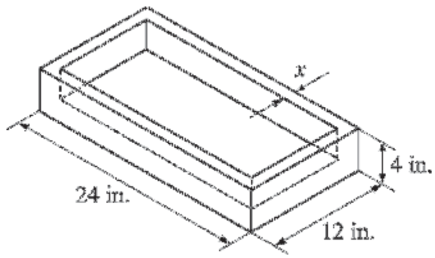
8.5 *EXAMPLES OF MATLAB APPLICATIONS*

Sample Problem 8-4: Determining wall thickness of a box

The outside dimensions of a rectangular box (bottom and four sides, no top), made of aluminum, are 24 by 12 by 4 inches. The wall thickness of the bottom and the sides is x. Derive an expression that relates the weight of the box and the wall thickness x. Determine the thickness x for a box that weighs 15 lb. The specific weight of aluminum is 0.101 lb/in.^{3}.



Solution

The volume of the aluminum V_{Al} is calculated from the weight W of the box by:

$$V_{Al} = \frac{W}{\gamma}$$

where γ is the specific weight. The volume of the aluminum based on the dimensions of the box is given by

$$V_{Al} = 24 \cdot 12 \cdot 4 - (24 - 2x)(12 - 2x)(4 - x)$$

where the inside volume of the box is subtracted from the outside volume. This equation can be rewritten as

$$(24 - 2x)(12 - 2x)(4 - x) + V_{Al} - 24 \cdot 12 \cdot 4 = 0$$

which is a third-degree polynomial. A root of this polynomial is the required thickness x. A program in a script file that determines the polynomial and solves for the roots is:

`W=15; gamma=0.101;`	Assign `W` and `gamma`.
`VAlum=W/gamma;`	Calculate the volume of the aluminum.
`a=[-2  24];`	Assign the polynomial $24 - 2x$ to `a`.
`b=[-2  12];`	Assign the polynomial $12 - 2x$ to `b`.
`c=[-1  4];`	Assign the polynomial $4 - x$ to `c`.
`Vin=conv(c, conv(a,b));`	Multiply the three polynomials above.
`polyeq=[0 0 0 (VAlum-24*12*4)]+Vin`	Add V_{Al}– 24*12*4 to `Vin`.
`x=roots(polyeq)`	Determine the roots of the polynomial.

Note in the second-to-last line that in order to add the quantity $V_{Al} - 24 \cdot 12 \cdot 4$ to the polynomial `Vin` it has to be written as a polynomial of the same order as `Vin` (`Vin` is a polynomial of third order). When the program (saved as Chap8SamPro4) is executed, the coefficients of the polynomial and the value of x are displayed:

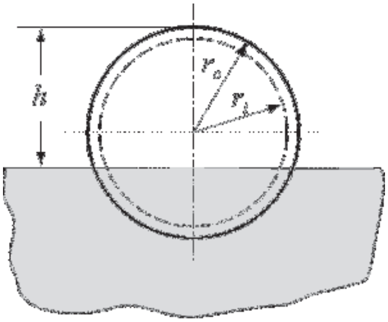
```
>> Chap8SamPro4
polyeq =
 -4.0000   88.0000 -576.0000   148.514
x =
  10.8656 + 4.4831i
  10.8656 - 4.4831i
  0.2687
```

The polynomial is: $-4x^3 + 88x^2 - 576x + 148.515$.

The polynomial has one real root, $x = 0.2687$ in., which is the thickness of the aluminum

Sample Problem 8-5: Floating height of a buoy

An aluminum thin-walled sphere is used as a marker buoy. The sphere has a radius of 60 cm and a wall thickness of 12 mm. The density of aluminum is $\rho_{Al} = 2690$ kg/m^3. The buoy is placed in the ocean, where the density of the water is 1030 kg/m^3. Determine the height h between the top of the buoy and the surface of the water.



Solution

According to Archimedes's law, the buoyancy force applied to an object that is placed in a fluid is equal to the weight of the fluid that is displaced by the object. Accordingly, the aluminum sphere will be at a depth such that the weight of the sphere is equal to the weight of the fluid displaced by the part of the sphere that is submerged.

The weight of the sphere is given by

$$W_{sph} = \rho_{Al} V_{Al}\, g = \rho_{Al}\frac{4}{3}\pi(r_o^3 - r_i^3)g$$

where V_{Al} is the volume of the aluminum; r_o and r_i are the outside and inside radii of the sphere, respectively; and g is the gravitational acceleration.

The weight of the water that is displaced by the spherical portion that is submerged is given by:

$$W_{wtr} = \rho_{wtr} V_{wtr}\, g = \rho_{wtr}\frac{1}{3}\pi(2r_o - h)^2(r_o + h)g$$

Setting the two weights equal to each other gives the following equation:

$$h^3 - 3r_o h^2 + 4r_o^3 - 4\frac{\rho_{Al}}{\rho_{wtr}}(r_o^3 - r_i^3) = 0$$

The last equation is a third-degree polynomial for h. The root of the polynomial is the answer.

A solution with MATLAB is obtained by writing the polynomials and using the `roots` function to determine the value of h.This is done in the following script file:

```
rout=0.60; rin=0.588;
rhoalum=2690; rhowtr=1030;
a0=4*rout^3-4*rhoalum*(rout^3-rin^3)/rhowtr;
p = [1 -3*rout 0 a0];
h = roots(p)
```

Assign the radii to variables.

Assign the densities to variables.

Assign the coefficient a_0.

Assign the coefficient vector of the polynomial.

Calculate the roots of the polynomial.

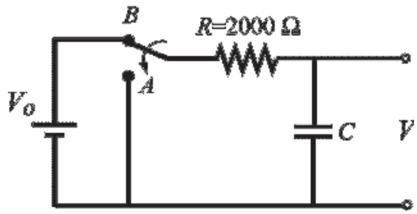
When the script file is executed in the Command Window, as shown below, the answer is three roots, since the polynomial is of the third degree. The only answer that is physically possible is the second, where $h = 0.9029$ m.

```
>> Chap8SamPro5
h =
    1.4542
    0.9029
   -0.5570
```

The polynomial has three roots. The only one that is physically possible for the problem is 0.9029 m.

Sample Problem 8-6: Determining the size of a capacitor

An electrical capacitor has an unknown capacitance. In order to determine its capacitance, the capacitor is connected to the circuit shown. The switch is first connected to B and the capacitor is charged. Then, the switch is connected to A and the capacitor discharges through the resistor. As the capacitor is discharging, the voltage across the capacitor is measured for 10 s in intervals of 1 s. The recorded measurements are given in the table below. Plot the voltage as a function of time and determine the capacitance of the capacitor by fitting an exponential curve to the data points.



t (s)	1	2	3	4	5	6	7	8	9	10
V (V)	9.4	7.31	5.15	3.55	2.81	2.04	1.26	0.97	0.74	0.58

Solution

When a capacitor discharges through a resistor, the voltage of the capacitor as a function of time is given by

$$V = V_0 e^{-t/RC}$$

where V_0 is the initial voltage, R the resistance of the resistor, and C the capacitance of the capacitor. As was explained in Section 8.2.2 the exponential function can be written as a linear equation for $\ln(V)$ and t in the form:

$$\ln(V) = \frac{-1}{RC}t + \ln(V_0)$$

This equation, which has the form $y = mx + b$, can be fitted to the data points by using the `polyfit(x,y,1)` function with t as the independent variable `x` and $\ln(V)$ as the dependent variable `y`. The coefficients m and b determined by the `polyfit` function are then used to determine C and V_0 by:

$$C = \frac{-t}{Rm} \quad \text{and} \quad V_0 = e^b$$

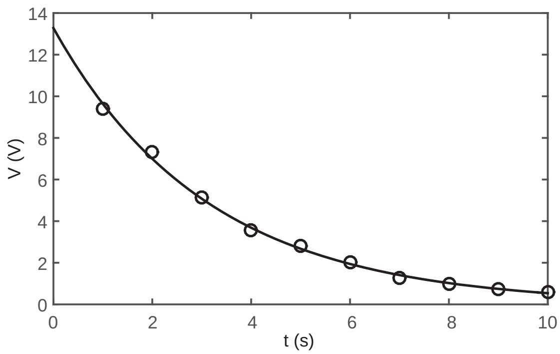
The following program written in a script file determines the best-fit exponential function to the data points, determines C and V_0, and plots the points and the fitted function.

```
R=2000;                                   Define R.
t=1:10;                                   Assign the data points to vectors t and v.
v=[9.4 7.31 5.15 3.55 2.81 2.04 1.26 0.97 0.74 0.58];
p=polyfit(t,log(v),1);                    Use the polyfit function with t and log(v).
C=-1/(R*p(1))                             Calculate C from p(1), which is m in the equation.
V0=exp(p(2))                              Calculate V0 from p(2), which is b in the equation.
tplot=0:0.1:10;                           Create vector tplot of time for plotting the function.
vplot=V0*exp(-tplot./(R*C));              Create vector vplot for plotting the function.
plot(t,v,'o',tplot,vplot)
```

When the script file is executed (saved as Chap8SamPro6) the values of C and V_0 are displayed in the Command Window as shown below:

```
>> Chap8SamPro6
C =                                       The capacitance of the capacitor is 1,600 μF.
    0.0016
V0 =
   13.2796
```

The program creates also the following plot (axis labels were added to the plot using the Plot Editor):

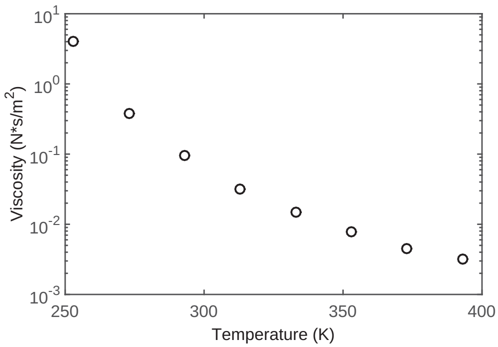


Sample Problem 8-7: Temperature dependence of viscosity

Viscosity, μ, is a property of gases and fluids that characterizes their resistance to flow. For most materials, viscosity is highly sensitive to temperature. Below is a table that gives the viscosity of SAE 10W oil at different temperatures (data from *B.R.* Munson, D.F. Young, and T.H. Okiishi, *Fundamentals of Fluid Mechanics*, 4th ed., John Wiley and Sons, 2002). Determine an equation that can be fitted to the data.

T (°C)	–20	0	20	40	60	80	100	120
μ (N s/m²) $(\times 10^{-5})$	4	0.38	0.095	0.032	0.015	0.007 8	0.004 5	0.003 2

Solution



To determine what type of equation might provide a good fit to the data, μ is plotted as a function of T (absolute temperature) with a linear scale for T and a logarithmic scale for μ. The plot, shown on the right, indicates that the data points do not appear to line up along a straight line. This means that a simple exponential function of the form $y = be^{mx}$, which models a straight line with these axes, will not provide the best fit. Since the points in the figure appear to lie along a curved line, a function that can possibly have a good fit to the data is:

$$\ln(\mu) = a_2 T^2 + a_1 T + a_0$$

This function can be fitted to the data by using MATLAB's `polyfit(x,y,2)` function (second-degree polynomial), where the independent variable is T and the dependent variable is ln(μ). The equation above can be solved for μ to give the viscosity as a function of temperature:

$$\mu = e^{(a_2 T^2 + a_1 T + a_0)} = e^{a_0} e^{a_1 T} e^{a_2 T^2}$$

The following program determines the best fit to the function and creates a plot that displays the data points and the function.

```
T=[-20:20:120];
mu=[4 0.38 0.095 0.032 0.015 0.0078 0.0045 0.0032];
TK=T+273;
p=polyfit(TK,log(mu),2)
```