```
>> C=[4 2 6; -2 8 10; 6 2 3]; Solving the form XC = D.

>> D=[8 4 0];

>> Xc=D/C Solving by using right division: X = D/C.

Xc =
    -1.8049 0.2927 2.6341

>> Xd=D*inv(C) Solving by using the inverse of C: X=DC<sup>-1</sup>.

Xd =
    -1.8049 0.2927 2.6341
```

3.4 ELEMENT-BY-ELEMENT OPERATIONS

In Sections 3.2 and 3.3 it was shown that when the regular symbols for multiplication and division (* and /) are used with arrays, the mathematical operations follow the rules of linear algebra. There are, however, many situations that require element-by-element operations. These operations are carried out on each of the elements of the array (or arrays). Addition and subtraction are by definition already element-by-element operations, since when two arrays are added (or subtracted) the operation is executed with the elements that are in the same position in the arrays. Element-by-element operations can be done only with arrays of the same size.

Element-by-element multiplication, division, or exponentiation of two vectors or matrices is entered in MATLAB by typing a period in front of the arithmetic operator.

Symbol	<u>Description</u>	Symbol	<u>Description</u>
.*	Multiplication	./	Right division
.^	Exponentiation	.\	Left Division

If two vectors a and b are $a=[a_1 \ a_2 \ a_3 \ a_4]$ and $b=[b_1 \ b_2 \ b_3 \ b_4]$, then element-by-element multiplication, division, and exponentiation of the two vectors gives:

$$a.*b = [a_1*b_1 \ a_2*b_2 \ a_3*b_3 \ a_4*b_4]$$

$$a./b = [a_1/b_1 \ a_2/b_2 \ a_3/b_3 \ a_4/b_4]$$

$$a.^b = [(a_1)^{b_1} (a_2)^{b_2} (a_3)^{b_3} (a_4)^{b_4}]$$

If two matrices A and B are

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \text{ and } B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

then element-by-element multiplication and division of the two matrices give:

$$A \cdot *B = \begin{bmatrix} A_{11}B_{11} & A_{12}B_{12} & A_{13}B_{13} \\ A_{21}B_{21} & A_{22}B_{22} & A_{23}B_{23} \\ A_{31}B_{31} & A_{32}B_{32} & A_{33}B_{33} \end{bmatrix} \qquad A \cdot /B = \begin{bmatrix} A_{11}/B_{11} & A_{12}/B_{12} & A_{13}/B_{13} \\ A_{21}/B_{21} & A_{22}/B_{22} & A_{23}/B_{23} \\ A_{31}/B_{31} & A_{32}/B_{32} & A_{33}/B_{33} \end{bmatrix}$$

Element-by-element exponentiation of matrix A gives:

$$A ^ n = \begin{bmatrix} (A_{11})^n & (A_{12})^n & (A_{13})^n \\ (A_{21})^n & (A_{22})^n & (A_{23})^n \\ (A_{31})^n & (A_{32})^n & (A_{33})^n \end{bmatrix}$$

Element-by-element multiplication, division, and exponentiation are demonstrated in Tutorial 3-2.

Tutorial 3-2: Element-by-element operations.

>> A=[2 6 3;	5 8 4]		Define a 2×3 array A.
A =			
2 6	3		
5 8	4		
>> B=[1 4 10;	3 2 7]		Define a 2 × 3 array B.
B =			
1 4	10		
3 2	7		
>> A.*B			Element-by-element multiplication of array A by B.
ans =			
2 24	30		
15 16	28		
>> C=A./B			Element-by-element division
C =		of array A by B. The result is	
2.0000	1.5000	0.3000	assigned to variable C.
1.6667	4.0000	0.5714	assigned to variable c.

Tutorial 3-2: Element-by-element operations. (Continued)

```
>> B.^3
                                            Element-by-element exponen-
                                            tiation of array B. The result
ans =
                                            is an array in which each term
              64
                                            is the corresponding term in B
      1
                    1000
                                            raised to the power of 3.
     27
               8
                     343
                                              Trying to multiply A*B gives
>> A*B
                                              an error, since A and B cannot
                                              be multiplied according to lin-
??? Error using ==> *
                                              ear algebra rules. (The number
Inner matrix dimensions must agree.
                                              of columns in A is not equal to
                                              the number of rows in B.)
```

Element-by-element calculations are very useful for calculating the value of a function at many values of its argument. This is done by first defining a vector that contains values of the independent variable, and then using this vector in element-by-element computations to create a vector in which each element is the corresponding value of the function. One example is:

```
>> x=[1:8]

x =

1 2 3 4 5 6 7 8

>> y=x.^2-4*x

y =

-3 -4 -3 0 5 12 21 32

Vector x is used in element-by-element calculations of the elements of vector y.
```

In the example above $y = x^2 - 4x$. Element-by-element operation is needed when x is squared. Each element in the vector y is the value of y that is obtained when the value of the corresponding element of the vector x is substituted in the equation. Another example is:

In the last example $y = \frac{z^3 + 5z}{4z^2 - 10}$. Element-by-element operations are used in this example three times: to calculate z^3 and z^2 , and to divide the numerator by the denominator.