

5.14 PROBLEMS

1. Use the `plot` command to plot the function $f(x) = x^2 - 10\sqrt{x} + 2$ for $0 \leq x \leq 5$.
2. Use the `plot` command to plot the function $f(x) = (0.5x^4 + 1.1x^3 - 0.9x^2)e^{-0.7x}$ for $-3 < x < 10$.
3. Use the `plot` command to plot the function $f(x) = 3\cos(1.7x)e^{-0.3x} + 2\sin(1.4x)e^{0.3x}$ for $-7 \leq x \leq 7$.
4. Plot the function $f(x) = x^2e^{-x}$ and its derivative for $0 \leq x \leq 10$ in one figure. Plot the function with a solid line, and the derivative with a dashed line. Add a legend and label the axes.
5. Make two separate plots of the function $f(x) = x^4 - 2x^3 + 1.3x^2 - 0.3x + 0.02$, one plot for $-3 \leq x \leq 4$ and one for $0 \leq x \leq 1$.
6. Use the `fplot` command to plot the function $f(x) = 5(e^{-0.5x} - e^{-0.8x})$ for $0 \leq x \leq 10$.
7. Plot the function $f(x) = \sin(2x)\cos^2(0.5x)$ and its derivative, both on the same plot, for $-\pi \leq x \leq 2\pi$. Plot the function with a solid line and the derivative with a dashed line. Add a legend and label the axes.
8. The orbit of the planet Mercury around the sun can be approximated by the equation $r = \frac{3.44 \times 10^7}{1 - 0.206 \cos \theta}$ miles. Make a plot of the orbit.

9. A parametric equation is given by

$$x = 0.7\sin(10t), \quad y = 1.2\sin(8t)$$

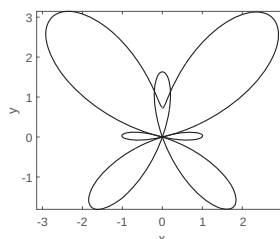
Plot the function for $0 \leq t \leq \pi$. Format the plot such that both axes will range from -1.5 to 1.5 .

10. The butterfly curve (Fay, T. H. "The Butterfly Curve." Amer. Math. Monthly 96, pp. 442-443, 1989) is given by the following parametric equations:

$$x = \sin t \left[e^{\cos t} - 2 \cos(4t) + \sin^5(t/12) \right]$$

$$y = \cos t \left[e^{\cos t} - 2 \cos(4t) + \sin^5(t/12) \right]$$

On one page make two plots of butterfly curves. One for $0 \leq t \leq 2\pi$ and the other for $0 \leq t \leq 10\pi$.

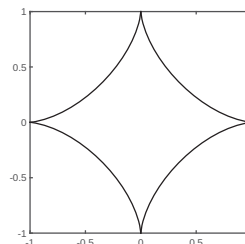


11. A plot of an astroid is shown in the figure on the right. Make the plot using the Cartesian equation:

$$x^{2/3} + y^{2/3} = 1$$

12. Make the plot of the astroid that is shown in the figure in the previous problem by using the parametric equation:

$$x = \cos^3(t) \quad \text{and} \quad y = \sin^3(t) \quad \text{for} \quad -\pi \leq x \leq \pi.$$



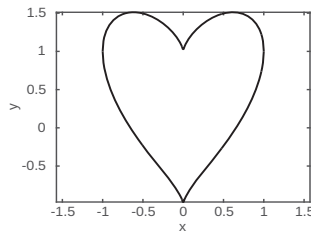
13. Plot the function $f(x) = \frac{x^2 - 6x + 7}{x^3 - 8}$ in the domain $0 < x < 4$. Notice that the function has a vertical asymptote at $x = 2$. Plot the function by creating two vectors for the domain of x . The first vector (name it $x1$) includes elements from 0 to 1.9, and the second vector (name it $x2$) includes elements from 2.1 to 4. For each x vector create a y vector (name them $y1$ and $y2$) with the corresponding values of y according to the function. To plot the function make two curves in the same plot ($y1$ vs. $x1$, and $y2$ vs. $x2$).

14. Plot the function $f(x) = x + \frac{1}{x^2 - 1}$ for $-4 \leq x \leq 4$. Notice that the function has two vertical asymptotes. Plot the function by dividing the domain of x into three parts: one from -4 to near the left asymptote, one between the two asymptotes, and one from near the right asymptote to 4. Set the range of the y axis from -15 to 15 .

15. The shape of the heart shown in the figure is given by the equation:

$$x^2 + \left(y - \sqrt[3]{x^2}\right)^2 = 1$$

Make a plot of the heart.



16. The shape of the pretzel shown is given by the following parametric equations:

$$x = (3.3 - 0.3t^2)\cos t \quad y = (3.3 - 0.4t^2)\sin t$$

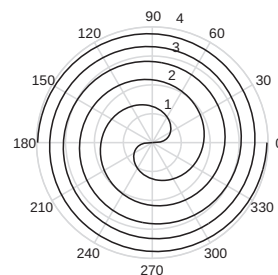
where $-4 < t < 4$. Make a plot of the pretzel.



17. Make a polar plot of the function:

$$r = \pm \sqrt{\theta} \quad \text{for } 0 \leq \theta \leq 5\pi$$

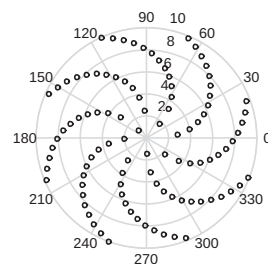
The plot, shown in the figure, is Fermat's spiral.



18. Make a polar plot of the function:

$$\theta = n 135.7^\circ \quad r = \sqrt{n} \quad \text{for } n = 1, 2, 3, \dots, 100$$

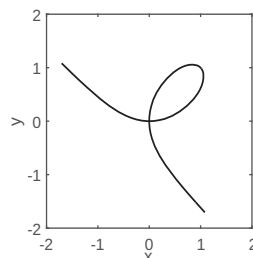
The plot is shown on the right.



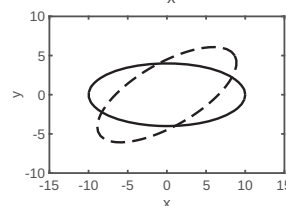
19. Make a plot (shown) of the function:

$$x^3 + y^3 = 2xy$$

(Hint: Rewrite the function in a polar form.)



20. Plot two ellipses in one figure (shown). The ellipse with the solid line has major axes of $a = 10$ and $b = 4$. The ellipse with the dashed line is the solid-line ellipse rotated by 30° .



21. The following data gives the height (in inches) of a sunflower plant as a function of time (days after it was planted).

Time (days)	10	20	30	40	50	60	70
Height (in.)	9	22	44	63	80	94	97

The height can be modeled by the logistic function:

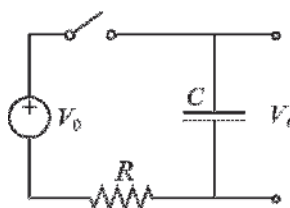
$$H = \frac{100.8}{1 + 23e^{-0.093t}}$$

where H is the height (in.) and t is the time (days). Make a plot of the height versus time. The figure should show the data from the table above as points and the height modeled by the equation as a solid line. Add a legend, and label the axes.

22. The voltage V_C t seconds after closing the switch in the circuit shown is given by:

$$V_C = V_0(1 - e^{-t/RC})$$

Plot V_C as a function of t for $0 \leq t \leq 15$ s. Label the axes. $V_0 = 36$ V, $R = 2,500 \, \Omega$, and $C = 1,200 \, \mu\text{F}$.

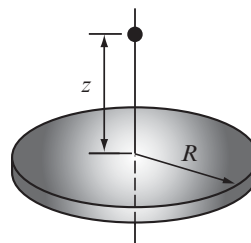


23. The force F (in N) acting between a particle with a charge q and a round disk with a radius R and a charge Q is given by the equation:

$$F = \frac{Qqz}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

where $\epsilon_0 = 0.885 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$ is the permittivity constant and z is the distance to the particle.

Consider the case where $Q = 9.4 \times 10^{-6} \text{ C}$, $q = 2.4 \times 10^{-5} \text{ C}$, and $R = 0.1 \text{ m}$. Make a plot of F as a function of z for $0 < z < 0.3 \text{ m}$. Use MATLAB's built-in function `max` to find the maximum value of F and the corresponding distance z .



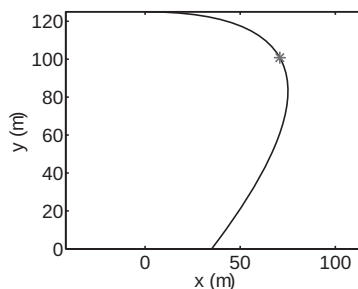
24. The curvilinear motion of a particle is defined by the following parametric equations:

$$x = 52t - 9t^2 \text{ m} \quad y = 125 - 5t^2 \text{ m}$$

The velocity of the particle is given by

$$v = \sqrt{v_x^2 + v_y^2}, \text{ where } v_x = \frac{dx}{dt} \text{ and } v_y = \frac{dy}{dt}.$$

For $0 < t < 5 \text{ s}$ make one plot that shows the position of the particle (y versus x) and a second plot (on the same page) of the velocity of the particle as a function of time. In addition, by using MAT-



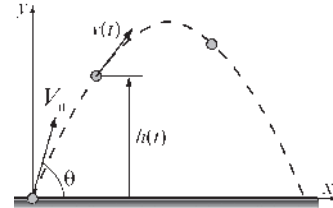
LAB's `min` function, determine the time at which the velocity is the lowest, and the corresponding position of the particle. Using an asterisk marker, show the position of the particle in the first plot. For time use a vector with spacing of 0.1 s.

25. The height and speed of a projectile shoot at a speed v_0 at an angle θ as a function of time are given by:

$$h(t) = v_0 t \sin \theta - gt^2 / 2$$

$$v(t) = \sqrt{v_0^2 - 2v_0 g t \sin \theta + g^2 t^2}$$

where $g = 9.81 \text{ m/s}^2$. Determine the time that the projectile will hit the ground and plot the height and the speed as a function of time (two plots on one page) for the case that $v_0 = 200 \text{ m/s}$ and $\theta = 70^\circ$. Add titles and label the axes.



26. The position x as a function of time of a particle that moves along a straight line is given by:

$$x(t) = 8 - 4t^3 e^{-0.4t} + 2t^2 \text{ ft}$$

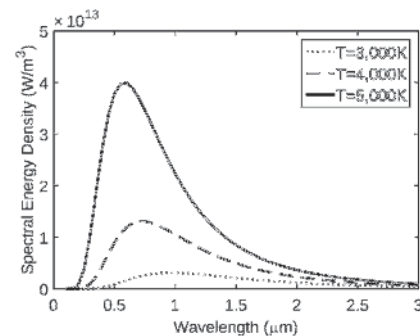
The velocity $v(t)$ of the particle is determined by the derivative of $x(t)$ with respect to t , and the acceleration $a(t)$ is determined by the derivative of $v(t)$ with respect to t .

Derive the expressions for the velocity and acceleration of the particle, and make plots of the position, velocity, and acceleration as functions of time for $0 \leq t \leq 8 \text{ s}$. Use the `subplot` command to make the three plots on the same page with the plot of the position on the top, the velocity in the middle, and the acceleration at the bottom. Label the axes appropriately with the correct units.

27. According to Planck's law of black-body radiation, the spectral energy density R as a function of wavelength λ (m) and temperature T (K) is given by:

$$R = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

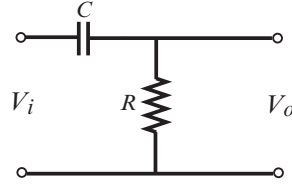
where $c = 3 \times 10^8 \text{ m/s}$ is the speed of light, $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$ is the Planck constant, and $k = 1.38 \times 10^{-23} \text{ J/K}$ is Boltzmann constant. Make the shown figure that contains plots of R as a function of λ for $0.1 \leq \lambda \leq 3 \mu\text{m}$ for three temperatures $T = 3,000 \text{ K}$, $T = 4,000 \text{ K}$, and $T = 5,000 \text{ K}$.



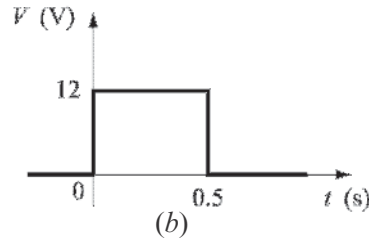
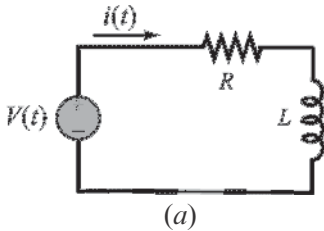
28. A high-pass filter passes signals with frequencies that are higher than a certain cutoff frequency. In this filter the ratio of the magnitudes of the voltages is given by:

$$\left| \frac{V_o}{V_i} \right| = \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}$$

where $\omega = 2\pi f$ is the frequency of the input signal. Given $R = 2,000$ and $C = 0.2 \mu\text{F}$, plot $\left| \frac{V_o}{V_i} \right|$ as a function of f for $10 \leq f \leq 50,000$ Hz. Use logarithmic scale for the horizontal (f) axis and linear scale for the vertical axis.



29. A resistor, $R = 4 \Omega$ and an inductor, $L = 1.3 \text{ H}$, are connected in a circuit to a voltage source as shown in Figure (a) (an RL circuit). When the voltage



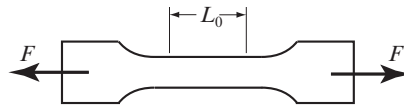
source applies a rectangular voltage pulse with an amplitude of $V = 12 \text{ V}$ and a duration of 0.5 s , as shown in Figure (b), the current $i(t)$ in the circuit as a function of time is given by:

$$i(t) = \frac{V}{R}(1 - e^{-Rt/L}) \quad \text{for } 0 \leq t \leq 0.5 \text{ s}$$

$$i(t) = e^{-Rt/L} \frac{V}{R} (e^{0.5R/L} - 1) \quad \text{for } 0.5 \leq t \text{ s}$$

Make a plot of the current as a function of time for $0 \leq t \leq 2 \text{ s}$.

30. In a typical tension test a dog-bone shaped specimen is pulled in a machine. During the test, the force F needed to pull the specimen and the length L of a gauge section are measured. This data is used for plotting a stress-strain diagram of the material. Two definitions, engineering and true, exist for stress and strain. The engineering stress σ_e and strain ϵ_e are defined by $\sigma_e = \frac{F}{A_0}$ and $\epsilon_e = \frac{L - L_0}{L_0}$, where L_0 and A_0 are the initial gauge length and the initial cross-sectional area of the specimen, respectively. The true stress σ_t and strain ϵ_t are defined by $\sigma_t = \frac{F}{A} \frac{L}{L_0}$ and $\epsilon_t = \ln \frac{L}{L_0}$.



The following are measurements of force and gauge length from a ten-

sion test with an aluminum specimen. The specimen has a round cross section with a radius of 0.25 in. (before the test). The initial gauge length is 0.5 in. Use the data to calculate and generate the engineering and true stress-strain curves, both on the same plot. Label the axes and use a legend to identify the curves.

Units: When the force is measured in pounds (lb) and the area is calculated in in.², the unit of the stress is psi (pounds per square inch).

F (lb)	0	4,390	7,250	10,780	11,710	12,520	12,800	13,340
L (in.)	0.5	0.50146	0.50226	0.50344	0.50423	0.50577	0.50693	0.51138
F (lb)	13,740	13,820	13,850	13,910	13,990	14,020	14,130	
L (in.)	0.52006	0.52169	0.52362	0.52614	0.53406	0.54018	0.56466	

31. According to special relativity, a rod of length L moving at velocity v will shorten by an amount δ , given by:

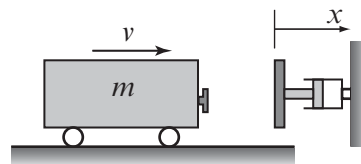
$$\delta = L \left(1 - \sqrt{1 - \frac{v^2}{c^2}} \right)$$

where c is the speed of light (about 300×10^6 m/s). Consider a rod of 2 m long, and make three plots of δ as a function of v for $0 \leq v \leq 300 \times 10^6$ m/s. In the first plot use linear scale for both axes. In the second plot use logarithmic scale for v and linear scale for δ , and in the third plot use logarithmic scale for both v and δ . Which of the plots is the most informative?

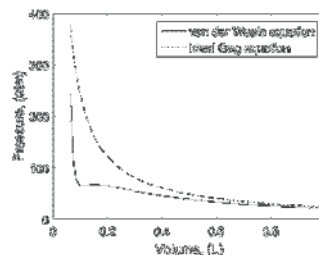
32. A railroad bumper is designed to slow down a rapidly moving railroad car. After a 20,000-kg railroad car traveling at 20 m/s engages the bumper, its displacement x (in meters) and velocity v (in m/s) as a function of time t (in seconds) is given by:

$$x(t) = 4.219(e^{-1.58t} - e^{-6.32t}) \quad \text{and} \quad v(t) = 26.67e^{-6.32t} - 6.67e^{-1.58t}$$

Plot the displacement and the velocity as a function of time for $0 < t < 4$ s. Make two plots on one page.



33. The ideal gas equation states that $\frac{PV}{RT} = n$, where P is the pressure, V is the volume, T is the temperature, $R = 0.08206$ (L atm)/(mol K) is the gas constant, and n is the number of moles. Real gases, especially at high pressures, deviate from this behavior. Their response can be modeled with the



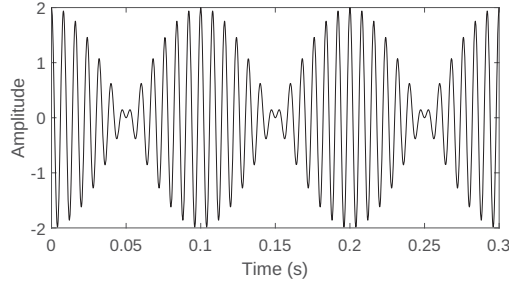
van der Waals equation $P = \frac{nRT}{V-nb} - \frac{n^2a}{V^2}$, where a and b are material constants. For CO_2 $a = 3.592 \text{ L}^2\text{atm/mol}^2$, and $b = 0.04267 \text{ L/mol}$. Make the shown figure that displays two plots of P versus V for $0.065 \leq V \leq 1 \text{ L}$. In one plot the pressure is calculated by using the ideal gas equation and the other by using the van der Waals equation. Label the axes and display a legend.

34. Two sound waves of slightly different frequencies f_1 and f_2 :

$$y_1 = \cos(2\pi f_1 t)$$

$$y_2 = \cos(2\pi f_2 t)$$

produce sound that is alternating loud and soft. This phenomenon, which is called beating, is described by the equation:

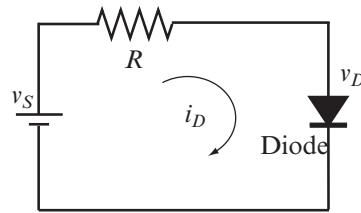


$$y = 2 \cos\left(2\pi \frac{f_1 + f_2}{2} t\right) \cos\left(2\pi \frac{f_1 - f_2}{2} t\right)$$

Make a plot of the beating sound (shown) for $0 \leq t \leq 0.3 \text{ s}$ for the case that $f_1 = 130 \text{ Hz}$ and $f_2 = 120 \text{ Hz}$.

35. Consider the diode circuit shown in the figure. The current i_D and the voltage v_D can be determined from the solution of the following system of equations:

$$i_D = I_0 \left(e^{\frac{q v_D}{k T}} - 1 \right), \quad i_D = \frac{v_S - v_D}{R}$$



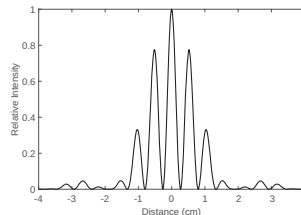
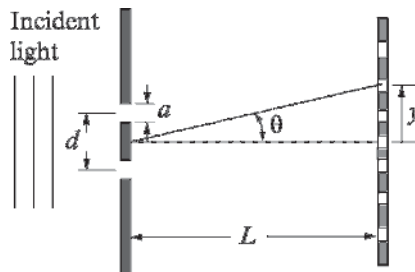
The system can be solved numerically or graphically. The graphical solution is found by plotting i_D as a function of v_D from both equations. The solution is the intersection of the two curves. Make the plots and estimate the solution for the case where $I_0 = 10^{-14} \text{ A}$, $v_S = 1.5 \text{ V}$, $R = 1200 \text{ } \Omega$, and $\frac{kT}{q} = 30 \text{ mV}$.

36. A monochromatic light that passes through a double slit produces on a screen a diffraction pattern consisting of bright and dark fringes. The intensity of the bright fringes, I , as a function of θ can be calculated by:

$$I = I_{\max}(\cos \beta)^2 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

where $\alpha = \frac{\pi a}{\lambda} \sin \theta$ and $\beta = \frac{\pi d}{\lambda} \sin \theta$, λ

is the light wave length, a is the width of the slits, and d is the distance between the slits. Plot (as shown) the relative intensity I / I_{\max} as a function of y (distance to fringes on the screen) for $-4 \leq y \leq 4$ cm given $\lambda = 480$ nm, $a = 0.025$ mm, and $d = 0.09$ mm.



37. A simply supported beam is loaded as shown. The shear force V and bending moment M as a function of x are given by the following equations:

$$V(x) = 400 - 200x \text{ lb}$$

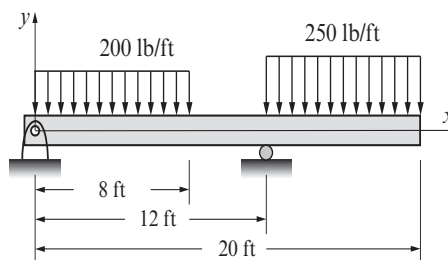
$$M(x) = -100x^2 + 400x \text{ lb-ft}$$

for $0 \leq x \leq 8$ ft.

$$V(x) = -1,200 \text{ lb}, \quad M(x) = -1,200x + 6,400 \text{ lb-ft} \quad \text{for } 8 \leq x \leq 12 \text{ ft}$$

$$V(x) = -250x + 5,000 \text{ lb}, \quad M(x) = -125(x - 12)^2 + 2,000x - 32,000 \text{ lb-ft}$$

for $12 < x < 20$ ft.



Plot the shear force and the bending moment as a function of x (two figures on one page such that the shear force diagram is displayed above the bending moment diagram).

38. Biological oxygen demand (BOD) is a measure of the relative oxygen depletion effect of a waste contaminant and is widely used to assess the amount of pollution in a water source. The BOD in the effluent (L_c in mg/L) of a rock filter without recirculation is given by:

$$L_c = \frac{L_0}{1 + \frac{(2.5D)^{2/3}}{\sqrt{Q}}}$$

where L_0 is the influent BOD (mg/L), D is the depth of the filter (m), and Q is the hydraulic flow rate [$\text{L}/(\text{m}^2\text{-day})$]. Assuming $Q = 300 \text{ L}/(\text{m}^2\text{-day})$ plot the effluent BOD as a function of the depth of the filter ($100 < D < 2000$ m)

for $L_0 = 5, 10$, and 20 mg/L. Make the three plots in one figure and estimate the depth of filter required for each of these cases to obtain drinkable water. Label the axes and display a legend.

39. The shape of a asymmetric four-digit series NACA airfoil is described by the equations:

$$x_U = x - y_t \sin \theta \quad y_U = y_c + y_t \cos \theta$$

$$x_L = x + y_t \sin \theta \quad y_L = y_c - y_t \cos \theta$$

where the subscripts U and L corresponds to the upper and lower airfoil surface, respectively. y_t is half the thickness of the foil given by:

$$y_t = 5tc \left[0.2969 \sqrt{\frac{x}{c}} - 0.126 \left(\frac{x}{c} \right) - 0.3516 \left(\frac{x}{c} \right)^2 + 0.2843 \left(\frac{x}{c} \right)^3 - 0.1015 \left(\frac{x}{c} \right)^4 \right]$$

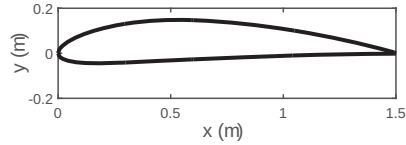
where c is the cord length, t is the maximum thickness (as a fraction of the cord length), and x is the position along the cord. y_c is the coordinate of the camber line given by:

$$y_c = m \frac{x}{p^2} \left(2p - \frac{x}{c} \right) \text{ for } 0 \leq x \leq pc, \text{ and } y_c = m \frac{c-x}{(1-p)^2} \left(1 + \frac{x}{c} - 2p \right) \text{ for } pc \leq x \leq c$$

where m and p are constants. The angle θ is given by:

$$\theta = \arctan \left[\frac{2m}{p^2} \left(p - \frac{x}{c} \right) \right] \text{ for } 0 \leq x \leq pc, \text{ and } \theta = \arctan \left[\frac{2m}{(1-p)^2} \left(p - \frac{x}{c} \right) \right] \text{ for } pc \leq x \leq c.$$

Plot the airfoil shown in the figure (NACA 4412) for which $t=0.12$, $p=0.4$, $m=0.04$, and $c=1.5$ m.



40. The Taylor series expansion for $\sin^2 x$ is:

$$x^2 - \frac{2^3 x^4}{4!} + \frac{2^5 x^6}{6!} - \frac{2^5 x^6}{6!} + \frac{2^7 x^8}{8!}$$

Plot the figure on the right, which shows, for $0 \leq x \leq 4.5$, the plot of the function $\sin^2 x$ and plots of the Taylor series expansion of $\sin^2 x$ with two, three, and five terms. Label the axes and display a legend.

