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• In the diff(S, var) command (which is used for differentiation of expressions with several symbolic variables) the differentiation is carried out with respect to the variable var.

• The second or higher (*n*th) derivative can be determined with the diff(S, n) or diff(S, var, n) command, where n is a positive number. *n* = 2 for the second derivative, *n* = 3 for the third, and so on.

Some examples are:

```
>> syms x y t
                                   Define x, y, and t as symbolic variables.
>> S=exp(x^4);
                                             Assign to S the expression e^{x^4}.
>> diff(S)
                              Use the diff(S) command to differentiate S.
ans =
                                           The answer 4x^3e^{x^4} is displayed.
4*x^3*exp(x^4)
\rightarrow diff((1-4*x)^3) Use the diff(S) command to differentiate (1-4x)^3.
                                    The answer -12(1-4x)^3 is displayed.
-12*(1-4*x)^2
>> R=5*y^2*cos(3*t);
                                      Assign to R the expression 5v^2\cos(3t)
>> diff(R)
                              Use the diff (R) command to differentiate R.
                      MATLAB differentiates R with respect to y (default
ans =
10*y*cos(3*t)
                      symbolic variable); the answer 10y\cos(3t) is displayed.
>> diff(R,t)
                     Use the diff (R,t) command to differentiate R w.r.t. t.
ans =
                                     The answer -15y^2\sin(3t) is displayed.
-15*y^2*sin(3*t)
>> diff(S,2)
                 Use diff (S, 2) command to obtain the second derivative of S.
ans =
                                              The answer 12x^2e^{x^4} + 16x^6e^{x^4}
12*x^2*exp(x^4)+16*x^6*exp(x^4)
                                              is displayed.
```

It is also possible to use the diff command by typing the expression to be differentiated as a string directly in the command without having the variables in the expression first created as symbolic objects. However, the variables in the differentiated expression do not exist as independent symbolic objects.

## 11.5 Integration

Symbolic integration can be carried out by using the int command. The command can be used for determining indefinite integrals (antiderivatives) and definite integrals. For indefinite integration the form of the command is:

$$\left[\begin{array}{c} \text{int(S)} \end{array}\right] \qquad \text{or} \qquad \left[\begin{array}{c} \text{int(S,var)} \end{array}\right]$$

- Either S can be the name of a previously created symbolic expression, or an expression can be typed in for S.
- In the int(S) command, if the expression contains one symbolic variable, the integration is carried out with respect to that variable. If the expression contains more than one variable, the integration is carried out with respect to the default symbolic variable (Section 11.1.3).
- In the int(S, var) command, which is used for integration of expressions
  with several symbolic variables, the integration is carried out with respect to the
  variable var.

Some examples are:

```
>> syms x y t
                                   Define x, y, and t as symbolic variables.
>> S=2*cos(x)-6*x;
                                  Assign to S the expression 2\cos x - 6x.
>> int(S)
                                  Use the int (S) command to integrate S.
ans =
                                     The answer 2 \sin x - 3x^2 is displayed.
2*sin(x) - 3*x^2
>> int(x*sin(x))
                            Use the int (S) command to integrate x\sin(x).
ans =
                                  The answer \sin x - x \cos x is displayed.
sin(x) - x*cos(x)
>>R=5*y^2*cos(4*t);
                                     Assign to R the expression 5y^2\cos(4t).
>> int(R)
                                 Use the int (R) command to integrate R.
                      MATLAB integrates R with respect to y (default sym-
(5*y^3*cos(4*t))/3 bolic variable); the answer 5y^3cos(4t)/3 is displayed.
>> int(R,t)
                         Use the int (R, t) command to integrate R w.r.t. t.
ans =
                                    The answer 5y^2\cos(4t)/4 is displayed.
(5*y^2*sin(4*t))/4
```

For definite integration the form of the command is:

$$[int(S,a,b)]$$
 or  $[int(S,var,a,b)]$ 

• a and b are the limits of integration. The limits can be numbers or symbolic variables.  $(\sin y - 5y^2)dy$ 

For example, determination of the definite integral  $\int_0^{\pi} (\sin y - 5y^2) dy$  with MATLAB is:

```
>> syms y
>> int(sin(y)-5*y^2,0,pi)
ans =
2 - (5*pi^3)/3
```