11.8 NUMERICAL CALCULATIONS WITH SYMBOLIC EXPRESSIONS

Once a symbolic expression is created by the user or by the output from any of MATLAB's symbolic operations, there may be a need to substitute numbers for the symbolic variables and calculate the numerical value of the expression. This can be done by using the `subs` command. The `subs` command has several forms and can be used in different ways. The following describes several forms that are easy to use and are suitable for most applications. In one form, the variable (or variables) for which a numerical value is substituted and the numerical value itself are typed inside the `subs` command. In another form, each variable is assigned a numerical value in a separate command and then the variable is substituted in the expression.

The `subs` command in which the variable and its value are typed inside the command is shown first. Two cases are presented—one for substituting a numerical value (or values) for one symbolic variable, and the other for substituting numerical values for two or more symbolic variables.

Substituting a numerical value for one symbolic variable:

A numerical value (or values) can be substituted for one symbolic variable when a symbolic expression has one or more symbolic variables. In this case the `subs` command has the form:

```
R = subs(S,var,number)
```

The name of the symbolic expression.

The variable for which a numerical value is substituted.

The numerical value (or values) assigned to `var`.

- `number` can be one number (a scalar), or an array with many elements (a vector or a matrix).
- The value of `S` is calculated for each value of `number` and the result is assigned to `R`, which will have the same size as `number` (scalar, vector, or matrix).
- If `S` has one variable, the output `R` is numerical. If `S` has several variables and a numerical value is substituted for only one of them, the output `R` is a symbolic expression.

An example with an expression that includes one symbolic variable is:

```
>> syms x
>> S=0.8*x^3+4*exp(0.5*x)
S =
4*exp(x/2) + (4*x^3)/5
>> SD=diff(S)
```

Define `x` as a symbolic variable.

Assign to `S` the expression $0.8x^3 + 4e^{(0.5x)}$.

Use the `diff(S)` command to differentiate `S`.

```
SD =
2*exp(x/2)+(12*x^2)/5)
```

The answer $2e^{x/2} + 12x^2/5$ is assigned to `SD`.

```
>> subs(SD, x, 2)
```

Use the `subs` command to substitute $x = 2$ in `SD`.

```
ans =
   15.0366
```

The value of `SD` is displayed.

```
>> SDU=subs(SD, x, [2:0.5:4])
```

Use the `subs` command to substitute x = [2, 2.5, 3, 3.5, 4] (vector) in `SD`.

```
SDU =
   15.0366   21.9807   30.5634   40.9092   53.1781
```

The values of `SD` (assigned to `SDU`) for each value of x are displayed in a vector.

In the last example, notice that when the numerical value of the symbolic expression is calculated, the answer is numerical (the display is indented). An example of substituting numerical values for one symbolic variable in an expression that has several symbolic variables is:

```
>> syms a g t v
```

Define `a`, `g`, `t`, and `v` as symbolic variables.

```
>> Y=v^2*exp(a*t)/g
Y =
v^2*exp(a*t)/g
```

Create the symbolic expression $v^2 e^{at}/g$ and assign it to `Y`.

```
>> subs(Y,t,2)
```

Use the `subs` command to substitute $t = 2$ in `SD`.

```
ans =
v^2*exp(2*a)/g
```

The answer $v^2 e^{2a}/g$ is displayed.

```
>> Yt=subs(Y,t,[2:4])
```

Use the `subs` command to substitute t = [2, 3, 4] (vector) in `Y`.

```
Yt =
[ v^2*exp(2*a)/g, v^2*exp(3*a)/g, v^2*exp(4*a)/g]
```

The answer is a vector with elements of symbolic expressions for each value of t.

Substituting a numerical value for two or more symbolic variables:

A numerical value (or values) can be substituted for two or more symbolic variables when a symbolic expression has several symbolic variables. In this case the `subs` command has the following form (it is shown for two variables, but it can be used in the same form for more):

```
R = subs(S,{var1,var2},{number1,number2})
```

The name of the symbolic expression.

The variables for which numerical values are substituted.

The numerical value (or values) assigned to `var1` and `var2`.

- The variables `var1` and `var2` are the variables in the expression `S` for which the numerical values are substituted. The variables are typed as a cell array (inside curly braces { }). A cell array is an array of cells where each cell can be an array of numbers or text.
- The numbers `number1,number2` substituted for the variables are also typed as a cell array (inside curly braces { }). The numbers can be scalars, vectors, or matrices. The first cell in the numbers cell array (`number1`) is substituted for the variable that is in the first cell of the variable cell array (`var1`), and so on.
- If all the numbers that are substituted for variables are scalars, the outcome will be one number or one expression (if some of the variables are still symbolic).
- If, for at least one variable, the substituted numbers are an array, the mathematical operations are executed element-by-element and the outcome is an array of numbers or expressions. It should be emphasized that the calculations are performed element-by-element even though the expression `S` is not typed in the element-by-element notation. This also means that all the arrays substituted for different variables must be of the same size.
- It is possible to substitute arrays (of the same size) for some of the variables and scalars for other variables. In this case, in order to carry out element-by-element operations, MATLAB expands the scalars (array of 1s times the scalar) to produce an array result.

The substitution of numerical values for two or more variables is demonstrated in the next examples.

```
>> syms a b c e x                      Define a, b, c, e, and x as symbolic variables.
>> S=a*x^e+b*x+c
S =                                    Create the symbolic expression
a*x^e+b*x+c                            ax^e + bx + c and assigned it to S.
>> subs(S,{a,b,c,e,x},{5,4,-20,2,3})   Substitute in S scalars for
                                       all the symbolic variables.
           Cell array.  Cell array.
ans =
   37                                  The value of S is displayed.
>> T=subs(S,{a,b,c},{6,5,7})           Substitute in S scalars for the
T =                                    symbolic variables a, b, and c.
5*x+ 6*x^e+7        The result is an expression with the variables x and
>> R=subs(S,{b,c,e},{[2 4 6],9,[1 3 5]})   Substitute in S a scalar for c,
                                           and vectors for b and e.
R =                                        The result is a vector of
[  2*x+a*x+9, a*x^3+4*x+9, a*x^5+6*x+9]    symbolic expressions.
>> W=subs(S,{a,b,c,e,x},{[4 2 0],[2 4 6],[2 2 2],[1 3 5],[3 2 1]})
                          Substitute in S vectors for all the variables.
```

```
W =
    20    26     8
```

The result is a vector of numerical values.

A second method for substituting numerical values for symbolic variables in a symbolic expression is to first assign numerical values to the variables and then use the `subs` command. In this method, once the symbolic expression exists (at which point the variables in the expression are symbolic) the variables are assigned numerical values. Then the subs command is used in the form:

```
R = subs(S)
```

The name of the symbolic expression.

Once the symbolic variables are redefined as numerical variables they can no longer be used as symbolic. The method is demonstrated in the following examples.

```
>> syms A c m x y
```

Define `A`, `c`, `m`, `x`, and `y` as symbolic variables

```
>> S=A*cos(m*x)+c*y
S =
c*y+A*cos(m*x)
```

Create the symbolic expression $A\cos(mx) + cy$ and assign it to `S`.

```
>> A=10; m=0.5; c=3;
```

Assign numerical values to variables `A`, `m`, and `c`.

```
>> subs(S)
```

Use the `subs` command with the expression `S`.

```
ans =
3*y + 10*cos(x/2)
```

The numerical values of variables `A`, `m`, and `c` are substituted in `S`.

```
>> x=linspace(0,2*pi,4);
```

Assign numerical values (vector) to variable `x`.

```
>> T = subs(S)
```

Use the `subs` command with the expression `S`.

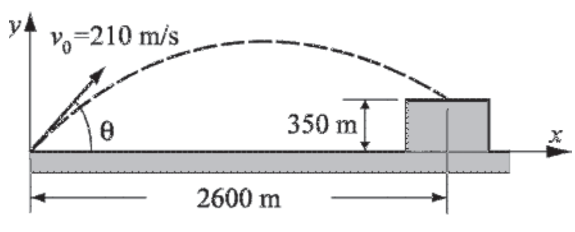
```
T =
[ 3*y+10, 3*y+5, 3*y-5, 3*y-10]
```

The numerical values of variables `A`, `m`, `c`, and `x` are substituted. The result is a vector of symbolic expressions.

11.9 Examples of MATLAB Applications

Sample Problem 11-2: Firing angle of a projectile

A projectile is fired at a speed of 210 m/s and an angle θ. The projectile's intended target is 2,600 m away and 350 m above the firing point.



(*a*) Derive the equation that has to be solved in order to determine the angle θ such that the projectile will hit the target.