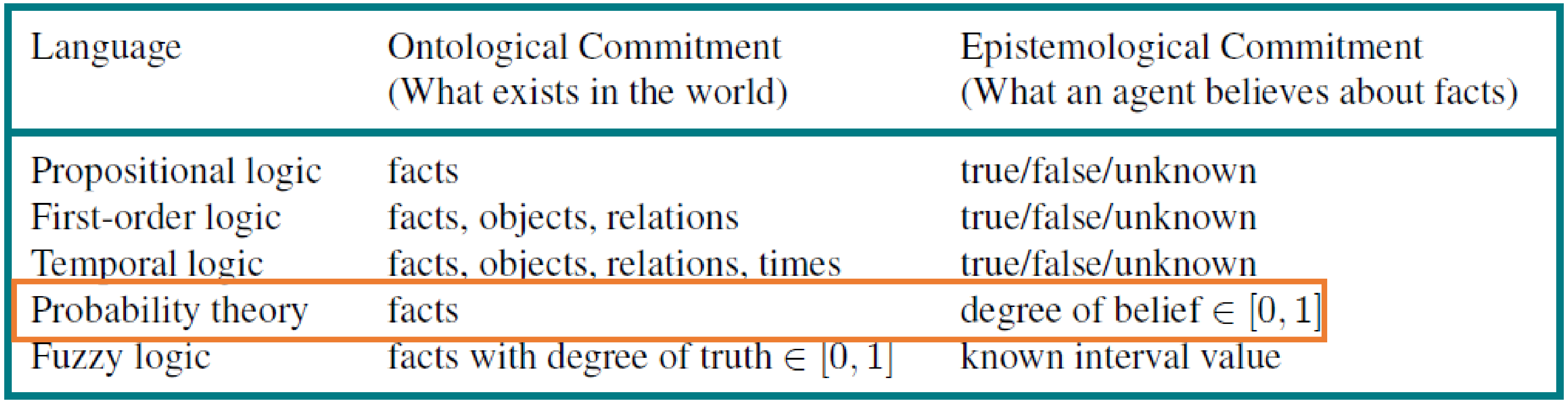
Probabilistic Reasoning



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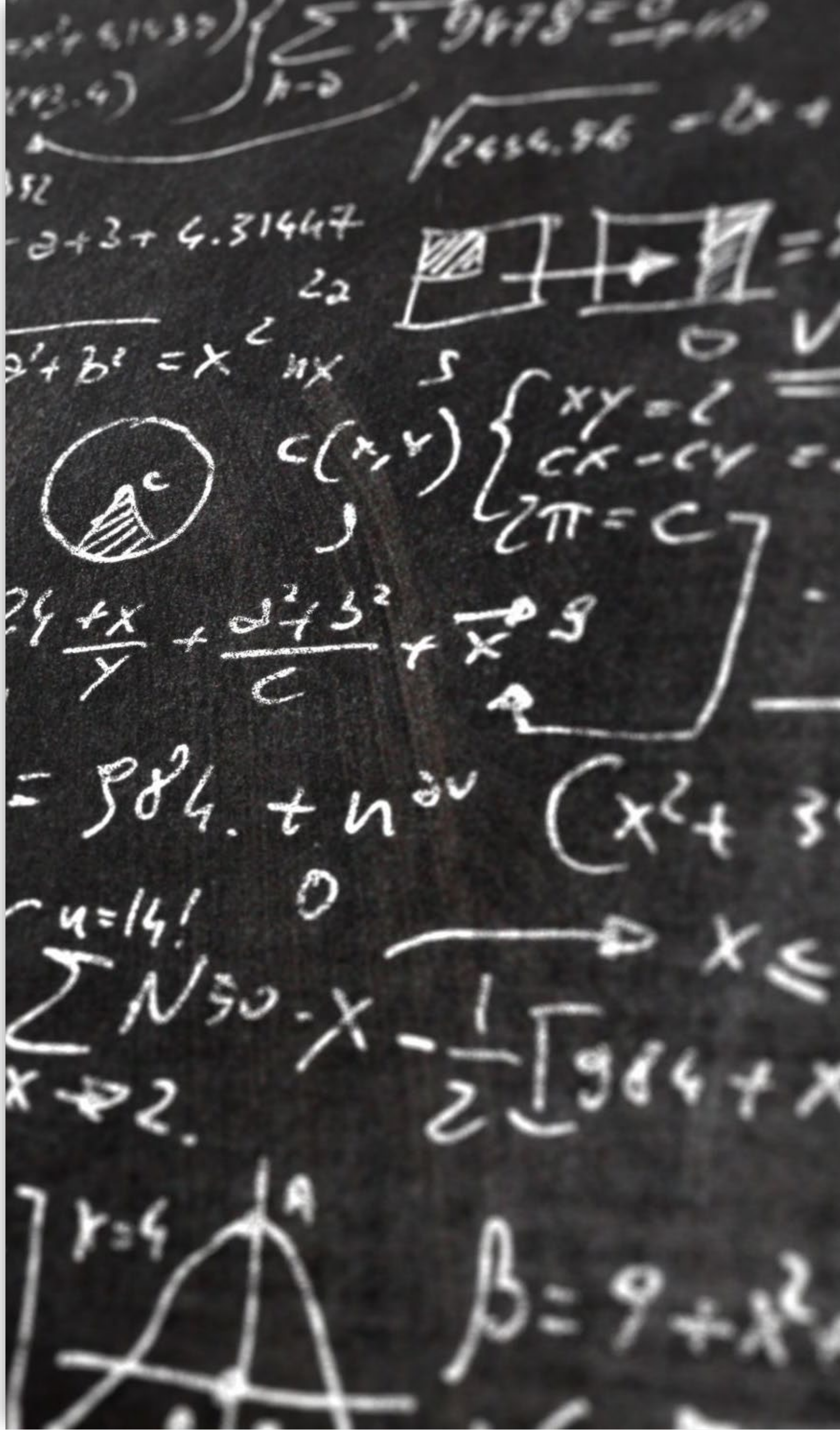
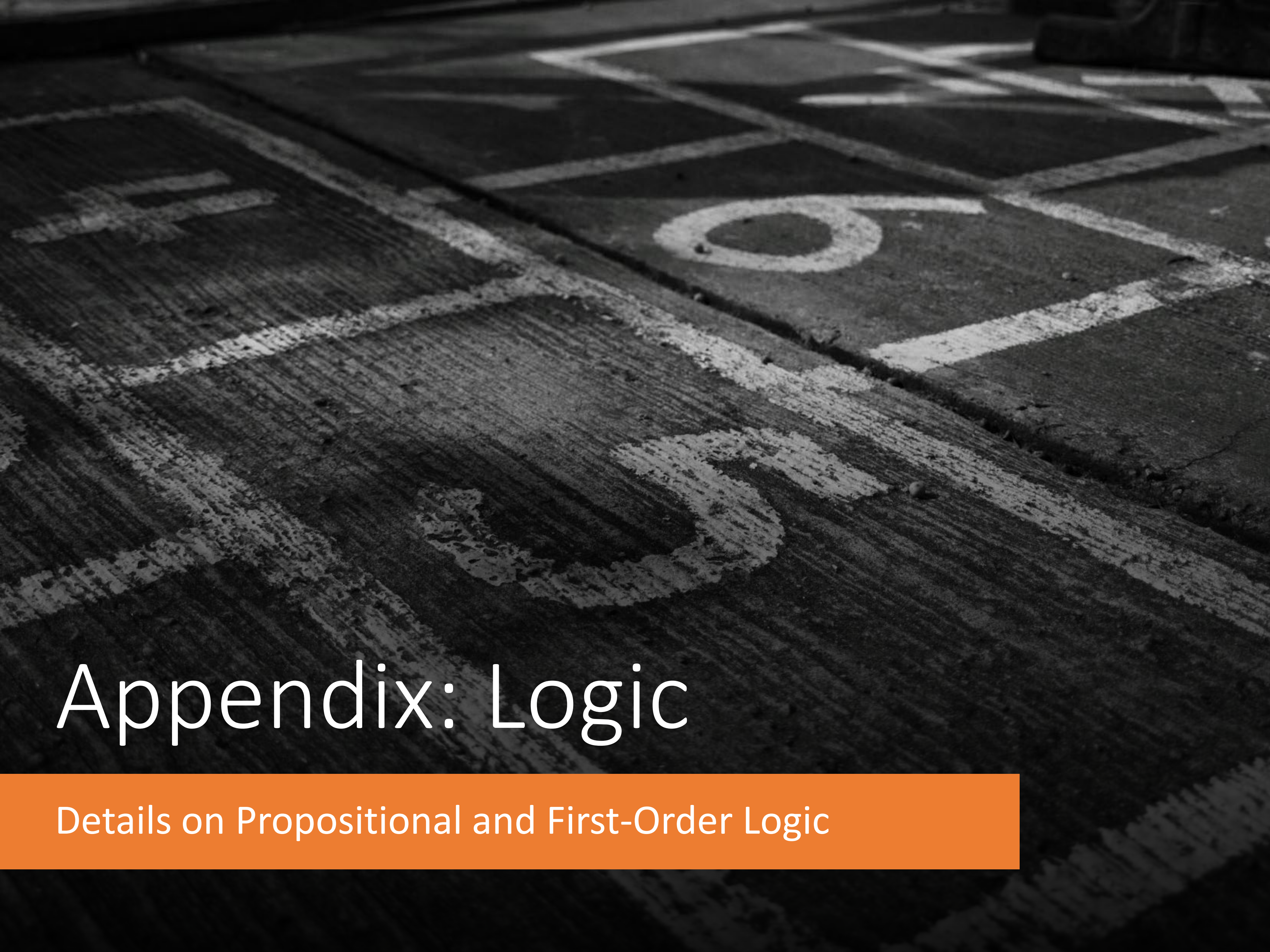
- Replaces true/false with a probability.
- This is the basis for
  - Probabilistic reasoning under uncertainty
  - Decision theory
  - Machine Learning

We will talk about these topics a lot more

Conclusion

- The **clear separation between knowledge and inference engine** is very useful.
- **Pure logic** is often not flexible enough. The fullest realization of knowledge-based agents using logic was in the field of expert systems or knowledge-based systems in the 1970s and 1980s.
- **Pretrained Large Language Models** are an interesting new application of knowledge-based agents based on natural language.
- Next, we will talk about **probability theory** which is the standard language to reason under uncertainty and forms the basis of machine learning.





Appendix: Logic

Details on Propositional and First-Order Logic

Logic to Represent Knowledge

Logic is a formal system for representing and manipulating facts (i.e., knowledge) so that true conclusions may be drawn

Syntax: rules for constructing valid sentences

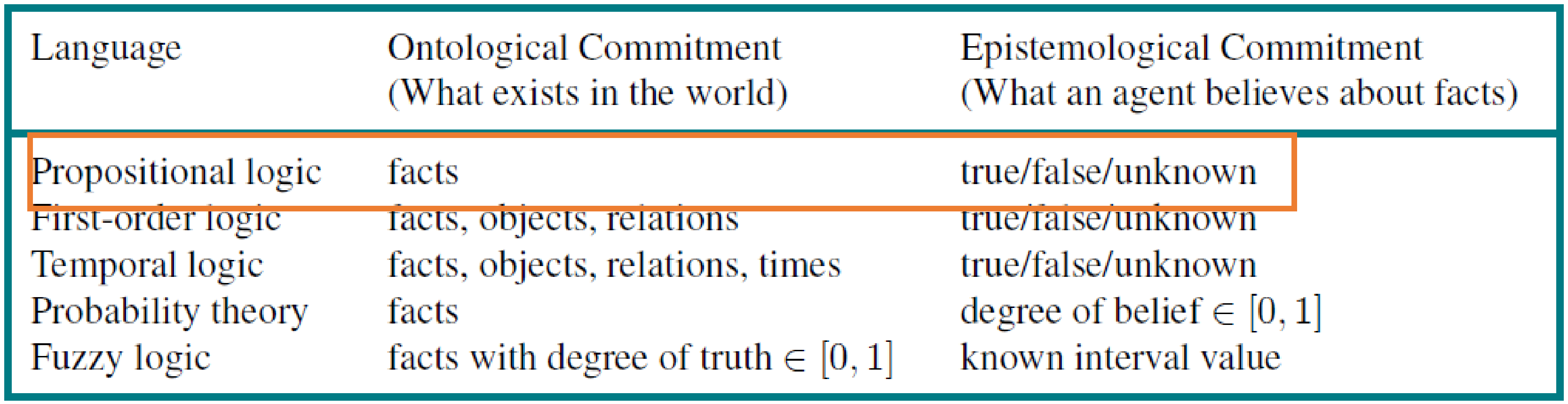
E.g., $x + 2 \geq y$ is a valid arithmetic sentence, $\geq x2y +$ is not

Semantics: "meaning" of sentences, or relationship between logical sentences and the real world

Specifically, semantics defines truth of sentences

E.g., $x + 2 \geq y$ is true in a world where $x = 5$
and $y = 7$

Propositional Logic



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Propositional Logic: Syntax in Backus-Naur Form

$$
\begin{array}{rcll}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} & \\
\mathit{AtomicSentence} & \rightarrow & \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots & \text{= Symbols} \\
\mathit{ComplexSentence} & \rightarrow & (\ \mathit{Sentence}\ ) & \\
 & \mid & \neg\ \mathit{Sentence} & \text{Negation} \\
 & \mid & \mathit{Sentence} \land \mathit{Sentence} & \text{Conjunction} \\
 & \mid & \mathit{Sentence} \lor \mathit{Sentence} & \text{Disjunction} \\
 & \mid & \mathit{Sentence} \;\Rightarrow\; \mathit{Sentence} & \text{Implication} \\
 & \mid & \mathit{Sentence} \;\Leftrightarrow\; \mathit{Sentence} & \text{Biconditional} \\
 & & & \\
\text{OPERATOR PRECEDENCE} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow &
\end{array}
$$

Validity and Satisfiability

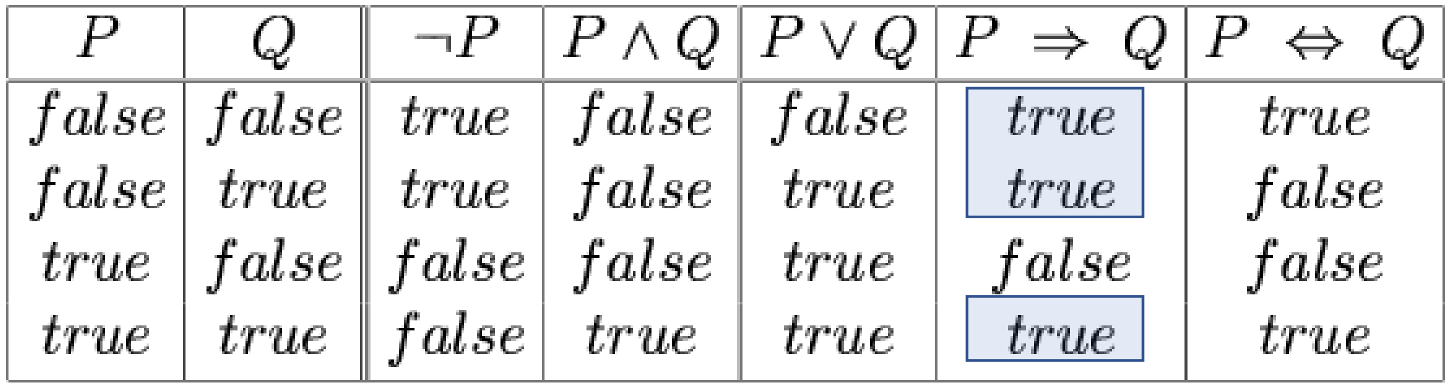
A sentence is **valid** if it is true in **all** models/worlds	e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$ are called tautologies and are useful to deduct new sentences.
A sentence is **satisfiable** if it is true in **some** model	e.g., $A \vee B$, C useful to find new facts that satisfy all current possible worlds.
A sentence is **unsatisfiable** if it is true in no models	e.g., $A \wedge \neg A$

Possible Worlds, Models and Truth Tables

A **model** specifies a "possible world" with the true/false status of each proposition symbol in the knowledge base

- E.g., **P** is true and **Q** is true
- With two symbols, there are $2^2 = 4$ possible worlds/models, and they can be enumerated exhaustively using:

A **truth table** specifies the truth value of a composite sentence for each possible assignments of truth values to its atoms. Each row is a model.



P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

We have 3 possible worlds for $P \Rightarrow Q = true$

Propositional Logic: Semantics

Rules for evaluating truth with respect to a model:

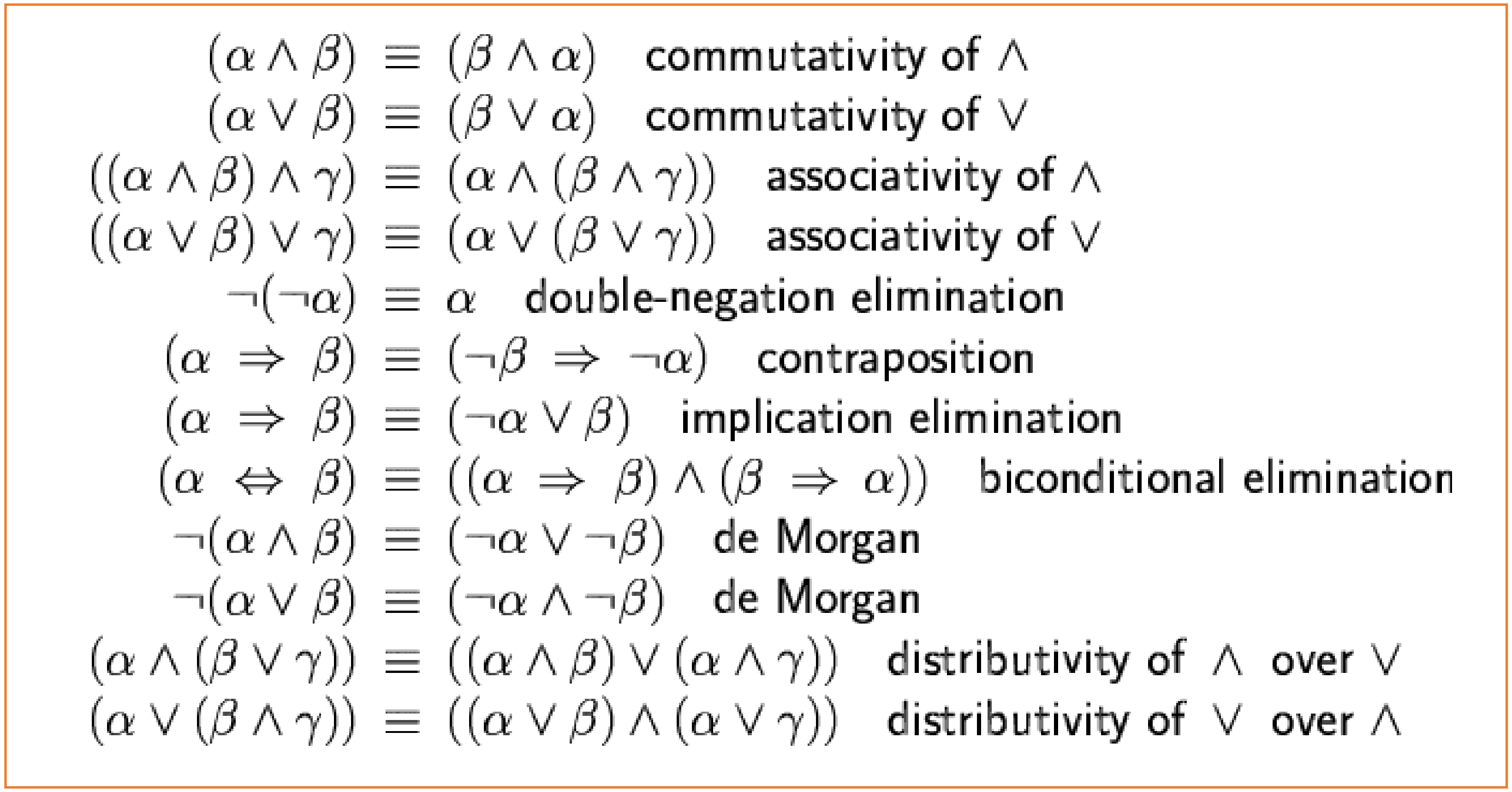
- $\neg$**P** is true iff **P** is false
- **P** $\wedge$ **Q** is true iff **P** is true and **Q** is true
- **P** $\vee$ **Q** is true iff **P** is true or **Q** is true
- **P** $\Rightarrow$ **Q** is true iff **P** is false or **Q** is true

Sentence

Model

Logical Equivalence

Two sentences are logically equivalent iff (read if, and only if) they are true in same models



$(\alpha \land \beta)$	$\equiv$	$(\beta \land \alpha)$	commutativity of $\land$
$(\alpha \lor \beta)$	$\equiv$	$(\beta \lor \alpha)$	commutativity of $\lor$
$((\alpha \land \beta) \land \gamma)$	$\equiv$	$(\alpha \land (\beta \land \gamma))$	associativity of $\land$
$((\alpha \lor \beta) \lor \gamma)$	$\equiv$	$(\alpha \lor (\beta \lor \gamma))$	associativity of $\lor$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \lor \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \land \beta)$	$\equiv$	$(\neg\alpha \lor \neg\beta)$	de Morgan
$\neg(\alpha \lor \beta)$	$\equiv$	$(\neg\alpha \land \neg\beta)$	de Morgan
$(\alpha \land (\beta \lor \gamma))$	$\equiv$	$((\alpha \land \beta) \lor (\alpha \land \gamma))$	distributivity of $\land$ over $\lor$
$(\alpha \lor (\beta \land \gamma))$	$\equiv$	$((\alpha \lor \beta) \land (\alpha \lor \gamma))$	distributivity of $\lor$ over $\land$

Entailment

- **Entailment** means that a sentence follows from the premises contained in the knowledge base:

$$\boldsymbol{KB \models \alpha}$$

- The knowledge base *KB* entails sentence α iff α is true in all models where *KB* is true
  - E.g., KB with x = 0 entails sentence x * y = 0
- Tests for entailment
  - ***KB*** $\models$ **α** iff **(*KB*** $\Rightarrow$ **α)** is *valid*
  - ***KB*** $\models$ **α** iff **(*KB*** $\wedge\neg$**α)** is *unsatisfiable*

Inference

- **Logical inference:** a procedure for generating sentences that follow from (ar entailed by) a knowledge base KB.
- An inference procedure is **sound** if it derives a sentence α iff KB $\models \alpha$. I.e, it only derives **true sentences**.
- An inference procedure is **complete** if it can derive **all** α for which $KB \models \alpha$.

Inference

- How can we check whether a sentence α is entailed by KB?
- How about we **enumerate all possible models of the KB** (truth assignments of all its symbols), and check that α is true in every model in which KB is true?
  - This is sound: All produced answer are correct.
  - This is complete: It will produce all correct answers.
  - **Problem**: if KB contains *n* symbols, the truth table will be of size 2^n
- Better idea: use ***inference rules***, or sound procedures to generate new sentences or *conclusions* given the *premises* in the KB.
- Look at the textbook for inference rules and resolution.

Inference Rules

- Modus Ponens

$$\begin{array}{cll} \alpha \Rightarrow \beta, \alpha & \longleftarrow & \text{premises} \\ \cline{1-1} \beta & \longleftarrow & \text{conclusion} \end{array}$$

This means: If the KB contains the sentences $\alpha \Rightarrow \beta$ and α then β is true.

- And-elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

Inference Rules

- And-introduction

$$\frac{\alpha,\ \beta}{\alpha \wedge \beta}$$

- Or-introduction

$$\frac{\alpha}{\alpha \vee \beta}$$

Inference Rules

- Double negative elimination

$$\frac{\neg\neg\alpha}{\alpha}$$

- Unit resolution

$$\frac{\alpha \vee \beta, \neg\beta}{\alpha}$$

Resolution

$$\frac{\alpha \vee \beta, \neg\beta \vee \gamma}{\alpha \vee \gamma} \quad \text{or} \quad \frac{\alpha \vee \beta, \beta \Rightarrow \gamma}{\alpha \vee \gamma}$$

- Example:
  - α: “The weather is dry”
  - β: “The weather is rainy”
  - γ: “I carry an umbrella”

Resolution is Complete

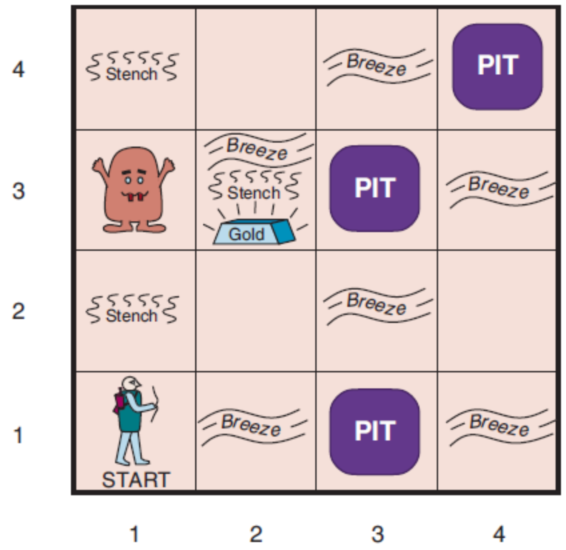
$$\frac{\alpha \vee \beta, \neg \beta \vee \gamma}{\alpha \vee \gamma}$$

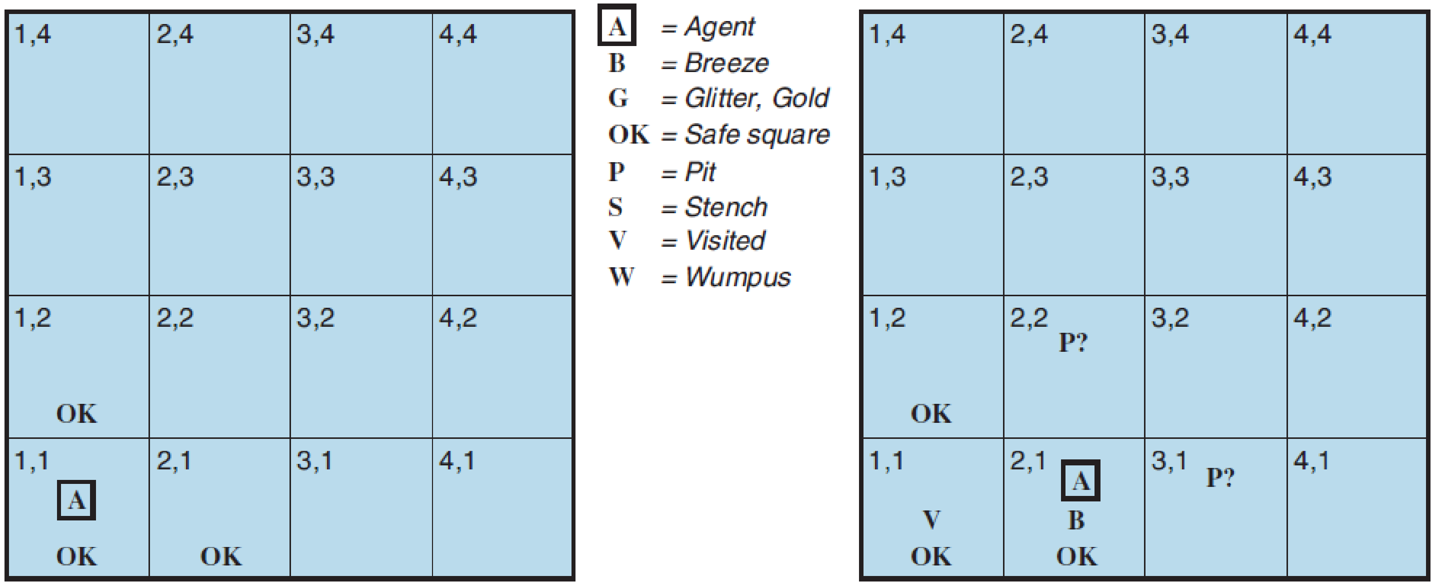
- To prove KB ⊨ α, assume KB ∧ ¬ α and derive a contradiction
- Rewrite KB ∧ ¬ α as a conjunction of *clauses*,
  or disjunctions of *literals*
  - *Conjunctive normal form* (CNF)
- Keep applying resolution to clauses that contain *complementary literals* and adding resulting clauses
  to the list
  - If there are no new clauses to be added, then KB does not entail α
  - If two clauses resolve to form an *empty clause*, we have a contradiction and KB ⊨ α

Complexity of Inference

- Propositional inference is ***co-NP-complete***
  - *Complement* of the SAT problem: $\alpha \models \beta$ if and only if the sentence $\alpha \land \neg \beta$ is *unsatisfiable*
  - Every known inference algorithm has worst-case exponential run time complexity.
- Efficient inference is only possible for restricted cases
  - e.g., Horn clauses are disjunctions of literals with at most one positive literal.

Example: Wumpus World





(a) (b)

Example: Wumpus World

Initial KB needs to contain rules like these for each square:

$Breeze(1,1) \Leftrightarrow Pit(1,2) \vee Pit(2,1)$
$Breeze(1,2) \Leftrightarrow Pit(1,1) \vee Pit(1,3) \vee Pit(2,2)$
$\text{Stench}(1,1) \Leftrightarrow W(1,2) \vee W(2,1)$
…

We have to enumerate all possible scenarios in propositional logic! First-order logic can help.

Percepts at (1,1) are no breeze or stench. Add the following facts to the KB:

$\neg Breeze(1,1)$
$\neg Stench(1,1)$

Inference will tell us that the following facts are entailed:

$\neg Pit(1,2), \neg Pit(2,1), \neg W(1,2), \neg W(2,1)$

This means that (1,2) and (2,1) are safe.

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions.
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences in models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
- Resolution is complete for propositional logic.
- Algorithms use forward, backward chaining, are linear in time, and complete for special clauses (definite clauses).

Limitations of Propositional Logic

Suppose you want to say “All humans are mortal”

- In propositional logic, you would need ~6.7 billion statements of the form:

  Michael_Is_Human and Michael_Is_Mortal,

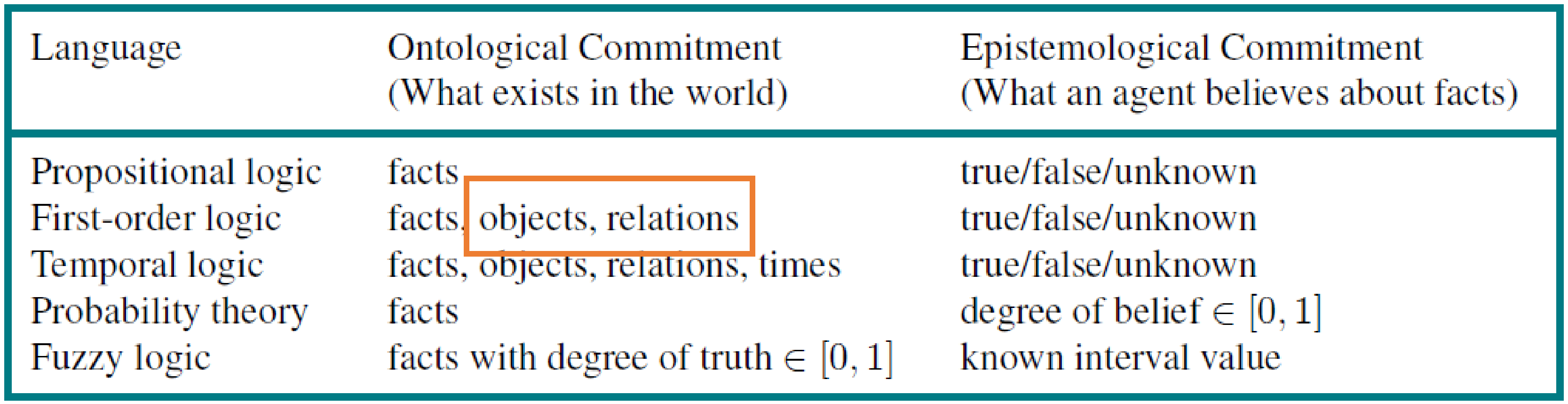
  Sarah_Is_Human and Sarah_Is_Mortal, …

Suppose you want to say “Some people can run a marathon”

- You would need a disjunction of ~6.7 billion statements:

  Michael_Can_Run_A_Marathon or … or Sarah_Can_Run_A_Marathon

First-Order Logic



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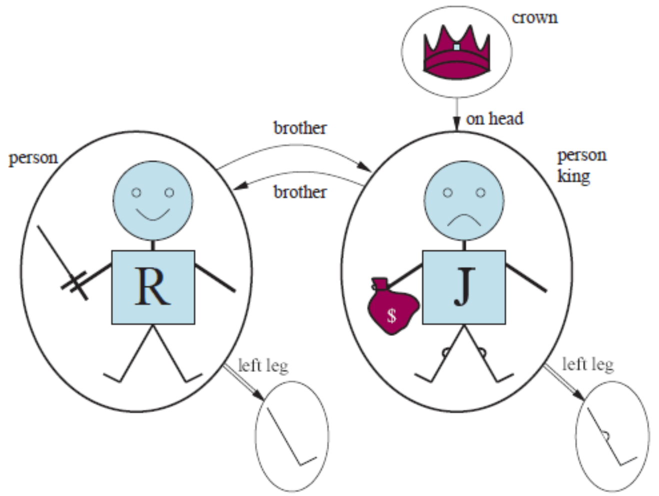
First-order Logic adds **objects** and **relations** to the facts of propositional logic.

This addresses the issues of propositional logic, which needs to store a fact for each instance of and object individually.

Syntax of FOL



Sentence	$\rightarrow$	*AtomicSentence* | *ComplexSentence*
AtomicSentence	$\rightarrow$	*Predicate* | *Predicate*(*Term*, ...) | *Term* = *Term*
ComplexSentence	$\rightarrow$	(*Sentence*)
	|	$\neg$ *Sentence*
	|	*Sentence* $\land$ *Sentence*
	|	*Sentence* $\lor$ *Sentence*
	|	*Sentence* $\Rightarrow$ *Sentence*
	|	*Sentence* $\Leftrightarrow$ *Sentence*
	|	*Quantifier* *Variable*, ... *Sentence*
Term	$\rightarrow$	*Function*(*Term*, ...)
	|	*Constant*
	|	*Variable*
Quantifier	$\rightarrow$	$\forall$ | $\exists$
Constant	$\rightarrow$	A | X_1 | *John* | ...
Variable	$\rightarrow$	a | x | s | ...
Predicate	$\rightarrow$	*True* | *False* | *After* | *Loves* | *Raining* | ...
Function	$\rightarrow$	*Mother* | *LeftLeg* | ...
OPERATOR PRECEDENCE	:	$\neg, =, \land, \lor, \Rightarrow, \Leftrightarrow$

Objects

Relations. Predicate is/returns True or False

Function returns an object

Universal Quantification

- $\forall$**x P(x)**
- Example: "Everyone at SMU is smart"
  $\forall$**x At(x,SMU)** $\Rightarrow$ **Smart(x)**
  Why not $\forall$**x At(x,SMU)** $\wedge$ **Smart(x)**?
- Roughly speaking, equivalent to the conjunction of all possible instantiations of the variable:
  [At(John, SMU) $\Rightarrow$ **Smart(John)]** $\wedge$ **...**
  [At(Richard, SMU) $\Rightarrow$ **Smart(Richard)]** $\wedge$ **...**
- $\forall$**x P(x)** is true in a model m iff **P(x)** is true with **x** being each possible object in the model

Existential Quantification

- **∃x P(x)**

- Example: "Someone at SMU is smart"
  ∃x At(x,SMU) ∧ Smart(x)
  Why not **∃x At(x,SMU) ⇒ Smart(x)**?

- Roughly speaking, equivalent to the disjunction of all possible instantiations:
  [At(John,SMU) ∧ Smart(John)] ∨
  [At(Richard,SMU) ∧ Smart(Richard)] ∨ …

- **∃x P(x)** is true in a model m iff **P(x)** is true with **x** being some possible object in the model

Properties of Quantifiers

- $\forall$**x** $\forall$**y** is the same as $\forall$**y** $\forall$**x**
- $\exists$**x** $\exists$**y** is the same as $\exists$**y** $\exists$**x**
- $\exists$**x** $\forall$**y** is not the same as $\forall$**y** $\exists$**x**
  - $\exists$**x** $\forall$**y Loves(x,y)**
    - "There is a person who loves everyone"
  - $\forall$**y** $\exists$**x Loves(x,y)**
    - "Everyone is loved by at least one person"

- **Quantifier duality:** each quantifier can be expressed using the other with the help of negation
  - $\forall$**x Likes(x,IceCream)** $\neg\exists$**x**
  - $\exists$**x Likes(x,Broccoli)** $\neg\forall$

Equality

- **Term_1 = Term_2** is true under a given model if and only if **Term_1** and **Term_2** refer to the same object

- E.g., definition of **Sibling** in terms of **Parent**:

  $\forall x,y\ \mathrm{Sibling}(x,y) \Leftrightarrow$
  $[\neg(x = y) \wedge\ \exists m,f \neg\ (m = f) \wedge \mathrm{Parent}(m,x) \wedge \mathrm{Parent}(f,x) \wedge$
  $\mathrm{Parent}(m,y) \wedge\ \mathrm{Parent}(f,y)]$

Example: The Kinship Domain

- Brothers are siblings

  $\forall$x,y Brother(x,y) $\Rightarrow$ Sibling(x,y)
- “Sibling” is symmetric

  $\forall$x,y Sibling(x,y) $\Leftrightarrow$ Sibling(y,x)
- One's mother is one's female parent

  $\forall$m,c (Mother(c) = m) $\Leftrightarrow$ (Female(m) $\wedge$ Parent(m,c))

Example: The Set Domain

- $\forall s\ Set(s) \Leftrightarrow (s = \{\}) \vee (\exists x,s_2\ Set(s_2) \wedge s = \{x|s_2\})$
- $\neg\exists x,s\ \{x|s\} = \{\}$
- $\forall x,s\ x \in s \Leftrightarrow s = \{x|s\}$
- $\forall x,s\ x \in s \Leftrightarrow [\ \exists y,s_2\ (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))]$
- $\forall s_1,s_2\ s_1 \subseteq s_2 \Leftrightarrow (\forall x\ x \in s_1 \Rightarrow x \in s_2)$
- $\forall s_1,s_2\ (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$
- $\forall x,s_1,s_2\ x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \wedge x \in s_2)$
- $\forall x,s_1,s_2\ x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \vee x \in s_2)$

Inference in FOL

Inference in FOL is complicated!

1. **Reduction to propositional logic** and then use propositional logic inference.

2. **Directly do inference on FOL (or a subset like definite clauses)**
   - Unification: Combine two sentences into one.
   - Forward Chaining for FOL
   - Backward Chaining for FOL
   - Logical programming (e.g., Prolog)