EE447 : In Class Problem Sets

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1 EE447 In Class Problems: Laplace Transforms and Linearization

1.1 Laplace Transform

1.1.1

$$\mathcal{L}\{0.01e^{-32t}\} =$$

1.1.2

$$\mathcal{L}\{10^4 \sin(0.5t)\} =$$

1.1.3

$$\mathcal{L}\{J\ddot{x} + (B_1 + 2B_2)\dot{x} + K_3 x\} =$$

1.1.4

$$\mathcal{L}\{75e^{+5t}\} =$$

1.2 Inverse Laplace Transform

1.2.1

$$\mathcal{L}^{-1}\{\frac{1}{(s+2)}\} =$$

1.2.2

$$\mathcal{L}^{-1}\{\frac{144}{s^2+144}\} =$$

1.2.3

$$\mathcal{L}^{-1}\{A_2 X(s)s^2 + A_1 X(s)s + A_0 X(s)\} =$$

1.2.4

$$\mathcal{L}^{-1}\{X(s)(s^3 + As^2 + C)\} =$$

1.3 Partial Fraction Expansion

1.3.1

Expand

$$G(s) = \frac{10(s+20)}{(s+10)(s+36)(s+100)}$$

1.3.2

Expand

$$G(s) = \frac{0.01}{(s+0.6)(s+1.6)}$$

and compute the inverse Laplace Transform, $g(t)$.

1.4 Linearization

1.4.1

Linearize the function

$$f(x) = kx + 0.5\sin(x)$$

about the point $x = 3$ (radians). Call the linearized function $\hat{f}(x)$.

1.4.2

Using the result of 1.4.1, graph $f(x)$ and $\hat{f}(x)$ for $k = 1$ and $0 \leq x \leq 6$. You may do the graph by hand or by computer.

1.5 Linearization

Linearize

$$A_2\ddot{x} + A_1(\dot{x} + 0.2\dot{x}^3) + A_0 x = f(t)$$

about the point $\dot{x} = 0$, so that it forms a LODE. **Hint:** The function is actually non linear in $\dot{x}$ so the thing we need to linearize is actually

$$A_2\ddot{x} + A_1(\dot{x} + 0.2\dot{x}^3) + A_0 x = f(\dot{x})$$

1.5.1

A spring is measured at several applied forces and the resulting data fits the function

$$f(x) = 0.02x^4 + 0.65x^2 + 16x$$

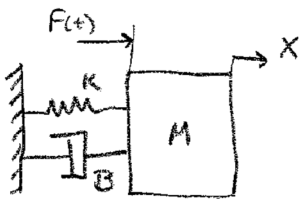
Linearize this function about the point $x = 0$.

1.5.2

Using the result of 1.5.1, for what range of x is the error between $f(x)$ and the linearized function $\hat{f}(x)$ less than 10%? **Hint:** find the result numerically.

2 Translational Dynamical Systems

2.1



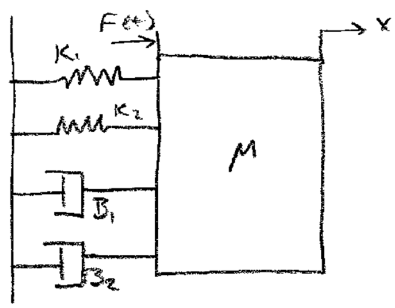
2.1.1

Find the equation of motion (EOM) in ODE form for $x(t)$.

2.1.2

Find $\frac{X(s)}{F(s)}$

2.2



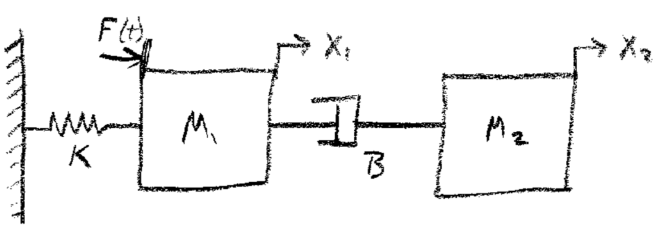
2.2.1

Find the differential equation.

2.2.2

Find the transfer function $\frac{X(s)}{F(s)}$

2.3



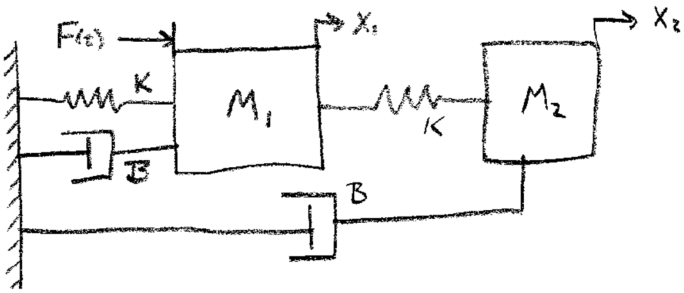
2.3.1

Find the equation of motion (EOM) in ODE form for M_1 and M_2.

2.3.2

Find $\frac{X_2(s)}{F(s)}$

2.4



Find the equations of motion (EOMs) and the transfer function $\frac{X_1(s)}{F(s)}$

2.5

A system has the transfer function

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{M_1 s^2 + (B_1 + B_2)S + K/M_1}$$

2.5.1

Normalize the transfer function $G(s)$.

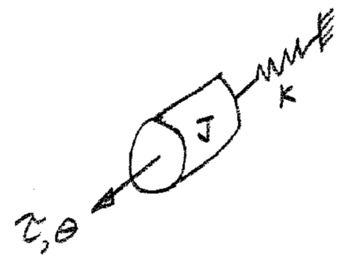
2.6

For the following transfer function, plot the poles and zero(s) in the complex plane.

$$H(s) = \frac{s + 20}{s^2 + 25s + 256.25}$$

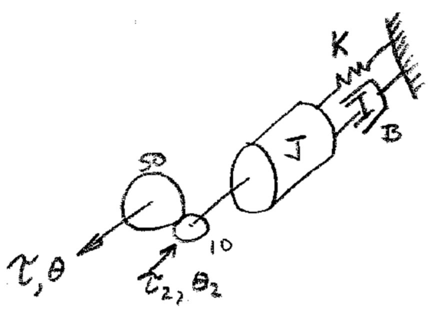
3 Rotational Dynamical Systems and State Space

3.1



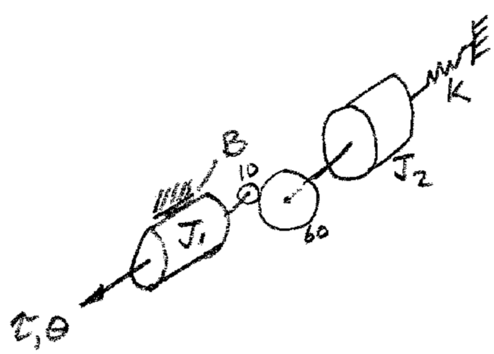
a) Find the equation of motion (EOM)
b) Find $\frac{\theta(s)}{\tau(s)}$

3.2



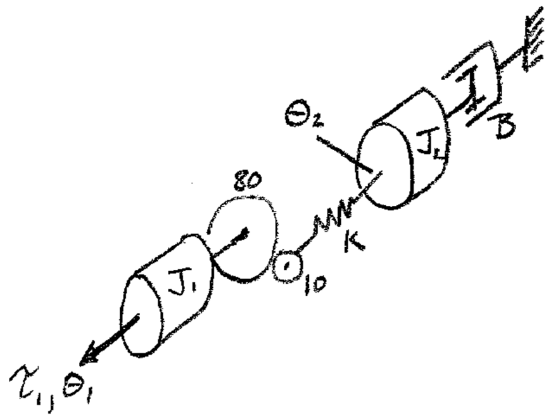
Write the EOM and get $\frac{\theta(s)}{\tau(s)}$.

3.3



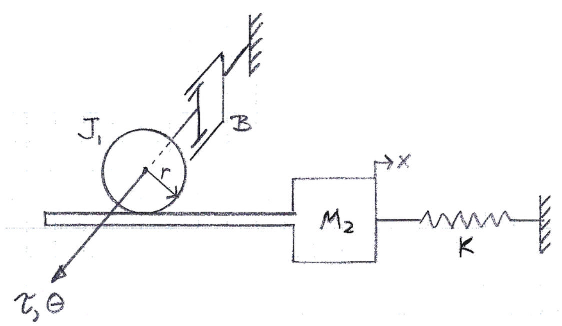
Find the equation of motion (EOM) and the transfer function $\frac{\theta(s)}{\tau(s)}$

3.4



Find the equations of motion (EOMs) and the transfer function $\frac{\theta_1(s)}{\tau_1(s)}$

3.5



Find the equations of motion (EOMs) and the transfer function $\frac{\theta(s)}{\tau(s)}$. (Note, unlike many problems, we will consider the inertia of the gear (J_1) in this problem.)

State Space

3.6

3.6.1

Represent the system of Problem 3.3 in state space. Do not worry about the second equation (the "output equation", $Y = CX + DU$).

3.6.2

For

$$J_1 = 10 \quad J_2 = 100 \quad K = 20 \quad B = 5$$

Find A,B, in numerical form

3.7

Find the State Space representation for the translational system of Problem 2.3 (also ignore the output equation).

Hint: the EOMs of that problem were:

$$M_1\ddot{x}_1 + Kx_1 + B(\dot{x}_1 - \dot{x}_2) = F(t)$$
$$M_2\ddot{x}_2 + B(\dot{x}_2 - \dot{x}_1) = 0$$

The state variables are all the motion variables from which we can get a kinetic (mass) or potential (spring) energy: $x_1, \dot{x}_1, x_2, \dot{x}_2$.

3.8

Find the output equation matrices C and D for the two previous ICPs. Assume the output variable for the first problem is θ and for the second problem there is a two-vector output:

$$Y = \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix}$$

4 EE447 In Class Problems: Plotting Frequency Response

4.1 Decibels

Express the following in dB:

1. $1000 =$ ______ dB
2. $\sqrt{10} \times 10^5 =$ ______ dB
3. $0.01 =$ ______ dB
4. $0.5 =$ ______ dB
5. $\frac{1}{\sqrt{2}} =$ ______ dB

4.2

Plot the poles of the following 2nd order transfer function:

$$G(s) = \frac{10}{s^2 + 6.32\zeta s + 10}$$

for $\zeta = \{1, 0.5, 0.1\}$

4.3

Let

$$G(s) = \frac{1000}{s + 1000}$$

Draw $|G(j\omega)|$ and $\angle G(j\omega)$.

4.4

Draw the Bode magnitude and phase plots of

$$G(s) = \frac{(s + 316)}{316}$$

4.5

Draw the Bode magnitude and phase plots of

$$G(s) = \frac{3.16 \times 10^4 (s + 1)}{(s + 31.6)(s + 1000)}$$

4.6

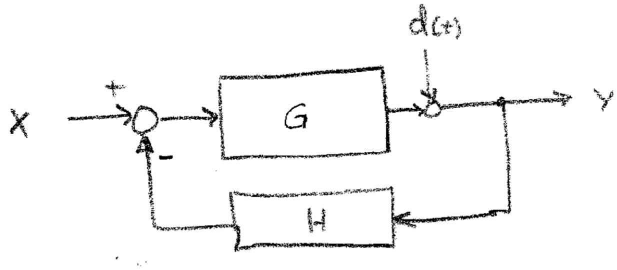
Draw the Bode magnitude and phase plots of

$$G(s) = \frac{1000}{s^2 + 63.2\zeta s + 1000}$$

for

$$\zeta = \{1.0, 0.5, 0.1\}$$

5 EE447 In Class Problems: Closed Loop Feedback



(ICPs 5.1 — 5.4 refer to this diagram:)

5.1

Find $\frac{Y(s)}{d(s)}$ for $G = 100, H = 2, X = 0$.

5.2

For the system in 5.1, find $\frac{Y}{X}$ for d(t) = 0 (D(s) = 0), and find $\frac{Y}{d}$ for $X(s) = 0$.
Compute both for $G = 90, 100, 110$ and $H = 2$.

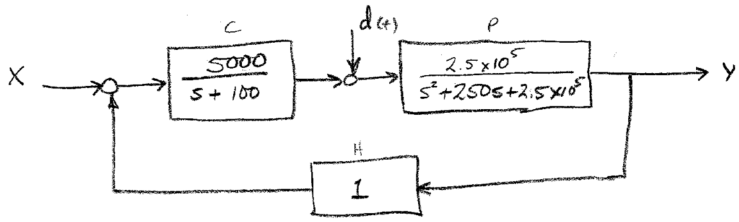
5.3

From the results of 5.1, find the sensitivity coefficients for the performance measures $P_1 = |\frac{Y}{X}|$ and $P_2 = |\frac{Y}{d}|$with respect to the parameter $p_1 = G$?

5.4

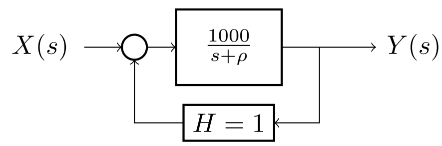
For $G = \frac{500}{s+270}, H = 1$, find disturbance gain $\frac{Y(s)}{d(s)}$ for $X(s) = 0$. Sketch Bode Magnitude Plot of $GH(s)$ and $\frac{Y(s)}{d(s)}$ on the same axes.

5.5



A) Find $\frac{Y(s)}{d(s)}$ assuming $CPH >> 1$. Make a Bode magnitude plot of $|Y(s)/d(s)|$. On the same axes, make a Bode Magnitude plot of $CPH(s)$.
B) For what range of frequencies is the loop gain magnitude > 1?
C) What do you notice about the relationship between $CPH(s)$ and $\frac{Y(s)}{d(s)}$?

5.6



A) Let ρ take on the values {90, 100, 110}. What is the 3dB frequency, ω_{3dB} of the closed loop transfer function $\frac{Y(s)}{X(s)}$ for each value of ρ.?

B) What is the sensitivity coefficient of ω_{3dB} with respect to ρ?

5.7

Referring back to the first block diagram (Prob 5.1), assume $G = \frac{100}{s+20}$, $H = 1$.

A) What is $\frac{Y(s)}{d(s)}$? Sketch Bode Magnitude Plot.

B) Evaluate $|\frac{Y(j\omega)}{d(j\omega)}|$ for $\omega = \{1, 500\}$ and $X = 0$.

C) What should you change to reduce disturbance output to about -15dB at $\omega = 500$?

6 EE447 In Class Problems: Root Locus

In all Root Locus problems, we are looking for a) correct number of diverging asymptotes, b) correct angles for the asymptotes, c) correct part of the real line occupied by the RL, d) correct intercept of the asymptotes with the real line, e) arrows correctly showing direction of pole movement as K is increased.

6.1

6.1.1

Plot the root locus for $K > 0$ where

$$CPH(s) = \frac{K}{(s+10)(s+20)}$$

6.1.2

Find the value of K at the point where the RL intersects the line $y = j10$. Hint: use the magnitude condition.

6.2

Plot the Root Locus of

$$CPH(s) = \frac{K(s+1)}{(s+5+j)(s+5-j)}$$

6.3

Plot the Root Locus of

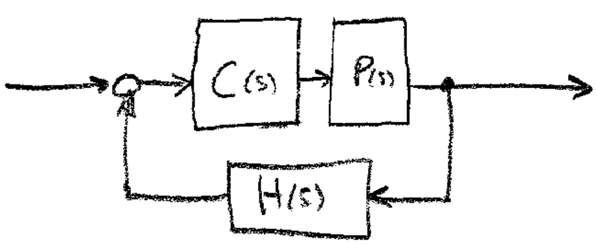
$$CPH(s) = \frac{K(s+6)}{s(s+3+j3)(s+3-j3)}$$

6.4

Plot the root locus for

$$CPH(s) = \frac{K(s^2+6s+73)}{s(s+4)(s+4+4j)(s+4-4j)} \qquad K > 0$$

6.5



For $C(s) = C_1(s) = \frac{K}{(s+10)}$ and $P(s) = \frac{4}{(s+0.5-2j)(s+0.5+2j)}$, $H(s) = 1$, plot the root locus by hand for $K \geq 0$.

6.6

Redo the root locus of Problem 6.5 for $C(s) = C_2(s) = \frac{K(s+4)}{(s+10)}$, P, H unchanged.

Comment on the stability of the closed loop system with this controller compared to $C_1(s)$ of Problem 6.5.

6.7

Plot the Root Locus for

$$G(s) = \frac{K(s+3)(s+1)}{(s+2)(s^2+6s+10)(s+1+j)(s+1-j)} \qquad K > 0$$

Be sure to give your calculations for number of asymptotes, intercept of asymptotes, angle of asymptotes, etc.

7 EE447 In Class Problems: SSE and Closed Loop Performance

In this ICP set there is a mix of hand and computer problems. Please bring a laptop to class. If you do not have a laptop, share with a friend! (In many classrooms there are very few power outlets so charge it up first!).

To receive credit for your hand work, turn in paper as usual. For your computer work there are multiple options, but only one will be used. We will announce which method will be used in class:

1. Show your work to the professor or a TA and they will note completion and initial on your hard copy.
2. Copy your graphic output and computer scripts into a word doc. Export to PDF. Upload into the "ICP 7" assignment in Canvas/Gradescope.

You may use Matlab if you prefer. However, ICP 8 will **require** Scilab use so this would be a good chance to get ready.

7.1 Steady State Error (SSE)

Find the system type for each of the systems below:

System	Type
$CPH(s) = \frac{250}{s^2(s+0.1)(s+10)}$	
$CPH(s) = \frac{20(s+0.2)}{s(s+0.1)(s^2+10s)}$	
$CPH(s) = \frac{5\times10^4}{(s^3+10s^2+126s+3000)}$	
$CPH(s) = \frac{20(s+2)}{s(s+0.2)}$	
$CPH(s) = \frac{100(s+0.01)}{s^4+5s^3+50s^2}$	

7.2

Let $C(s) = 50$, $P(s) = \frac{200(s+1)}{s(s+3)(s+50)}$, $H(s) = 1$.

7.2.1

What is the system type?

7.2.2

If $x(t) = 0.2t^2$, what is the SSE?

7.2.3

What is the SSE of this system for $x(t) = 100u(t)$?

7.2.4

What is the SSE of this system for $x(t) = 0.01t$?

7.3

Which systems have zero steady state error for $x(t) = 10t$?

A) $CPH(s) = \frac{K_A}{s(s+5)}$	B) $CPH(s) = \frac{K_B}{s^2(s+50)}$
C) $CPH(s) = \frac{K_C}{s^3(s+100)}$	D) $CPH(s) = \frac{K_D}{s^2(s-10)}$

7.4 S-Plane Regions

Find the s-plane region for 2nd order poles corresponding to the following step response performance requirements;

7.4.1

$1 < T_s < 3 \qquad 2\% < \%OS < 5\%$.

7.4.2

$T_s = 2sec$, $\%OS = 2\%$.

7.4.3

$2 < \omega_n < 3.5$, $1sec < T_s < 2sec$.

7.5

Identify the point or region of the s-plane corresponding to:

7.5.1

$T_S = 0.8\text{sec}$, $\%\text{OS} = 5\%$

7.5.2

$T_S \leq 10\text{sec}$, $\%\text{OS} = 10\%$

7.5.3

$T_S \leq 5\text{sec}$, $\%\text{OS} \leq 2\%$

7.6 Manual PID Design

Manually estimate PID controller parameters, K_P, K_I, K_D for the following system (you may use the computer for Root Locus).

$$P(s) = \frac{100(s + 0.1)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where

$$\omega_n = 2.0, \qquad \zeta = 0.875$$

to achieve the following specs:

$$T_s = 0.075, \quad \%OS = 5\%$$

7.7

Manually estimate PID controller parameters, K_P, K_I, K_D for

$$P(s) = \frac{26.7}{(s+1)(s+4)}$$

to achieve:

$$T_s = 0.75, \quad \%OS = 10\%$$

(you may use the computer for Root Locus).

7.8 Gain Margin and Phase Margin

(if time) Consider a closed loop system where

$$C(s) = \frac{K}{(s+10)} \qquad P(s) = \frac{500(s+1)}{(s+0.1)(s+25)(s+200)} \qquad H(s) = 1$$

7.8.1

Plot the Bode Magnitude and Phase Diagrams of $CPH(s)$ by hand for $K = 100$.

7.8.2

Using your Bode plot from the previous section, compute the Phase and Gain Margins.

7.8.3

Find a value of K to get a Gain Margin of $15dB$. What is the new Phase Margin?

7.8.4

Verify your answer with the Scilab "Margins" command.

8 PID Controller Design with Scilab

These ICP problems require the use of Scilab and the following scripts (provided by class website):

- `setup.sce` (rename, save, and customize this for each individual problem)
- `stepperf.sce` (routines which compute $T_s, \%OS$, etc.)
- `optigainX.sce` (script which searches the PID design space and finds best design).
- `quickplot.sce` (Optional, easily get step response of your plant with any K_P, K_I, K_D).

To report your result, give the best output you get as it is output by the software. Example:

```
[       Balanced] Kp:  7.632634919  Ki:  0.0015567  Kd:  0.008297948
Settling Time:  0.04  Overshoot:  7.1 percent  SSE:  0.029  Ctl Effort:  0.78
  Search boundary reached:  Kp min Ki max Kd min
```

8.1

Enter the following systems into Scilab:
(don't forget to initialize s first)
A)

$$\frac{10^4(s+4)}{(s+50)(s+800)}$$

B)

$$\frac{3200}{(s^2+34s+3200)}$$

Use the `csim()` command to plot the step responses.
(This is just to verify that you have Scilab set up and you know how to properly enter system equations.)

8.2

Using the provided Scilab tools, find PID controller parameters, K_P, K_I, K_D for the following system

$$P(s) = \frac{100(s+0.1)}{s^2+2\zeta\omega_n s+\omega_n^2}$$

where

$$\omega_n = 2.0, \qquad \zeta = 0.875$$

to achieve the following specs:

$$T_s = 0.075, \quad \%OS = 5\%, \quad Cu = 1.0$$

Initialize your search range using the results of ICP 7.6.

8.3

Using the provided Scilab tools, find a PID controller for

$$P(s) = \frac{26.7}{(s+1)(s+4)}$$

For

$$T_s = 0.75, \quad \%OS = 10\%, \quad SSE \leq 3\%, \quad Cu = 50.0$$

Initialize your search range using the results of ICP 7.7.

8.4

Wind turbine speed via blade pitch control (Dorf, Example 9.10, p675)

$$P(s) = \frac{7000\omega_n^2}{(\tau s + 1)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

Where

$$\tau = 5.0, \quad \omega_n = 20, \quad \zeta = 0.005$$

to achieve:

$$T_s = 20sec, \quad \%OS = 20.0\%, \quad Cu = 0.001$$

Hint: start with Kp = 0.0001 and go even smaller with Kp, Kd.

8.5

Flexible Robot Arm: (Dorf Design Problem 9.2 p737)

$$P(s) = \frac{1}{s^2 + 9s + 12}$$

For

$$T_s = 2.0sec, \quad \%OS = 20.0\%, \quad Cu = 40$$

8.6

Radar Tracking System: (Phillips and Parr, p275))

$$P(s) = \frac{1}{s(s+2)}$$

To get

$$T_s = 8sec, \quad \%OS = 5\%, \quad Cu = 10$$

8.7

Voltage control system (Phillips and Parr, p276)

$$P(s) = \frac{1}{(s+1)(s+2)}$$

To get

$$T_s = 1.0sec, \quad \%OS = 5\%, \quad Cu = 150$$

9 EE447 ICP 9: Discrete time implementation of controllers.

9.1 Sampling Theorem

An analog signal has bandwidth 25 Hz. What is the minimum necessary sampling frequency necessary to reconstruct the signal without distortion?

9.2 Tustin's Method

A controller has the continuous time transfer function:

$$C_1(s) = \frac{1000}{(s + 7.5)}$$

Apply Tustin's method to convert it into a discrete time controller with sampling time $T = 0.01$.

9.3 Tustin's Method

A controller has the continuous time transfer function:

$$C_2(s) = \frac{50(s + 2)}{(s + 0.1)(s + 50)}$$

Apply Tustin's method to convert it into a discrete time controller with sampling time $T = 0.001$.

9.4 Conversion to digital filter

Convert the following discrete time controller to a digital filter:

$$C_3(z) = 30\frac{(z + 1)}{(z + 0.997)}$$

Express your digital filter in a form which can be directly coded into software.

9.5 Conversion to digital filter

Convert the following discrete time controller to a digital filter:

$$C_4(z) = 10^3\frac{(z + 1)}{(z + 0.76)(z + 0.997)}$$