Methods for Measuring Vertical Tire Stiffness

R. K. Taylor, L. L. Bashford, M. D. Schrock

Abstract. *Vertical stiffness was measured for a 260/80R20 radial ply agricultural drive tire using five methods; load-deflection, non-rolling vertical free vibration, non-rolling equilibrium load-deflection, rolling vertical free vibration, and rolling equilibrium load-deflection. Tests were conducted at three inflation pressures (41, 83, and 124 kPa). Non-rolling free vibration resulted in the highest stiffness for all inflation pressures. Load-deflection and non-rolling equilibrium load-deflection results were similar at all inflation pressures. Rolling vertical free vibration and rolling equilibrium load-deflection results were similar at inflation pressures of 83 and 124 kPa. Non-rolling free vibration is not an adequate method for determining vertical tire stiffness when the tire is represented by a spring and viscous damper in parallel. The discrepancy between load-deflection and free vibration test results was attributed to hysteresis in the tire. Tire properties should be measured at the desired forward velocity when the tire is modeled as a spring and viscous damper in parallel.*
Keywords. *Tire properties, Vertical stiffness, Vibration.*

Simulating ride dynamics of unsprung vehicles, such as an agricultural tractor, requires accurate knowledge of tire properties. However, a review of ride vibration behavior of off-road vehicles (Crolla and Maclaurin, 1985) found no adequate tire model. Lines and Murphy (1991) indicated that little information on tire properties is available to adequately simulate tractor ride vibration.

The vertical tire model has long been considered to be a spring and damper in parallel (Thompson et al., 1972; Volfson and Estrin, 1983; and Crolla et al., 1990). The spring portion has been proposed to be both linear and nonlinear, whereas the damper typically is assumed to be linear. However, Stayner et al. (1984) stated that a linear spring and a viscous damper are inadequate for representing the tire on unsprung vehicles, but they also suggested that more complex systems have not been substantiated in simulating ride vibration.

Researchers have long used load-deflection tests to determine vertical tire stiffness. Kising and Gohlich (1989) reported using three other methods for determining tire properties: free oscillation, harmonic excitation, and harmonic/stochastic excitation. The first two methods are used commonly in analyzing tire properties, whereas the third is better known in the field of signal analysis in identifying the transfer function. Kising and Gohlich (1989) reported that the three methods resulted in similar values but that the harmonic/stochastic excitation method was the most reliable at the higher travel speeds of interest to them. Matthews and Talamo (1965) used load-deflection tests to determine static stiffness and free vibration to determine dynamic stiffness. They found that the average dynamic stiffness exceeded static stiffness by about 10%. Brassart et al. (1995) reported that vertical stiffness determined from forced vibration exceeded that measured from load-deflection tests by about 40%.

Pottinger et al. (1985) found that stiffness of rolling tires was about 13% less than stiffness of non-rolling tires with a coefficient of determination of 0.945. Static non-rolling stiffness was determined from load-deflection tests, whereas rolling stiffness was determined from free oscillation. Lines and Murphy (1991) and Kising and Gohlich (1989) found that tire stiffness decreased with increased forward velocity. Lines and Murphy (1991) also reported that no simple relationship occurred between stiffness of non-rolling and rolling tires. Kising and Gohlich (1989) found that rolling stiffness was 15 to 25% less than non-rolling stiffness and that the vertical stiffness of a rolling tire becomes constant above 30 km/h.

The objective of this research was to determine the vertical stiffness of a 260/80R20 radial ply agricultural drive tire using the following methods:

1. Load-deflection (LD)
2. Non-rolling vertical free vibration (NR-FV)
3. Non-rolling equilibrium load-deflection (NR-LD)
4. Rolling vertical free vibration (R-FV)
5. Rolling equilibrium load-deflection (R-LD)

Equipment and Procedures

A small radial-ply drive tire was chosen for this study based on the limited size of available test equipment. Tire specifications and dimensions are shown in table 1.

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Table 1. Tire specifications

Manufacturer	Goodyear
Tire	Super Traction Radial
Tire size	260/80R20
Overall diameter (m)	0.95
Section width (m)	0.26
Rolling circumference (m)	2.85
Static loaded radius (m)	0.43
Maximum rated capacity	1018 kg @ 165 kPa
Unloaded circumference (m*)	2.99

* Dimension measured by author, all others from Goodyear, 1995.

A pivoting test frame was constructed with a radius (l) of 2.34 m from the pivot point to the axle center (fig. 1). The drop mechanism used in the vertical free vibration tests consisted of a steel base plate with an inverted U-shaped bracket with two tabs to support the axle. The distance from the base plate to the tabs was 0.45 m, which placed the axle center 0.47 m from the base plate. The tabs were just long enough to support the axle without causing initial upward movement of the tire when the mechanism pivoted to drop the tire. The axle, when resting on the tabs, was in contact with a switch that initiated data acquisition at the instant the tire was dropped.

A similar frame was constructed for the rolling tests. The distance from the pivot point to the axle center was 1.52 m. A hydraulic motor drove the test carriage through a set of cables along a hard surface approximately 15.5 m in length.

A rotary potentiometer at the pivot point was used to measure angular displacement of the test frame. Vertical displacement was approximated by angular displacement since angular displacement was small compared to the distance (l) to the axle. The potentiometer was powered by a 5 VDC power supply, and data were recorded as a function of time by a PC equipped with a data acquisition card.

The mass moments of inertia for both test frames were needed for various load configurations. The center of gravity and natural frequency were measured to determine the mass moment of inertia for each frame. The center of gravity was measured by suspending each frame from two different points and measuring the angle of the frame from the vertical plane. The natural frequency was measured by timing the small oscillations of each frame while they were suspended from the pivot point. Oscillations were initiated by pushing the frame, which was then allowed to move freely.

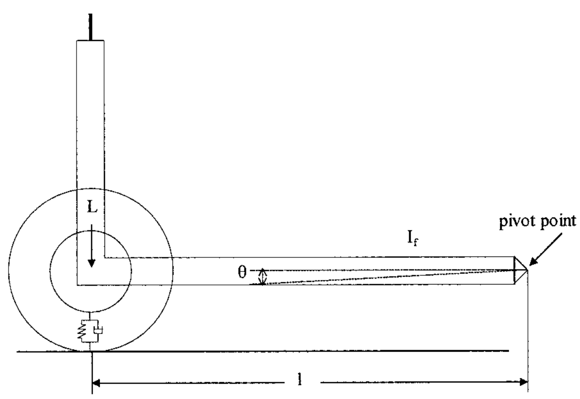

Figure 1–Diagram of the test frame during vertical free vibration.

STATIC LOAD-DEFLECTION TESTS

Load-deflection data were measured at three inflation pressures (41, 83, and 124 kPa) by increasing the static load and measuring static loaded radius (SLR) as the tire rested on a smooth metal plate (ASAE S296.4, 1999). The unloaded radius (UR) of the tire was determined from the circumference of the tire. As load was applied to the tire, SLR was measured and tire deflection was calculated as the difference between the UR and SLR. The vertical load applied by the test frame was determined by weighing the frame at the axle with a load cell. Cast iron suitcase weights (approximately 0.4 kN each) were added to the test frame in pairs, and axle height was measured each time. This process was repeated until the load on the tire exceeded the rated load for the inflation pressure. At this time, load was reduced in the same increments by removing suitcase weights in the reverse order and axle heights were measured. This process was repeated until the tire returned to the initial load condition. The rated and vertical loads for each series of tests and inflation pressures are shown in table 2.

The load-deflection tests were conducted twice for each inflation pressure. A load-deflection test was conducted at each inflation pressure before and after the series of non-rolling vertical free vibration tests at that pressure. Taylor (1996) examined many potential equations to describe the relationship between load and deflection. Equation 1 was selected based on simplicity and goodness of fit. Both data sets were combined and regressed to fit equation 1:

$$L = a\delta + b\delta^2 \tag{1}$$

where
- L = vertical load (N)
- δ = vertical deflection (m)
- a = regression coefficient (N/m)
- b = regression coefficient (N/m^2)

Vertical stiffness was determined by taking the first derivative of load with respect to deflection. This resulting stiffness increases linearly with deflection (eq. 2):

$$k = a + 2b\delta \tag{2}$$

where
- k = vertical stiffness (N/m)
- d = vertical deflection (m)
- a = regression coefficient (N/m)
- b = regression coefficient (N/m^2)

Table 2. Vertical loads (kN) used for each inflation pressure

	Inflation Pressure (kPa)		
	41	83	124
Rated Load*	5.3	6.8	8.3
Test Loads	2.3	2.3	2.3
	3.2	3.2	3.2
	4.0	4.0	4.0
	4.8	4.8	4.8
	5.7	5.7	5.7
	6.6	6.6	6.6
	7.4	7.4	7.4
		8.3	8.3
		9.1	9.1

* For single tires used in field service with speeds up to 40 km/h.

NON-ROLLING FREE VIBRATION TESTS

Non-rolling vertical stiffness was determined with two methods. One method measured the rate of decay of free oscillation by dropping the tire from a known height and regressing the vertical position versus time data. The second method used the equilibrium load-deflection data from each free vibration test. The equilibrium deflection for a given vertical load was calculated based on a SLR measurement made after vibration ceased.

Non-rolling free vibration tests were conducted at various vertical loads and the same three inflation pressures as the static load-deflection tests (table 2). Non-rolling free vibration tests were performed three times for each load condition. The angular position of the test frame was measured with a rotary potentiometer as a function of time at a rate of 400 samples/s for 5 s after the tire was dropped. The angular position of the frame was used to determine the actual vertical position of the axle. After oscillations had fully decayed, the final axle height was measured for each test to determine the equilibrium deflection. Vertical load and equilibrium deflection were regressed to fit equation 1. Stiffness was determined from the non-rolling equilibrium load deflection data in the same manner as the static load-deflection data.

The generally accepted vertical model for a tire consists of a spring and viscous damper in parallel. Assuming that stiffness and damping coefficients are constant values and that damping is entirely viscous, the equation of motion for the test frame and tire/wheel system shown in figure 1 can be represented by equation 3. This equation is valid for small values of θ, where $\sin\theta$ can be approximated by θ. For oscillatory (underdamped) motion, the general solution to the homogenous portion of this equation of motion is shown in equation 4, where the angular displacement is a function of time (Thomson, 1988). The equilibrium position is determined by the vertical load (L) on the tire for the given inflation pressure. The measured angular position versus time data were regressed to equation 4, which typically yielded coefficients of determination exceeding 0.95.

$$I_f \ddot{\theta} + l^2 c \dot{\theta} + l^2 k \dot{\theta} = Ll \tag{3}$$

$$\theta(t) = \Theta e^{-\zeta \omega_n t} \sin\left(\sqrt{1-\zeta^2}\ \omega_n t + \beta\right) + \theta_{eq} \tag{4}$$

where

θ = angular position of the test frame (rad)
$\dot{\theta}$ = angular velocity of the test frame (rad/s)
$\ddot{\theta}$ = angular acceleration of the test frame (rad/s^2)
I_f = mass moment of inertia of the test frame (kg m^2)
l = radius of the test frame (m)
c = damping coefficient (N s/m)
k = stiffness coefficient (N/m)
L = vertical load (N)
t = time (s)
Θ = maximum angular position of the frame at t_0 (rad)
ω_n = natural frequency (rad/s)
ζ = damping ratio
β = initial phase angle (rad)
θ_{eq} = equilibrium position (rad)

Data for each inflation pressure were analyzed over the same approximate vertical range of motion. The minimum range of motion for each inflation pressure determined the range used for analysis. For each inflation pressure, this occurred for the tests with the smallest load on the tire. When the vertical load on the tire was increased, the range of motion for the tire increased (i.e., the tire was deflected more and rebounded higher). To equalize the range of motion, oscillations with the greatest magnitude at the front of each data set were removed in half-cycle intervals until a vertical range of motion was similar for all tests at a given inflation pressure.

The vertical stiffness (k) for a given condition was determined from the natural frequency (ω_n), the mass moment of inertia of the frame assembly (I_f), and the distance from the pivot point of the frame to the axle (l) (eq. 5):

$$k = \frac{I_f\, \omega_n^2}{l^2} \tag{5}$$

Limiting the range of deflection over which data are analyzed should result in a stiffness value that is representative of the stiffness for the equilibrium deflection. The vertical stiffness determined for each repetition was regressed as a function of the equilibrium deflection that was measured for each repetition to fit equation 2. The resulting equation defines the relationship between vertical stiffness and deflection over a wide range of deflections for each inflation pressure.

ROLLING FREE VIBRATION TESTS

Rolling vertical stiffness was determined with the same two methods as non-rolling vertical stiffness. Free oscillation was initiated by rolling the tire over a 0.09 m high obstacle at approximately 4 km/h. The angular position of the test frame was recorded as a function of time for 5 s using a similar measurement apparatus described for the non-rolling free vibration tests. Linear speed of the test carriage and rotational speed of the tire were recorded by counting pulses from optical encoders over the duration of each test. Rolling vertical free vibration tests were conducted at the same inflation pressures as other tests. The vertical loads used for the non-rolling tests varied from those shown in table 2 because of the different test frame. A minimum of five loads representing a similar range as those in table 2 was used at each inflation pressure. The procedure that was used to determine the stiffness for the non-rolling free vibration tests also was used for the rolling tests. Tests at each load and inflation pressure were repeated five times. Position versus time data were analyzed in the same manner as the non-rolling free vibration data. A rolling axle height was determined from the equilibrium position of the regression for each rolling test. Rolling equilibrium deflection was obtained from this value and the measured circumference of the tire. The load on the tire and the rolling equilibrium deflection were used to determine the rolling load-deflection stiffness.

RESULTS

As expected with a rubber product, some hysteresis was experienced during the load-deflection tests. For a given load and inflation pressure, the tire deflection was greater during unloading than it was during loading. More hysteresis was observed at lower inflation pressures. Load data were regressed as a function of deflection using equation 1 for load-deflection, non-rolling equilibrium load-deflection, and rolling equilibrium load-deflection measurement methods. Stiffness was determined by taking the first derivative of load with respect to deflection. The coefficients for equation 2 and the r^2 are shown in table 3 for the three inflation pressures and load-deflection measurement methods. These r^2 values are from the regression of load data as a function of deflection (eq. 1). Non-rolling equilibrium load-deflection and rolling equilibrium load-deflection measurement methods had greater r^2 values than the static load-deflection method because hysteresis was absent in the two former methods (table 3).

Stiffnesses obtained from non-rolling and rolling free vibration tests were regressed as linear functions of equilibrium deflection (eq. 2). The model coefficients and statistical parameters are shown in table 3. The regression results show that the vertical stiffness from rolling free vibration for an inflation pressure of 41 kPa was independent of deflection.

The stiffness-deflection relationships determined with different methods are shown in figure 2 for an inflation pressure of 41 kPa. Stiffness determined from non-rolling free vibration is higher than stiffness with any of the other measurement methods. Stiffnesses determined from load-deflection, non-rolling equilibrium load-deflection, and rolling equilibrium load-deflection methods appear similar. Stiffness determined from rolling free vibration is essentially constant with changing deflection. This observation coupled with a low coefficient of determination indicates that stiffness is independent of deflection for this test condition.

The differences in vertical stiffness clearly indicate that the measurement method impacts the results. This leads the authors to believe that the methods are causing bias in the results.

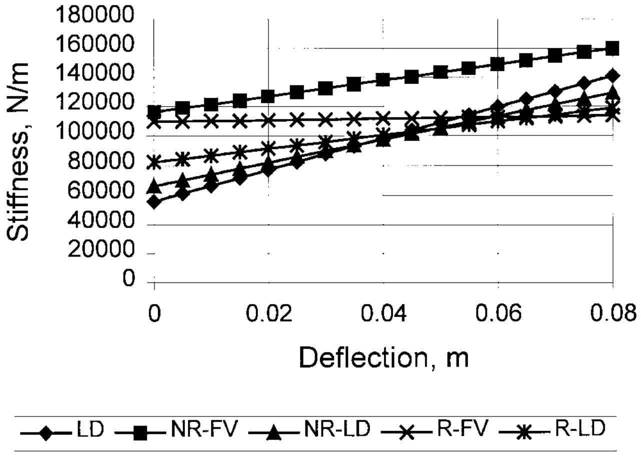

Figure 2–Vertical stiffness versus deflection for the five measurement methods. Inflation pressure is 41 kPa.

The stiffness-deflection relationships determined with different measurement methods at inflation pressures of 83 and 124 kPa are shown in figures 3 and 4, respectively. Stiffness determined from non-rolling free vibration is typically higher than stiffness with any of the other measurement methods. Stiffnesses determined from load-deflection and non-rolling equilibrium load-deflection methods appear similar. Stiffness determined from rolling free vibration and rolling equilibrium load-deflection methods also appear similar.

These results show that different measurement methods for vertical tire stiffness result in different stiffness values. Though the differences in stiffness determined by non-rolling free vibration and load-deflection methods are similar to that found by other researchers (Matthews and Talamo, 1965 and Brassart et al., 1995) it raises the question of which value is correct. If the mathematical model (nonlinear spring and viscous damper in parallel) of the tire was correct, the stiffness determined from all methods should be the same.

Table 3. Stiffness-deflection model coefficients ($k = a + 2b\delta$) and statistical parameters for each inflation pressure and measurement method.

Measurement Method	Inflation Pressure (kPa)	a N/m	2b N/m^2	r^2
LD	41	55628	1066160	0.962
	83	87092	1506772	0.980
	124	101815	3176060	0.989
NR-FV	41	116036	547130	0.711
	83	156934	821204	0.852
	124	209444	1185690	0.819
NR-LD	41	65661	798010	0.996
	83	89071	1652488	0.994
	124	106534	3195760	0.998
R-FV	41	109579	50927	0.074
	83	131445	470473	0.946
	124	160773	702653	0.951
R-LD	41	82040	459748	0.999
	83	142076	411308	0.999
	124	186633	292616	0.995

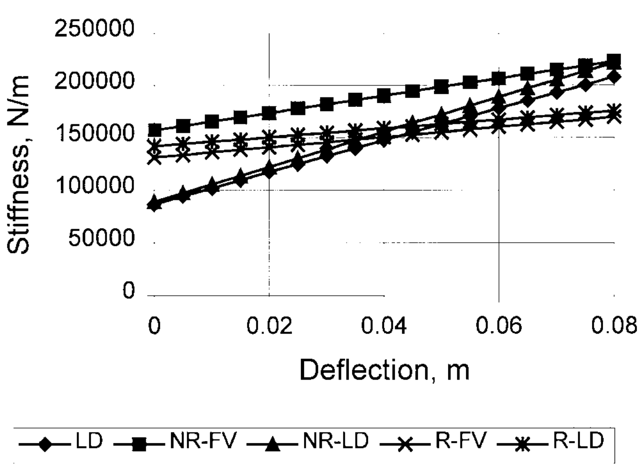

Figure 3–Vertical stiffness versus deflection for the five measurement methods. Inflation pressure is 83 kPa.

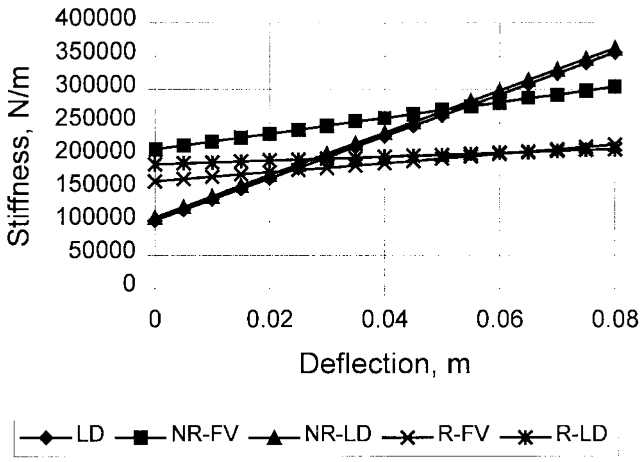

Figure 4–Vertical stiffness versus deflection for the five measurement methods. Inflation pressure is 124 kPa.

Results from the equilibrium conditions may indicate how to choose the correct stiffness value. If the spring force is determined by integrating stiffness with respect to deflection, then the stiffness determined from non-rolling free vibration cannot be correct. The restoring force from the spring does not equal the load on the tire for these test conditions, just as the stiffness determined from the two methods differs. The same is true for the stiffness determined from rolling free vibration at an inflation pressure of 41 kPa.

Hysteresis in the tire is likely causing the tire to appear stiffer during free vibration. Hysteretic damping sometimes is considered as a complex stiffness (Steidel, 1989). The tire is allowed to relax in the time between applying loads for the load-deflection method, so the complex stiffness component from the hysteretic damping is not measured. As the frequency of vibration increases, the rubber in the tire carcass has less time to relax and thus deflect. The tire appears stiffer because it is deflecting less for a given load. As inflation pressure is reduced, the tire carcass plays a larger role in vertical stiffness, which means hysteresis becomes more significant.

CONCLUSIONS

Vertical stiffness was measured for a 260/80R20 radial ply agricultural drive tire using five methods; load-deflection, non-rolling vertical free vibration, non-rolling equilibrium load-deflection, rolling vertical free vibration, and rolling equilibrium load-deflection. Tests were conducted at three inflation pressures (41, 83, and 124 kPa). The following conclusions can be made based on the data presented here.

1. Non-rolling free vibration resulted in the greatest stiffness for all inflation pressures.
2. Load-deflection and non-rolling equilibrium load-deflection results were similar at all inflation pressures.
3. Rolling vertical free vibration and rolling equilibrium load-deflection results were similar at inflation pressures of 83 and 124 kPa.
4. If the tire is represented by a spring and viscous damper in parallel, non-rolling free vibration is not an adequate method for determining vertical tire stiffness.
5. The discrepancy between load-deflection and non-rolling free vibration test results was attributed to hysteresis in the tire.
6. If the tire is modeled as a spring and viscous damper in parallel, tire properties should be measured at the desired forward velocity.
7. If the tire is represented by a spring and viscous damper in parallel, rolling free vibration is not an adequate method for determining vertical tire stiffness at low inflation pressures.

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