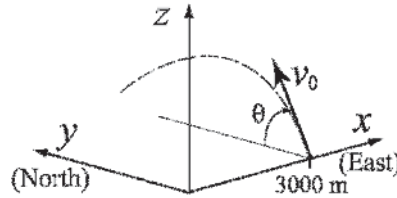


- The view command can also set a default view:
  - `view(2)` sets the default to the top view, which is a projection onto the  $x$ - $y$  plane with  $az = 0^\circ$ , and  $el = 90^\circ$ .
  - `view(3)` sets the default to the standard 3-D view with  $az = -37.5^\circ$  and  $el = 30^\circ$ .
- The viewing direction can also be set by selecting a point in space from which the plot is viewed. In this case the `view` command has the form `view([x,y,z])`, where  $x$ ,  $y$ , and  $z$  are the coordinates of the point. The direction is determined by the direction from the specified point to the origin of the coordinate system and is independent of the distance. This means that the view is the same with point  $[6, 6, 6]$  as with point  $[10, 10, 10]$ . Top view can be set up with  $[0, 0, 1]$ . A side view of the  $xz$  plane from the negative  $y$  direction can be set with  $[0, -1, 0]$ , and so on.

### 10.5 EXAMPLES OF MATLAB APPLICATIONS

#### Sample Problem 10-1: 3-D projectile trajectory

A projectile is fired with an initial velocity of 250 m/s at an angle of  $\theta = 65^\circ$  relative to the ground. The projectile is aimed directly north. Because of a strong wind blowing to the west, the projectile also moves in this direction at a constant speed of 30 m/s. Determine and plot the trajectory of the projectile until it hits the ground. For comparison, plot also (in the same figure) the trajectory that the projectile would have had if there was no wind.



#### Solution

As shown in the figure, the coordinate system is set up such that the  $x$  and  $y$  axes point in the east and north directions, respectively. Then the motion of the projectile can be analyzed by considering the vertical direction  $z$  and the two horizontal components  $x$  and  $y$ . Since the projectile is fired directly north, the initial velocity  $v_0$  can be resolved into a horizontal  $y$  component and a vertical  $z$  component:

$$v_{0y} = v_0 \cos(\theta) \quad \text{and} \quad v_{0z} = v_0 \sin(\theta)$$

In addition, due to the wind the projectile has a constant velocity in the negative  $x$  direction,  $v_x = -30$  m/s.

The initial position of the projectile  $(x_0, y_0, z_0)$  is at point  $(3000, 0, 0)$ . In the vertical direction the velocity and position of the projectile are given by:

$$v_z = v_{0z} - gt \quad \text{and} \quad z = z_0 + v_{0z}t - \frac{1}{2}gt^2$$

The time it takes the projectile to reach the highest point ( $v_z = 0$ ) is  $t_{hmax} = \frac{v_{0z}}{g}$ . The total flying time is twice this time,  $t_{tot} = 2t_{hmax}$ . In the horizontal direction the velocity is constant (both in the  $x$  and  $y$  directions), and the position of the projectile is given by:

$$x = x_0 + v_x t \quad \text{and} \quad y = y_0 + v_{0y} t$$

The following MATLAB program written in a script file solves the problem by following the equations above.

```
v0=250; g=9.81; theta=65;
x0=3000; vx=-30;
v0z=v0*sin(theta*pi/180);
v0y=v0*cos(theta*pi/180);
t=2*v0z/g;
tplot=linspace(0,t,100);
z=v0z*tplot-0.5*g*tplot.^2;
y=v0y*tplot;
x=x0+vx*tplot;
xnowind(1:length(y))=x0;
plot3(x,y,z,'k-','xnowind',y,z,'k--')
grid on
axis([0 6000 0 6000 0 2700])
xlabel('x (m)'); ylabel('y (m)'); zlabel('z (m)')
```

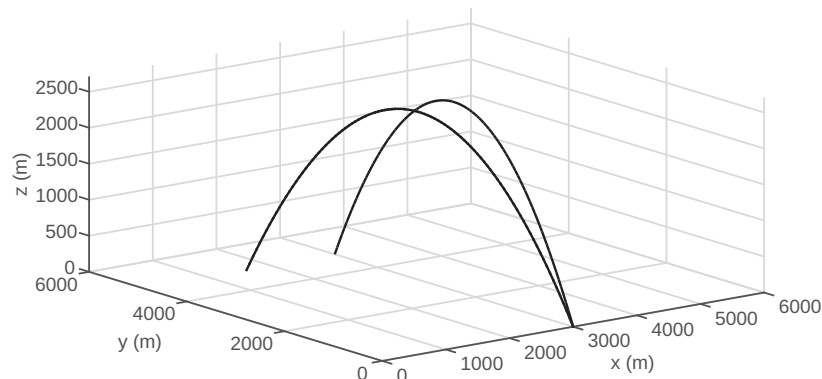
Creating a time vector with 100 elements.

Calculating the x, y, and z coordinates of the projectile at each time.

Constant x coordinate when no wind.

Two 3-D line plots.

The figure generated by the program is shown below.



### Sample Problem 10-2: Electric potential of two point charges

The electric potential  $V$  around a charged particle is given by

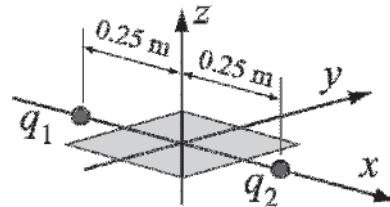
$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

where  $\epsilon_0 = 8.8541878 \times 10^{-12} \frac{\text{C}}{\text{N m}^2}$  is the permittivity constant,  $q$  is the magnitude of the charge in coulombs, and  $r$  is the distance from the particle in meters. The electric field of two or more particles is calculated by using superposition. For example, the electric potential at a point due to two particles is given by

$$V = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right)$$

where  $q_1$ ,  $q_2$ ,  $r_1$ , and  $r_2$  are the charges of the particles and the distance from the point to the corresponding particle, respectively.

Two particles with a charge of  $q_1 = 2 \times 10^{-10} \text{ C}$  and  $q_2 = 3 \times 10^{-10} \text{ C}$  are positioned in the  $xy$  plane at points  $(0.25, 0, 0)$  and  $(-0.25, 0, 0)$ , respectively, as shown. Calculate and plot the electric potential due to the two particles at points in the  $xy$  plane that are located in the domain  $-0.2 < x < 0.2$  and  $-0.2 \leq y \leq 0.2$  (the units in the  $xy$  plane are meters). Make the plot such that the  $xy$  plane is the plane of the points, and the  $z$  axis is the magnitude of the electric potential.



#### Solution

The problem is solved by following these steps:

- A grid is created in the  $xy$  plane with the domain  $-0.2 \leq x \leq 0.2$  and  $-0.2 \leq y \leq 0.2$ .
- The distance from each grid point to each of the charges is calculated.
- The electric potential at each point is calculated.
- The electric potential is plotted.

The following is a program in a script file that solves the problem.

```
eps0=8.85e-12; q1=2e-10; q2=3e-10;
```

```
k=1/(4*pi*eps0);
```

```
x=-0.2:0.01:0.2;
```

```
y=-0.2:0.01:0.2;
```

```
[X,Y]=meshgrid(x,y);
```

```
r1=sqrt((X+0.25).^2+Y.^2);
```

Creating a grid in the  $xy$  plane.

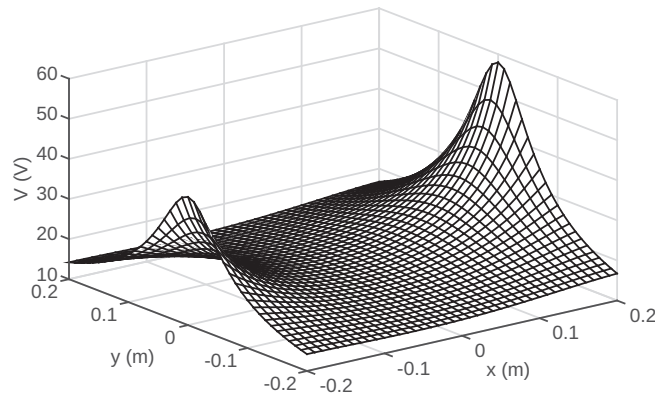
Calculating the distance  $r_1$  for each grid point.

```

r2=sqrt((X-0.25).^2+Y.^2);    Calculating the distance  $r_2$  for each grid point.
V=k*(q1./r1+q2./r2);          Calculating the electric potential  $V$  at each grid point.
mesh(X,Y,V)
xlabel('x (m)'); ylabel('y (m)'); zlabel('V (V)')

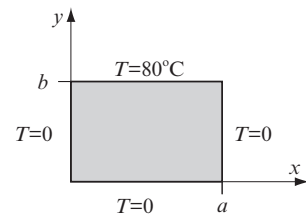
```

The plot generated when the program runs is:



### Sample Problem 10-3: Heat conduction in a square plate

Three sides of a rectangular plate ( $a = 5$  m,  $b = 4$  m) are kept at a temperature of  $0^\circ\text{C}$  and one side is kept at a temperature  $T_1 = 80^\circ\text{C}$ , as shown in the figure. Determine and plot the temperature distribution  $T(x, y)$  in the plate.



#### Solution

The temperature distribution,  $T(x, y)$  in the plate can be determined by solving the two-dimensional heat equation. For the given boundary conditions  $T(x, y)$  can be expressed analytically by a Fourier series (Erwin Kreyszig, *Advanced Engineering Mathematics*, John Wiley and Sons, 1993):

$$T(x, y) = \frac{4T_1}{\pi} \sum_{n=1}^{\infty} \frac{\sin \left[ (2n-1) \frac{\pi x}{a} \right]}{(2n-1)} \frac{\sinh \left[ (2n-1) \frac{\pi y}{a} \right]}{\sinh \left[ (2n-1) \frac{\pi b}{a} \right]}$$

A program in a script file that solves the problem is listed below. The program follows these steps:

- Create an  $X, Y$  grid in the domain  $0 \leq x \leq a$  and  $0 \leq y \leq b$ . The length of the plate,  $a$ , is divided into 20 segments, and the width of the plate,  $b$ , is divided into 16 segments.
- Calculate the temperature at each point of the mesh. The calculations are

done point by point using a double loop. At each point the temperature is determined by adding  $k$  terms of the Fourier series.

(c) Make a surface plot of  $T$ .

```

a=5; b=4; na=20; nb=16; k=5; T0=80;
clear T
x=linspace(0,a,na);
y=linspace(0,b,nb);
[X,Y]=meshgrid(x,y);
for i=1:nb
    for j=1:na
        T(i,j)=0;
        for n=1:k
            ns=2*n-1;
            T(i,j)=T(i,j)+sin(ns*pi*X(i,j)/a).*sinh(ns*pi*Y(i,j)/
a)/(sinh(ns*pi*b/a)*ns);
        end
        T(i,j) = T(i,j)*4*T0/pi;
    end
end
mesh(X,Y,T)
xlabel('x (m)'); ylabel('y (m)'); zlabel('T ( ^oC)')

```

Creating a grid in the  $x y$  plane.

First loop,  $i$ , is the index of the grid's row.

Second loop,  $j$ , is the index of the grid's column.

Third loop,  $n$ , is the  $n^{\text{th}}$  term of the Fourier series,  $k$  is the number of terms.

The program was executed twice, first using five terms ( $k = 5$ ) in the Fourier series to calculate the temperature at each point, and then with  $k = 50$ . The mesh plots created in each execution are shown in the figures below. The temperature should be uniformly  $80^\circ\text{C}$  at  $y = 4$  m. Note the effect of the number of terms ( $k$ ) on the accuracy at  $y = 4$  m.

