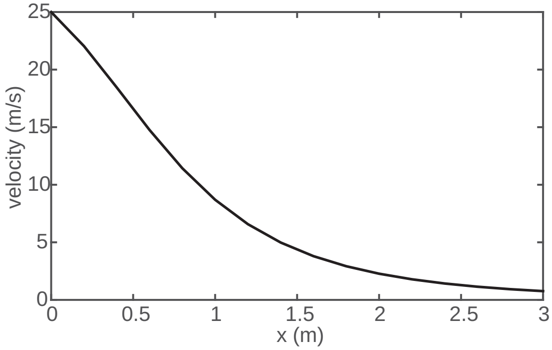
```
3.0000        0.7607
```

The plot generated by the program of the velocity as a function of distance is:



9.6 PROBLEMS

1. Determine the two solutions of the equation $x^3 - e^{0.8x} = 20$ between $x = 0$ and $x = 8$.

2. Determine the solution of the equation $3\sin(0.5x) - 0.5x + 2 = 0$.

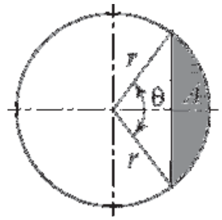
3. Determine the three roots of the equation $x^3 - x^2 e^{-0.5x} - 3x = -1$.

4. Determine the positive roots of the equation $\cos^2 x - 0.5xe^{0.3x} + 5 = 0$.

5. The area A of a circle segment is given by:

$$A = \frac{1}{2}r^2(\theta - \sin\theta)$$

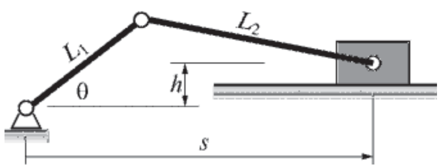
Determine the angle θ (in degrees) if $r = 7$ in. and $A = 21.2$ in^2.



6. The position s of the slider as a function of θ in the crank-slider mechanism shown is given by:

$$s = L_1\cos\theta + \sqrt{L_2^2 - (L_1\sin\theta - h^2)^2}$$

Given $L_1 = 5$ in., $L_2 = 8$ in., and $h = 1.5$ in., determine the angle θ, when $s = 9$ in. (There are two solutions.)



7. The van der Waals equation gives a relationship between the pressure p (atm), volume V (L), and temperature T (K) for a real gas:

$$P = \frac{nRT}{V-b} - \frac{n^2 a}{V^2}$$

where n is the number of moles, $R = 0.08206$(L atm)/(mol K) is the gas constant, and a (L^2 atm/mol^2) and b (L/mol) are material constants.

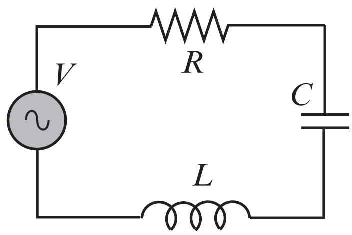
Determine the volume of 1.5 mol of nitrogen ($a = 1.39$ L^2 atm/mol^2, $b = 0.03913$ L/mol) at temperature of 350 K and pressure of 70 atm.

8. An estimate of the minimum velocity required for a round flat stone to skip when it hits the water is given by (Lyderic Bocquet, "The Physics of Stone Skipping," Am. J. Phys., vol. 71, no. 2, February 2003):

$$V = \frac{\sqrt{\dfrac{16Mg}{\pi C \rho_w d^2}}}{\sqrt{1 - \dfrac{8M\tan^2\beta}{\pi d^3 C \rho_w \sin\theta}}}$$

where M and d are the stone mass and diameter, ρ_w is the water density, C is a coefficient, θ is the tilt angle of the stone, β is the incidence angle, and $g = 9.81$ m/s^2. Determine d if $V = 0.8$ m/s. (Assume that $M = 0.1$ kg, $C = 1$, $\rho_w = 1{,}000$ kg/m^3, and $\beta = \theta = 10°$.)

9. A series RLC circuit with an AC voltage source is shown. The amplitude of the current, I, in this circuit is given by:



$$I = \frac{v_m}{\sqrt{R^2 + [\omega_d L - 1/(\omega_d C)]^2}}$$

where $\omega_d = 2\pi f_d$ in which f_d is the driving frequency; R and C are the resistance of the resistor and capacitance of the capacitor, respectively; and v_m is the amplitude of V. For the circuit in the figure $R = 80\ \Omega$, $C = 18 \times 10^{-6}$ F, $L = 260 \times 10^{-3}$ H, and $v_m = 10$ V.

Determine f_d for which $I = 0.1$ A. (There are two solutions.)

10. For fluid flow in a pipe, the Colebrook–White (or Colebrook) equation gives a relationship between the friction coefficient, f, and the Reynolds number:

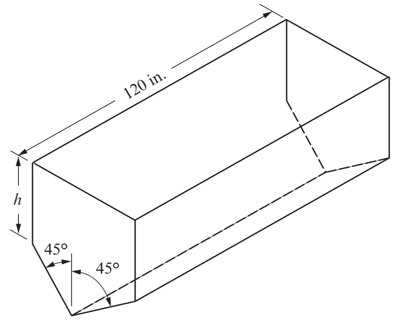
$$\sqrt{\frac{1}{f}} = -0.86\ \ln\left(\frac{k/d}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

where k/d is the pipe relative roughness. Determine f if $k/d = 0.0004$, and $\mathrm{Re} = 2 \times 10^6$.

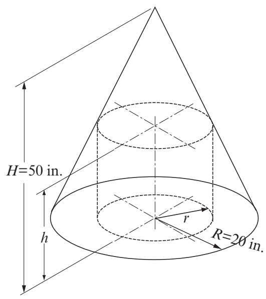
11. Using MATLAB's built-in function `fminbnd`, determine the minimum and the maximum of the function

$$f(x) = \frac{2+(x-1.45)^2}{3+3.5(0.8x^2-0.6x+2)}$$

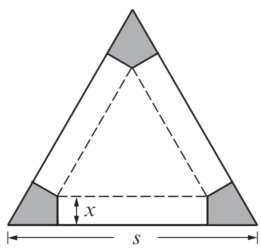
12. A flat rectangular sheet of metal that is 70 in. wide and 120 in. long is formed to make a container with the geometry shown in the figure. (Additional flat metal pieces are attached at the ends.) Using MATLAB's built-in function `fminbnd`, determine the value of h such that the container will have the maximum possible volume, and determine the corresponding volume.



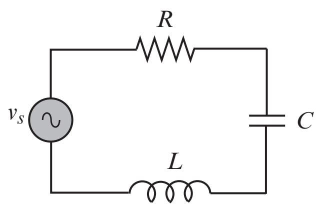
13. Using MATLAB's built-in function `fminbnd`, determine the dimensions (radius r and height h) and the volume of the cylinder with the largest volume that can be made inside of a cone with a radius R of 20 in. and height H of 50 in.



14. A prismatic box with equilateral triangular base is made from a equilateral triangular sheet with sides s by cutting off the corners and folding the edges along the dashed lines. For $s = 25$ in., use MATLAB's built-in function `fminbnd` to determine the value of x such that the box will have the maximum possible volume, and determine the corresponding volume.



15. An RLC circuit with an alternating voltage source is shown. The source voltage v_s is given by $v_s = v_m \sin(\omega_d t)$, where $\omega_d = 2\pi f_d$, in which f_d is the driving frequency. The amplitude of the current, I, in this circuit is given by:

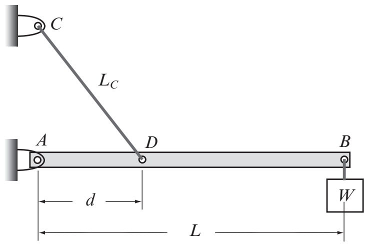


$$I = \frac{v_m}{\sqrt{R^2+[\omega_d L-1/(\omega_d C)]^2}}$$

where R and C are the resistance of the resistor and capacitance of the capacitor, respectively. For the circuit in the figure $C = 15 \times 10^{-6}$ F,

$L = 240 \times 10^{-3}$ H, $R = 22\ \Omega$, and $v_m = 26$V. Plot I as a function of f for $60 \le f \le 110$ Hz. Using MATLAB's built-in function `fminbnd`, determine the frequency where I is maximum and the corresponding value of I.

16. A 108-in.-long beam AB is attached to the wall with a pin at point A and to a 68-in.-long cable CD. A load $W = 250$lb is attached to the beam at point B. The tension in the cable T is given by:



$$T = \frac{WLL_C}{d\sqrt{L_C^2 - d^2}}$$

where L and L_C are the lengths of the beam and the cable, respectively, and d is the distance from point A to point D, where the cable is attached. Make a plot of T versus d. Determine the distance d where the tension in the cable is the smallest.

17. Use MATLAB to calculate the following integrals:

(*a*) $\displaystyle\int_1^{11} \frac{x^3 e^{-0.2x}}{1+x^2}dx$ (*b*) $\displaystyle\int_2^{7} \frac{4x+3\cos(4x)}{2+\sin x}dx$

18. Use MATLAB to calculate the following integrals:

(*a*) $\displaystyle\int_0^{3} \sqrt[4]{1+0.5x^3 - x^2}\, dx$ (*b*) $\displaystyle\int_0^{8} \frac{x\cos x + 2x^2}{e^x}dx$

19. The speed of a race car during the first 7 s of a race is given by:

t (s)	0	1	2	3	4	5	6	7
v (mi/h)	0	14	39	69	95	114	129	139

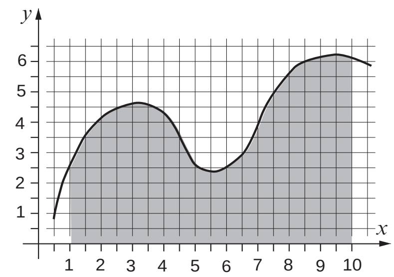
Determine the distance the car traveled during the first 7 s.

20. A rubber band is stretched by fixing one end pulling the other end. Measurements of the applied force at different displacements are given in the following table:

x (in.)	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8
F (lb)	0	0.85	1.30	1.60	1.87	2.14	2.34	2.52

Determine the work done by the force while stretching the rubber band.

21. Use numerical integration to approximate the size of the shaded area shown in the figure. Create a vector with values of x from 1 through 10 and estimate the corresponding y coordinate. Then, determine the area by using MATLAB's built-in function `trapz`.



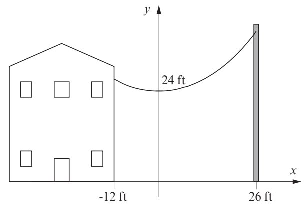
22. The electric wire that connects the house to the pole has the shape of a catenary:

$$y = a\cosh\left(\frac{x}{a}\right)$$

By using the equation:

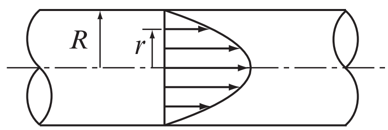
$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

determine the length of the wire.



23. The flow rate Q (volume of fluid per second) in a round pipe can be calculated by:

$$Q = \int_0^r 2\pi v r\, dr$$



For turbulent flow the velocity profile can be estimated by: $v = v_{\max}\left(1 - \frac{r}{R}\right)^{1/n}$. Determine Q for $R = 0.25$ in., $n = 7$, $v_{max} = 80$ in./s.

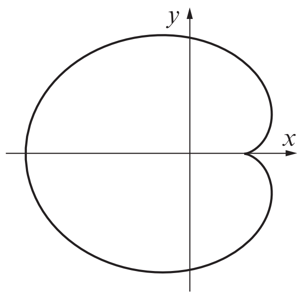
24. The length of a curve given by a parametric equation $x(t)$, $y(t)$ is given by:

$$\int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\ dt$$

The cardioid curve shown in the figure is given by:

$x = 2b\cos t - b\cos 2t$ and $y = 2b\sin t - b\sin 2t$

with $0 \le t \le 2\pi$. Plot the cardioid with $b = 5$ and determine the length of the curve.



25. The variation of gravitational acceleration g with altitude y is given by:

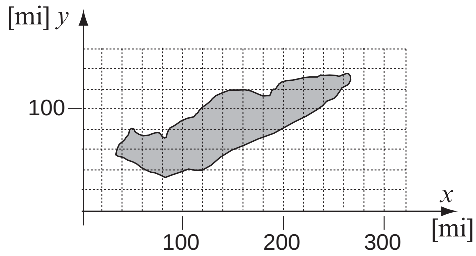
$$g = \frac{R^2}{(R+y)^2}g_0$$

where $R = 6371$ km is the radius of the Earth, and $g_0 = 9.81$ m/s^2 is the gravitational acceleration at sea level. The change in the gravitational potential energy, ΔU, of an object that is raised from the Earth is given by:

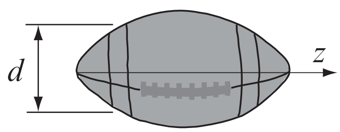
$$\Delta U = \int_0^h mg\,dy$$

Determine the change in the potential energy of a satellite with a mass of 500 kg that is raised from the surface of the Earth to a height of 800 km.

26. An approximate map of Lake Erie is shown in the figure. Use numerical integration to estimate the area of the lake. Make a list of the width of the lake as a function of x. Start with $x = 40$ mi and use increments of 20 mi, such that the last point is $x = 260$. Compare the result with the actual area Lake Erie, which is 9,940 square miles.



27. To estimate the surface area and volume of a football, the diameter of the ball is measured at different points along the ball. The surface area, S, and volume, V, can be determined by:



$$S = 2\pi\int_0^L r\,dz \quad \text{and} \quad V = \pi\int_0^L r^2\,dz$$

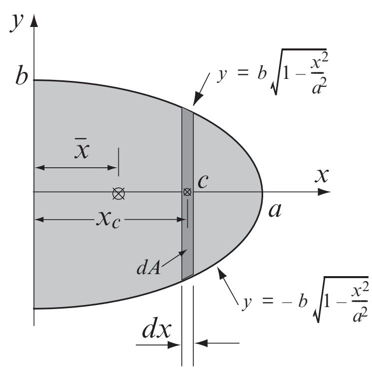
Use the data given in the table to determine the volume and surface area of the ball.

z (in.)	0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
d (in.)	0	2.6	3.2	4.8	5.6	6	6.2	6.0	5.6	4.8	3.3	2.6	0

28. A cross-sectional area has the geometry of half an ellipse, as shown in the figure to the right. The coordinate $\bar{x}$ of the centroid of the area can be calculated by:

$$\bar{x} = \frac{M_y}{A}$$

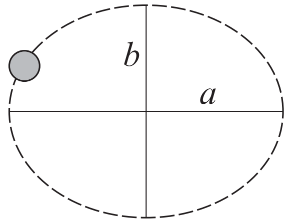
where A is the area given by $A = \frac{1}{2}\pi ab$, and M_y is the moment of the area about the y axis, given by:



$$M_y = \int_A x_c\,dA = 2b\int_0^a x\sqrt{1-\frac{x^2}{a^2}}\,dx$$

Determine $\bar{x}$ when $a = 40$ mm and $b = 15$ mm.

29. The orbit of Mercury is elliptical in shape, with $a = 5.7909 \times 10^7$ km and $b = 5.1614 \times 10^7$ km. The perimeter of an ellipse can be calculated by



$$P = 4a\int_0^{\pi/2} \sqrt{1 - k^2 \sin^2\theta}\, d\theta$$

where $k = \frac{\sqrt{a^2 - b^2}}{a}$. Determine the distance Mercury travels in one orbit. Calculate the average speed at which Mercury travels (in km/s) if one orbit takes about 88 days.

30. The Fresnel integrals are:

$$S(x) = \int_0^x \sin(t^2)\, dt \quad \text{and} \quad C(x) = \int_0^x \cos(t^2)\, dt$$

Calculate $S(x)$ and $C(x)$ for $0 \le x \le 4$ (use spacing of 0.05). In one figure plot two graphs—one of $S(x)$ versus x and the other of $C(x)$ versus x. In a second figure plot $S(x)$ versus $C(x)$.

31. Use a MATLAB built-in function to numerically solve:

$$\frac{dy}{dx} = \frac{x^2\sqrt{y}}{5} - 2x \quad \text{for} \quad 0 \le x \le 5 \quad \text{with} \quad y(0) = 3$$

Plot the numerical solution.

32. Use a MATLAB built-in function to numerically solve:

$$\frac{dy}{dx} = -yx^2 + 1.5y \text{ for } \quad 0 \le x \le 3 \quad \text{with} \quad y(0) = 2$$

In one figure plot the numerical solution as a solid line and the exact solution as discrete points.

Exact solution: $y = 2e^{-(2x^3 - 9x)/6}$.

33. Use a MATLAB built-in function to numerically solve:

$$\frac{dy}{dx} = (1 + y^2)\tan x \quad \text{for} \quad 0 \le x \le 0.8 \quad \text{with} \quad y(0) = \sqrt{3}$$

In one figure plot the numerical solution as a solid line and the exact solution as discrete points (10 equally spaced points).

Exact solution: $y = -\tan\left[\ln\,[\cos(x)] - \frac{\pi}{3}\right]$.

34. Use a MATLAB built-in function to numerically solve:

$$\frac{dy}{dx} = -x^2 + \frac{x^3 e^{-y}}{4} \quad \text{for} \quad 1 \le x \le 5 \quad \text{with} \quad y(1) = 1$$

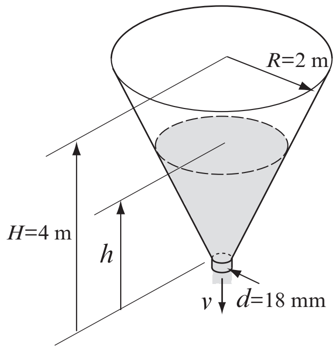
Plot the solution.

35. The growth of a fish is often modeled by the von Bertalanffy growth model:

$$\frac{dw}{dt} = aw^{2/3} - bw$$

where w is the weight and a and b are constants. Solve the equation for w for the case $a = 5\,\text{lb}^{1/3}$, $b = 2\,\text{day}^{-1}$, and $w(0) = 0.5$ lb. Make sure that the selected time span is just long enough so that the maximum weight is approached. What is the maximum weight for this case? Make a plot of w as a function of time.

36. A water tank shaped as a cone ($R = 2$ m, $H = 4$ m) has a circular hole at the bottom ($d = 18$ mm), as shown. According to Torricelli's law, the speed v of the water that is discharging from the hole is given by:



$$v = \sqrt{2gh}$$

where h is the height of the water and $g = 9.81\ \text{m/s}^2$. The rate at which the height, h, of the water in the tank changes as the water flows out through the hole is given by:

$$\frac{dh}{dt} = -\frac{H^2 d^2}{4R^2}\frac{\sqrt{2gh}}{h^2}$$

Solve the differential equation for h. The initial height of the water is $h = 3$ m. Solve the problem for different times and find an estimate for the time when $h = 0.1$ m. Make a plot of h as a function of time.

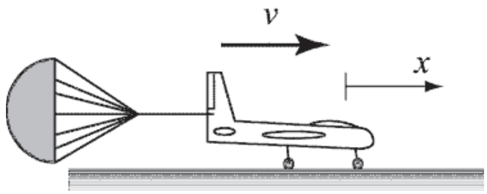
37. The sudden outbreak of an insect population can be modeled by the equation

$$\frac{dN}{dt} = RN\left(1 - \frac{N}{C}\right) - \frac{rN^2}{N_c^2 - N^2}$$

The first term relates to the well-known logistic population growth model where N is the number of insects, R is an intrinsic growth rate, and C is the carrying capacity of the local environment. The second term represents the effects of bird predation. Its effect becomes significant when the population reaches a critical size N_c; r is the maximum value that the second term can reach at large values of N.

Solve the differential equation for $0 \le t \le 50$ days and two growth rates, $R = 0.55$ and $R = 0.58\ \text{day}^{-1}$, and with $N(0) = 10{,}000$. The other parameters are $C = 10^4$, $N_c = 10^4$, $r = 10^4\ \text{day}^{-1}$. Make one plot comparing the two solutions and discuss why this model is called an "outbreak" model.

38. An airplane uses a parachute and other means of braking as it slows down on the runway after landing. Its acceleration is given by $a = -0.0035v^2 - 3\text{m/s}^2$. Since $a = \frac{dv}{dt}$, the rate of change of the velocity is given by:



$$\frac{dv}{dt} = -0.0035v^2 - 3$$

Consider an airplane with a velocity of 300 km/h that opens its parachute and starts decelerating at $t = 0$ s.

(*a*) By solving the differential equation, determine and plot the velocity as a function of time from $t = 0$ s until the airplane stops.

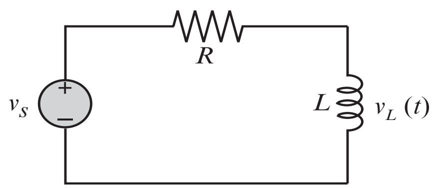
(*b*) Use numerical integration to determine the distance x the airplane travels as a function of time. Make a plot of x versus time.

39. The population growth of species with limited capacity can be modeled by the equation:

$$\frac{dN}{dx} = kN\left(1 - \frac{N}{N_{\max}}\right) - r\frac{N^2}{N_c}$$

where N is the population size, N_{max} is the limiting number for the population, and k, r, and N_c are constants. The second term in the equation represent the effect of predation. Consider the case where $N_{max} = 6{,}000$, $k = 0.196$ 1/yr, $r = 40$ 1/yr, $N_c = 3{,}000$, and $N(0) = 50$. Determine N for $0 \le t \le 50$ yr. Make a plot of N as a function of t.

40. An RL circuit includes a voltage source v_s, a resistor $R = 1.8\ \Omega$, and an inductor $L = 0.4$H, as shown in the figure. The differential equation that describes the response of the circuit is



$$\frac{L}{R}\frac{di_L}{dt} + i_L = \frac{v_s}{R}$$

where i_L is the current in the inductor. Initially $i_L = 0$, and then at $t = 0$ the voltage source is changed. Determine the response of the circuit for the following three cases:

(*a*) $v_S = 10\sin(30\pi t)$ V for $t \ge 0$.

(*b*) $v_S = 10e^{-t/0.06}\sin(30\pi t)$ V for $t \ge 0$.

Each case corresponds to a different differential equation. The solution is the current in the inductor as a function of time. Solve each case for $0 \le t \le 0.4$ s. For each case plot v_s and i_L versus time (make two separate plots on the same page).

41. Growth of many organisms can be modeled with the equation:

$$\frac{dm}{dt} = k\,m^{3/4}\left[1 - \left(\frac{m}{m_{\max}}\right)^{1/4}\right]$$

where $m(t)$ is the mass of the organism, m_{max} is the assumed maximum mas, and k is a constant. Solve the equation for $0 \le t \le 400$ days, given $k = 0.3\ \text{kg}^{1/4}/\text{day}$, $m_{max} = 300$ kg and $m(0) = 1$ kg. Make a plot of m as a function of time.

42. The velocity, v, of an object that falls freely due to the Earth gravity can be modeled with the equation:

$$m\frac{dv}{dt} = -mg + kv^2$$

where m is the mass of the object, $g = 9.81\ \text{m/s}^2$, and k is a constant. Solve the equation for v for the case that $m = 5$ kg, $k = 0.05$ kg/m, $0 \le t \le 15$ s, and $v(0) = 0$ m/s. Make a plot of v as a function of time.