Chapter 8
Polynomials, Curve Fitting, and Interpolation

Polynomials are mathematical expressions that are frequently used for problem solving and modeling in science and engineering. In many cases an equation that is written in the process of solving a problem is a polynomial, and the solution of the problem is the zero of the polynomial. MATLAB has a wide selection of functions that are specifically designed for handling polynomials. How to use polynomials in MATLAB is described in Section 8.1.

Curve fitting is a process of finding a function that can be used to model data. The function does not necessarily pass through any of the points, but models the data with the smallest possible error. There are no limitations to the type of the equations that can be used for curve fitting. Often, however, polynomial, exponential, and power functions are used. In MATLAB curve fitting can be done by writing a program or by interactively analyzing data that is displayed in the Figure Window. Section 8.2 describes how to use MATLAB programming for curve fitting with polynomials and other functions. Section 8.4 describes the basic fitting interface that is used for interactive curve fitting and interpolation.

Interpolation is the process of estimating values between data points. The simplest kind of interpolation is done by drawing a straight line between the points. In a more sophisticated interpolation, data from additional points is used. How to interpolate with MATLAB is discussed in Sections 8.3 and 8.4.

8.1 POLYNOMIALS

Polynomials are functions that have the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

The coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers, and n which is a nonnegative integer, is the degree, or order, of the polynomial.

Examples of polynomials are:

$f(x) = 5x^5 + 6x^2 + 7x + 3$ polynomial of degree 5.

$f(x) = 2x^2 - 4x + 10$ polynomial of degree 2.

$f(x) = 11x - 5$ polynomial of degree 1.

A constant (e.g., $f(x) = 6$) is a polynomial of degree 0.

In MATLAB, polynomials are represented by a row vector in which the elements are the coefficients $a_n, a_{n-1}, ..., a_1, a_0$. The first element is the coefficient of the x with the highest power. The vector has to include all the coefficients, including the ones that are equal to 0. For example:

Polynomial	**MATLAB representation**
$8x + 5$	p = [8 5]
$2x^2 - 4x + 10$	d = [2 –4 10]
$6x^2 - 150$, MATLAB form: $6x^2 + 0x - 150$	h = [6 0 –150]
$5x^5 + 6x^5 - 7x$, MATLAB form: $5x^5 + 0x^4 + 0x^3 + 6x^5 - 7x + 0$	c = [5 0 0 6 7 0]

8.1.1 Value of a Polynomial

The value of a polynomial at a point x can be calculated with the function `polyval` that has the form:

```
polyval(p,x)
```

`p` is a vector with the coefficients of the polynomial.

`x` is a number, or a variable that has an assigned value, or a computable expression.

`x` can also be a vector or a matrix. In such a case the polynomial is calculated for each element (element-by-element), and the answer is a vector, or a matrix, with the corresponding values of the polynomial.

Sample Problem 8-1: Calculating polynomials with MATLAB

For the polynomial $f(x) = x^5 - 12.1x^4 + 40.59x^3 - 17.015x^2 - 71.95x + 35.88$:

(a) Calculate $f(9)$.

(b) Plot the polynomial for $-1.5 \le x \le 6.7$.

Solution

The problem is solved in the Command Window.

(a) The coefficients of the polynomials are assigned to vector `p`. The function `polyval` is then used to calculate the value at $x = 9$.

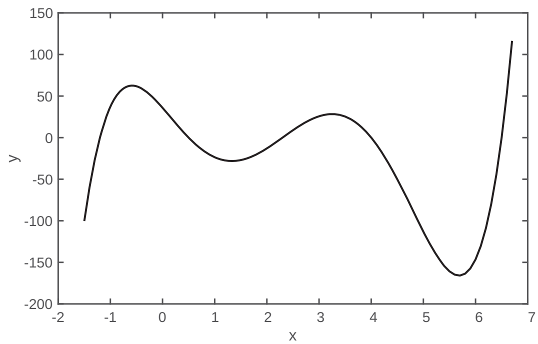
```
>> p = [1 -12.1 40.59 -17.015 -71.95 35.88];
>> polyval(p,9)
ans =
  7.2611e+003
```

(*b*) To plot the polynomial, a vector `x` is first defined with elements ranging from –1.5 to 6.7. Then a vector `y` is created with the values of the polynomial for every element of `x`. Finally, a plot of `y` vs. `x` is made.

```
>> x=-1.5:0.1:6.7;
>> y=polyval(p,x);
>> plot(x,y)
```

Calculating the value of the polynomial for each element of the vector `x`.

The plot created by MATLAB is presented below (axis labels were added with the Plot Editor).



8.1.2 Roots of a Polynomial

The roots of a polynomial are the values of the argument for which the value of the polynomial is equal to zero. For example, the roots of the polynomial $f(x) = x^2 - 2x - 3$ are the values of x for which $x^2 - 2x - 3 = 0$, which are $x = -1$ and $x = 3$.

MATLAB has a function, called `roots`, that determines the root, or roots, of a polynomial. The form of the function is:

```
r = roots(p)
```

`r` is a column vector with the roots of the polynomial.

`p` is a row vector with the coefficients of the polynomial.

For example, the roots of the polynomial in Sample Problem 8-1 can be determined by:

```
>> p= 1 -12.1 40.59 -17.015 -71.95 35.88];
>> r=roots(p)
r =
    6.5000
    4.0000
    2.3000
   -1.2000
    0.5000
```

When the roots are known, the polynomial can actually be written as:
$f(x) = (x+1.2)(x-0.5)(x-2.3)(x-4)(x-6.5)$

The `roots` command is very useful for finding the roots of a quadratic equation. For example, to find the roots of $f(x) = 4x^2 + 10x - 8$, type:

```
>> roots([4 10 -8])
ans =
   -3.1375
    0.6375
```

When the roots of a polynomial are known, the `poly` command can be used for determining the coefficients of the polynomial. The form of the `poly` command is:

```
p = poly(r)
```

`p` is a row vector with the coefficients of the polynomial.

`r` is a vector (row or column) with the roots of the polynomial.

For example, the coefficients of the polynomial in Sample Problem 8-1 can be obtained from the roots of the polynomial (see above) by:

```
>> r=[6.5 4 2.3 -1.2 0.5];
>> p=poly(r)
p =
    1.0000  -12.1000   40.5900  -17.0150  -71.9500   35.8800
```

8.1.3 Addition, Multiplication, and Division of Polynomials

Addition:

Two polynomials can be added (or subtracted) by adding (subtracting) the vectors of the coefficients. If the polynomials are not of the same order (which means that the vectors of the coefficients are not of the same length), the shorter vector has to be modified to be of the same length as the longer vector by adding zeros (called padding) in front. For example, the polynomials $f_1(x) = 3x^6 + 15x^5 - 10x^3 - 3x^2 + 15x - 40$ and $f_2(x) = 3x^3 - 2x - 6$ can be added by:

```
>> p1=[3 15 0 -10 -3 15 -40];
>> p2=[3 0 -2 -6];
>> p=p1+[0 0 0 p2]
p =
     3    15     0    -7    -3    13   -46
```

Three 0s are added in front of p2, since the order of p1 is 6 and the order of p2 is 3.

Multiplication:

Two polynomials can be multiplied using the MATLAB built-in function `conv`, which has the form:

`c = conv(a,b)`

`c` is a vector of the coefficients of the polynomial that is the product of the multiplication.

`a` and `b` are the vectors of the coefficients of the polynomials that are being multiplied.

- The two polynomials do not have to be of the same order.
- Multiplication of three or more polynomials is done by using the `conv` function repeatedly.

For example, multiplication of the polynomials $f_1(x)$ and $f_2(x)$ above gives:

```
>> pm=conv(p1,p2)
pm =
     9    45    -6   -78   -99    65   -54   -12   -10   240
```

which means that the answer is:

$$9x^9 + 45x^8 - 6x^7 - 78x^6 - 99x^5 + 65x^4 - 54x^3 - 12x^2 - 10x + 240$$

Division:

A polynomial can be divided by another polynomial with the MATLAB built-in function `deconv`, which has the form:

`[q,r] = deconv(u,v)`

`q` is a vector with the coefficients of the quotient polynomial.
`r` is a vector with the coefficients of the remainder polynomial.

`u` is a vector with the coefficients of the numerator polynomial.
`v` is a vector with the coefficients of the denominator polynomial.

For example, dividing $2x^3 + 9x^2 + 7x - 6$ by $x + 3$ is done by:

```
>> u=[2 9 7 -6];
>> v=[1 3];
```

```
>> [a b]=deconv(u,v)
a =
     2     3    -2
b =
     0     0     0     0
```

The answer is: $2x^2 + 3x - 2$.

Remainder is zero.

An example of division that gives a remainder is $2x^6 - 13x^5 + 75x^3 + 2x^2 - 60$ divided by $x^2 - 5$:

```
>> w=[2 -13 0 75 2 0 -60];
>> z=[1 0 -5];
>> [g h]=deconv(w,z)
g =
    2   -13    10    10    52
h =
    0     0     0     0     0    50   200
```

The quotient is: $2x^4 - 13x^3 + 10x^2 + 10x + 52$.

The remainder is: $50x + 200$.

The answer is: $2x^4 - 13x^3 + 10x^2 + 10x + 52 + \frac{50x+200}{x^2-5}$.

8.1.4 Derivatives of Polynomials

The built-in function `polyder` can be used to calculate the derivative of a single polynomial, a product of two polynomials, or a quotient of two polynomials, as shown in the following three commands.

`k = polyder(p)` Derivative of a single polynomial. `p` is a vector with the coefficients of the polynomial. `k` is a vector with the coefficients of the polynomial that is the derivative.

`k = polyder(a,b)` Derivative of a product of two polynomials. `a` and `b` are vectors with the coefficients of the polynomials that are multiplied. `k` is a vector with the coefficients of the polynomial that is the derivative of the product.

`[n d] = polyder(u,v)` Derivative of a quotient of two polynomials. `u` and `v` are vectors with the coefficients of the numerator and denominator polynomials. `n` and `d` are vectors with the coefficients of the numerator and denominator polynomials in the quotient that is the derivative.

The only difference between the last two commands is the number of output arguments. With two output arguments MATLAB calculates the derivative of the quotient of two polynomials. With one output argument, the derivative is of the product.