```
Tplot=273+[-20:120];
muplot = exp(p(1)*Tplot.^2 + p(2)*Tplot + p(3));
semilogy(TK,mu,'o',Tplot,muplot)
```

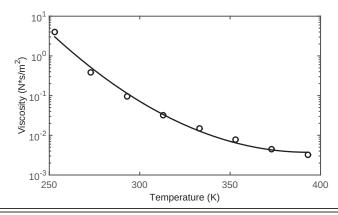
When the program executes (saved as Chap8SamPro7), the coefficients that are determined by the polyfit function are displayed in the Command Window (shown below) as three elements of the vector p.

```
>> Chap8SamPro7
p = 0.0003 -0.2685 47.1673
```

With these coefficients the viscosity of the oil as a function of temperature is:

$$\mu = Ae^{(0.0003T^{\frac{\pi}{2}} - 0.2685T + 47.1673)} = e^{47.1673}e^{-0.2685T}e^{0.0003T^{\frac{\pi}{2}}}$$

The plot that is generated shows that the equation correlates well to the data points (axis labels were added with the Plot Editor).



## 8.6 PROBLEMS

- 1. Plot the polynomial  $y = 0.9x^5 0.3x^4 15.5x^3 + 7x^2 + 36x 7$  in the domain  $-4 \le x \le 4$ . First create a vector for x, next use the polyval function to calculate y, and then use the plot function.
- 2. Plot the polynomial  $y = 0.7x^4 13.5x^2 + 6x 37$  in the domain -5 < x < 5. First create a vector for x, next use the polyval function to calculate y, and then use the plot function.
- 3. Determine the polynomial y(x) that has roots at x = -0.7, x = 0.5, x = 3.4, and x = 5.8. Make a plot of the polynomial in the domain -1 < x < 6.

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4. Use MATLAB to carry out the following multiplication of two polynomials:

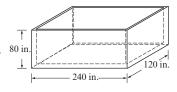
$$(2x^2-3x+6)(-5x^3+4x-7)$$

5. Use MATLAB to carry out the following multiplication of polynomials:

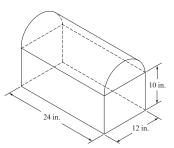
$$x(x+1.8)(x-0.4)(x-1.6)$$

Plot the polynomial in the domain -2 < x < 2.

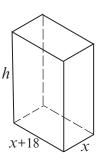
- 6. Use MATLAB to divide the polynomial  $-9x^6 + 12x^5 + 5x^4 9x^3 + 17x^2 7x 15$  by the polynomial  $3x^2 2x 3$ .
- 7. Use MATLAB to divide the polynomial  $0.9x^5 5.96x^4 + 20.85x^3 24.1x^2 + 3x + 7.5$  by the polynomial  $0.5x^3 2.2x^2 + 6x + 3$ .
- 8. The product of four consecutive even integers is 1,488,384. Using MAT-LAB's built-in function for operations with polynomials, determine the two integers.
- 9. The product of three integers with spacing of 3 between them (e.g., 9, 12, 15) is 11,960. Using MATLAB's built-in functions for operations with polynomials, determine the three integers.
- 10. The product of three distinct integers is 6,240. The sum of the numbers is 85. The difference between the largest and the smallest is 57. Using MAT-LAB's built-in functions for operations with polynomials, determine the three integers.
- 11. A rectangular steel container (no top) has the outside dimensions shown in the figure. The thickness of the bottom surface is t, and the thickness of side walls is t/2. Determine t if the weight of the container is 1,300 lb. The specific weight of steel is 0.284 lb/in.<sup>3</sup>.



12. An aluminum container has the geometry shown in the figure (the bottom part is a rectangular box and the top is half a cylinder). The outside dimensions are shown. The wall thickness of the bottom and all the vertical walls is 2t, and the walls thickness of the cylindrical section is t. Determine t if the tank weight is 30 lb. The specific weight of aluminum is 0.101 lb/ in.<sup>3</sup>.



- 13. A rectangular box (no top) is welded together using sheet metal. The length of the box's base is 18 in. longer than its width. The total surface area of the sheet metal that is used is 2,500 in.<sup>2</sup>.
  - (a) Using polynomials write an expression for the volume V in terms of x.
  - (b) Make a plot of V versus x for  $5 \le x \le 35$  in.
  - (c) Determine the dimensions of the box that maximizes the volume and determine that volume.



14. The probability *P* of selecting three distinct numbers out of *n* numbers is calculated by:

$$P = \frac{2 \cdot 3}{n(n-1)(n-2)}$$

Determine how many numbers, n, should be in a lottery game such that the probability of matching three numbers out of n numbers will be at least 1/100,000, but not greater than 1/95,000.

15. Write a user-defined function that adds or subtracts two polynomials of any order. Name the function p=polyadd(p1,p2,operation). The first two input arguments p1 and p2 are the vectors of the coefficients of the two polynomials. (If the two polynomials are not of the same order, the function adds the necessary zero elements to the shorter vector.) The third input argument operation is a string that can be either 'add' or 'sub', for adding or subtracting the polynomials, respectively, and the output argument is the resulting polynomial.

Use the function to add and subtract the following polynomials:

$$f_1(x) = 8x^6 + 10x^5 - 5x^3 + 13x^2 - 4x - 2$$
 and  $f_2(x) = 4x^4 + 7x^2 + 6$ 

16. Write a user-defined function that multiplies two polynomials. Name the function p=polymult(p1, p2). The two input arguments p1 and p2 are vectors of the coefficients of the two polynomials. The output argument p is the resulting polynomial.

Use the function to multiply the following polynomials:

$$f_1(x) = -2x^6 + 3x^4 + 4x^3 - 7x + 8$$
 and  $f_2(x) = 5x^4 - 4x^2 + 3x - 5$   
Check the answer with MATLAB's built-in function conv.

17. Write a user-defined function that calculates the maximum (or minimum) of a quadratic equation of the form:

$$f(x) = ax^2 + bx + c$$

Name the function [x, y, w] = maxormin(a, b, c). The input arguments are the coefficients a, b, and c. The output arguments are x, the coordinate of the maximum (or minimum); y, the maximum (or minimum) value; and w, which is equal to 1 if y is a maximum and equal to 2 if y is a minimum.

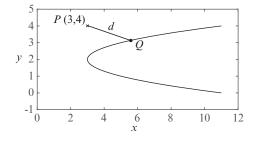
Use the function to determine the maximum or minimum of the following functions:

(a) 
$$f(x) = 3x^2 - 7x + 14$$
 (b)  $f(x) = -5x^2 - 11x + 15$ 

- 18. A cylinder with base radius r and height h is constructed inside a sphere such that it is in contact with the surface of a sphere, as shown in the figure. The radius of the sphere is R = 11 in.
  - (a) Create a polynomial expression for the volume V of the cylinder in terms of h.
  - (b) Make a plot of V versus h for  $0 \le h \le 11$  in.
  - (c) Using the roots command determine h if the volume of the cylinder is 2,000 in.<sup>3</sup>.
  - (d) Determine the value of h that corresponds to the cylinder with the largest possible volume, and determine that volume.
- 19. Consider the parabola:

$$x = 2(y-2)^2 + 3$$
, and the point  $P(3, 4)$ .

- (a) Write a polynomial expression for the distance d from point P to an arbitrary point Q on the parabola.
- (b) Make a plot of d versus y for  $0 \le y \le 4$ .



- (c) Determine the coordinates of Q if d = 3 (there are two points).
- (d) Determine the coordinates of Q that correspond to the smallest d, and calculate the corresponding value of d.
- (e) Make a plot that shows the parabola, point P, the two points from part (c), and the point from part (d).

20. The following data is given:

$\chi$	-5	-4	-1	1	4	6	9	10
у	12	10	6	2	-3	-6	-11	-12

- (a) Use linear least-squares regression to determine the coefficients m and b in the function y = mx + b that best fits the data.
- (b) Make a plot that shows the function and the data points.
- 21. The boiling temperature of water  $T_B$  at various altitudes h is given in the following table. Determine a linear equation in the form  $T_B = mh + b$  that best fits the data. Use the equation for calculating the boiling temperature at 5,000 m. Make a plot of the points and the equation.

h (ft)	-1,000	0	3,000	8,000	15,000	22,000	28,000
$T(^{\circ}\mathrm{F})$	213.9	212	206.2	196.2	184.4	172.6	163.1

22. The U.S. population in selected years between 1815 and 1965 is listed in the table below. Determine a quadratic equation in the form  $P = a_2t^2 + a_1t + a_0$ , where t is the number of years after 1800 and P is the population in millions, that best fits the data. Use the equation to estimate the population in 1915 (the population was 98.8 millions). Make a plot of the population versus the year that shows the data points and the equation.

Year	1815	1845	1875	1905	1935	1965
Population (millions)	8.3	19.7	44.4	83.2	127.1	190.9

23. The number of bacteria  $N_B$  measured at different times t is given in the following table. Determine an exponential function in the form  $N_B = Ne^{\alpha t}$  that best fits the data. Use the equation to estimate the number of bacteria after 5 h. Make a plot of the points and the equation.

t (h)	0	1	3	4	6	7	9
$N_B$	500	600	1,000	1,400	2,100	2,700	4,100

24. Growth data of a sunflower plant is given in the following table:

Day	7	21	35	49	63	77	91
Height (in.)	8.5	21	50	77	89	98	99

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The data can be modeled with a function in the form  $y = \frac{H}{1+e^{-(a+bt)}}$ , where y is the height, H is a maximum height, a and b are constants, and t is the number of days. By using the method described in Section 8.2.2, and assuming that H = 102 in., determine the constants a and b such that the function best fits the data. Use the function to estimate the height in day 40. In one figure, plot the function and the data points.

- 25. Use the growth data from Problem 24 for the following:
  - (a) Curve-fit the data with a third-order polynomial. Use the polynomial to estimate the height in day 40.
  - (b) Fit the data with linear and spline interpolations and use each interpolation to estimate the height in day 40.

In each part make a plot of the data points (circle markers) and the fitted curve or the interpolated curves. Note that part (b) has two interpolation curves.

## 26. The following points are given:

X	1	2.2	3.7	6.4	9	11.5	14.2	17.8	20.5	23.2
У	12	9	6.6	5.5	7.2	9.2	9.6	8.5	6.5	2.2

- (a) Fit the data with a first-order polynomial. Make a plot of the points and the polynomial.
- (b) Fit the data with a second-order polynomial. Make a plot of the points and the polynomial.
- (c) Fit the data with a third-order polynomial. Make a plot of the points and the polynomial.
- (d) Fit the data with an fifth-order polynomial. Make a plot of the points and the polynomial.

## 27. The standard air density, D (average of measurements made), at different heights, h, from sea level up to a height of 33 km is given below.

h (km)	0	3	6	9	12	15
$D (kg/m^3)$	1.2	0.91	0.66	0.47	0.31	0.19
h (km)	18	21	24	27	30	33
$D (kg/m^3)$	0.12	0.075	0.046	0.029	0.018	0.011

(a) Make the following four plots of the data points (density as a function of height): (1) both axes with linear scale; (2) h with log axis, D with linear axis; (3) h with linear axis, D with log axis; (4) both log axes. According to the plots, choose a function (linear, power, exponential, or

- logarithmic) that best fits the data points and determine the coefficients of the function.
- (b) Plot the function and the points using linear axes.
- 28. Write a user-defined function that determines the best fit of an exponential function of the form  $y = be^{mx}$ . Name the function [b, m] = expofit(x,y), where the input arguments x and y are vectors with the coordinates of the data points, and the output arguments b and m are the constants of the fitted exponential equation. Use expofit to fit the data below. Make a plot that shows the data points and the function.

Х	0.4	2.2	3.1	5.0	6.6	7.6
у	1.7	10.1	26.9	61.2	158	398

29. Estimated values of thermal conductivity of silicon at different temperatures are given in the following table.

T (K)	2	4	6	8	10	20	40	60
k (W/m-K)	46	300	820	1,560	2,300	5,000	3,500	2,100
T (K)	80	100	150	250	350	500	1,000	1,400
k (W/m-K)	1,350	900	400	190	120	75	30	20

- (a) Make a plot of k versus T using log scale on both axes.
- (b) Curve-fit the data with a second-order polynomial  $y = ax^2 + bx + c$  in which  $x = \log(T)$  and  $y = \log(k)$ . Once the coefficients a, b, and c are determined, write an equation for k as a function of  $\log(T)$ . Use this equation for curve-fitting the data. Make a second plot that shows the data points with markers and the curve-fitted equation with a solid line.
- (c) Repeat part (b) using a third-order polynomial.
- 30. Measurements of the concentration, *C*, of a substance during a chemical reaction at different times *t* are shown in the table.

t (h	(	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
C(g/g)	L) [	1.7	3.1	5.7	9.1	6.4	3.7	2.8	1.6	1.2	0.8	0.7	0.6

(a) Suppose that the data can be modeled with an equation in the form:

$$C(t) = \frac{1}{a_2 t^2 + a_1 t + a_0}$$

Determine the coefficients  $a_0$ ,  $a_1$ , and  $a_2$  such that the equation best fits the data. Use the equation to estimate the concentration at t = 2h.

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Make a plot of the data points and the equation.

(b) Suppose that the data can be modeled with an equation in the form:

$$C(t) = \frac{1}{a_3 t^3 + a_2 t^2 + a_1 t + a_0}$$

Determine the coefficients  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_3$  such that the equation best fits the data. Use the equation to estimate the concentration at t = 2 h. Make a plot of the data points and the equation.

- 31. Use the data from Problem 30 for the following:
  - (a) Fit the data with linear interpolation. Estimate the concentration at t = 2.25. Make a plot that shows the data points and curve made of interpolated points.
  - (b) Fit the data with spline interpolation. Estimate the concentration at t = 2.25 h. Make a plot that shows the data points and a curve made of interpolated points.
- 32. The relationship between two variables *y* and *x* is known to be:

$$y = a \frac{x}{b+x}$$

The following data points are given:

X	5	10	15	20	25	30	35	40	45	50
у	15	25	32	33	37	35	38	39	41	42

Determine the constants a and b by curve-fitting the equation to the data points. Make a plot of y versus x. In the plot show the data points with markers and the curve-fitted equation with a solid line. Use the equation to estimate y at x = 23. (The curve fitting can be done by writing the reciprocal of the equation and using a first-order polynomial.)

- 33. Curve-fit the data from the previous problem with a third-order polynomial. Use the polynomial to estimate y at x = 23. Make a plot of the points and the polynomial.
- 34. When rubber is stretched, its elongation is initially proportional to the applied force, but as it reaches about twice its original length, the force required to stretch the rubber increases rapidly. The force, as a function of elongation, that was required to stretch a rubber specimen that was initially 3 in. long is displayed in the following table.
  - (a) Curve-fit the data with a fourth-order polynomial. Make a plot of the data points and the polynomial. Use the polynomial to estimate the force when the rubber specimen was 11.5 in. long.
  - (b) Fit the data with spline interpolation (use MATLAB's built-in function interp1). Make a plot that shows the data points and a curve made by interpolation. Use interpolation to estimate the force when the rubber

Force (lb)	0	0.6	0.9	1.16	1.18	1.19	1.24	1.48
Elongation (in.)	0	1.2	2.4	3.6	4.8	6.0	7.2	8.4
Force (lb)	1.92	3.12	4.14	5.34	6.22	7.12	7.86	8.42
Elongation (in.)	9.6	10.8	12.0	13.2	14.4	15.6	16.8	18

specimen was 11.5 in. long.

35. The transmission of light through a transparent solid can be described by the equation:

$$I_T = I_0 (1 - R)^2 e^{-\beta L}$$

where  $I_T$  is the transmitted intensity,  $I_0$  is the intensity of the incident beam,  $\beta$  is the absorption coefficient, L is the length of the transparent solid, and R is the fraction of light which is reflected at the interface. If the light is normal to the interface and the beams are transmitted through air,

$$R = \left(\frac{n-1}{n+1}\right)^2$$
 where *n* is the index of refraction for the transparent solid.

Experiments measuring the intensity of light transmitted through specimens of a transparent solid of various lengths are given in the following table. The intensity of the incident beam is 5 W/m<sup>2</sup>.

L (cm)	0.5	1.2	1.7	2.2	4.5	6.0
$I_T$ (W/m <sup>2</sup> )	4.2	4.0	3.8	3.6	2.9	2.5

Use this data and curve fitting to determine the absorption coefficient and index of refraction of the solid.