```
vcar=v0car+acar*t;                    Calculate the car's velocity.
speed_trainRcar=sqrt(vcar.^2+v0train^2);
                          Calculate the speed of the train relative to the car.
table=[t' y' x' d' vcar' speed_trainRcar']
                                    Create a table (see note below).
```

**Note:** In the commands above, `table` is the name of the variable that is a matrix containing the data to be displayed.

When the script file is executed, the following is displayed in the Command Window:

```
table =
        0 -400.0000 -200.0000  447.2136   41.0667   89.2139
   1.0000 -320.8000 -156.9333  357.1284   45.0667   91.1243
   2.0000 -241.6000 -109.8667  265.4077   49.0667   93.1675
   3.0000 -162.4000  -58.8000  172.7171   53.0667   95.3347
   4.0000  -83.2000   -3.7333   83.2837   57.0667   97.6178
   5.0000   -4.0000   55.3333   55.4777   61.0667  100.0089
   6.0000   75.2000  118.4000  140.2626   65.0667  102.5003
   7.0000  154.4000  185.4667  241.3239   69.0667  105.0849
   8.0000  233.6000  256.5333  346.9558   73.0667  107.7561
   9.0000  312.8000  331.6000  455.8535   77.0667  110.5075
  10.0000  392.0000  410.6667  567.7245   81.0667  113.3333
```

Time (s) | Train position (ft) | Car position (ft) | Car-train distance (ft) | Car speed (ft/s) | Train speed relative to the car (ft/s)

In this problem the results (numbers) are displayed by MATLAB without any text. Instructions on how to add text to output generated by MATLAB are presented in Chapter 4.

## 3.9 Problems

***Note:*** *Additional problems for practicing mathematical operations with arrays are provided at the end of Chapter 4.*

1. For the function $y = x^2 - \frac{x}{x+3}$, calculate the value of $y$ for the following values of $x$ using element-by-element operations: $0, 1, 2, 3, 4, 5, 6, 7$ .

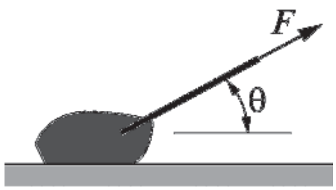
2. For the function $y = x^4 e^{-x}$, calculate the value of $y$ for the following values of $x$ using element-by-element operations: $1.5, 2, 2.5, 3, 3.5, 4$ .

3. For the function $y = (x + x\sqrt{x+3})(1+2x^2) - x^3$, calculate the value of $y$ for the following values of $x$ using element-by-element operations: –2, –1.5, –1, –0.5, 0, 0.5, 1, 1.5, 2.

4. For the function $y = \frac{4\sin x + 6}{(\cos^2 x + \sin x)^2}$, calculate the value of $y$ for the following values of $x$ using element-by-element operations: $15°, 25°, 35°, 45°, 55°, 65°$.

5. The radius, $r$, of a sphere can be calculated from its volume, $V$, by:
$$r = \sqrt[3]{\frac{3V}{4\pi}}$$
The surface area of a sphere, $S$, is given by:
$$S = 4\pi r^2$$
Determine the radius and surface area of spheres with volumes of 4,000, 3,500, 3,000, 2,500, 2,000, 1,500, and 1,000 in.$^3$. Display the results in a three-column table where the values of $r$, $V$, and $S$ are displayed in the first, second, and third columns, respectively. The values of $r$ and $S$ that are displayed in the table should be rounded to the nearest tenth of an inch.

6. A 70 lb-bag of rice is being pulled by a person by applying a force $F$ at an angle $\theta$ as shown. The force required to drag the bag is given by:




$$F(\theta) = \frac{70\mu}{\mu\sin\theta + \cos\theta}$$
where $\mu = 0.35$ is the friction coefficient.
(*a*) Determine $F(\theta)$ for $\theta = 5°, 10°, 15°, 20°, 25°, 30°$, and $35°$.
(*b*) Determine the angle $\theta$ where $F$ is minimum. Do it by creating a vector $\theta$ with elements ranging from $5°$ to $35°$ and spacing of 0.01. Calculate $F$ for each value of $\theta$ and then find the maximum $F$ and associated $\theta$ with MATLAB's built-in function `max`.

7. The remaining loan balance, $B$, of a fixed payment $n$ years mortgage after $x$ years is given by:
$$B = \frac{L\left[\left(1+\frac{r}{12}\right)^{12n} - \left(1+\frac{r}{12}\right)^{12x}\right]}{\left(1+\frac{r}{12}\right)^{12n} - 1}$$
where $L$ is the loan amount, and $r$ is the annual interest rate. Calculate the balance of a 30-year, \$100,000 mortgage, with annual interest rate of 6% (use 0.06 in the equation) after 0, 5, 10, 15, 20, 25, and 30 years. Create a seven-element vector for $x$ and use element-by-element operations. Display the results in a two-row table where the values of years and balance are displayed in the first and second rows, respectively.

8. The length $|\mathbf{u}|$ (magnitude) of a vector $\mathbf{u} = x\mathbf{i} + y\mathbf{j} - z\mathbf{k}$ is given by $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$. Given the vector $\mathbf{u} = -5.6\,\mathbf{i} + 11\,\mathbf{j} - 14\mathbf{k}$, determine its length by writing one MATLAB command in which the vector is multiplied by itself using element-by-element operation and the MATLAB built-in functions `sum` and `sqrt` are used.

9. A unit vector $\mathbf{u}_n$ in the direction of the vector $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is given by $\mathbf{u}_n = \mathbf{u} / |\mathbf{u}|$ where $|\mathbf{u}|$ is the length (magnitude) of the vector, given by $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$. Given the vector $\mathbf{u} = 4\mathbf{i} + 13\mathbf{j} - 7\mathbf{k}$, determine the unit vector in the direction of $\mathbf{u}$ using the following steps:
   (*a*) Assign the vector to a variable `u`.
   (*b*) Using element-by-element operation and the MATLAB built-in functions `sum` and `sqrt` calculate the length of $\mathbf{u}$ and assign it to the variable `Lu`.
   (*c*) Use the variables from parts (*a*) and (*b*) to calculate $\mathbf{u}_n$.
   (*d*) Verify that the length of $\mathbf{u}_n$ is 1 using the same operations as in part (*b*).

10. The angle between two vectors $\mathbf{u}_1 = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $\mathbf{u}_2 = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$ can be determined by $\cos\theta = \dfrac{x_1x_2 + y_1y_2 + z_1z_2}{|\mathbf{u}_1||\mathbf{u}_2|}$, where $|\mathbf{u}_i| = \sqrt{x_i^2 + y_i^2 + z_i^2}$.
    Given the vectors $\mathbf{u}_1 = 3.2\mathbf{i} - 6.8\mathbf{j} + 9\mathbf{k}$ and $\mathbf{u}_2 = -4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$, determine the angle between them (in degrees) by writing one MATLAB command that uses element-by-element multiplication and the MATLAB built-in functions `acosd`, `sum`, and `sqrt`.

11. The following vector is defined in MATLAB:

    `d=[2 4 3]`

    By hand (pencil and paper) write what will be displayed if the following commands are executed by MATLAB. Check your answers by executing the commands with MATLAB.
    (*a*) `d+d` (*b*) `d.^d` (*c*) `d.*d` (*d*) `d.^2`

12. The following two vectors are defined in MATLAB:

    `v=[3 -1 2] , u=[6 4 -3]`

    By hand (pencil and paper) write what will be displayed if the following commands are executed by MATLAB. Check your answers by executing the commands with MATLAB.
    (*a*) `v.*u` (*b*) `v.^u` (*c*) `v*u'`

13. Define the vector $v = [1\ 3\ 5\ 7]$. Then use the vector in a mathematical expression to create the following vectors:
    (*a*) $a = [3\ 9\ 15\ 21]$ (*b*) $b = [1\ 9\ 25\ 49]$

(*c*) $c = [1\ 1\ 1\ 1]$ (*d*) $d = [6\ \ 6\ \ 6\ \ 6]$

14. Define the vector $v = [5\ \ 4\ \ 3\ \ 2]$. Then use the vector in a mathematical expression to create the following vectors:

(*a*) $a = \left[\frac{1}{5+5}\ \ \frac{1}{4+4}\ \ \frac{1}{3+3}\ \ \frac{1}{2+2}\right]$ (*b*) $b = [5^5\ 4^4\ 3^3\ 2^2]$

(*c*) $c = \left[\frac{5}{\sqrt{5}}\ \ \frac{4}{\sqrt{4}}\ \ \frac{3}{\sqrt{3}}\ \ \frac{2}{\sqrt{2}}\right]$ (*d*) $d = \left[\frac{5^2}{5^5}\ \ \frac{4^2}{4^4}\ \ \frac{3^2}{3^3}\ \ \frac{2^2}{2^2}\right]$

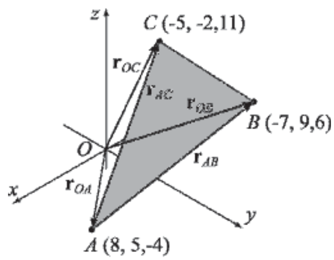
15. Define $x$ and $y$ as the vectors $x = [0.5,\ 1,\ 1.5,\ 2,\ 2.5]$ and $y = [0.8,\ 1.6,\ 2.4,\ 3.2,\ 4.0]$. Then use them in the following expressions to calculate $z$ using element-by-element calculations.

(*a*) $z = x^2 + 2xy$ (*b*) $z = xye^{y/x} - \sqrt[3]{x^4y^3 + 8.5}$

16. Define $r$ and $s$ as scalars $r = 1.6 \times 10^3$ and $s = 14.2$, and, $t$, $x$, and $y$ as vectors $t = [1,\ 2,\ 3,\ 4,\ 5]$, $x = [2,\ 4,\ 6,\ 8,\ 10]$, and $y = [3,\ 6,\ 9,\ 12,\ 15]$. Then use these variables to calculate the following expressions using element-by-element calculations for the vectors.

(*a*) $G = xt + \frac{r}{s^2}(y^2 - x)t$ (*b*) $R = \frac{r(-xt + yt^2)}{15} - s^2(y - 0.5x^2)t$

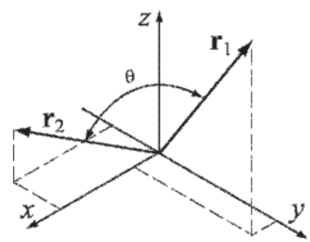
17. The area of a triangle $ABC$ can be calculated by $|\mathbf{r}_{AB} \times \mathbf{r}_{AC}|/2$, where $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$ are vectors connecting the vertices $A$ and $B$, and $A$ and $C$, respectively. Determine the area of the triangle shown in the figure. Use the following steps in a script file to calculate the area. First, define the vectors $\mathbf{r}_{OA}$, $\mathbf{r}_{OB}$, and $\mathbf{r}_{OC}$ from knowing the coordinates of points $A$, $B$, and $C$. Then determine the vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$ from $\mathbf{r}_{OA}$, $\mathbf{r}_{OB}$, and $\mathbf{r}_{OC}$. Finally, determine the area by using MATLAB's built-in functions `cross`, `sum`, and `sqrt`.



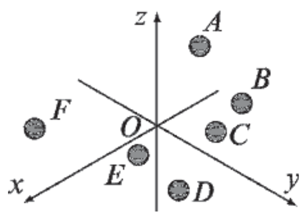

18. The cross product of two vectors can be used for determining the angle between two vectors:

$$\theta = \sin^{-1}\left(\frac{|\mathbf{r}_1 \times \mathbf{r}_2|}{|\mathbf{r}_1||\mathbf{r}_2|}\right)$$

Use MATLAB's built-in functions `asind`, `cross`, `sqrt`, and `dot` to find the angle (in degrees) between $\mathbf{r}_1 = 2.5\mathbf{i} + 8\mathbf{j} - 5\mathbf{k}$ and $\mathbf{r}_2 = -1\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$. Recall that $|\mathbf{r}| = \sqrt{\mathbf{r} \cdot \mathbf{r}}$.



19. The center of mass, $(\bar{x}, \bar{y}, \bar{z})$, of $n$ particles can be calculated by:




$$\bar{x} = \frac{\sum_{i=1}^{i=n} m_i x_i}{\sum_{i=1}^{i=n} m_i}, \quad \bar{y} = \frac{\sum_{i=1}^{i=n} m_i y_i}{\sum_{i=1}^{i=n} m_i}, \quad \bar{z} = \frac{\sum_{i=1}^{i=n} m_i z_i}{\sum_{i=1}^{i=n} m_i}$$

where $x_i$, $y_i$, and $z_i$ and $m_i$ are the coordinates and the mass of particle $i$, respectively. The coordinates and mass of six particles are listed in the following table. Calculate the center of mass of the particles.

| Particle | Mass (kg) | Coordinate $x$ (mm) | Coordinate $y$ (mm) | Coordinate $z$ (mm) |
|---|---|---|---|---|
| *A* | 0.5 | -10 | 8 | 32 |
| *B* | 0.8 | -18 | 6 | 19 |
| *C* | 0.2 | -7 | 11 | 2 |
| *D* | 1.1 | 5 | 12 | -9 |
| *E* | 0.4 | 0 | -8 | -6 |
| *F* | 0.9 | 25 | -20 | 8 |

20. Define the vectors:

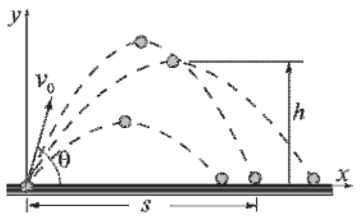
$$\mathbf{a} = 7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}, \quad \mathbf{b} = -4\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}, \text{ and } \mathbf{c} = 5\mathbf{i} - 6\mathbf{j} + 8\mathbf{k}$$

Use the vectors to verify the identity:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$$

Using MATLAB's built-in functions `cross` and `dot`, calculate the value of the left and right sides of the identity.

21. The maximum distance $s$ and the maximum height $h$ that a projectile shot at an angle $\theta$ are given by:




$$s = \frac{v_0^2}{g}\sin 2\theta \quad \text{and} \quad h = \frac{v_0^2 \sin^2\theta}{2g}$$

where $v_0$ is the shooting velocity and $g = 9.81$ m/s$^2$. Determine $s(\theta)$ and $h(\theta)$ for $\theta = 15°, 25°, 35°, 45°, 55°, 65°, 75°$ if $v_0 = 260$ m/s.

22. Use MATLAB to show that the sum of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges to $\pi^2/6$. Do this by computing the sum for:
(*a*) $n = 5$, (*b*) $n = 50$, (*c*) $n = 5000$
For each part create a vector $n$ in which the first element is 1, the increment is 1 and the last term is 5, 50, or 5,000. Then use element-by-element calculations to create a vector in which the elements are $\frac{1}{n^2}$. Finally, use MAT-

LAB's built-in function `sum` to sum the series. Compare the values to $\pi^2/6$. Use `format long` to display the numbers.

23. Use MATLAB to show that the sum of the infinite series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges to 6. Do this by computing the sum for
(*a*) $n = 5$, (*b*) $n = 15$, (*c*) $n = 30$
For each part, create a vector `n` in which the first element is 1, the increment is 1 and the last term is 5, 15, or 30. Then use element-by-element calculations to create a vector in which the elements are $\frac{n^2}{2^n}$. Finally, use MATLAB's built-in function `sum` to sum the series. Use `format long` to display the numbers.

24. The natural exponential function can be expressed by $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. Determine $e^2$ by calculating the sum of the series for:
(*a*) $n = 5$, (*b*) $n = 15$, (*c*) $n = 25$
For each part create a vector `n` in which the first element is 0, the increment is 1, and the last term is 5, 15, or 25. Then use element-by-element calculations to create a vector in which the elements are $\frac{x^n}{n!}$. Finally, use the MATLAB built-in function `sum` to add the terms of the series. Compare the values obtained in parts (*a*), (*b*), and (*c*) with the value of $e^2$ calculated by MATLAB.

25. Show that $\lim_{x \to \pi/3} \frac{\sin(x - \pi/3)}{4\cos^2 x - 1} = \frac{-\sqrt{3}}{6}$. Do this by first creating a vector $x$ that has the elements $\pi/3 - 0.1$, $\pi/3 - 0.01$, $\pi/3 - 0.0001$, $\pi/3 + 0.0001$, $\pi/3 + 0.01$, and $\pi/3 + 0.1$. Then, create a new vector $y$ in which each element is determined from the elements of $x$ by $\frac{\sin(x - \pi/3)}{4\cos^2 x - 1}$. Compare the elements of $y$ with the value $\frac{-\sqrt{3}}{6}$. Use `format long` to display the numbers.

26. Show that $\lim_{x \to 0} \frac{5\sin(6x)}{4x} = 7.5$. Do this by first creating a vector $x$ that has the elements 1.0, 0.1, 0.01, 0.001, and 0.0001. Then, create a new vector $y$ in which each element is determined from the elements of $x$ by $\frac{5\sin(6x)}{4x}$. Compare the elements of $y$ with the value 7.5. Use `format long` to display the numbers.

27. The Hazen Williams equation can be used to calculate the pressure drop, $P_d$ (psi/ft of pipe) in pipes due to friction:

$$P_d = 4.52Q^{1.85} / (C^{1.85}d^{4.87})$$

where $Q$ is the flow rate (gpm), $C$ is a design coefficient determined by the type of pipe, and $d$ is pipe diameter in inches. Consider a 3.5-in.-diameter steel pipe with $C = 120$ . Calculate the pressure drop in a 1000-ft-long pipe for flow rates of 250, 300, 350, 400, and 450 gpm. To carry out the calculation, first create a five-element vector with the values of the flow rates (250, 300, ...). Then use the vector in the formula using element-by-element operations.

28. The monthly lease payment, *Pmt*, of a new car can be calculated by:

$$Pmt = \frac{\left[Pv - \dfrac{Fv}{(1+i/12)^N}\right]}{\dfrac{1 - \dfrac{1}{1+(1+i/12)^N}}{i/12}}$$

where *Pv* and *Fv* are the present value and the future value (at the end of the lease) of the car, respectively. $N$ is the duration of the lease in months, and $i$ is the interest rate per year. Consider a 36-months-lease of a car with a present value of \$38,000 and a future value of \$23,400. Calculate the monthly payments if the yearly interest rates are 3, 4, 5, 6, 7, and 8%. To carry out the calculation, first create a five-element vector with the values of the interest rates (0.03, 0.04, ...). Then use the vector in the formula using element-by-element operations.

29. Create the following three matrices:

$$A = \begin{bmatrix} 5 & -3 & 7 \\ 1 & 0 & -6 \\ -4 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 & -1 \\ 6 & 8 & -7 \\ 4 & 4 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -9 & 8 & 3 \\ 1 & 7 & -5 \\ 3 & 3 & 6 \end{bmatrix}$$

(*a*) Calculate $A + B$ and $B + A$ to show that addition of matrices is commutative.

(*b*) Calculate $A*(B*C)$ and $(A*B)*C$ to show that multiplication of matrices is associative.

(*c*) Calculate $5(B+C)$ and $5B+5C$ to show that, when matrices are multiplied by a scalar, the multiplication is distributive.

(*d*) Calculate $(A+B)*C$ and $A*C+B*C$ to show that matrix multiplication is distributive.

30. Use the matrices $A$, $B$, and $C$ from the previous problem to answer the following:

    ($a$) Does $A*B = B*A$? ($b$) Does $(B*C)^{-1} = B^{-1}*C^{-1}$?

    ($c$) Does $(A^{-1})^t = (A^t)^{-1}$? ($^t$ means transpose) ($d$) Does $(A + B)^t = A^t + B^t$?

31. Create a $3 \times 3$ matrix `A` having random integer values between 1 and 5. Call the matrix $A$ and, using MATLAB, perform the following operations. For each part explain the operation.

    ($a$) `A.^A` ($b$) `A.*A` ($c$) `A*A-1`

    ($d$) `A./A` ($e$) `det(A)` ($f$) `inv(A)`

32. The magic square is an arrangement of numbers in a square grid in such a way that the sum of the numbers in each row, and in each column, and in each diagonal is the same. MATLAB has a built-in function `magic(n)` that returns an $n \times n$ magic square. In a script file create a $(5 \times 5)$ magic square, and then test the properties of the resulting matrix by finding the sum of the elements in each row, in each column and in both diagonals. In each case, use MATLAB's built-in function `sum`. (Other functions that can be useful are `diag` and `fliplr`.)

33. Solve the following system of three linear equations:

$$\begin{aligned} -2x + 5y + 7z &= -17.5 \\ 3x - 6y + 2z &= 40.6 \\ 9x - 3y + 8z &= 56.2 \end{aligned}$$

34. Solve the following system of six linear equations:

$$\begin{aligned} 2a - 4b + 5c - 3.5d + 1.8e + 4f &= 52.52 \\ -1.5a + 3b + 4c - d - 2e + 5f &= -21.1 \\ 5a + b - 6c + 3d - 2e + 2f &= -27.6 \\ 1.2a - 2b + 3c + 4d - e + 4f &= 9.16 \\ 4a + b - 2c - 3d - 4e + 1.5f &= -17.9 \\ 3a + b - c + 4d - 2e - 4f &= -16.2 \end{aligned}$$

35. A football stadium has 100,000 seats. In a game with full capacity people with the following ticket and associated cost attended the game:

| | Student | Alumni | Faculty | Public | Veterans | Guests |
|---|---|---|---|---|---|---|
| Cost | \$25 | \$40 | \$60 | \$70 | \$32 | \$0 |

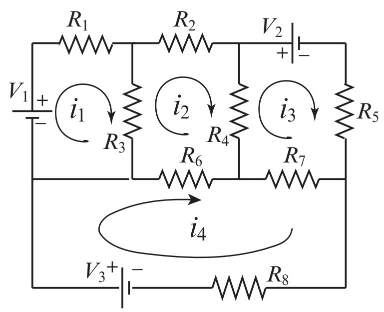
Determine the number of people that attended the game in each cost category if the total revenue was \$4,897,000, there were 11,000 more alumni than faculty, the number of public plus alumni together was 10 times the number of veterans, the number of faculty plus alumni together was the same as the number of students, and the number of faculty plus students together was four times larger than the number of guests and veterans together.

36. A food company manufactures five types of 8-oz trail mix packages using different mixtures of peanuts, almonds, walnuts, raisins, and M&Ms. The mixtures have the following compositions:

| | Peanuts (oz) | Almonds (oz) | Walnuts (oz) | Raisins (oz) | M&Ms (oz) |
|---|---|---|---|---|---|
| Mix 1 | 3 | 1 | 1 | 2 | 1 |
| Mix 2 | 1 | 2 | 1 | 3 | 1 |
| Mix 3 | 1 | 1 | 0 | 3 | 3 |
| Mix 4 | 2 | 0 | 3 | 1 | 2 |
| Mix 5 | 1 | 2 | 3 | 0 | 2 |

How many packages of each mix can be manufactured if 105 lb of peanuts, 74 lb of almonds, 102 lb of walnuts, 118 lb of raisins, and 121 lb of M&Ms are available? Write a system of linear equations and solve.

37. The electrical circuit shown consists of resistors and voltage sources. Determine $i_1, i_2, i_3$ and $i_4$, using the mesh current method based on Kirchhoff's voltage law (see Sample Problem 3-4).




$V_1 = 28$ V, $V_2 = 36$ V, $V_3 = 42$ V
$R_1 = 16\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 6\,\Omega$
$R_4 = 12\,\Omega$, $R_5 = 8\,\Omega$, $R_6 = 14\,\Omega$
$R_7 = 4\,\Omega$, $R_8 = 5\,\Omega$.