```
>> v=4*randn(1,6)+50
v =
   42.7785   57.4344   47.5819   50.4134   52.2527   50.4544
```

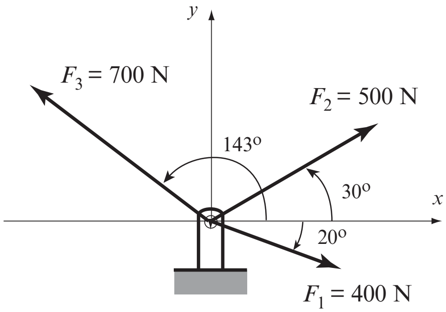
Integers of normally distributed numbers can be obtained by using the `round` function.

```
>> w=round(4*randn(1,6)+50)
w =
    51    49    46    49    50    44
```

3.8 EXAMPLES OF MATLAB APPLICATIONS

Sample Problem 3-2: Equivalent force system (addition of vectors)

Three forces are applied to a bracket as shown. Determine the total (equivalent) force applied to the bracket.



Solution

A force is a vector (a physical quantity that has a magnitude and direction). In a Cartesian coordinate system a two-dimensional vector **F** can be written as:

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} = F\cos\theta\mathbf{i} + F\sin\theta\mathbf{j} = F(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

where F is the magnitude of the force and θ is its angle relative to the x axis, F_x and F_y are the components of **F** in the directions of the x and y axes, respectively, and **i** and **j** are unit vectors in these directions. If F_x and F_y are known, then F and θ can be determined by:

$$F = \sqrt{F_x^2 + F_y^2} \quad \text{and} \quad \tan\theta = \frac{F_y}{F_x}$$

The total (equivalent) force applied on the bracket is obtained by adding the forces that are acting on the bracket. The MATLAB solution below follows three steps:

- Write each force as a vector with two elements, where the first element is the x component of the vector and the second element is the y component.
- Determine the vector form of the equivalent force by adding the vectors.
- Determine the magnitude and direction of the equivalent force.

The problem is solved in the following program, written in a script file.

```
% Sample Problem 3-2 solution (script file)
clear
F1M=400; F2M=500; F3M=700;
Th1=-20; Th2=30; Th3=143;
F1=F1M*[cosd(Th1) sind(Th1)]
F2=F2M*[cosd(Th2) sind(Th2)]
F3=F3M*[cosd(Th3) sind(Th3)]
Ftot=F1+F2+F3
FtotM=sqrt(Ftot(1)^2+Ftot(2)^2)
Th=atand(Ftot(2)/Ftot(1))
```

Define variables with the magnitude of each vector.

Define variables with the angle of each vector.

Define the three vectors.

Calculate the total force vector.

Calculate the magnitude of the total force vector.

Calculate the angle of the total force vector.

When the program is executed, the following is displayed in the Command Window:

```
F1 =
  375.8770 -136.8081
F2 =
  433.0127  250.0000
F3 =
 -559.0449  421.2705
Ftot =
  249.8449  534.4625
FtotM =
  589.9768
Th =
   64.9453
```

The components of F_1.

The components of F_2.

The components of F_3.

The components of the total force.

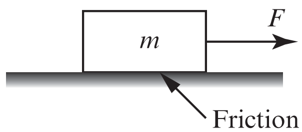
The magnitude of the total force.

The direction of the total force in degrees.

The equivalent force has a magnitude of 589.98 N, and is directed 64.95° (ccw) relative to the x axis. In vector notation, the force is $\mathbf{F} = 249.84\mathbf{i} + 534.46\mathbf{j}$ N.

Sample Problem 3-3: Friction experiment (element-by-element calculations)

The coefficient of friction, μ, can be determined in an experiment by measuring the force F required to move a mass m. When F is measured and m is known, the coefficient of friction can be calculated by:



$$\mu = F/(mg) \quad (g = 9.81 \text{ m/s}^2).$$

Results from measuring F in six tests are given in the table below. Determine the coefficient of friction in each test, and the average from all tests.

Test	1	2	3	4	5	6
Mass m (kg)	2	4	5	10	20	50
Force F (N)	12.5	23.5	30	61	117	294

Solution

A solution using MATLAB commands in the Command Window is shown below.

```
>> m=[2 4 5 10 20 50];
>> F=[12.5 23.5 30 61 117 294];
>> mu=F./(m*9.81)

mu =

   0.6371   0.5989   0.6116   0.6218   0.5963   0.5994

>> mu_ave=mean(mu)

mu_ave =
    0.6109
```

Enter the values of `m` in a vector.

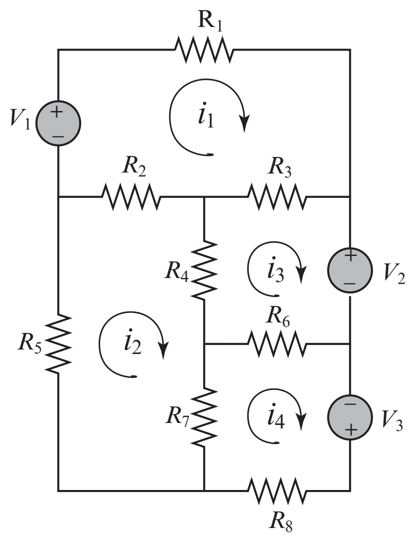
Enter the values of `F` in a vector.

A value for `mu` is calculated for each test, using element-by-element calculations.

The average of the elements in the vector `mu` is determined by using the function `mean`.

Sample Problem 3-4: Electrical resistive network analysis (solving a system of linear equations)

The electrical circuit shown consists of resistors and voltage sources. Determine the current in each resistor using the mesh current method, which is based on Kirchhoff's voltage law.



$V_1 = 20\,\text{V},\ V_2 = 12\,\text{V},\ V_3 = 40\,\text{V}$
$R_1 = 18\Omega,\ R_2 = 10\Omega,\ R_3 = 16\Omega$
$R_4 = 6\Omega,\ R_5 = 15\Omega,\ R_6 = 8\Omega$
$R_7 = 12\Omega,\ R_8 = 14\Omega$

Solution

Kirchhoff's voltage law states that the sum of the voltage around a closed circuit is zero. In the mesh current method a current is first assigned for each mesh (i_1, i_2, i_3, i_4 in the figure). Then Kirchhoff's voltage law is applied for each mesh. This results in a system of linear equations for the currents (in this case four equations). The solution gives the values of the mesh currents. The current in a resistor that belongs to two meshes is the sum of the currents in the corresponding meshes. It is convenient to assume that all the currents are in the same direction (clockwise in this case). In the equation for each mesh, the voltage source is positive if the current flows to the – pole, and the voltage of a resistor is negative for current in the direction of the mesh current.

The equations for the four meshes in the current problem are:

$$V_1 - R_1 i_1 - R_3(i_1 - i_3) - R_2(i_1 - i_2) = 0$$

$$-R_5 i_2 - R_2(i_2 - i_1) - R_4(i_2 - i_3) - R_7(i_2 - i_4) = 0$$

$$-V_2 - R_6(i_3 - i_4) - R_4(i_3 - i_2) - R_3(i_3 - i_1) = 0$$

$$V_3 - R_8 i_4 - R_7(i_4 - i_2) - R_6(i_4 - i_3) = 0$$

The four equations can be rewritten in matrix form $[A][x] = [B]$:

$$\begin{bmatrix} -(R_1+R_2+R_3) & R_2 & R_3 & 0 \\ R_2 & -(R_2+R_4+R_5+R_7) & R_4 & R_7 \\ R_3 & R_4 & -(R_3+R_4+R_6) & R_6 \\ 0 & R_7 & R_6 & -(R_6+R_7+R_8) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -V_1 \\ 0 \\ V_2 \\ -V_3 \end{bmatrix}$$

The problem is solved in the following program, written in a script file:

```
V1=20; V2=12; V3=40;
R1=18; R2=10; R3=16; R4=6;
R5=15; R6=8; R7=12; R8=14;
A=[-(R1+R2+R3) R2 R3 0
R2 -(R2+R4+R5+R7) R4 R7
R3 R4 -(R3+R4+R6) R6
0 R7 R6 -(R6+R7+R8)]
>> B=[-V1; 0; V2; -V3]
>> I=A\B
```

Define variables with the values of the V's and R's.

Create the matrix A.

Create the vector B.

Solve for the currents by using left division.

When the script file is executed, the following is displayed in the Command Window:

```
A =
   -44    10    16     0
    10   -43     6    12
    16     6   -30     8
     0    12     8   -34
B =
   -20
     0
    12
   -40
I =
    0.8411
    0.7206
    0.6127
    1.5750
>>
```

The numerical value of the matrix A.

The numerical value of the vector B.

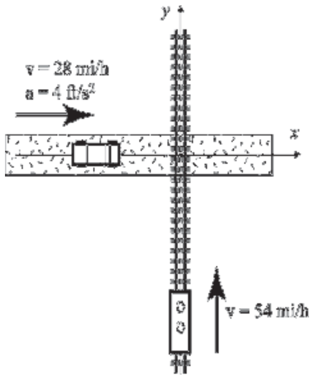
$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

The solution.

The last column vector gives the current in each mesh. The currents in the resistors R_1, R_5, and R_8 are $i_1 = 0.8411$ A, $i_2 = 0.7206$ A, and $i_4 = 1.5750$ A, respectively. The other resistors belong to two meshes and their current is the sum of the currents in the meshes.

The current in resistor R_2 is $i_1 - i_2 = 0.1205$ A.
The current in resistor R_3 is $i_1 - i_3 = 0.2284$ A.
The current in resistor R_4 is $i_2 - i_3 = 0.1079$ A.
The current in resistor R_6 is $i_4 - i_3 = 0.9623$ A.
The current in resistor R_7 is $i_4 - i_2 = 0.8544$ A.

Sample Problem 3-5: Motion of two particles



A train and a car are approaching a road crossing. At time $t = 0$ the train is 400 ft south of the crossing traveling north at a constant speed of 54 mi/h. At the same time the car is 200 ft west of the crossing traveling east at a speed of 28 mi/h and accelerating at 4 ft/s^2. Determine the positions of the train and the car, the distance between them, and the speed of the train relative to the car every second for the next 10 seconds.

To show the results, create an 11×6 matrix in which each row has the time in the first column and the train position, car position, distance between the train and the car, car speed, and the speed of the train relative to the car in the next five columns, respectively.

Solution

The position of an object that moves along a straight line at a constant acceleration is given by $s = s_0 + v_0 t + \frac{1}{2}at^2$ where s_0 and v_0 are the position and velocity at $t = 0$, and a is the acceleration. Applying this equation to the train and the car gives:

$$y = -400 + v_{0train} t \qquad \text{(train)}$$

$$x = -200 + v_{0car} t + \frac{1}{2} a_{car} t^2 \qquad \text{(car)}$$

The distance between the car and the train is: $d = \sqrt{x^2 + y^2}$. The velocity of the train is constant and in vector notation is given by $\mathbf{v}_{train} = v_{otrain}\mathbf{j}$. The car is accelerating and its velocity at time t is given by $\mathbf{v}_{car} = (v_{ocar} + a_{car}t)\mathbf{i}$. The velocity of the train relative to the car, $\mathbf{v}_{t/c}$, is given by $\mathbf{v}_{t/c} = \mathbf{v}_{train} - \mathbf{v}_{car} = -(v_{0\,car} + a_{car}t)\mathbf{i} + v_{0\,train}\mathbf{j}$. The magnitude (speed) of this velocity is the length of the vector.

The problem is solved in the following program, written in a script file. First a vector `t` with 11 elements for the time from 0 to 10 s is created, then the positions of the train and the car, the distance between them, and the speed of the train relative to the car at each time element are calculated.

```
v0train=54*5280/3600; v0car=28*5280/3600; acar=4;
```
Create variables for the initial velocities (in ft/s) and the acceleration.

```
t=0:10;
```
Create the vector `t`.

```
y=-400+v0train*t;
x=-200+v0car*t+0.5*acar*t.^2;
```
Calculate the train and car positions.

```
d=sqrt(x.^2+y.^2);
```
Calculate the distance between the train and car.