

Figure 5-14: Using the PLOTS Toolstrip.

As an example, two different figures, one with line plot and the other with bar plot, were created using the two vectors yr and sle. The two figures are displayed in Figure 5-14 and the commands that created the plots are displayed in the Command Window in Figure 5-13.

#### **Additional notes:**

• When selecting variables for the plot (in the Workspace Window), the first to be selected will be the independent variable (horizontal axis) and the second will be the dependent variable (vertical axis). After the selection, the variables can be switched by clicking on the Switch icon.

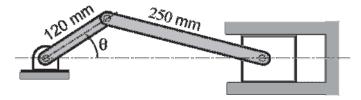


• If only one variable (vector) is selected for a figure, the values of the vector elements will be plotted versus the number of the element.

## 5.13 Examples of MATLAB Applications

# Sample Problem 5-2: Piston-crank mechanism

The piston-rod-crank mechanism is used in many engineering applications. In the mechanism shown in the following figure, the crank is rotating at a constant speed of 500 rpm.



Calculate and plot the position, velocity, and acceleration of the piston for one

revolution of the crank. Make the three plots on the same page. Set  $\theta = 0^{\circ}$  when t = 0.

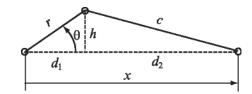
#### **Solution**

The crank is rotating with a constant angular velocity  $\dot{\theta}$ . This means that if we set  $\theta = 0^{\circ}$  when t = 0, then at time t the angle  $\theta$  is given by  $\theta = \dot{\theta}t$ , and means that  $\ddot{\theta} = 0$  at all times.

The distances  $d_1$  and h are given by:

$$d_1 = r\cos\theta$$
 and  $h = r\sin\theta$ 

With h known, the distance  $d_2$  can be calculated using the Pythagorean Theorem:



$$d_2 = (c^2 - h^2)^{1/2} = (c^2 - r^2 \sin^2 \theta)^{1/2}$$

The position x of the piston is then given by:

$$x = d_1 + d_2 = r \cos \theta + (c^2 - r^2 \sin^2 \theta)^{1/2}$$

The derivative of x with respect to time gives the velocity of the piston:

$$\dot{x} = -r\dot{\theta}\sin\theta - \frac{r^2\dot{\theta}\sin 2\theta}{2(c^2 - r^2\sin^2\theta)^{1/2}}$$

The second derivative of x with respect to time gives the acceleration of the piston:

$$\ddot{x} = -r\dot{\theta}^2 \cos \theta - \frac{4r^2\dot{\theta}^2 \cos 2\theta (c^2 - r^2 \sin^2 \theta) + (r^2\dot{\theta}\sin 2\theta)^2}{4(c^2 - r^2 \sin^2 \theta)^{3/2}}$$

In the equation above,  $\ddot{\theta}$  was taken to be zero.

A MATLAB program (script file) that calculates and plots the position, velocity, and acceleration of the piston for one revolution of the crank is shown below.

THDrpm=500; r=0.12; c=0.25; Define 
$$\dot{\theta}$$
, r, and c.

THD=THDrpm\*2\*pi/60; Change the units of  $\dot{\theta}$  from rpm to rad/s.

tf=2\*pi/THD; Calculate the time for one revolution of the crank.

t=linspace(0,tf,200); Create a vector for the time with 200 elements.

TH=THD\*t; Calculate  $\theta$  for each t.

d2s=c^2-r^2\*sin(TH).^2; Calculate  $d_2$  squared for each  $\theta$ .

x=r\*cos(TH)+sqrt(d2s); Calculate x for each  $\theta$ .

xd=-r\*THD\*sin(TH)-(r^2\*THD\*sin(2\*TH))./(2\*sqrt(d2s));

```
xdd = -r*THD^2*cos(TH) - (4*r^2*THD^2*cos(2*TH).*d2s +
(r<sup>2</sup>*sin(2*TH)*THD).<sup>2</sup>)./(4*d2s.<sup>(3/2)</sup>);
                                             Calculate \dot{x} and \ddot{x} for each \theta.
subplot(3,1,1)
                                                           Plot x vs. t.
plot(t,x)
grid
                                                      Format the first plot.
xlabel('Time (s)')
ylabel('Position (m)')
subplot(3,1,2)
plot(t,xd)
                                                           Plot \dot{x} vs. t.
grid
                                              Format the second plot.
xlabel('Time (s)')
ylabel('Velocity (m/s)')
subplot(3,1,3)
plot(t,xdd)
                                                         Plot \ddot{x} vs. t.
                                               Format the third plot.
grid
xlabel('Time (s)')
ylabel('Acceleration (m/s^2)')
```

When the script file runs it generates the three plots on the same page as shown in Figure 5-13. The figure nicely shows that the velocity of the piston is zero at the end points of the travel range where the piston changes the direction of the motion. The acceleration is maximum (directed to the left) when the piston is at the right end.

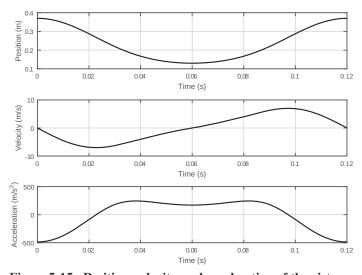


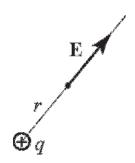
Figure 5-15: Position, velocity, and acceleration of the piston vs. time.

## Sample Problem 5-3: Electric Dipole

The electric field at a point due to a charge is a vector **E** with magnitude *E* given by Coulomb's law:

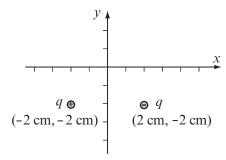
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

where  $\epsilon_0 = 8.8541878 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$  is the permittivity constant, q is the magnitude of the charge, and r is the distance between the charge and the point. The direction of E is along the line that connects the charge with the point. E points outward from q if q is positive, and



toward q if q is negative. An electric dipole is created when a positive charge and a negative charge of equal magnitude are placed some distance apart. The electric field,  $\mathbf{E}$ , at any point is obtained by superposition of the electric field of each charge.

An electric dipole with  $q = 12 \times 10^{-9}$  C is created, as shown in the figure. Determine and plot the magnitude of the electric field along the x axis from x = -5 cm to x = 5 cm.

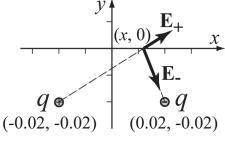


## **Solution**

The electric field  $\mathbf{E}$  at any point (x, 0) along the x axis is obtained by adding the electric field vectors due to each of the charges.

$$\mathbf{E} = \mathbf{E}^- + \mathbf{E}^+$$

The magnitude of the electric field is the length of the vector **E**.



The problem is solved by following these steps:

- **Step 1:** Create a vector x for points along the x axis.
- **Step 2:** Calculate the distance (and distance squared) from each charge to the points on the *x* axis.

$$r_{\text{minus}} = \sqrt{(0.02 - x)^2 + 0.02^2}$$
  $r_{plus} = \sqrt{(0.02 + x)^2 + 0.02^2}$ 

Step 3: Write unit vectors in the direction from each charge to the points on the x axis.

$$\mathbf{E}_{\text{minus}UV} = \frac{1}{r_{\text{minus}}} \left[ (0.02 - x)\mathbf{i} - 0.02\mathbf{j} \right] \qquad \mathbf{E}_{phsUV} = \frac{1}{r_{phs}} \left[ (0.02 + x)\mathbf{i} + 0.02\mathbf{j} \right]$$

**Step 4:** Calculate the magnitude of the vector  $\mathbf{E}_{-}$  and  $\mathbf{E}_{+}$  at each point by using Coulomb's law.

$$E_{\text{minusMAG}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r_{\text{minus}}^2} \qquad E_{plusMAG} = \frac{1}{4\pi\epsilon_0} \frac{q}{r_{plus}^2}$$

- **Step 5:** Create the vectors  $\mathbf{E}_{-}$  and  $\mathbf{E}_{+}$  by multiplying the unit vectors by the magnitudes.
- **Step 6:** Create the vector **E** by adding the vectors  $\mathbf{E}_{-}$  and  $\mathbf{E}_{+}$ .
- **Step 7:** Calculate E, the magnitude (length) of E.
- **Step 8:** Plot E as a function of x.

A program in a script file that solves the problem is:

```
q=12e-9;
epsilon0=8.8541878e-12;
x=[-0.05:0.001:0.05]';
                                          Create a column vector x.
rminusS=(0.02-x).^2+0.02^2;
                                              Step 2. Each variable
rminus=sgrt(rminusS);
                                               is a column vector.
rplusS=(x+0.02).^2+0.02^2;
rplus=sqrt(rplusS);
                                              Steps 3 & 4. Each vari-
                                              able is a two column
EminusUV=[((0.02-x)./rminus), (-0.02./rminus)];
                                              matrix. Each row is
EplusUV=[((x+0.02)./rplus), (0.02./rplus)];
                                              the vector for the cor-
                                              responding x.
EminusMAG=(q/(4*pi*epsilon0))./rminusS;
EplusMAG=(q/(4*pi*epsilon0))./rplusS;
Eminus=[EminusMAG.*EminusUV(:,1), EminusMAG.*EminusUV(:,2)];
Eplus=[EplusMAG.*EplusUV(:,1), EplusMAG.*EplusUV(:,2)];
E=Eminus+Eplus;
                                          Step 6.
                                                          Step 5.
EMAG=sgrt(E(:,1).^2+E(:,2).^2);
                                          Step 7.
plot(x,EMAG,'k','linewidth',1)
xlabel('Position along the x-axis (m)','FontSize',12)
ylabel('Magnitude of the electric field (N/C)','FontSize',12)
title('ELECTRIC FIELD DUE TO AN ELECTRIC DIPOLE', 'FontSize',12)
```

When this script file is executed in the Command Window, the following figure is created in the Figure Window: