Measurement of Equivalent Stiffness and Damping of Shock Absorbers

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INTRODUCTION

This paper describes a new testing and analysis methodology for obtaining equivalent linear stiffness and damping of automotive shock absorbers for use in system-level chassis and vehicle computer aided engineering (CAE) models for noise and vibration prediction. Since most of the system-level CAE models in a vehicle are linear in nature, equivalent linear parameters of chassis components are much more useful than comprehensive non-linear models. Also, a hydraulic actuated elastomer test machine which is the current industry standard, is not suitable for testing shock absorbers in the mid-to-high frequency range where the typical road input displacements fall within the noise floor of the hydraulic machine. Hence, an electrodynamic shaker was used for exciting the shock absorbers under displacements less than 0.05 mm up to 500 Hz. Furthermore, instead of the swept sine technique, actual road data were used to excite the shocks. Equivalent linear spring-damper models were developed based on least-squares curve-fitting of the test data.

There are two approaches to modeling shock absorbers: analytical (or physical) modeling based on physical and geometrical data, and parametric modeling based on experimental data. An exhaustive review of physical models to date is presented by Duym, et. al [1]. The physical models [2-4] attempt to calculate the shock absorber force as a function of displacement, velocity and acceleration from a system of differential equations. Other simplified models using springs and dashpots in various combinations have been built. Attempts have been made to include non-linearities due to hysteresis and backlash, which lead to a set of non-linear differential equations requiring numerical solution [5].

The parametric modeling approach involving development of an input/output relation of the shock absorber based on experimental data is ideal for computer aided engineering (CAE) models . In this approach, a shock absorber is characterized by a **"black-box"** system for a specific range of test conditions. The shock absorber is subjected to a known input and the output force is measured. A model is then developed from these measurements, which describes the input-output relationship. The parameters of the model may or may not have any physical meaning, but are strongly correlated with measurements. One limitation of parametric modeling approach is that the model is valid only within the boundary of test conditions. This means, a model that has been developed using smooth road test data may not be accurate for use under rough road conditions.

A comprehensive physical model of the shock absorber is necessary to study the effects of design changes and to tune the shock absorber to obtain the desired performance. The vendors have used physical models in the design stage. If the objective, however, is to characterize the performance of the shock absorbers for CAE simulations and benchmarking, the parametric modeling approach similar to the one presented in this paper is appropriate.

TEST PROCEDURE

An electrodynamic shaker was used for exciting the shock absorbers under displacements less than 0.05 mm up to 500 Hz. Furthermore, instead of the swept sine technique as used in standard hydraulic actuated testing, actual road data were used to excite the shocks. This enables the development of both non-linear as well as equivalent linear parametric models from the measured data.

Figure 1 shows a picture of the experimental set-up. As seen, the shock absorber is fixed at the tube end using a U-shaped clamp to a massive plate on a test bed. The rod end of the shock is connected to a shaker (50-lbf shaker from MB dynamics) through an impedance head (PCB Model No. 288C01). The impedance head has an accelerometer and a force transducer, both integrated into the same unit for measuring the displacement and force. The LMS Time Waveform Replicator (TWR Revision 3.4

under TMON) software and DIFA Scadas II (with QDAC) front-end hardware were used to generate, apply and control the input to the shaker in order to reproduce road excitations in the lab.

For testing under actual stoke lengths (pre-loads), a pair of thin cords and a thin aluminum plate was used as shown in the picture. The aluminum plate (size= 2 X 2 X 1/8 inch) with a hole was bonded to the rod end near the step, and a pair of thin cotton cords (about 3 mm in diameter) was attached to the plate using S-hooks. The other ends of the cords were fixed to the bottom plate. First, the cords were tied by pushing the rod to its approximate stroke length, and then the exact stroke length was adjusted and maintained by using turnbuckles in the middle of the cords. Based on trials with other materials we decided to use the cotton cord for pre-loading the shock.

DATA ANALYSIS

A linear spring-damper model of the form, $f(t)= K\,x(t) + c\,v(t)$, where x= input displacement, v= input velocity, and f (t)= output force was developed based on test data in the time domain. The term K is the spring stiffness (N/m) and c= viscous damping coefficient (Ns/m). In the frequency domain, for a simple harmonic excitation, the above model is interpreted as:
$F(\omega)= K\,X(\omega) + j\,c\,\omega\,X(\omega)$, where ω is the frequency in rad/sec. The input-output relationship in the frequency domain is: $F(\omega) / X(\omega) = K_R + j\,K_I$,
where K_R is the real part and K_I is the imaginary part of the dynamic stiffness. We can also write
$F(\omega) / X(\omega) = K_M\, e^{j\phi}$, where, $K_M= \sqrt{(K^2 + (c\omega)^2)}$,
and $\phi = \arctan(c\omega/K)$.
Here K_M is the magnitude of the Dynamic Stiffness and ϕ is its phase at the frequency of excitation. Values of K_M and ϕ at various frequencies of interest can be obtained from the above linear model.

It should be noted that K & c are treated as constants (independent of displacement amplitude & frequency) in the time domain, while the complex dynamic stiffness is a function of frequency if the excitation is assumed as simple harmonic. Care must be exercised in the correct use and interpretation of these models, as they are not applicable for all cases.

RESULTS & DISCUSSION

Table 1 is a summary of all results obtained from the shaker tests. Figures 2 through 5 refer to test No. 1, for the front shock absorber under smooth road excitation. Figure 2 shows the raw data; input displacement and output force in the time domain before post-processing. The r.m.s. values of the measured displacement and force in this case are 0.013 mm and 0.91 Newtons. Note the rapid decrease in the original PSDs with increase in frequency as shown in Figure 3. The sampling frequency for all road data was 2000 Hz, hence data up to half its value are theoretically useful. The filtered response, however, shows data only in the range 25-300 Hz. This is the frequency range in which we were able to generate valid control algorithms in all our tests without either over-loading or under-loading the shaker. The shaker displacements were either too large (below 25 Hz) or too small (above 300 Hz) outside of this frequency range.

Figure 4 shows the force vs. displacement and force vs. velocity curves obtained by plotting the filtered time histories. The contribution of many frequencies to the stiffness and damping of the shock absorber as well the presence of strong non-linearities are quite evident from these shapes. In fact, one can easily extract the bi-linear damping exhibited by most shocks under low frequencies from these hysteresis loops.

Next, a comparison of the measured vs. model force is shown in Figure 5 in two different formats. The model force is generated from the curve-fitting constants. The accuracy of the model varies with each test. It is seen that an ideal linear model is one in which all the dots lie on the straight line in Figure 5.

CONCLUSIONS

It is demonstrated that we can use electrodynamic shakers to obtain the equivalent dynamic properties of shock absorbers for NVH applications. Application of pre-load is an issue which needs further investigation. Although a simple fixture using cotton cord worked well in this case, some shakers are capable of withstanding static pre-loads which may be suitable for testing shock absorbers under larger displacements and lower frequencies.

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**Table 1. Equivalent Stiffness (K) and Damping (c) for each test case:
Frequency Range: 25-300 Hz**

Shock Absorber	Excitation	RMS Disp., mm	RMS Force, N	Stiffness K, N/mm	Damping c, Ns/mm
Front	Rough Road	0.025	1.30	32.73	0.170
Front	Smooth Road	0.013	0.91	35.19	0.231
Rear	Rough Road	0.031	1.64	24.16	0.200
Rear	Smooth Road	0.019	1.15	25.42	0.231

Figures Captions:

Figure 1. Electrodynamic Shaker Experimental Set-up

Figure 2. Input and Output Time Histories

Figure 3. Power Spectral Densities (PSD) of Input and Output

Figure 4. Force vs Displacement and Force vs. Velocity Plots

Figure 5. Equivalent Linear Model

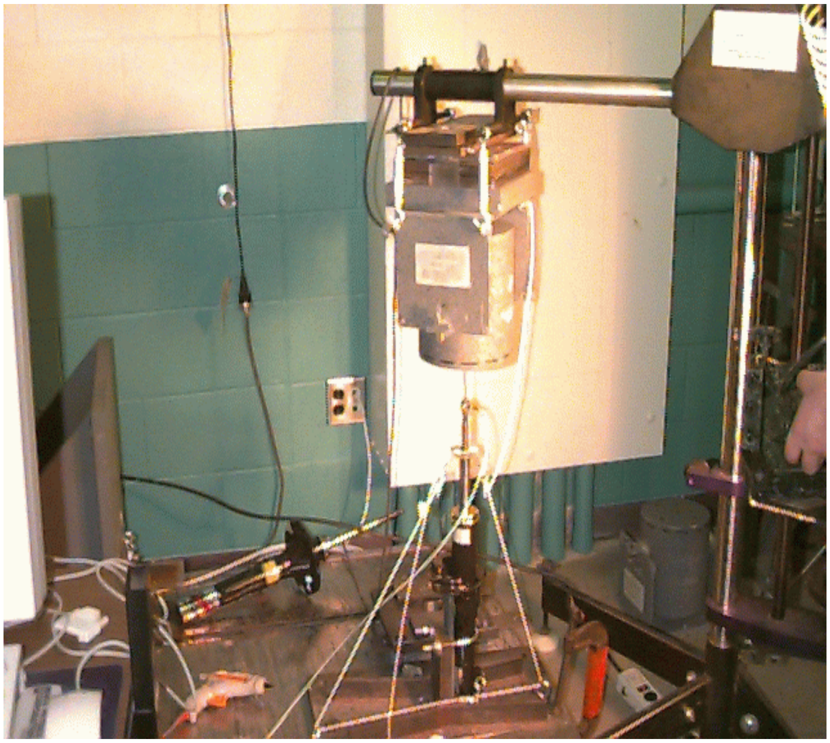


Figure 1. (Rao)

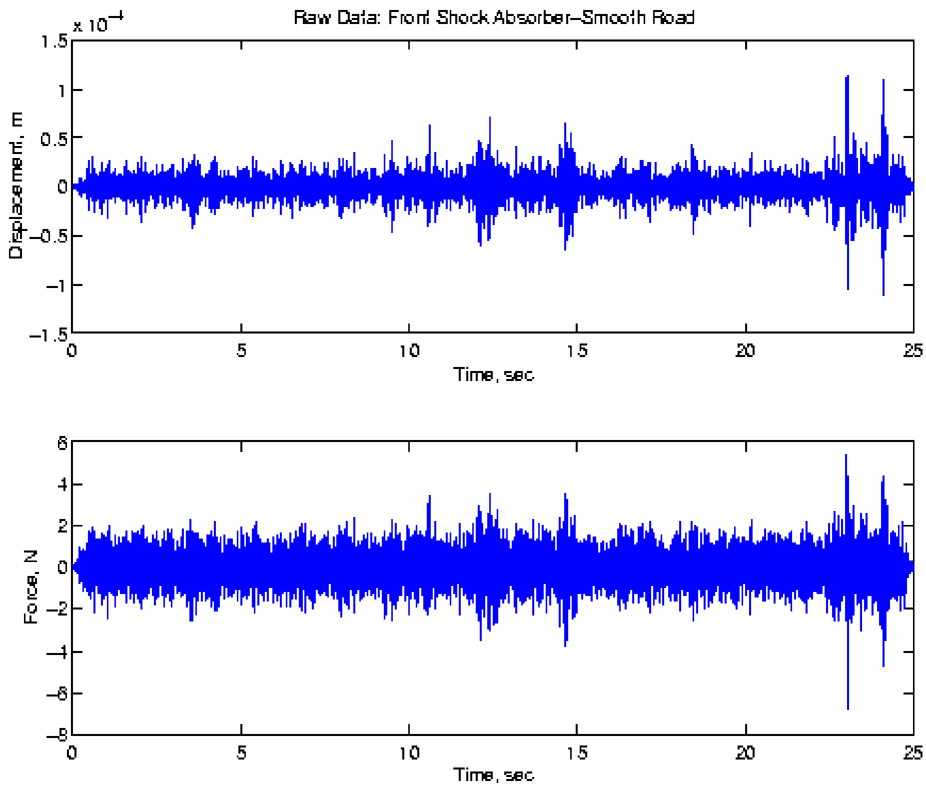

Raw Data: Front Shock Absorber–Smooth Road
x 10⁻⁴
Displacement, m
Time, sec
Force, N
Time, sec

Figure 2. (Rao)

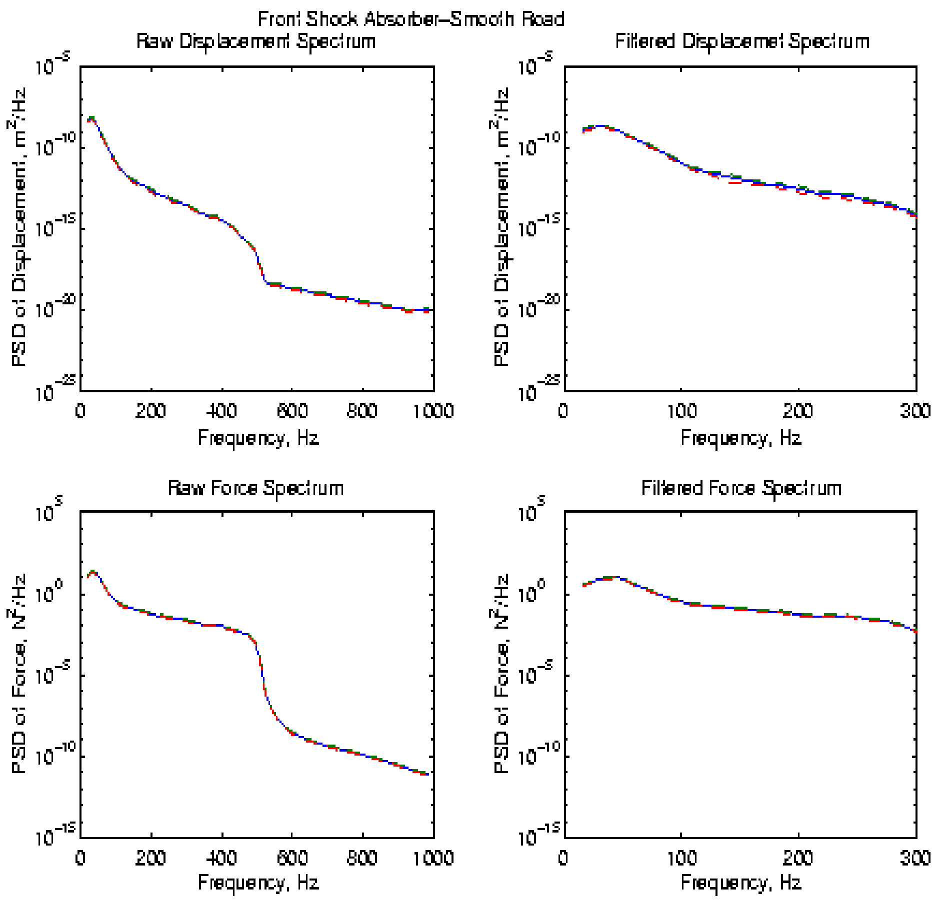

Front Shock Absorber–Smooth Road
Raw Displacement Spectrum
Filtered Displacemet Spectrum
PSD of Displacement, m²/Hz
Frequency, Hz
Raw Force Spectrum
Filtered Force Spectrum
PSD of Force, N²/Hz
Frequency, Hz

Figure 3. (Rao)

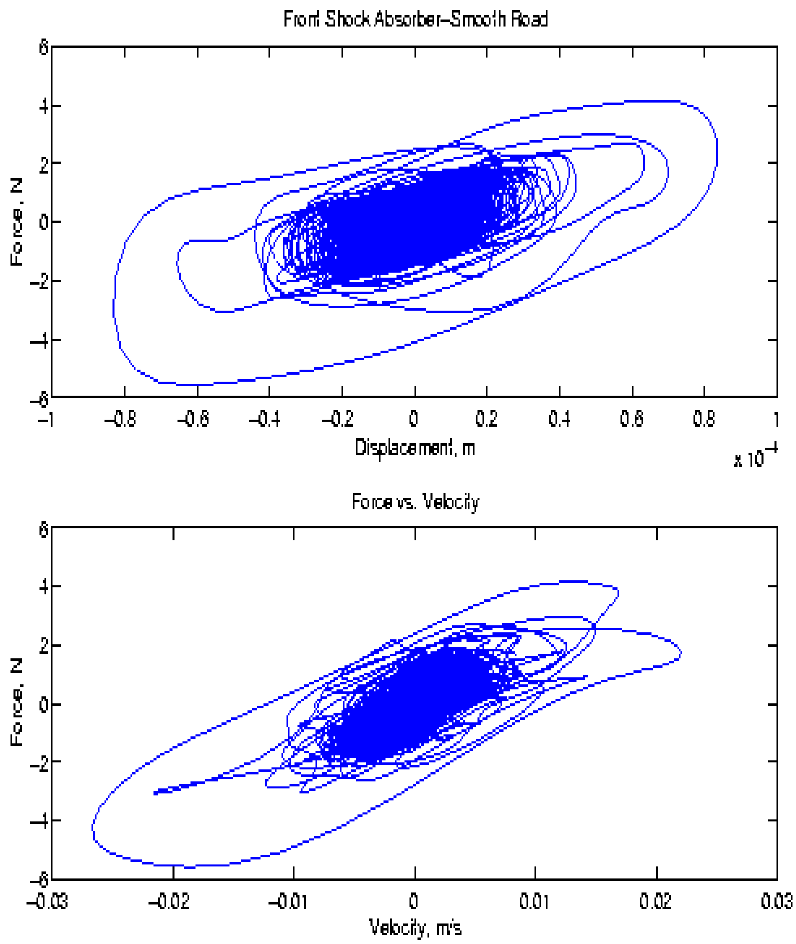
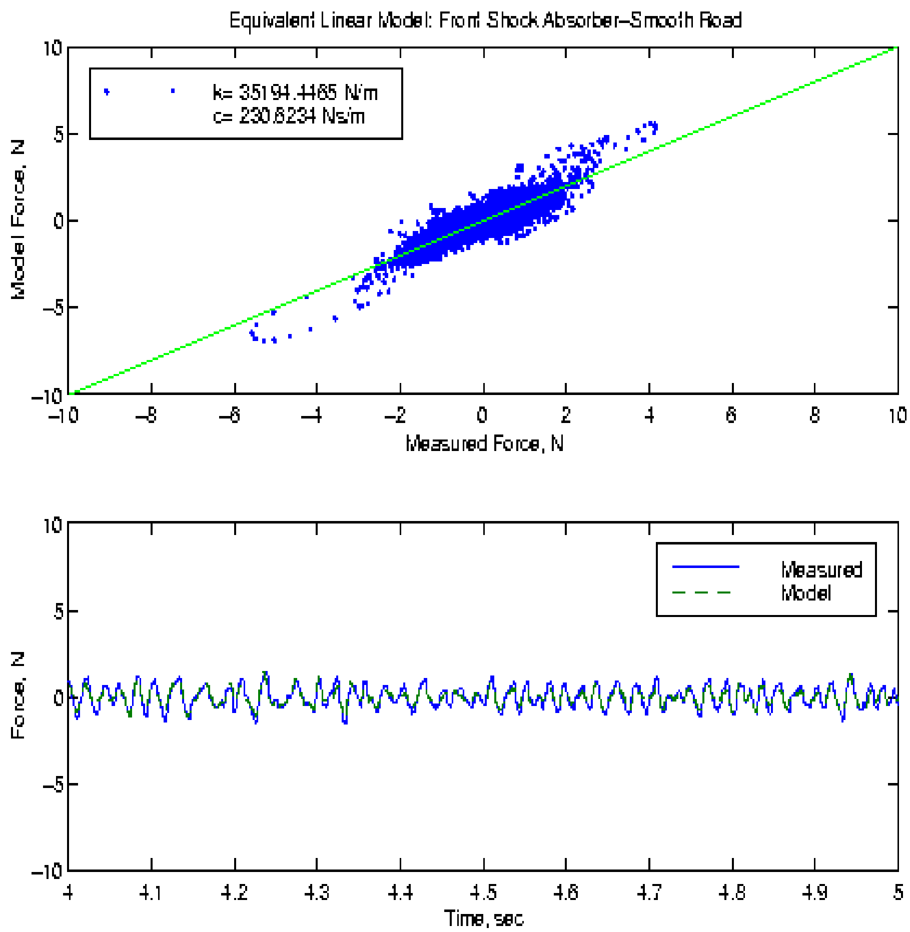

Front Shock Absorber–Smooth Road
Force, N
Displacement, m
x 10^-4
Force vs. Velocity
Force, N
Velocity, m/s

Figure 4. (Rao)



Equivalent Linear Model: Front Shock Absorber–Smooth Road
k= 35194.4465 N/m
c= 230.6234 Ns/m
Model Force, N
Measured Force, N
Measured
Model
Force, N
Time, sec

Figure 5. (Rao)