Gram Matrix. Problem 10.14

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Functions and their examples

```
library(magrittr)

vnorm = function(x) {
  # euclidean norm of vector
  ( t(x) %*% x ) %>% sqrt
}
angle = function(x,y) acos(crossprod(x,y)/(vnorm(x) * vnorm(y))) * (180/pi)

a1 = c(0,1,0,0.7)
a2 = c(0.2,0,0,1.7805)
vnorm(a1)
```

```
##          [,1]
## [1,] 1.220656
```

```
vnorm(a2)
```

```
##          [,1]
## [1,] 1.791698
```

```
angle(a1,a2)
```

```
##          [,1]
## [1,] 55.25826
```

Problem and solution

The problem states that all columns have a vector norm of 1, i.e. $for\ all\ i,\ a_i^T a_i = 1$. It also states that each column has an angle of 60 degrees with every other column, i.e. $for\ i \neq j,\ a_i^T a_j = cos(\pi/3)$.

Describe the gram matrix of A, $G = A^T A$. The gram matrix describes, in a compact way, the value of the inner product for each pair of columns.

```
G = diag(x=1,nrow=3,ncol=3)
G[row(G) != col(G)] = cos(pi/3)
G
```

```
##      [,1] [,2] [,3]
## [1,]  1.0  0.5  0.5
## [2,]  0.5  1.0  0.5
## [3,]  0.5  0.5  1.0
```