

# Gram Matrix. Problem 10.14

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## Functions and their examples

```
library(magrittr)

vnorm = function(x) {
  # euclidean norm of vector
  ( t(x) %*% x ) %>% sqrt
}
angle = function(x,y) acos(crossprod(x,y)/(vnorm(x) * vnorm(y))) * (180/pi)

a1 = c(0,1,0,0.7)
a2 = c(0.2,0,0,1.7805)
vnorm(a1)

##           [,1]
## [1,]  1.220656

vnorm(a2)

##           [,1]
## [1,]  1.791698

angle(a1,a2)

##           [,1]
## [1,]  55.25826
```

## Problem and solution

The inner product of the columns with themselves, *for*  $i \neq j$ ,  $a_i a_j = 1$ , because all column vectors have Frobenius norm of 1. The problem states that each column has an angle of 60 degrees with every other column, i.e. *for all*  $i$ ,  $a_i^T a_i = \cos(\pi/3)$ .

Describe the gram matrix of  $\mathbf{A}$ ,  $G = A^T A$ .

```
G = diag(x=1,nrow=3,ncol=3)
G[row(G) != col(G)] = cos(pi/3)
G

##           [,1] [,2] [,3]
## [1,]  1.0  0.5  0.5
## [2,]  0.5  1.0  0.5
## [3,]  0.5  0.5  1.0
```