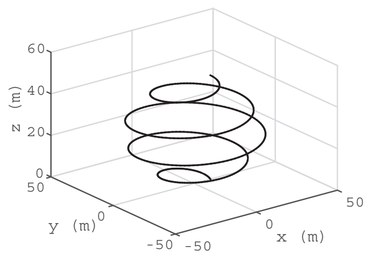
10.6 PROBLEMS

1. The position of a moving particle as a function of time is given by:

$$x = 0.01(30 - t)^2 \sin(2t) \qquad y = 0.01(30 - t)^2 \cos(2t) \qquad z = 0.5\,t^{1.5}$$

Plot the position of the particle for $0 \le t \le 20$ s.

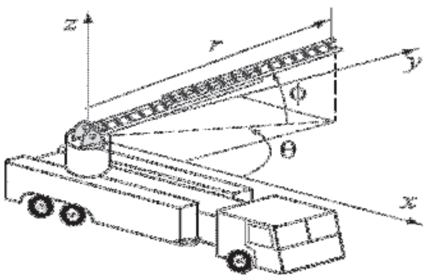
2. A staircase of height h is modeled by the parametric equations:



$$x = r\cos(t) \qquad y = r\sin(t) \qquad z = \frac{ht}{2\pi n}$$

where $r = h[2 + 5\sin(t/8)]/10$, $n = 4$, and $h = 50$ m is the staircase height. Make a 3-D plot (shown) of the staircase. (Create a vector t for the domain 0 to $2\pi n$, and use the `plot3` command.)

3. The ladder of a fire truck can be elevated (increase of angle ϕ), rotated about the z axis (increase of angle θ), and extended (increase of r). Initially the ladder rests on the truck ($\phi = 0$, $\theta = 0$, and $r = 8$ m). Then the ladder is moved to a new position by raising the ladder at a rate of 5 deg/s, rotating at a rate of 8 deg/s, and extending the ladder at a rate of 0.6 m/s. Determine and plot the position of the tip of the ladder for 10 s.



4. Make a 3-D surface plot of the function $z = \frac{x^2}{4} + 2\sin^2(0.7y)$ in the domain $-4 \le x \le 4$ and $-3 \le y \le 3$.

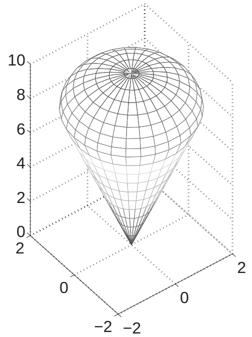
5. Make a 3-D surface plot of the function $z = -0.7x^4 - 0.7y^4$ in the domain $-2 \le x \le 2$ and $-2 \le y \le 2$.

6. Make a 3-D surface plot of the function $z = -1.4xy^3 + 1.4yx^3$ in the domain $-2 \le x \le 2$ and $-2 \le y \le 2$.

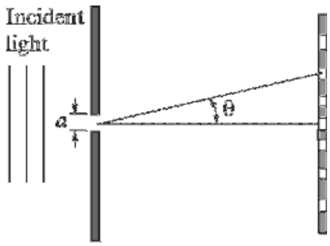
7. Make a 3-D mesh plot of the function $z = \frac{-\cos 2R}{e^{0.2R}}$, where $R = \sqrt{x^2 + y^2}$ in the domain $-5 \le x \le 5$ and $-5 \le y \le 5$.

8. Make a 3-D surface plot of the function $z = \cos(0.7x + 0.7y)\cos(0.7x - 0.7y)$ in the domain $-\pi \le x \le \pi$ and $-\pi \le y \le \pi$.

9. Make a plot of the ice cream cone shown in the figure. The cone is 8 in. tall with a 4-in. diameter base. The ice cream at the top is a 4-in. diameter hemisphere.
A parametric equation for the cone is:
$$x = r\cos\theta\,,\quad y = r\sin\theta\,,\quad z = 4r$$
with $0 \le \theta \le 2\pi$ and $0 \le r \le 2$
A parametric equation for a sphere is:
$$x = r\cos\theta\sin\phi\,,\quad y = r\sin\theta\sin\phi\,,\quad z = r\cos\phi$$
with $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$

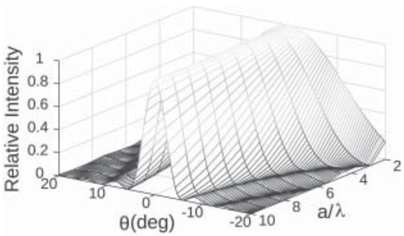


10. A monochromatic light that passes through a slit produces on a screen a diffraction pattern consisting of bright and dark fringes. The intensity of the bright fringes, I, as a function of θ can be calculated by:



$$I = I_{max}\left(\frac{\sin\alpha}{\alpha}\right)^2$$

where $\alpha = \frac{\pi a}{\lambda}\sin\theta$, λ is the light wave length, a is the width of the slits. Make a 3-D plot (shown) that shows the relative intensity I/I_{max} as a function of θ for $-20° \le \theta \le 20°$, and a function of a/λ for $2 \le a/\lambda \le 10$.

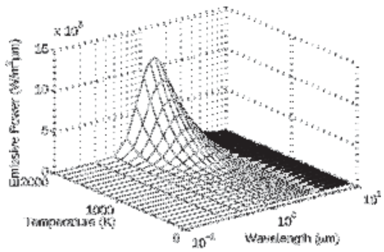


11. Molecules of a gas in a container are moving around at different speeds. Maxwell's speed distribution law gives the probability distribution $P(v)$ as a function of temperature and speed:
$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}$$
where M is the molar mass of the gas in kg/mol, $R = 8.31$ J/(mol K), is the gas constant, T is the temperature in kelvins, and v is the molecule's speed in m/s.

Make a 3-D plot of $P(v)$ as a function of v and T for $0 \le v \le 1{,}000$ m/s and $70 \le T \le 320$ K for oxygen (molar mass 0.032 kg/mol).

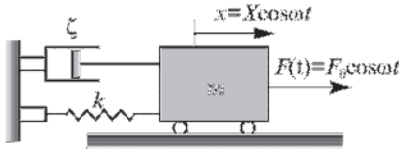
12. Plank's distribution law gives the black-body emissive power (amount of radiation energy emitted) as a function of temperature and wavelength:

$$E = \frac{C_1}{\lambda^5\left[e^{C_2/\lambda T}-1\right]} \qquad \left(\frac{\text{W}}{\text{m}^2\mu\text{m}}\right)$$

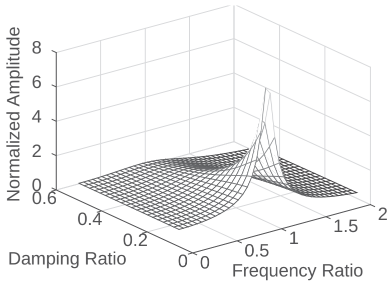


where $C_1 = 3.742 \times 10^8$ Wμm^4/m^2, $C_2 = 1.439 \times 10^4$ μmK, T is the temperature in degrees K, and λ is the wavelength in μm. Make a 3-D plot (shown in the figure) of E as a function of λ ($0.1 \le \lambda \le 10$ μm) and T for $100 \le T \le 2000$ K. Use a logarithmic scale for λ. This can be done with the command: `set(gca,'xscale','log')`.

13. Consider steady-state vibration of a friction-free spring-mass-damper system subjected to harmonic applied force. The normalized amplitude of the mass is given by:

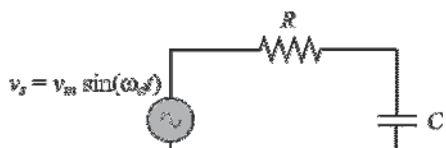


$$\frac{Xk}{F_0} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$



where $r = \omega/\omega_n$ is the frequency ratio, and ζ is the damping ratio. Make a 3-D plot (shown) of the normalized amplitude (z axis) as a function of the frequency ratio for $0 \le r \le 2$, and a function of the damping ratio for $0.05 \le \zeta \le 0.5$.

14. An *RLC* circuit with an alternating voltage source is shown. The source voltage v_s is given by $v_s = v_m \sin(\omega_d t)$, where $\omega_d = 2\pi f_d$, in which f_d is the driving frequency. The amplitude of the current, *I*, in this circuit is given by:



$$I = \frac{v_m}{\sqrt{R^2+[\omega_d L - 1/(\omega_d C)]^2}}$$

where R and C are the resistance of the resistor and capacitance of the capacitor, respectively. For the circuit in the figure $C = 15 \times 10^{-6}$F, $L = 240 \times 10^{-3}$H, and $v_m = 24$V.

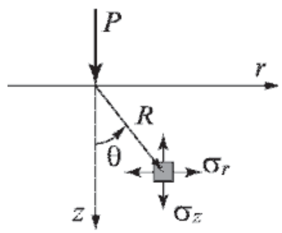
(*a*) Make a 3-D plot of I (z axis) as a function of ω_d (x axis) for

$60 \le f \le 110$ Hz, and as a function of R (y axis) for $10 \le R \le 40\ \Omega$.

(*b*) Make a plot that is a projection on the $x\,z$ plane. Estimate from this plot the natural frequency of the circuit (the frequency at which I is maximum). Compare the estimate with the calculated value of $1/(2\pi\sqrt{LC})$.

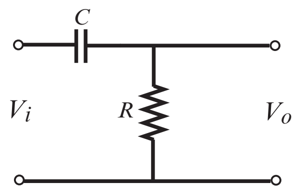
15. In the solution of elasticity problem of a normal point load applied to the surface of a half plane that was solved by Boussinesq in 1878, the stresses σ_r and σ_z are given by:

$$\sigma_z = -\frac{3Pz^3}{2\pi R^5} \quad \text{and} \quad \sigma_r = \frac{P}{2\pi}\left[\frac{1-2\nu}{R(R+z)} - \frac{3r^2 z}{R^5}\right]$$



where ν is Poisson's ratio. For $P = 2,000$ lb and $\nu = 0.3$, plot the stress components (each in a separate figure) as a function of r and z in the domain $0 \le \theta \le 90°$ and $0.02 \le R \le 0.1$ in. Plot the coordinates r and z in the horizontal plane and the stresses in the vertical direction.

16. A high-pass filter passes signals with frequencies that are higher than a certain cutoff frequency. In this filter the ratio of the magnitudes of the voltages is given by:



$$\left|\frac{V_o}{V_i}\right| = \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}$$

where $\omega = 2\pi f$ is the frequency of the input signal.

(*a*) Make a 3-D mesh plot of $\left|\frac{V_o}{V_i}\right|$ (z axis) as a function of f (x axis) for $1 \le f \le 10^6$ Hz, and as a function of RC (y axis) for $0.4 \times 10^{-4} \le RC \le 6 \times 10^{-3}$ s. Use a logarithmic scale for the x axis. This can be done by typing the MATLAB command `set(gca,'Xscale','log')` following the `mesh` command. A vector with constant spacing on a logarithmic scale can be created with the command `logspace(a,b,n)`.

(*b*) Make a plot that is a projection on the $x\,z$ plane.

17. The equation for the streamlines for uniform flow over a cylinder is

$$\psi(x, y) = y - \frac{y}{x^2+y^2}$$

where ψ is the stream function. For example, if $\psi = 0$, then $y = 0$. Since the equation is satisfied for all x, the x axis is the zero ($\psi = 0$) streamline. Observe that the collection of points where $x^2 + y^2 = 1$ is also a streamline. Thus, the stream function above is for a cylinder of radius 1. Make a 2-D contour plot of the streamlines around a cylinder with 1 in. radius. Set up the domain for x and y to range between –3 and 3. Use 100 for the number of contour levels. Add to the figure a plot of a circle with a radius of 1. Note that MATLAB also plots streamlines inside the cylinder. This is a mathematical artifact.

18. The deflection w of a clamped circular membrane of radius r_d subjected to pressure P is given by (small deformation theory):

$$w(r) = \frac{Pr_d^4}{64K}\left[1-\left(\frac{r}{r_d}\right)^2\right]^2$$

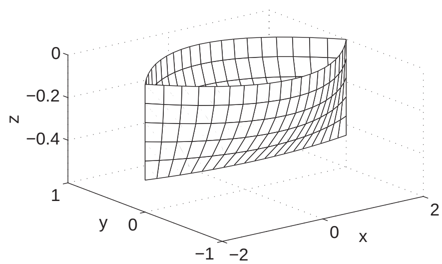
where r is the radial coordinate, and $K = \frac{Et^3}{12(1-\nu^2)}$, where E, t, and ν are the elastic modulus, thickness, and Poisson's ratio of the membrane, respectively. Consider a membrane with $P = 15\,\text{psi}$, $r_d = 15\,\text{in.}$, $E = 18 \times 10^6$ psi, $t = 0.08$ in., and $\nu = 0.3$. Make a surface plot of the membrane.

19. The Verhulst model, given in the following equation, describes the growth of a population that is limited by various factors such as overcrowding and lack of resources:

$$N(t) = \frac{N_\infty}{1+\left(\frac{N_\infty}{N_0}-1\right)e^{-rt}}$$

where $N(t)$ is the number of individuals in the population, N_0 is the initial population size, N_∞ is the maximum population size possible due to the various limiting factors, and r is a rate constant. Make a surface plot of $N(t)$ versus t and N_∞ assuming $r = 0.1\,\text{s}^{-1}$, and $N_0 = 10$. Let t vary between 0 and 100 and N_∞ between 100 and 1,000.

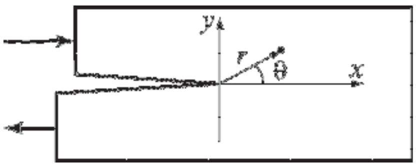
20. The geometry of a ship hull (Wigley hull) can be modeled by the equation:



$$y = \pm\frac{B}{2}\left[1-\left(\frac{2x}{L}\right)^2\right]\left[1-\left(\frac{2z}{T}\right)^2\right]$$

where x, y, and z are the length, width, and height, respectively. Use MATLAB to make a 3-D figure of the hull as shown. Use $B = 1.2$, $L = 4$, $T = 0.5$, $-2 \le x \le 2$, and $-0.5 \le z \le 0$.

21. The stress fields near a crack tip of a linear elastic isotropic material for mode II loading are given by:



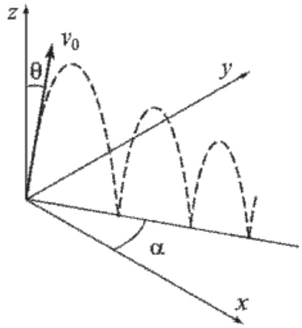
$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_{xy} = \frac{K_{II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

For K_{II} = 300 ksi$\sqrt{\text{in.}}$ plot the stresses (each in a separate figure) in the domain $0 \le \theta \le 90°$ and $0.02 \le r \le 0.2$ in. Plot the coordinates x and y in the horizontal plane, and the stresses in the vertical direction.

22. A ball thrown up falls back to the floor and bounces many times. For a ball thrown up in the direction shown in the figure, the position of the ball as a function of time is given by:



$$x = v_x t \qquad y = v_y t \qquad z = v_z t - \frac{1}{2}gt^2$$

The velocities in the x and y directions are constants throughout the motion and are given by $v_x = v_0\sin(\theta)\cos(\alpha)$ and $v_y = v_0\sin(\theta)\sin(\alpha)$. In the vertical z direction the initial velocity is $v_z = v_0\cos(\theta)$, and when the ball impacts the floor its rebound velocity is 0.8 of the vertical velocity at the start of the previous bounce. The time between bounces is given by $t_b = 2v_z / g$, where v_z is the vertical component of the velocity at the start of the bounce. Make a 3-D plot (shown in the figure) that shows the trajectory of the ball during the first five bounces. Take $v_0 = 20$ m/s, $\theta = 30°$, $\alpha = 25°$, and $g = 9.81$ m/s^2.