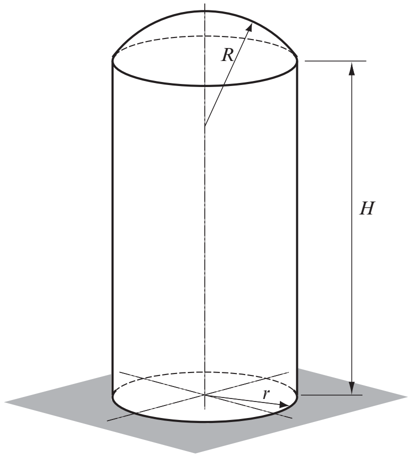
4.6 EXAMPLES OF MATLAB APPLICATIONS

Sample Problem 4-1: Height and surface area of a silo

A cylindrical silo with radius r has a spherical cap roof with radius R. The height of the cylindrical portion is H. Write a program in a script file that determines the height H for given values of r, R, and the volume V. In addition, the program calculates the surface area of the silo.



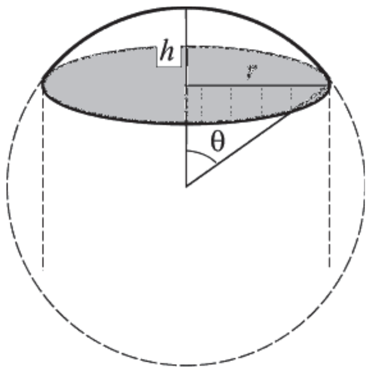
Use the program to calculate the height and surface area of a silo with r = 30 ft, R = 45 ft, and a volume of 200,000 ft^3. Assign values for r, R, and V in the Command Window.

Solution

The total volume of the silo is obtained by adding the volume of the cylindrical part and the volume of the spherical cap. The volume of the cylinder is given by

$$V_{cyl} = \pi r^2 H$$

and the volume of the spherical cap is given by:



$$V_{cap} = \frac{1}{3}\pi h^2(3R - h)$$

where $h = R - R\cos\theta = R(1 - \cos\theta)$,

and θ is calculated from $\sin\theta = \frac{r}{R}$.

Using the equations above, the height, H, of the cylindrical part can be expressed by

$$H = \frac{V - V_{cap}}{\pi r^2}$$

The surface area of the silo is obtained by adding the surface areas of the cylindrical part and the spherical cap.

$$S = S_{cyl} + S_{cap} = 2\pi rH + 2\pi Rh$$

A program in a script file that solves the problem is presented below:

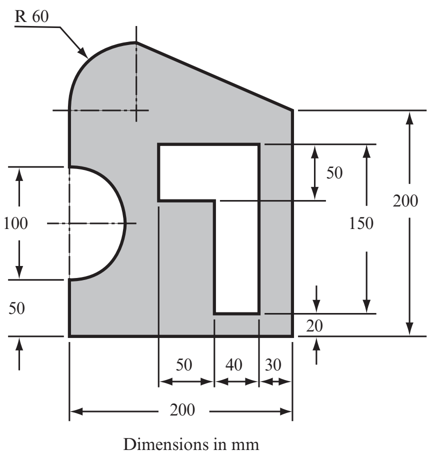
```
theta=asin(r/R);                    Calculating θ.
h=R*(1-cos(theta));                 Calculating h.
```

```
Vcap=pi*h^2*(3*R-h)/3;                    Calculating the volume of the cap.
H=(V-Vcap)/(pi*r^2);                      Calculating H.
S=2*pi*(r*H + R*h);                       Calculating the surface area
fprintf('The height H is: %f ft.',H)
fprintf('\nThe surface area of the silo is: %f square ft.',S)
```

The Command Window where the script file, named silo, was executed is:

```
>> r=30; R=45; V=200000;                  Assigning values to r, R, and V.
>> silo                                   Running the script file named
The height H is: 64.727400 ft.
The surface area of the silo is: 15440.777753 square ft.
```

Sample Problem 4-2: Centroid of a composite area



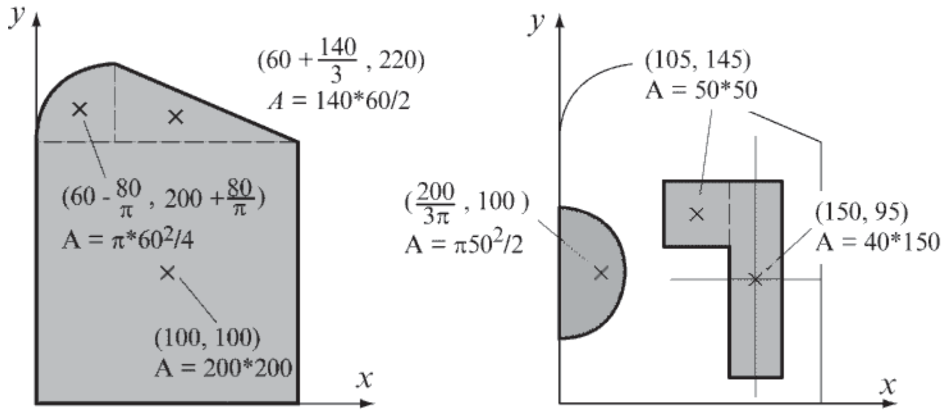
Write a program in a script file that calculates the coordinates of the centroid of a composite area. (A composite area can easily be divided into sections whose centroids are known.) The user needs to divide the area into sections and know the coordinates of the centroid (two numbers) and the area of each section (one number). When the script file is executed, it asks the user to enter the three numbers as a row in a matrix. The user enters as many rows as there are sections. A section that represents a hole is taken to have a negative area. For output, the program displays the coordinates of the centroid of the composite area. Use the program to calculate the centroid of the area shown in the figure.

Solution

The area is divided into six sections as shown in the following figure. The total area is calculated by adding the three sections on the left and subtracting the three sections on the right. The location and coordinates of the centroid of each section are marked in the figure, as well as the area of each section.

The coordinates $\bar{X}$ and $\bar{Y}$ of the centroid of the total area are given by $\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}$ and $\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$, where $\bar{x}$, $\bar{y}$, and A are the coordinates of the centroid and area of each section, respectively.

A script file with a program for calculating the coordinates of the centroid



Units: coordinates mm, area mm^2

of a composite area is provided below.

```
% The program calculates the coordinates of the centroid
% of a composite area.
clear C xs ys As
C=input('Enter a matrix in which each row has three ele-
ments.\nIn each row enter the x and y coordinates of the
centroid and the area of a section.\n');
xs=C(:,1)';        Creating a row vector for the x coordinate
                   of each section (first column of C).
ys=C(:,2)';        Creating a row vector for the y coordinate
                   of each section (second column of C).
As=C(:,3)';        Creating a row vector for the area of
                   each section (third column of C).
A=sum(As);         Calculating the total area.
x=sum(As.*xs)/A;   Calculating the coordinates of the
y=sum(As.*ys)/A;   centroid of the composite area.
fprintf('The coordinates of the centroid are: ( %f, %f
)\n',x,y)
```

The script file was saved with the name Centroid. The following shows the Command Window where the script file was executed.

```
>> Centroid
Enter a matrix in which each row has three elements.
In each row enter the x and y coordinates of the centroid
and the area of a section.
```

```
[100 100 200*200
60-80/pi 200+80/pi pi*60^2/4
60+140/3 220 140*60/2
200/(3*pi) 100 -pi*50^2/2
105 145 -50*50
150 95 -40*150]
The coordinates of the centroid are: ( 85.387547 , 131.211809 )
```

Entering the data for matrix `C`. Each row has three elements: the x, y, and A of a section.

Sample Problem 4-3: Voltage divider

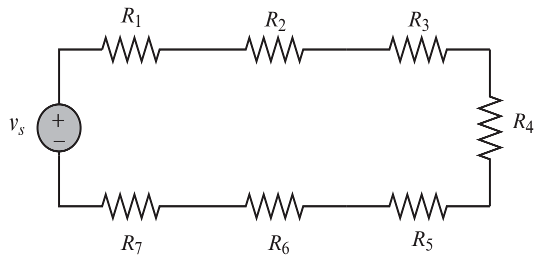
When several resistors are connected in an electrical circuit in series, the voltage across each of them is given by the voltage divider rule:

$$v_n = \frac{R_n}{R_{eq}} v_s$$

where v_n and R_n are the voltage across resistor n and its resistance, respectively, $R_{eq} = \Sigma R_{eq}$ is the equivalent resistance, and v_s is the source voltage. The power dissipated in each resistor is given by:

$$P_n = \frac{R_n}{R_{eq}^2} v_s^2$$

The figure below shows a circuit with seven resistors connected in series.



Write a program in a script file that calculates the voltage across each resistor, and the power dissipated in each resistor, in a circuit that has resistors connected in series. When the script file is executed, it requests that the user first enter the source voltage and then to enter the resistances of the resistors in a vector. The program displays a table with the resistance listed in the first column, the voltage across the resistor in the second column, and the power dissipated in the resistor in the third column. Following the table, the program displays the current in the circuit and the total power.

Execute the file and enter the following data for v_s and the R's.

$v_s = 24\text{V}$, $R_1 = 20\Omega$, $R_2 = 14\Omega$, $R_3 = 12\Omega$, $R_4 = 18\Omega$, $R_5 = 8\Omega$, $R_6 = 15\Omega$, $R_7 = 10\Omega$.

Solution

A script file that solves the problem is shown below.

```
% The program calculates the voltage across each resistor
% in a circuit that has resistors connected in series.
vs=input('Please enter the source voltage ');
Rn=input('Enter the values of the resistors as elements in a
row vector\n');
Req=sum(Rn);                          Calculate the equivalent resistance.
vn=Rn*vs/Req;                         Apply the voltage divider rule.
Pn=Rn*vs^2/Req^2;                     Calculate the power in each resistor.
i = vs/Req;                           Calculate the current in the circuit.
Ptotal = vs*i;                        Calculate the total power in the circuit.
Table = [Rn', vn', Pn'];              Create a variable table with the
disp(' ')                             vectors Rn, vn, and Pn as columns.
disp(' Resistance Voltage   Power')   Display headings for
disp('    (Ohms)     (Volts)    (Watts)')   the columns.
disp(' ')                             Display an empty line.
disp(Table)                           Display the variable Table.
disp(' ')
fprintf('The current in the circuit is %f Amps.',i)
fprintf('\nThe total power dissipated in the circuit is %f
Watts.',Ptotal)
```

The Command Window where the script file was executed is:

```
>> VoltageDivider                               Name of the script file.
Please enter the source voltage 24  <--  Voltage entered by the user.
Enter the value of the resistors as elements in a row vector

[20  14  12  18  8  15  10] <-- Resistor values entered as a vector.
  Resistance  Voltage    Power
     (Ohms)    (Volts)   (Watts)
   20.0000      4.9485    1.2244
   14.0000      3.4639    0.8571
   12.0000      2.9691    0.7346
   18.0000      4.4536    1.1019
    8.0000      1.9794    0.4897
```