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Self-adaptive Differential Evolution Algorithm using Population Size Reduction and Three Strategies

Janez Brest · **Mirjam Sepesy Maučec**

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Abstract Many real-world optimization problems are large-scale in nature. In order to solve these problems, an optimization algorithm is required that is able to apply a global search regardless of the problems' particularities. This paper proposes a self-adaptive differential evolution algorithm, called jDElscop, for solving large-scale optimization problems with continuous variables. The proposed algorithm employs three strategies and a population size reduction mechanism. The performance of the jDElscop algorithm is evaluated on a set of benchmark problems provided for the Special Issue on the Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems. Non-parametric statistical procedures were performed for multiple comparisons between the proposed algorithm and three well-known algorithms from literature. The results show that the jDElscop algorithm can deal with large-scale continuous optimization effectively. It also behaves significantly better than other three algorithms used in the comparison, in most cases.

Keywords Differential evolution · Self-adaptation · Large-scale optimization · Multiple statistical comparison

1 Introduction

The problem of global optimization naturally arises over many applications, e.g. in advanced engineering design, data analysis, financial planning, risk management, scientific modelling, etc. (Mladenovic et al 2008). Most cases of practical interest are characterized by multiple local optima. The heuristic methods do not offer a guarantee of locating the global optimum, but give satisfactory results for a wide range of global optimization problems.

Many real-world problems can be formulated as continuous optimization problems (COP). The goal of COP is to find variables of vector $\mathbf{x} = \{x_1, x_2, ..., x_D\}$, such that objective function $f(\mathbf{x})$ is optimized. D denotes the dimensionality of the function. The domains of the variables are defined by their lower and upper bounds $[x_{j,low}, x_{j,upp}]$ for $j = 1, 2, ..., D$.

This paper considers a large scale unconstrained COP, and proposes a self-adaptive differential evolution algorithm jDElscop to solve COPs.

The paper is structured as follows. Section 2 surveys the latest work on large-scale optimization. Section 3 reviews already defined mechanisms of self-adapted differential evolution, which are also used in our new algorithm. Section 4 describes the novel variation of the algorithm, called jDElscop. Section 5 shows the experimental results on the benchmark functions. A comparative statistical analysis of multiple algorithms using non-parametric tests is given and discussed. Section 6 summarizes the important conclusions of this paper.

2 Related Work

A differential Evolution (DE) algorithm belongs to Evolutionary Algorithms (EAs). The DE algorithm was proposed by Storn and Price (1997), and since then the DE algorithm has been used in different areas. The original DE was modified, and many new versions proposed in (Price et al 2005; Zhang and Sanderson 2009; Das et al 2009; Rahnamayan et al 2008; Feoktistov 2006). Neri and Tirronen (2010) give a survey of recent advances in differential evolution.

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J. Brest, M. S. Maučec
Faculty of Electrical Engineering and Computer Science, University of Maribor, Smetanova ulica 17, SI-2000 Maribor, Slovenia
E-mail: janez.brest@uni-mb.si

The original DE algorithm has three control parameters, which are being fixed during the optimization process. Later, various adaptation mechanisms were proposed to overcome the hand-tuning problems of the DE control parameters (Qin et al 2009; Qin and Suganthan 2005; Brest et al 2006a; Zhang and Sanderson 2009; Teng et al 2009).

Teo (2006) proposed a DE algorithm with a dynamic population sizing strategy based on self-adaptation, while F and CR control parameters are also self-adapted. Brest et al (2007) compared some versions of adaptive and self-adaptive algorithms. Qin and Suganthan in (Qin and Suganthan 2005; Qin et al 2009) proposed a Self-adaptive Differential Evolution algorithm (SaDE), where the choice of learning strategy and the two control parameters F and CR are not required to be pre-defined. During evolution, the suitable learning strategy and parameter settings are gradually self-adapted, according to the learning experience.

Brest et al (2006a) proposed a self-adaptive jDE algorithm. The self-adaptive mechanism was used in algorithms for unconstrained optimization (Brest et al 2006a, 2007; Brest and Maučec 2008). Another version of the jDE algorithm with population size reduction was used for large scale global optimization on CEC 2008 (Brest et al 2008). Yet another version, based on the jDE algorithm, was used for dynamic optimization problems (Brest et al 2009). The jDE algorithm was also adopted for the solving of constrained optimization problems (Brest et al 2006b; Brest 2009). The self-adaptation mechanism proposed in (Brest et al 2006a) has been widely-used in many practical cases (Caponio et al 2009) and have shown excellent results. The population size reduction mechanism has a good performance in terms of robustness (Neri and Tirronen 2010).

Historically, scaling EAs to large scale problems has attracted much interest, including both theoretical and practical studies (Tang et al 2007, 2009; Molina et al 2009a). To solve high-dimensional problems (MacNish 2007; Tang et al 2007) cooperative coevolution (Potter and De Jong 1994; Sofge et al 2002; Zamuda et al 2008) is often used. Liu et al (2001) used FEP (fast evolutionary programming) with cooperative coevolution (FEPCC) to speedup convergence rates on large-scale problems, van den Bergh and Engelbrecht (2004) used a cooperative approach to particle swarm optimisation (PSO). Yang et al (2007) used differential evolution with cooperative coevolution (DECC). Molina et al (2009b); Gao and Wang (2007); Muelas et al (2009) used a memetic DE algorithm for high-dimensional problem optimization. Other techniques using local search (Molina et al 2009a; Muelas et al 2009; Zhao et al 2008), neighbourhood search (Muelas et al 2009; Yang et al 2008c), Estimation of Distribution Algorithm (EDA) (Wang and Li 2008) are used to tackle those problems.

Recently, Yang, Tang, and Yao proposed DECC-G (Yang et al 2008a), and MLCC (Yang et al 2008b) algorithms. Both algorithms use cooperative coevolution for solving large-scale optimization problems. In (Brest et al 2010) a comparison is presented of experimental results for *jDElsgo*, DECC-G, DECC-G* [1], and MLCC algorithms.

3 Differential evolution and *jDE* algorithms

This section provides an overview of those algorithms and mechanisms necessary for a better understand of our new algorithm, as proposed in the next section.

3.1 Differential evolution

The population of the original DE algorithm (Storn and Price 1997) contains NP individuals. An individual is defined as a D-dimensional vector. If G denotes the generation, the population at generation G consists of:

$$\mathbf{x}_i^{(G)} = \{x_{i,1}^{(G)}, x_{i,2}^{(G)}, ..., x_{i,D}^{(G)}\},\ i = 1, 2, ..., NP. \tag{1}$$

During one generation for each vector $\mathbf{x}_i^{(G)}$, DE employs mutation and crossover operations to produce a trial vector:

$$\mathbf{u}_i^{(G)} = \{u_{i,1}^{(G)}, u_{i,2}^{(G)}, ..., u_{i,D}^{(G)}\},\ i = 1, 2, ..., NP. \tag{2}$$

Then a selection operation is used to choose vectors for the next generation $(G+1)$.

The initial population is usually selected uniformly randomly between the lower $x_{j,low}$ and upper $x_{j,upp}$ bounds defined for each variable x_j. These bounds are specified according to the nature of the problem.

3.1.1 Mutation operation

Mutation for each population vector $\mathbf{x}_i^{(G)}$ creates a mutant vector $\mathbf{v}_i^{(G)}$:

$$\mathbf{v}_i^{(G)} = \{v_{i,1}^{(G)}, v_{i,2}^{(G)}, ..., v_{i,D}^{(G)}\},\ i = 1, 2, ..., NP. \tag{8}$$

A new mutant vector can be created using one of the mutation strategies. The most useful strategies (Storn and Price 1997; Price et al 2005; Feoktistov 2006) are presented in Fig. 1, where the indexes $r_d, d = 1, ..., 5$ represent the random and mutually different integers generated within the range $[1, NP]$ and also different from index i. F is a mutation scale factor within the range $[0, 2]$, usually less than 1. Vector $\mathbf{x}_{best}^{(G)}$ is the best vector in generation G. Some authors use the name 'target to best' for the strategy in eq. (5).

[1] DECC-G*: The same as DECC-G, except that the grouping structure was used as prior knowledge. The parameter group size was set to s=50. The adaptive weighting strategy of DECC-G was not used.

$$DE/rand/1: \quad \mathbf{v}_i^{(G)} = \mathbf{x}_{r_1}^{(G)} + F \cdot (\mathbf{x}_{r_2}^{(G)} - \mathbf{x}_{r_3}^{(G)}) \tag{3}$$

$$DE/best/1: \quad \mathbf{v}_i^{(G)} = \mathbf{x}_{best}^{(G)} + F \cdot (\mathbf{x}_{r_1}^{(G)} - \mathbf{x}_{r_2}^{(G)}) \tag{4}$$

$$DE/current\,to\,best/1: \quad \mathbf{v}_i^{(G)} = \mathbf{x}_i^{(G)} + F \cdot (\mathbf{x}_{best}^{(G)} - \mathbf{x}_i^{(G)}) + F \cdot (\mathbf{x}_{r_1}^{(G)} - \mathbf{x}_{r_2}^{(G)}) \tag{5}$$

$$DE/best/2: \quad \mathbf{v}_i^{(G)} = \mathbf{x}_{best}^{(G)} + F \cdot (\mathbf{x}_{r_1}^{(G)} - \mathbf{x}_{r_2}^{(G)}) + F \cdot (\mathbf{x}_{r_3}^{(G)} - \mathbf{x}_{r_4}^{(G)}) \tag{6}$$

$$DE/rand/2: \quad \mathbf{v}_i^{(G)} = \mathbf{x}_{r_1}^{(G)} + F \cdot (\mathbf{x}_{r_2}^{(G)} - \mathbf{x}_{r_3}^{(G)}) + F \cdot (\mathbf{x}_{r_4}^{(G)} - \mathbf{x}_{r_5}^{(G)}) \tag{7}$$

Fig. 1 The most useful DE's strategies

3.1.2 Crossover operation

After mutation, a 'binary' crossover operation forms the trial vector $\mathbf{u}_i^{(G)}$ according to the target vector $\mathbf{x}_i^{(G)}$ and its corresponding mutant vector $\mathbf{v}_i^{(G)}$.

$$u_{i,j}^{(G)} = \begin{cases} v_{i,j}^{(G)} & \text{if } rand(0,1) \leq CR \text{ or } j = j_{rand}, \\ x_{i,j}^{(G)} & \text{otherwise} \end{cases} \tag{9}$$
$$i = 1,2,...,NP \text{ and } j = 1,2,...,D.$$

CR is a crossover control parameter or factor within the range $[0,1)$ and presents the probability of creating parameters for a trial vector from the mutant vector. Index j_{rand} is a randomly chosen integer within the range $[1,NP]$. It ensures that the trial vector contains at least one parameter from the mutant vector. Here we have described the binary crossover operation (*bin*). The other DE crossover operation that could also be used in optimizations is exponential (*exp*).

If some components of the trial vector are out of bounds, the proposed solutions in literature Storn and Price (1997, 1995); Rönkkönen et al (2005); Price et al (2005) are: they are reflected from bounds, set on bounds or used as they are (out of bounds).

3.1.3 Selection operation

The DE algorithm uses a greedy selection. The selection operator selects between the target and corresponding trial vectors. A member of the next generation becomes the fittest vector, i.e. vector with the better fitness value. For example, if we have a minimization problem, we will use the following selection rule:

$$\mathbf{x}_i^{(G+1)} = \begin{cases} \mathbf{u}_i^{(G)} & \text{if } f(\mathbf{u}_i^{(G)}) \leq f(\mathbf{x}_i^{(G)}), \\ \mathbf{x}_i^{(G)} & \text{otherwise.} \end{cases} \tag{10}$$

The DE is depicted in Algorithm 1. As can be seen in Alg. 1, it is population-based algorithm, where three operations: mutation (line 5), crossover (lines 6-13), and selection (lines 14-18) are used to generate the new vector $\mathbf{x}_i^{(G+1)}$.

```
1: Initialization(); {Generate uniformly distributed random population}
2: while not termination condition met do
3:    for (i = 0; i < NP; i++) do
4:       Select random indexes r1, r2, and r3 to be different from each
         other and from the index i.
5:       v_i^(G) = x_r1^(G) + F × (x_r2^(G) − x_r3^(G))
6:       j_rand = rand{1, D}
7:       for (j = 0; j < D; j++) do
8:          if (rand(0,1) ≤ CR or j == j_rand) then
9:             u_ij^(G) = v_ij^(G)
10:         else
11:            u_ij^(G) = x_ij^(G)
12:         end if
13:      end for
14:      if (f(u_i^(G)) ≤ f(x_i^(G))) then
15:         x_i^(G+1) = u_i^(G)
16:      else
17:         x_i^(G+1) = x_i^(G)
18:      end if
19:   end for
20: end while
```

Algorithm 1: Differential Evolution algorithm

3.2 Self-adaptive differential evolution

The self-adaptive differential evolution (jDE) algorithm, based on the self-adapting control parameter mechanism, was proposed by (Brest et al 2006a). The self-adaptive control mechanism is used to change control parameters, i.e. select weighting factor F and crossover constant *CR*, during the run. The self-adaptive control parameters $F_i^{(G+1)}$ and $CR_i^{(G+1)}$ are calculated as follows (Brest et al 2006a):

$$F_i^{(G+1)} = \begin{cases} F_l + rand_1 \cdot F_u & \text{if } rand_2 < \tau_1, \\ F_i^{(G)} & \text{otherwise,} \end{cases}$$

$$CR_i^{(G+1)} = \begin{cases} rand_3 & \text{if } rand_4 < \tau_2, \\ CR_i^{(G)} & \text{otherwise.} \end{cases}$$

They produce control parameters F and *CR* in a new parent vector. The quantities $rand_j, j \in \{1,2,3,4\}$ represent uniform random values within the range $[0,1]$. τ_1 and τ_2 are probabilities to adjust control parameters F and *CR*, respectively. Constants τ_1, τ_2, F_l, F_u are assigned fixed values to $0.1, 0.1, 0.1, 0.9$, respectively. The new F takes value from $[0.1, 1.0]$ and the new *CR* from $[0,1]$ randomly. The control parameter values $F_i^{(G+1)}$ and $CR_i^{(G+1)}$ are obtained before the mutation operation is performed. This means, they influence the mutation, crossover, and selection operations of the new vector $\mathbf{x}_i^{(G+1)}$.

In (Brest et al 2006a) the third control parameter, population size *NP*, was not changed during the evolutionary process.

Some ideas, on how to improve the jDE algorithm, are reported by Brest et al (2007), but here we used the original jDE algorithm and mechanisms, as described hereinafter.

3.3 *jDEdynNP-F* algorithm

The *jDEdynNP-F* algorithm (Brest et al 2008) was used for solving high-dimensional real-parameter optimization problems on CEC 2008. In this subsection we give an overview of the *jDEdynNP-F* algorithm. In this algorithm, F and CR control parameters are self-adapted (as in the *jDE* algorithm), and two mechanisms are used: population size reduction and F sign changing.

3.3.1 Population size reduction

In the *jDEdynNP-F* algorithm population size NP was decreased during the evolutionary process. If we assume that we have to optimize a minimization problem then one step of the population size reduction is calculated as follows (Brest and Maučec 2008; Brest et al 2008):

$$\mathbf{x}_i^{(G)} = \begin{cases} \mathbf{x}_{\frac{NP}{2}+i}^{(G)} & \text{if } f(\mathbf{x}_{\frac{NP}{2}+i}^{(G)}) < f(\mathbf{x}_i^{(G)}) \ \wedge \ G = G_R, \\ \mathbf{x}_i^{(G)} & \text{otherwise,} \end{cases} \tag{11}$$

$$NP^{(G+1)} = \begin{cases} \frac{NP^{(G)}}{2} & \text{if } G = G_R, \\ NP^{(G)} & \text{otherwise,} \end{cases} \tag{12}$$

for $i = 1, 2, ..., \frac{NP}{2}$.

Note that only a small part of the generations are exposed to reduction. Let G_R be the generation, whose population is to be reduced. The reduction is performed based on the sequence of comparisons between two individuals. One vector (individual) from the first half $\mathbf{x}_i^{(G)}$ of the current population and a corresponding individual from the second half $\mathbf{x}_{\frac{NP}{2}+i}^{(G)}$ are compared with regard to their fitness values and the better one is placed (as a survivor) at position i in the first half of the current population. The first part of the current population is the population of the next generation. The new population size is equal to half of the latest one.

The number of different population sizes, denoted as *pmax*, is the parameter of the evolutionary process. For a given value of *pmax*, $pmax - 1$ reductions need to be performed. Commonly, a predefined number of evaluations, denoted as *Max_FEs*, is set as a termination criteria. Using these two parameters (*pmax* and *Max_FEs*), and considering that an equal number of evaluations for each population size should be performed, the calculation of generations exposed to reduction is self-evident.

Special care is needed to preserve the minimal requirements of the original DE algorithm population size. Actually, the *DE/rand/1/bin* strategy requires at least four population members (since indexes i, r_1, r_2, and r_3 must be mutually different).

The properties of the population size reduction are:

- it follows the inspiration of the original DE selection operation and
- it does not require any additional operations, e.g. sorting of all individuals according on their fitness value.

3.3.2 Control parameter F sign changing

The *jDEdynNP-F* algorithm uses a mechanism that changes the sign of the control parameter $F' = -F$ with some probability ($prob = 0.75$) when $f(\mathbf{x}_{r_2}^{(G)}) > f(\mathbf{x}_{r_3}^{(G)})$ during the mutation operation as presented in Fig. 2. One can look at the proposed mechanism of sign changing also as swapping indexes r_2 and r_3.

```
prob = 0.75;
if (rand(0,1) < prob && f(x_{r2}^{(G)}) > f(x_{r3}^{(G)}))
    F' = -F;  // sign change
else
    F' = F;
F' is used in the mutation operation
```

Fig. 2 The control parameter F sign changing

4 The proposed algorithm

This section presents a new jDElscop algorithm for solving large scale COP consisting of:

- three strategies:
  - *DE/rand/1/bin* strategy, called *jDEbin*,
  - *DE/rand/1/exp* strategy, called *jDEexp*, and
  - *DE/best/1/bin* strategy, called *jDEbest*,
- new population size reduction mechanism, and
- control parameter F sign-change mechanism as described in Section 3.3.2.

The self-adaptive F and CR control parameters are applied by each strategy.

The structure of the jDElscop is given in Alg. 2. The algorithm performs the maximal number of iterations on the *NP* individuals. In each iteration, only one strategy is active and is applied to the mutation, crossover, and selection operations. The strategies and parameter encodings are described in Sect. 4.1.

1: {*pop* ... population}
2: {*imin* ... index of currently best individual}
3: {$\mathbf{x}_i$ ... i-th individual of population}
4: {*MAX_FEs* ... maximum number of function evaluations}
5: {*s* ... one of strategy (*jDEbin*, *jDEexp*, *jDEbest*)}
6: Initialization() {Generate uniformly distributed random population}
7: **for** ($it = 0$; $it < MAX\_FEs$; $it++$) **do**
8: $i = it$ **mod** *NP* {mod ... modulo operation}
9: **if** ($rand(0,1) < 0.1$ **and** ($it > \frac{MAX\_FEs}{2}$)) **then**
10: $s = 1$; {*jDEbest* strategy}
11: **else**
12: **if** ($i < \frac{NP}{2}$) **then**
13: $s = 2$; {*jDEbin* strategy}
14: **else**
15: $s = 3$; {*jDEexp* strategy}
16: **end if**
17: **end if**
18: {Perform one iteration of the jDE using strategy *s* on $\mathbf{x}_i^{(G)}$}
19: {The jDE applies mutation and crossover to generate trial vector $\mathbf{u}^{(G)}$}
20: $\mathbf{u}^{(G)}$ =jDE(*i*, *pop*, *s*)
21: {Fitness evaluation and selection}
22: **if** ($f(\mathbf{u}^{(G)}) \leq f(\mathbf{x}_i^{(G)})$) **then**
23: $\mathbf{x}_i^{(G+1)} = \mathbf{u}^{(G)}$
24: **end if**
25: **end for**

Algorithm 2: jDElscop algorithm

1: {*pop* ... population of *NP* individuals}
2: {**c** ... vector of individuals' fitness values}
3: {k_1, k_2, m ... temporary variables}
4: **for** ($i = 0$; $i < \frac{NP}{4}$; $i++$) **do**
5: $k_2 = \frac{NP}{4} + i$
6: **if** $\mathbf{c}[i] < \mathbf{c}[k_2]$ **then**
7: $pop[i] = pop[k_2]$
8: $\mathbf{c}[i] = \mathbf{c}[k_2]$
9: **end if**
10: **end for**
11: **for** ($i = 0$; $i < \frac{NP}{4}$; $i++$) **do**
12: $k_1 = \frac{NP}{2} + i$
13: $k_2 = \frac{3NP}{4} + i$
14: $m = \frac{NP}{4} + i$
15: **if** $\mathbf{c}[k_1] < \mathbf{c}[k_2]$ **then**
16: $pop[m] = pop[k_1]$
17: $\mathbf{c}[m] = \mathbf{c}[k_1]$
18: **else**
19: $pop[m] = pop[k_2]$
20: $\mathbf{c}[m] = \mathbf{c}[k_2]$
21: **end if**
22: **end for**
23: $NP = \lceil \frac{NP}{2} \rceil$

Algorithm 3: Population size reduction when two parts are used.

4.1 DE strategies

Our algorithm uses three DE strategies:

1. *jDEbin* (self-adaptive *DE/rand/1/bin*),
2. *jDEexp* (self-adaptive *DE/rand/1/exp*), and
3. *jDEbest* (self-adaptive *DE/best/1/bin*).

In each iteration of the evolutionary process only one strategy is active, as presented in lines 9–17 in Alg. 2. All strategies have their own self-adaptive control parameters F and CR:

$$\mathbf{x}_i^{(G)} = (x_{i,1}^{(G)}, x_{i,2}^{(G)}, ..., x_{i,D}^{(G)}, F_i^{(G),1}, CR_i^{(G),1}, ..., F_i^{(G),3}, CR_i^{(G),3})$$

The control parameters of three DE's strategies are encoded into each individual. The first pair of self-adaptive control parameters $F_i^{(G),1}$ and $CR_i^{(G),1}$ (e.g. $s = 1$) belong to the *jDEbest* strategy, the second pair (e.g. $s = 2$) belong to the *jDEbin* strategy, and the third pair (e.g. $s = 3$) belong to the *jDEexp* strategy.

The *jDEbest* strategy is used with a probability of 0.1, when the counter of iterations *it* exceed half of the maximal number of fitness evaluations. The main idea of using low probability for the *jDEbest* strategy is to keep others strategies as the leading strategies during the evolutionary process. Otherwise, the *jDEbin* and *jDEexp* are used, the first one on the first half of *NP* individuals and the second on the remaining individuals.

The self-adaptive control parameters $F_i^{(G+1),s}$ and $CR_i^{(G+1),s}$ are calculated as follows:

$$F_i^{(G+1),s} = \begin{cases} F_l^s + rand_1 \cdot F_u^s & \text{if } rand_2 < \tau_1, \\ F_i^{(G),s} & \text{otherwise,} \end{cases}$$

$$CR_i^{(G+1),s} = \begin{cases} CR_l^s + rand_3 \cdot CR_u^s & \text{if } rand_4 < \tau_2, \\ CR_i^{(G),s} & \text{otherwise.} \end{cases}$$

The values $F_l^s, F_u^s, CR_l^s, CR_u^s$ for $s \in \{1,2,3\}$ can be calculated on the intervals presented in Sect. 5.2. The *jDEbin* strategy uses ranges similar to those proposed in literature (Brest et al 2006a). The only exception, in this study parameter $F_l^{(2)} = 0.1 + \sqrt{\frac{1.0}{NP}}$ is used, while in (Brest et al 2006a) the constant $F_l^{(2)} = 0.1$ was applied. The reason for this is described in Sect. 4.4 in more detail.

We recommend to use of higher values for the F and CR control parameters for the *jDEexp* strategy than for the *jDEbin* strategy.

The mechanism of control parameter F sign change (see Sect. 3.3.2) is applied in the *jDEbin* and *jDEexp* strategies.

4.2 Population size reduction

Our algorithm proposes a new population size mechanism as presented in Alg. 3. The *jDEbin* strategy is applied to the first half of *NP* individuals. Lines 4–10 in Alg. 3 corresponds to the first part where reduction occurs; while lines 11–22 corresponds to the second reduction part that belongs

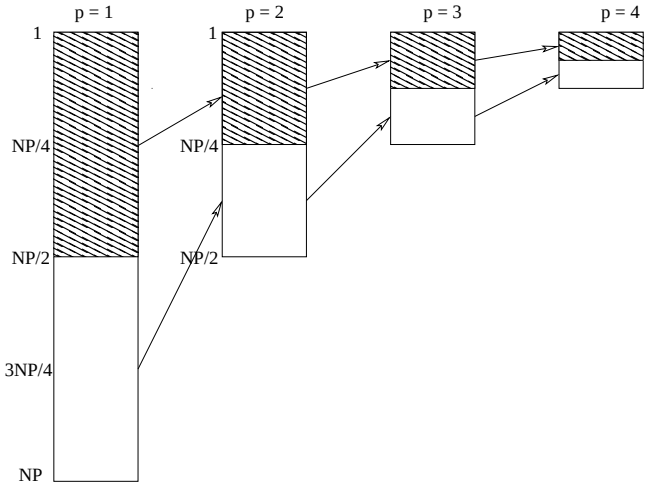

Fig. 3 Population size reduction when two parts are used. The arrows show how each part is reduced and its place after reduction.

to the *jDEexp* strategy. For example, Fig. 3 depicts the population size reduction when $pmax = 4$, e.g. three reductions are being performed during the evolutionary process.

In the proposed population size reduction mechanism, individuals, which compete with each other during the population reduction, belong to the same strategy. The algorithm for population size reduction is not computationally expensive and is performed only $(pmax - 1)$ times. Therefore, it causes only a slight increase of the computational time.

4.3 Initialization and population size

The initial population is selected uniform randomly between the lower $x_{j,low}$ and upper $x_{j,upp}$ bounds defined for each variable x_j. If some components of the trial vector are out of bounds after the mutation operation is applied, then they are set on bounds.

Table 1 Typical run, when Max_FEs =500,000 and $pmax = 4$ are assumed

p	1	2	3	4
NP_p	100	50	25	13
gen_p	1250	2500	5000	9615
$NP_p \times gen_p$	125,000	125,000	125,000	125,000

Population size NP seems to be important DE's parameter. Much more researches relating to F and CR control parameters can be found in literature. All three control parameters depend on each other and also the *good* parameter setting is dependable on the nature of the problem that is being optimized. In the experiments that follow, the number of allowed fitness evaluations is predefined to $Max\_FEs = 5{,}000D$, and the dimension of variables is large. Therefore the small value for population size parameter NP is highly recommended for DE algorithms.

In this paper, the population size NP is set to 100, number of reductions is set to $pmax = 4$, and an equal number of evaluations $\frac{Max\_FEs}{pmax}$ for each population size are performed.

An example that is valid for a typical run is illustrated in Table 1, assuming $Max\_FEs = 500{,}000$, $D = 100$, and $pmax = 4$. For $NP_4 = 13$, i.e. $p = pmax$, the number of generations is equal to $\lfloor \frac{125{,}000}{13} \rfloor = 9615$ and remains 5. The algorithm performs no more than Max_FEs function e.g. fitness evaluations.

Our algorithm optimizes a function as a black box solver for high dimensional problems optimization, however, a well-known cooperative coevolution with any divide-and-conquer strategy is not used.

4.4 Relationship between control parameters

Zaharie (2002) researched about critical values for the control parameters of DE. Premature convergence can be prevented if mutation and crossover induce an increase in the population variance. The values of the parameters which satisfy

$$2F^2 - \frac{2}{NP} + \frac{CR}{NP} = 0 \tag{13}$$

can be considered as critical (Zaharie 2002).

The *jDEbin* strategy uses $CR_l^{(2)} = 0.0$, which then using eq. (13) implies that $F_l^{(2)} = \sqrt{\frac{1.0}{NP}}$. We additionally increase $F_l^{(2)}$ by 0.1.

5 Experiments and Results

5.1 Benchmark functions

The jDElscop algorithm was tested on 19 benchmark functions prepared for a Special Issue on Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems (`http://sci2s.ugr.es/eamhco/CFP.php`). The benchmark functions are scalable. The dimensions of functions were 50, 100, 200, 500, and 1000, respectively, and 25 runs of an algorithm were needed for each function. The optimal solution results, $f(\mathbf{x}^*)$, were known for all benchmark functions.

Properties of these functions are presented in Tables 2, 3, and 4. Note, the separability of a function presents a measure of difficulty when solving it. However, a function $f(\mathbf{x})$ is separable if its parameters x_i are independent. In general, separable problems are considered as easiest, while the fully-nonseparable are usually the most difficult problems to

Table 2 Functions F_1–F_{11}

Function	Name	Definition
F_1	Shifted Sphere Function	$\sum_{i=1}^{D} z_i^2 + f\_bias, z = x - o$
F_2	Shifted Schwefel Problem 2.21	$\max_i\{\|z_i\|, 1 \le i \le D\} + f\_bias, z = x - o$
F_3	Shifted Rosenbrock's Function	$\sum_{i=1}^{D-1}(100(z_i^2 + z_{i+1})^2 + (z_i - 1)^2) + f\_bias, z = x - o$
F_4	Shifted Rastrigin's Function	$\sum_{i=1}^{D}(z_i^2 - 10\cos(2\pi z_i) + 10) + f\_bias, z = x - o$
F_5	Shifted Griewank's Function	$\sum_{i=1}^{D} \frac{z_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 + f\_bias, z = x - o$
F_6	Shifted Ackley's Function	$-20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} z_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_i))$ $+20 + e + f\_bias$
F_7	Shifted Schwefel's Problem 2.22	$\sum_{i=1}^{D} \|x_i\| + \prod_{i=1}^{D} \|x_i\|$
F_8	Shifted Schwefel's Problem 1.2	$\sum_{i=1}^{D}(\sum_{j=1}^{i} x_j)^2$
F_9	Shifted Extended f_{10}	$\left(\sum_{i=1}^{D-1} f_{10}(x_i, x_{i+1})\right) + f_{10}(x_D, x_1)$ $f_{10} = (x^2 + y^2)^{0.25} \cdot (\sin^2(50 \cdot (x^2 + y^2)^{0.1}) + 1)$
F_{10}	Shifted Bohachevsky	$\sum_{i=1}^{D}(x_i^2 + 2x_{i+1}^2 - 0.3\cos(3\pi x_i) - 0.4\cos(4\pi x_{i+1}) + 0.7)$
F_{11}	Shifted Schaffer	$\sum_{i=1}^{D-1}(x_i^2 + x_{i+1}^2)^{0.25}(\sin^2(50 \cdot (x_i^2 + x_{i+1}^2)^{0.1}) + 1)$

Table 3 Properties of F_1–F_{11}. U/M…Unimodal/Multimodal.

	Range	Fitness optimum	U/M	Shifted	Separable	Easily optimized dimension by dimension
F_1	$[-100, 100]^D$	-450	U	✓	✓	✓
F_2	$[-100, 100]^D$	-450	U	✓		
F_3	$[-100, 100]^D$	390	M	✓		✓
F_4	$[-5, 5]^D$	-330	M	✓	✓	✓
F_5	$[-600, 600]^D$	-180	M	✓		
F_6	$[-32, 32]^D$	-140	M	✓	✓	✓
F_7	$[-10, 10]^D$	0	U	✓	✓	✓
F_8	$[-65.536, 65.536]^D$	0	U	✓		
F_9	$[-100, 100]^D$	0	U	✓		✓
F_{10}	$[-15, 15]^D$	0	U	✓		
F_{11}	$[-100, 100]^D$	0	U	✓		✓

Table 4 Properties of F_{12}–$F_{19}*$. *NS*-…Non-shifted

	F_{ns}	F'	m_{ns}	Range	$f(\mathbf{x}*)$
F_{12}	*NS*-F_9	F_1	0.25	$[-100, 100]^D$	0
F_{13}	*NS*-F_9	F_3	0.25	$[-100, 100]^D$	0
F_{14}	*NS*-F_9	F_4	0.25	$[-5, 5]^D$	0
F_{15}	*NS*-F_{10}	*NS*-F_7	0.25	$[-10, 10]^D$	0
$F_{16}*$	*NS*-F_9	F_1	0.5	$[-100, 100]^D$	0
$F_{17}*$	*NS*-F_9	F_3	0.75	$[-100, 100]^D$	0
$F_{18}*$	*NS*-F_9	F_4	0.75	$[-5, 5]^D$	0
$F_{19}*$	*NS*-F_{10}	*NS*-F_7	0.75	$[-10, 10]^D$	0

Table 5 Experimental Results with dimension $D = 50$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Best	0.00e+00	6.95e-03	1.41e+01	0.00e+00	0.00e+00	5.68e-14	0.00e+00
Median	0.00e+00	2.44e-02	1.86e+01	0.00e+00	0.00e+00	8.53e-14	0.00e+00
Worst	0.00e+00	7.66e-02	7.60e+01	0.00e+00	0.00e+00	1.42e-13	0.00e+00
Mean	0.00e+00	3.15e-02	2.28e+01	0.00e+00	0.00e+00	9.55e-14	0.00e+00
Std	0.00e+00	2.09e-02	1.60e+01	0.00e+00	0.00e+00	2.58e-14	0.00e+00
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Best	2.10e-04	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1.09e+01	0.00e+00
Median	5.76e-03	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1.36e+01	0.00e+00
Worst	5.99e-02	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1.80e+01	0.00e+00
Mean	9.97e-03	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1.36e+01	0.00e+00
Std	1.29e-02	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1.48e+00	0.00e+00
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Best	0.00e+00	0.00e+00	2.51e-06	0.00e+00	0.00e+00		
Median	0.00e+00	0.00e+00	1.07e-02	2.09e-14	0.00e+00		
Worst	0.00e+00	0.00e+00	4.12e-02	7.70e-14	0.00e+00		
Mean	0.00e+00	0.00e+00	7.43e-03	2.41e-14	0.00e+00		
Std	0.00e+00	0.00e+00	9.40e-03	1.60e-14	0.00e+00		

Table 6 Experimental Results with dimension $D = 100$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Best	0.00e+00	7.34e-01	5.33e+01	0.00e+00	0.00e+00	1.42e-13	0.00e+00
Median	0.00e+00	1.13e+00	5.60e+01	0.00e+00	0.00e+00	1.99e-13	0.00e+00
Worst	0.00e+00	2.17e+00	1.17e+02	0.00e+00	0.00e+00	2.84e-13	0.00e+00
Mean	0.00e+00	1.21e+00	6.13e+01	0.00e+00	0.00e+00	2.00e-13	0.00e+00
Std	0.00e+00	3.80e-01	1.65e+01	0.00e+00	0.00e+00	3.33e-14	0.00e+00
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Best	1.50e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	3.99e+01	0.00e+00
Median	3.54e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	4.33e+01	0.00e+00
Worst	3.14e+01	1.20e-07	0.00e+00	1.20e-07	0.00e+00	9.87e+01	0.00e+00
Mean	5.57e+00	7.18e-09	0.00e+00	8.17e-09	0.00e+00	5.11e+01	0.00e+00
Std	5.98e+00	2.63e-08	0.00e+00	2.87e-08	0.00e+00	2.02e+01	0.00e+00
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Best	0.00e+00	0.00e+00	1.51e-04	2.10e-14	0.00e+00		
Median	0.00e+00	0.00e+00	4.30e-02	5.03e-14	0.00e+00		
Worst	0.00e+00	0.00e+00	2.78e+00	2.41e-13	0.00e+00		
Mean	0.00e+00	0.00e+00	3.21e-01	6.33e-14	0.00e+00		
Std	0.00e+00	0.00e+00	7.88e-01	4.48e-14	0.00e+00		

solve (Tang et al 2009). Although some of the used functions were separable in their original form, applying techniques such as Salomon's random coordinate rotation technique, makes them non-separable. In addition, the global optimum can be shifted.

5.2 Parameter settings

In the experiments, control parameters of the jDElscop algorithm were set as follows:

1. *NP* was reducible in a deterministic way, initial value $NP_1 = 100$,
2. *pmax* was fixed during the optimization, $pmax = 4$.
3. *jDEbin* strategy:
   - F was self-adaptive on interval $[0.1 + \sqrt{\frac{1.0}{NP}}, 1.0]$, initially set to 0.5,
   - CR was self-adaptive on interval $[0.0, 1.0]$, initially set to 0.9,
4. *jDEexp* strategy:

Table 7 Experimental Results with dimension $D = 200$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Best	0.00e+00	5.88e+00	1.31e+02	0.00e+00	0.00e+00	3.13e-13	0.00e+00
Median	0.00e+00	7.49e+00	1.36e+02	0.00e+00	0.00e+00	4.55e-13	0.00e+00
Worst	0.00e+00	9.38e+00	1.99e+02	0.00e+00	0.00e+00	5.97e-13	0.00e+00
Mean	0.00e+00	7.54e+00	1.40e+02	0.00e+00	0.00e+00	4.52e-13	0.00e+00
Std	0.00e+00	8.27e-01	1.71e+01	0.00e+00	0.00e+00	7.05e-14	0.00e+00
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Best	9.10e+01	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1.01e+02	0.00e+00
Median	2.32e+02	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1.04e+02	0.00e+00
Worst	8.58e+02	1.70e-07	0.00e+00	1.20e-07	0.00e+00	1.60e+02	1.03e-14
Mean	2.52e+02	4.30e-08	0.00e+00	9.58e-09	0.00e+00	1.10e+02	4.11e-16
Std	1.61e+02	6.71e-08	0.00e+00	3.31e-08	0.00e+00	1.82e+01	2.05e-15
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Best	0.00e+00	0.00e+00	9.16e+00	1.05e-13	0.00e+00		
Median	0.00e+00	0.00e+00	1.29e+01	1.84e-13	0.00e+00		
Worst	0.00e+00	0.00e+00	7.31e+01	4.60e-13	0.00e+00		
Mean	0.00e+00	0.00e+00	2.39e+01	2.04e-13	0.00e+00		
Std	0.00e+00	0.00e+00	2.32e+01	8.53e-14	0.00e+00		

Table 8 Experimental Results with dimension $D = 500$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Best	0.00e+00	2.65e+01	3.88e+02	0.00e+00	0.00e+00	9.66e-13	0.00e+00
Median	0.00e+00	3.06e+01	3.95e+02	0.00e+00	0.00e+00	1.17e-12	0.00e+00
Worst	0.00e+00	3.80e+01	4.56e+02	9.97e-01	0.00e+00	1.42e-12	0.00e+00
Mean	0.00e+00	3.06e+01	4.06e+02	1.59e-01	0.00e+00	1.18e-12	0.00e+00
Std	0.00e+00	2.52e+00	2.44e+01	3.72e-01	0.00e+00	1.18e-13	0.00e+00
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Best	3.89e+03	0.00e+00	0.00e+00	0.00e+00	0.00e+00	2.94e+02	0.00e+00
Median	5.65e+03	0.00e+00	0.00e+00	0.00e+00	0.00e+00	3.00e+02	0.00e+00
Worst	8.10e+03	3.39e-07	0.00e+00	2.89e-07	0.00e+00	3.53e+02	9.95e-01
Mean	5.66e+03	6.10e-08	0.00e+00	4.40e-08	0.00e+00	3.14e+02	8.00e-02
Std	1.28e+03	1.08e-07	0.00e+00	7.84e-08	0.00e+00	2.36e+01	2.75e-01
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Best	0.00e+00	0.00e+00	6.29e+01	5.30e-13	0.00e+00		
Median	0.00e+00	0.00e+00	6.64e+01	9.50e-13	0.00e+00		
Worst	0.00e+00	0.00e+00	1.22e+02	2.54e-12	0.00e+00		
Mean	0.00e+00	0.00e+00	7.65e+01	1.11e-12	0.00e+00		
Std	0.00e+00	0.00e+00	2.19e+01	4.85e-13	0.00e+00		

- F was self-adaptive on interval $[0.5, 1.0]$, initially set to 0.5,
- CR was self-adaptive on interval $[0.3, 1.0]$, initially set to 0.9,
- this strategy is used with probability of 0.1 when the counter of iterations *it* exceed half of the maximal number of fitness evaluations.

5. *jDEbest* strategy:
   - F was self-adaptive on interval $[0.4, 1.0]$, initially set to 0.5,
   - CR was self-adaptive on interval $[0.7, 0.95]$, initially set to 0.9,

PC Configure:
System: GNU/Linux CPU: 2.5 GHz RAM: 4 GB
Language: C/C++ Algorithm: jDElscop
Compiler: GNU Compiler

Table 9 Experimental Results with dimension $D = 1000$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Best	0.00e+00	5.37e+01	8.36e+02	0.00e+00	0.00e+00	2.25e-12	0.00e+00
Median	0.00e+00	6.21e+01	8.44e+02	0.00e+00	0.00e+00	2.61e-12	0.00e+00
Worst	0.00e+00	6.98e+01	9.00e+02	1.99e+00	0.00e+00	3.47e-12	0.00e+00
Mean	0.00e+00	6.14e+01	8.48e+02	1.99e-01	0.00e+00	2.67e-12	0.00e+00
Std	0.00e+00	4.05e+00	1.57e+01	4.97e-01	0.00e+00	3.24e-13	0.00e+00
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Best	2.41e+04	0.00e+00	0.00e+00	0.00e+00	0.00e+00	6.33e+02	0.00e+00
Median	3.15e+04	5.97e-08	0.00e+00	1.20e-07	0.00e+00	6.41e+02	0.00e+00
Worst	5.17e+04	1.10e-01	0.00e+00	1.07e-02	0.00e+00	6.86e+02	9.95e-01
Mean	3.21e+04	4.40e-03	0.00e+00	8.58e-04	0.00e+00	6.57e+02	3.98e-02
Std	5.49e+03	2.20e-02	0.00e+00	2.97e-03	0.00e+00	2.30e+01	1.99e-01
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Best	0.00e+00	0.00e+00	1.56e+02	1.65e-12	0.00e+00		
Median	0.00e+00	0.00e+00	1.67e+02	2.84e-12	0.00e+00		
Worst	0.00e+00	2.01e+01	2.18e+02	1.14e+00	0.00e+00		
Mean	0.00e+00	8.04e-01	1.72e+02	1.65e-01	0.00e+00		
Std	0.00e+00	4.02e+00	1.67e+01	3.86e-01	0.00e+00		

5.3 Experimental results

The obtained results (error values $f(\mathbf{x}) - f(\mathbf{x}^*)$) for dimensions $D = 50, 100, 200, 500$, and 1000 are presented in Tables 5–9, respectively. 25 runs were performed for each function. The best obtained result at the end of each optimization process is recorded during each run. The best, median, worst, mean values and standard deviations are shown in Tables 5–9. As suggested for this special issue, we approximate all values below $1e-14$ to 0.0.

Note that the obtained median value is much better than the corresponding mean value, which means that some outliers exists with a worse fitness value but, on average, the algorithm found a solution with good fitness value within the prescribed Max_FEs. For example, the obtained median value in Table 9 for dimension $D = 1000$ is equal to zero in 10 cases, lower than 10^{-6} in 4 cases, and it is worse than 10^{-6} in 5 cases.

The jDElscop algorithm seems to have difficulties when solving functions F_8 (Shifted Schwefel's Problem 1.2) and F_3 (Shifted Rosenbrock's Function) and hybrid composition functions F_{13} and F_{17} where F_3 is involved.

Tables 11–15 show the average running time in seconds on the 25 runs of the jDElscop algorithm. We used power regression $y = ax^b$ to analyze the time complexity of our algorithm. In our case x is dimensionality D, and y are the times in Tables 11–15. The results of power regression are presented in Table 16. It can be seen that our algorithm has a time complexity lower than $O(D^2)$.

Table 16 Time complexity analysis using power regression $y = ax^b$. rss denotes Residual Sum of Squares

Fun.	a	b	rss
F_1	3.4003E-4	1.8570	89.0667
F_2	4.2207E-4	1.8545	56.4348
F_3	3.6457E-4	1.8695	124.0883
F_4	5.3578E-4	1.9402	92.9612
F_5	6.7970E-4	1.9225	144.3762
F_6	4.9230E-4	1.9145	104.7618
F_7	3.7420E-4	1.8837	15.8602
F_8	2.8459E-4	1.8815	280.8795
F_9	2.3326E-3	1.9474	94.2575
F_{10}	5.8788E-4	1.9635	9.4240
F_{11}	1.9603E-3	1.9840	39.4422
F_{12}	7.3089E-4	1.9577	101.7376
F_{13}	8.0106E-4	1.9510	201.2616
F_{14}	9.2232E-4	1.9657	195.8741
F_{15}	4.4827E-4	1.9206	66.9233
$F_{16}*$	1.1245E-3	1.9730	253.3440
$F_{17}*$	1.6323E-3	1.9732	556.6561
$F_{18}*$	1.6087E-3	1.9785	525.2190
$F_{19}*$	5.7719E-4	1.9479	204.5267

5.4 Comparison of different algorithms

We compared our jDElscop algorithm with the following algorithms:

- the original DE algorithm with *DE/rand/1/exp* strategy, $F = 0.5$, and $CR = 0.9$, (Storn and Price 1997)
- the real-coded CHC (Eshelman and Schaffer 1993) algorithm, and
- the G-CMA-ES (Auger and Hansen 2005) algorithm.

Table 10 Average Values of Experimental Results.

	D	jDElscop	DE	CHC	G-CMA-ES
F_1	50	**0.00e+00**	**0.00E+00**	1.67E-11	**0.00E+00**
	100	**0.00e+00**	**0.00E+00**	3.56E-11	**0.00E+00**
	200	**0.00e+00**	**0.00E+00**	8.34E-01	**0.00E+00**
	500	**0.00e+00**	**0.00E+00**	2.84E-12	**0.00E+00**
	1000	**0.00e+00**	**0.00E+00**	1.36E-11	
F_2	50	3.15e-02	3.60E-01	6.19E+01	**2.75E-11**
	100	1.21e+00	4.45E+00	8.58E+01	**1.51E-10**
	200	7.54e+00	1.92E+01	1.03E+02	**1.16E-09**
	500	3.06e+01	5.35E+01	1.29E+02	**3.48E-04**
	1000	**6.14e+01**	8.46E+01	1.44E+02	
F_3	50	2.28e+01	2.89E+01	1.25E+06	**7.97E-01**
	100	6.13e+01	8.01E+01	4.19E+06	**3.88E+00**
	200	1.40e+02	1.78E+02	2.01E+07	**8.91E+01**
	500	4.06e+02	4.76E+02	1.14E+06	**3.58E+02**
	1000	**8.48e+02**	9.69E+02	8.75E+03	
F_4	50	**0.00e+00**	3.98E-02	7.43E+01	1.05E+02
	100	**0.00e+00**	1.27E-01	5.40E+02	6.48E+02
	500	**1.59e-01**	3.20E-01	1.91E+03	2.10E+03
	1000	**1.99e-01**	1.44E+00	4.76E+03	
F_5	50	**0.00e+00**	**0.00E+00**	1.67E-03	2.96E-04
	100	**0.00e+00**	**0.00E+00**	3.83E-03	1.58E-03
	200	**0.00e+00**	**0.00E+00**	8.76E-03	**0.00E+00**
	500	**0.00e+00**	**0.00E+00**	6.98E-03	2.96E-04
	1000	**0.00e+00**	**0.00E+00**	7.02E-03	
F_6	50	**9.55e-14**	1.43E-13	6.15E-07	2.09E+01
	100	**2.00e-13**	3.10E-13	4.10E-07	2.12E+01
	200	**4.52e-13**	6.54E-13	1.23E+00	2.14E+01
	500	**1.18e-12**	1.65E-12	5.16E+00	2.15E+01
	1000	**2.67e-12**	3.29E-12	1.38E+01	
F_7	50	**0.00e+00**	**0.00E+00**	2.66E-09	1.01E-10
	100	**0.00e+00**	**0.00E+00**	1.40E-02	4.22E-04
	200	**0.00e+00**	**0.00E+00**	2.59E-01	1.17E-01
	500	**0.00e+00**	**0.00E+00**	1.27E-01	7.21E+153
	1000	**0.00e+00**	**0.00E+00**	3.52E-01	
F_8	50	9.97e-03	3.44E+00	2.24E+02	**0.00E+00**
	100	5.57e+00	3.69E+02	1.69E+03	**0.00E+00**
	200	2.52e+02	5.53E+03	9.38E+03	**0.00E+00**
	500	5.66e+03	6.09E+04	7.22E+04	**2.36E-06**
	1000	**3.21e+04**	2.46E+05	3.11E+05	
F_9	50	**0.00E+00**	2.73E+02	3.10E+02	1.66E+01
	100	**7.18e-09**	5.06E+02	5.86E+02	1.02E+02
	200	**6.10e-08**	2.52E+03	3.00E+03	1.74E+03
	1000	**4.40e-03**	5.13E+03	6.11E+03	
F_{10}	50	**0.00e+00**	**0.00E+00**	7.30E+00	6.81E+00
	100	**0.00e+00**	**0.00E+00**	3.30E+01	1.66E+01
	200	**0.00e+00**	**0.00E+00**	7.13E+01	4.43E+01
	500	**0.00e+00**	**0.00E+00**	1.86E+02	1.27E+02
	1000	**0.00e+00**	**0.00E+00**	3.83E+02	

	D	jDElscop	DE	CHC	G-CMA-ES
F_{11}	50	**0.00e+00**	6.23E-05	2.16E+00	3.01E+01
	100	**8.17e-09**	1.28E-04	7.32E+01	1.64E+02
	200	**9.58e-09**	2.62E-04	3.85E+02	8.03E+02
	500	**4.40e-08**	6.76E-04	1.81E+03	4.16E+03
	1000	**8.58e-04**	1.35E-03	4.82E+03	
F_{12}	50	**0.00e+00**	5.35E-13	9.57E-01	1.88E+02
	100	**0.00e+00**	5.99E-11	1.03E+01	4.17E+02
	200	**0.00e+00**	9.76E-10	7.44E+01	9.06E+02
	500	**0.00e+00**	7.07E-09	4.48E+02	2.58E+03
	1000	**0.00e+00**	1.68E-08	1.05E+03	
F_{13}	50	**1.36e+01**	2.45E+01	2.08E+06	1.97E+02
	100	**5.11e+01**	6.17E+01	2.70E+06	4.21E+02
	200	**1.10e+02**	1.36E+02	5.75E+06	9.43E+02
	500	**3.14e+02**	3.59E+02	3.22E+07	2.87E+03
	1000	**6.57e+02**	7.30E+02	6.66E+07	
F_{14}	50	**0.00e+00**	4.16E-08	6.17E+01	1.09E+02
	100	**0.00e+00**	4.79E-02	1.66E+02	2.55E+02
	200	**4.11e-16**	1.38E-01	4.29E+02	6.09E+02
	500	**8.00e-02**	1.35E-01	1.46E+03	1.95E+03
	1000	**3.98e-02**	6.90E-01	3.62E+03	
F_{15}	50	**0.00e+00**	**0.00E+00**	3.98E-01	9.79E-04
	100	**0.00e+00**	**0.00E+00**	8.13E+00	6.30E-01
	200	**0.00e+00**	**0.00E+00**	2.14E+01	1.75E+00
	500	**0.00e+00**	**0.00E+00**	6.01E+01	2.82E+262
	1000	**0.00e+00**	**0.00E+00**	8.37E+01	
$F_{16}*$	50	**0.00e+00**	1.56E-09	2.95E-09	4.27E+02
	100	**0.00e+00**	3.58E-09	2.23E+01	8.59E+02
	200	**0.00e+00**	7.46E-09	1.60E+02	1.92E+03
	500	**0.00e+00**	2.04E-08	9.55E+02	5.45E+03
	1000	8.04e-01	**4.18E-08**	2.32E+03	
$F_{17}*$	50	**7.43e-03**	7.98E-01	2.26E+04	6.89E+02
	100	**3.21e-01**	1.23E+01	1.47E+05	1.51E+03
	200	**2.39e+01**	3.70E+01	1.75E+05	3.36E+03
	500	**7.65e+01**	1.11E+02	8.40E+05	9.59E+03
	1000	**1.72e+02**	2.36E+02	2.04E+07	
$F_{18}*$	50	**2.41e-14**	1.22E-04	1.58E+01	1.31E+02
	100	**6.33e-14**	2.98E-04	7.00E+01	3.07E+02
	200	**2.04e-13**	4.73E-04	2.12E+02	6.89E+02
	500	**1.11e-12**	1.22E-03	7.32E+02	2.05E+03
	1000	1.65e-01	**2.37E-03**	1.72E+03	
$F_{19}*$	50	**0.00e+00**	**0.00E+00**	3.59E+02	4.76E+00
	100	**0.00e+00**	**0.00E+00**	5.45E+02	2.02E+01
	200	**0.00e+00**	**0.00E+00**	2.06E+03	7.52E+02
	500	**0.00e+00**	**0.00E+00**	1.76E+03	2.44E+06
	1000	**0.00e+00**	**0.00E+00**	4.20E+03	

Table 17 Average Rankings of the algorithms when $D = 50$

Algorithm	Ranking
jDElscop	3.6578947368421035
DE	2.9210526315789473
CHC	1.368421052631579
G-CMA-ES	2.052631578947368

Table 18 Average Rankings of the algorithms when $D = 100$

Algorithm	Ranking
jDElscop	3.6578947368421035
DE	2.9210526315789473
CHC	1.368421052631579
G-CMA-ES	2.052631578947368

Table 11 Execution times with dimension $D = 50$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Mean	5.02e-01	6.15e-01	5.75e-01	1.07e+00	1.28e+00	9.05e-01	5.19e-01
Std	2.39e-03	2.22e-03	2.01e-03	1.59e-02	6.85e-03	4.45e-03	2.88e-03
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Mean	5.11e-01	3.99e+00	1.46e+00	3.90e+00	1.54e+00	1.64e+00	1.99e+00
Std	1.94e-03	2.17e-02	6.80e-03	1.50e-02	3.90e-02	5.19e-03	2.24e-02
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Mean	8.90e-01	2.51e+00	3.62e+00	3.68e+00	1.29e+00		
Std	2.71e-03	4.24e-02	1.74e-02	4.03e-02	4.45e-03		

Table 12 Execution times with dimension $D = 100$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Mean	1.65e+00	2.05e+00	1.87e+00	3.96e+00	4.67e+00	3.19e+00	1.71e+00
Std	1.10e-02	5.71e-03	6.18e-03	7.68e-02	2.12e-02	1.27e-02	1.09e-02
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Mean	1.66e+00	1.56e+01	5.50e+00	1.54e+01	5.76e+00	6.13e+00	7.59e+00
Std	5.60e-03	7.42e-02	1.56e-02	5.94e-02	1.23e-01	2.06e-02	9.09e-02
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Mean	3.04e+00	9.48e+00	1.37e+01	1.40e+01	4.63e+00		
Std	8.00e-03	1.06e-01	3.60e-02	1.84e-01	1.52e-02		

Table 13 Execution times with dimension $D = 200$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Mean	5.85e+00	7.32e+00	6.72e+00	1.49e+01	1.75e+01	1.19e+01	6.07e+00
Std	3.33e-02	2.39e-02	1.76e-02	2.48e-01	7.65e-02	5.48e-02	3.65e-02
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Mean	5.88e+00	6.19e+01	2.15e+01	6.12e+01	2.22e+01	2.33e+01	2.98e+01
Std	1.65e-02	6.43e-01	2.15e-01	2.19e-01	3.62e-01	5.34e-02	2.85e-01
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Mean	1.13e+01	3.73e+01	5.40e+01	5.51e+01	1.79e+01		
Std	4.06e-02	4.24e-01	1.63e-01	3.83e-01	5.23e-02		

Table 19 Average Rankings of the algorithms when $D = 200$

Algorithm	Ranking
jDElscop	3.6315789473684195
DE	2.894736842105263
CHC	1.368421052631579
G-CMA-ES	2.1052631578947367

Table 20 Average Rankings of the algorithms when $D = 500$

Algorithm	Ranking
jDElscop	3.6578947368421035
DE	2.9210526315789473
CHC	1.5263157894736836
G-CMA-ES	1.8947368421052628

The average error values of 25 runs for each algorithm are shown in Table 10. These are results for all dimensions (50, 100, 200, 500, 1000) for the original DE, CHC, and jDElscop algorithms, while for the G-CMA-ES algorithm the results are given for dimensions 50, 100, 200, and 500. The results for the original DE, CHC, G-CMA-ES algorithms were prepared for comparison purposes and

Table 14 Execution times with dimension $D = 500$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Mean	3.40e+01	4.20e+01	3.89e+01	9.05e+01	1.03e+02	7.11e+01	3.50e+01
Std	3.48e-01	2.28e-01	1.10e-01	1.53e+00	3.38e-01	2.59e-01	2.17e-01
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Mean	3.36e+01	3.82e+02	1.30e+02	3.80e+02	1.35e+02	1.41e+02	1.80e+02
Std	8.46e-02	1.10e+00	3.27e-01	1.15e+00	1.32e+00	3.32e-01	1.43e+00
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Mean	6.89e+01	2.29e+02	3.31e+02	3.39e+02	1.12e+02		
Std	2.70e-01	1.90e+00	9.52e-01	1.82e+00	2.08e-01		

Table 15 Execution times with dimension $D = 1000$.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Mean	1.33e+02	1.60e+02	1.56e+02	3.58e+02	4.01e+02	2.80e+02	1.40e+02
Std	1.12e+00	1.55e+00	3.40e-01	3.69e+00	1.16e+00	2.07e+00	8.59e-01
	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Mean	1.33e+02	1.52e+03	5.15e+02	1.52e+03	5.35e+02	5.55e+02	7.15e+02
Std	3.08e-01	3.75e+00	2.91e+00	3.59e+00	4.44e+00	1.55e+00	4.26e+00
	F_{15}	$F_{16}*$	$F_{17}*$	$F_{18}*$	$F_{19}*$		
Mean	2.69e+02	9.06e+02	1.30e+03	1.38e+03	4.33e+02		
Std	7.01e-01	4.19e+00	2.73e+00	4.54e+00	9.61e-01		

Table 22 Adjusted p-values when $D = 50$

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Shaf}	p_{Berg}
1	jDElscop vs. CHC	4.601914E-8	2.761148E-7 ✓	2.761148E-7 ✓	2.761148E-7 ✓	2.761148E-7 ✓
2	jDElscop vs. G-CMA-ES	1.268407E-4	7.610445E-4 ✓	6.342037E-4 ✓	3.805222E-4 ✓	3.805222E-4 ✓
3	DE vs. CHC	2.098472E-4	0.00125908 ✓	8.393891E-4 ✓	6.295418E-4 ✓	6.295418E-4 ✓
4	DE vs. G-CMA-ES	0.03814187	0.22885123	0.11442561	0.11442561	0.03814187 ✓
5	jDElscop vs. DE	0.07854585	0.47127510	0.15709170	0.15709170	0.15709170
6	CHC vs. G-CMA-ES	0.10235752	0.61414515	0.15709170	0.15709170	0.15709170

Table 21 Average Rankings of the algorithms when $D = 1000$

Algorithm	Ranking
jDElscop	2.736842105263158
DE	2.263157894736842
CHC	0.9999999999999996

were taken from `http://sci2s.ugr.es/eamhco/`.[2] The obtained results show that our algorithm performed well against all benchmark functions for all dimensions. The jDElscop algorithm found global optimum, i.e. the error value is zero, in many cases, also when $D = 1000$.

Statistical analyses introduced by Demšar (2006); García and Herrera (2008); García et al (2009), and García et al (2010) were used to analyse the obtained results of different algorithms.

Under the null hypothesis, the k algorithms are equivalent. If the null hypothesis is rejected, then at least one of the k algorithms performed significant differently from at least one other algorithm. However, this does not indicate which one. A *post-hoc* test needs to be done to obtain this information,

Performance comparisons of multiple algorithms can be arranged after ranking the algorithms according to their mean value of each benchmark function. Tables 17–21 show the average ranking of the algorithms when the dimension varies from 50 through 1000. Regardless of dimensions, the algorithms can be sorted by average ranking into the following order: jDElscop, DE, G-CMA-ES, and CHC. The best average ranking was obtained by the jDElscop algorithm, which outperformed the other three algorithms.

[2] It is interesting to note that the results obtained on the test suite by using *DE/rand/1/exp* strategy are clearly better than those obtained by employing the *DE/rand/1/bin* strategy.

Table 23 Adjusted p-values when $D = 100$

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Shaf}	p_{Berg}
1	jDElscop vs. CHC	4.601914E-8	2.761148E-7 ✓	2.761148E-7 ✓	2.761148E-7 ✓	2.761148E-7 ✓
2	jDElscop vs. G-CMA-ES	1.268407E-4	7.610445E-4 ✓	6.342037E-4 ✓	3.805222E-4 ✓	3.805222E-4 ✓
3	DE vs. CHC	2.098472E-4	0.00125908 ✓	8.393891E-4 ✓	6.295418E-4 ✓	6.295418E-4 ✓
4	DE vs. G-CMA-ES	0.03814187	0.22885123	0.11442561	0.11442561	0.03814187 ✓
5	jDElscop vs. DE	0.078545850	0.47127510	0.15709170	0.15709170	0.15709170
6	CHC vs. G-CMA-ES	0.10235752	0.61414515	0.15709170	0.15709170	0.15709170

Table 24 Adjusted p-values when $D = 200$

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Shaf}	p_{Berg}
1	jDElscop vs. CHC	6.545677E-8	3.927406E-7 ✓	3.927406E-7 ✓	3.927406E-7 ✓	3.927406E-7 ✓
2	DE vs. CHC	2.684031E-4	0.00161041 ✓	0.00134201 ✓	8.052094E-4 ✓	8.052094E-4 ✓
3	jDElscop vs. G-CMA-ES	2.684031E-4	0.00161041 ✓	0.00134201 ✓	8.052094E-4 ✓	8.052094E-4 ✓
4	DE vs. G-CMA-ES	0.05945109	0.35670655	0.17835327	0.17835327	0.05945109
5	CHC vs. G-CMA-ES	0.07854585	0.47127510	0.17835327	0.17835327	0.15709170
6	jDElscop vs. DE	0.07854585	0.47127510	0.17835327	0.17835327	0.15709170

Table 25 Adjusted p-values when $D = 500$

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Shaf}	p_{Berg}
1	jDElscop vs. CHC	3.598143E-7	2.158885E-6 ✓	2.158885E-6 ✓	2.158885E-6 ✓	2.158885E-6 ✓
2	jDElscop vs. G-CMA-ES	2.559570E-5	1.535742E-4 ✓	1.279785E-4 ✓	7.678712E-5 ✓	7.678712E-5 ✓
3	DE vs. CHC	8.688074E-4	0.00521284 ✓	0.00347522 ✓	0.00260642 ✓	0.00260642 ✓
4	DE vs. G-CMA-ES	0.01427390	0.08564344	0.04282172 ✓	0.04282172 ✓	0.01427390 ✓
5	jDElscop vs. DE	0.07854585	0.47127510	0.15709170	0.15709170	0.15709170
6	CHC vs. G-CMA-ES	0.37907971	2.27447831	0.37907971	0.37907971	0.37907971

Tables 17–21 show the average ranking of the for the jDElscop, CHC, and G-CMA-ES algorithms (without the G-CMA-ES algorithm) on a set of all dimensions. The best average ranking was obtained by the jDElscop algorithm, which outperformed the other algorithms.

5.4.1 Comparisons with a control algorithm

The Bonferroni-Dunn test (Dunn 1961) was used for the post-hoc test to detect significant difference for the jDElscop algorithm.

The performance of the jDElscop algorithm is significantly different if the corresponding ranks differ by at least the critical distance CD that is defined as follows:

$$CD = Q_\alpha \sqrt{\frac{k(k+1)}{6N}},$$

where Q_α is the critical value for a multiple non-parametric comparison with a control (Zar 1999). Four algorithms ($k = 4$) are used in statistical analysis for dimensions $D = 50, 100, 200$, and 500, while $k = 3$ for $D = 1000$, and $N = 19$ denotes the number of functions. Critical difference represents the threshold for the best performing algorithm, i.e. that with the highest ranking. Those rankings which do not exceed the threshold are associated with an algorithm with worse performance than the best performing algorithm.

Bonferroni-Dunn's graphics are depicted in Figures 4–8 for all problem dimensions (denoted by D). On each figure a horizontal cut line is drawn for two levels of significance,

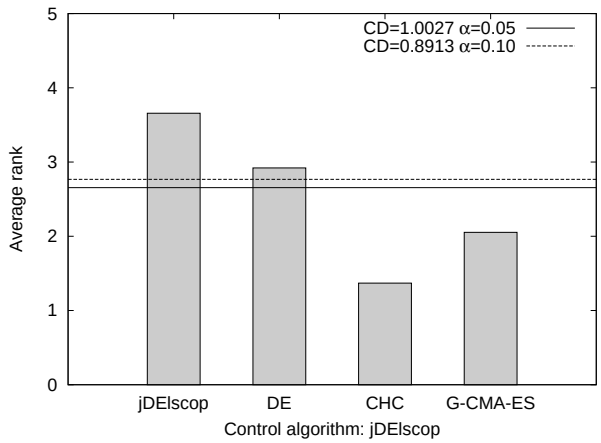

Fig. 4 Bonferroni-Dunn's graphic corresponding to the results when $D = 50$.

$\alpha = 0.05$ and $\alpha = 0.10$. The application of Bonferroni-Dunn's test informs us that jDElscop is significantly better than CHC and G-CMA-ES.

Bonferroni-Dunn's graphic is depicted in Figure 9 on a set of all dimensions for the jDElscop, CHC, and DE algorithms (without the G-CMA-ES algorithm). The application of Bonferroni-Dunn's test informs us that jDElscop is significantly better than CHC and DE.

5.4.2 Comparisons among all algorithms

Post-hoc tests were extended by the performing of other tests: Nemenyi's, Holm's, Shaffer's, and Bergmann-

Table 26 Adjusted p-values when $D = 1000$

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Shaf}	p_{Berg}
1	jDElscop vs. CHC	8.636119E-8	2.590835E-7 ✓	2.590835E-7 ✓	2.590835E-7 ✓	2.590835E-7 ✓
2	DE vs. CHC	9.888398E-5	2.966519E-4 ✓	1.977679E-4 ✓	9.888398E-5 ✓	9.888398E-5 ✓
3	jDElscop vs. DE	0.14429205	0.43287616	0.14429205	0.14429205	0.14429205

Table 27 Adjusted p-values on a set of all dimensions

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Shaf}	p_{Berg}
1	jDElscop vs. CHC	3.938167E-36	1.181450E-35 ✓	1.181450E-35 ✓	1.181450E-35 ✓	1.181450E-35 ✓
2	DE vs. CHC	4.461321E-16	1.338396E-15 ✓	8.922643E-16 ✓	4.461321E-16 ✓	4.461321E-16 ✓
3	jDElscop vs. DE	9.626145E-6	2.887843E-5 ✓	9.626145E-6 ✓	9.626145E-6 ✓	9.626145E-6 ✓

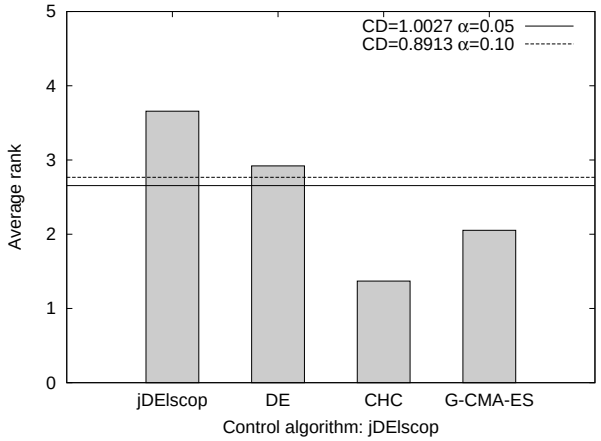

Fig. 5 Bonferroni-Dunn's graphic corresponding to the results when $D = 100$.

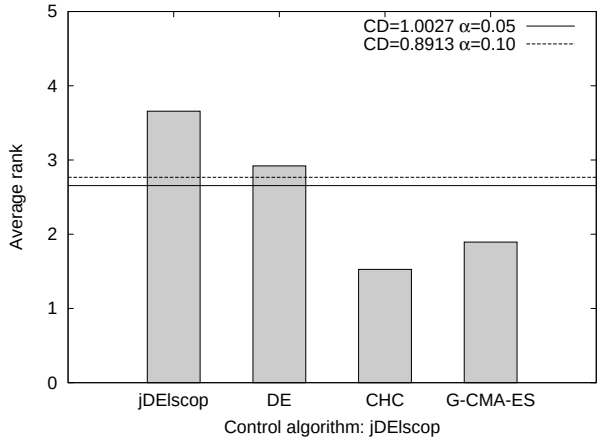

Fig. 7 Bonferroni-Dunn's graphic corresponding to the results when $D = 500$.

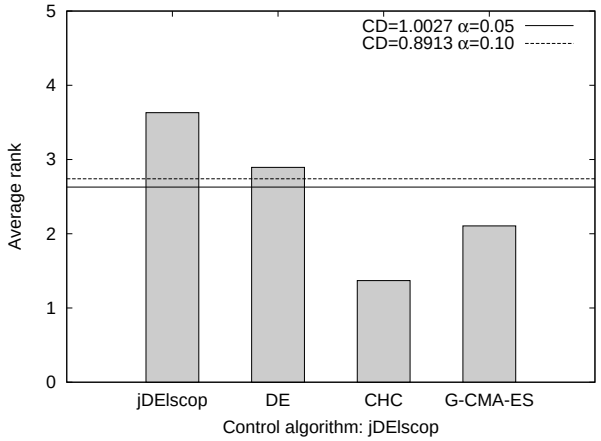

Fig. 6 Bonferroni-Dunn's graphic corresponding to the results when $D = 200$.

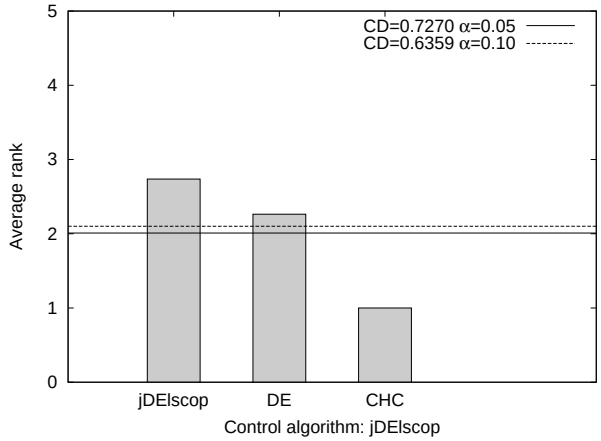

Fig. 8 Bonferroni-Dunn's graphic corresponding to the results when $D = 1000$.

Hommel's. They are based on computation of the adjusted p-values. Pairwise comparisons of all algorithms were done between each other.

Tables 22–26 report adjusted p-values, which take into account that multiple tests were conducted. We applied the following methods (García and Herrera 2008; García et al 2010) to acquire adjusted p-values: Nemenyi's test, Holm's procedure, Shaffer's static procedure, and Bergmann-Hommel's dynamic procedure. The mark ✓ indicates the case when a particular test rejects hypothesis for $\alpha = 0.05$.

The adjusted p-values for $D = 50$ are presented in Table 22. For $\alpha = 0.10$ and for $\alpha = 0.05$ Nemenyi's, Holm's, Shaffer's reject hypotheses 1–3 and Bergmann-Hommel's procedures rejects hypotheses 1–4.

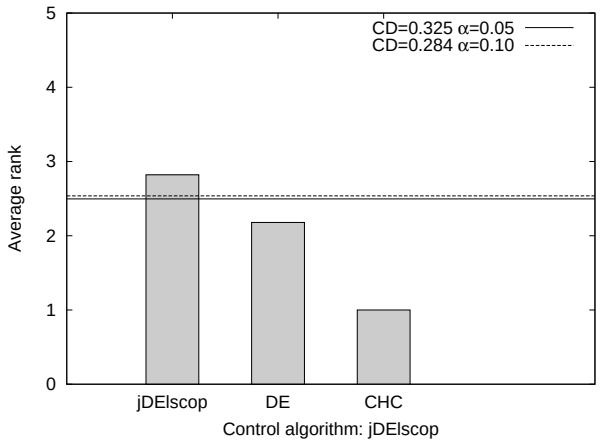

Fig. 9 Bonferroni-Dunn's graphic corresponding to the results on a set of all dimensions.

Table 23 shows the adjusted p-values for dimension $D = 100$. The Nemenyi's, Holm's, Shaffer's, and procedures reject hypotheses 1–3 for $\alpha = 0.10$ and for $\alpha = 0.05$. Bergmann-Hommel's procedures rejects hypotheses 1–4 for $\alpha = 0.10$ and for $\alpha = 0.05$.

The adjusted p-values for $D = 200$ are presented in Table 24. For $\alpha = 0.10$ and also for $\alpha = 0.05$ Nemenyi's, Holm's, Shaffer's, and Bergmann-Hommel's procedure reject hypotheses 1–3, and for $\alpha = 0.10$ Bergmann-Hommel's procedure rejects hypothesis 4, too.

Table 25 shows the adjusted p-values for dimension $D = 500$. For $\alpha = 0.05$ Nemenyi's procedure rejects hypotheses 1–3, while Holm's, Shaffer's, and Bergmann-Hommel's procedures reject hypotheses 1–4. The Nemenyi's, Holm's, Shaffer's, and Bergmann-Hommel's procedures reject hypotheses 1–4 for $\alpha = 0.10$. And for $\alpha = 0.10$ Nemenyi's procedure rejects hypothesis 4, too.

The adjusted p-values for $D = 1000$ of three algorithms only (without the G-CMA-ES algorithm) are presented in Table 26. For $\alpha = 0.10$ and $\alpha = 0.05$ Nemenyi's, Holm's, Shaffer's, and Bergmann-Hommel's procedures reject hypotheses 1–2.

Finally, statistical analysis of the average error was conducted. These well-known statistical tests show that the proposed jDElscop algorithm performs significantly better than the CHC and DE algorithms and performs better or comparable (not significantly better) to the original DE algorithm using the *DE/rand/1/exp* strategy.

The adjusted p-values on a set of all dimensions are presented in Table 27 for the jDElscop, CHC, and G-CMA-ES algorithms (without the G-CMA-ES algorithm). For $\alpha = 0.10$ and $\alpha = 0.05$ Nemenyi's, Holm's, Shaffer's, and Bergmann-Hommel's procedures reject hypotheses 1–3, e.g. all hypotheses are rejected.

The statistical procedures in this analysis were based on mean values. Apart from these tests, a comparison through the use of the median measure would also be informative. It might provide another aspect of differences between the algorithms.

6 Conclusions

This paper proposed a jDElscop algorithm. This algorithm was evaluated on a set of benchmark functions provided for the Special Issue on Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems.

The algorithm uses three DE's strategies, a newly proposed population size reduction mechanism, and a mechanism for changing the sign of F control parameter. The self-adaptive control mechanism for changing the control parameters (F and CR) is applied on all three DE strategies.

The obtained results and statistical analysis give evidence that the jDElscop algorithm is a highly competitive algorithm for large scale continuous optimization problems. To summarize the results of the tests, the jDElscop algorithm presents significantly better results than the remaining algorithms, in most cases.

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