```
>> b*A

ans =
     6    15    21     0
    30     3     9    12
    18     6    33    15
>> C=A*5
C =
    10    25    35     0
    50     5    15    20
    30    10    55    25
```

Multiply the matrix A by b. This can be done by either typing b*A or A*b.

Multiply the matrix A by 5 and assign the result to a new variable C. (Typing C=5*A gives the same result.)

Linear algebra rules of array multiplication provide a convenient way for writing a system of linear equations. For example, the system of three equations with three unknowns

$$A_{11}x_1 + A_{12}x_2 + A_{13}x_3 = B_1$$

$$A_{21}x_1 + A_{22}x_2 + A_{23}x_3 = B_2$$

$$A_{31}x_1 + A_{32}x_2 + A_{33}x_3 = B_3$$

can be written in a matrix form as:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

and in matrix notation as

$$AX = B \quad \text{where} \quad A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}, \ X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ and } B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}.$$

3.3 *ARRAY DIVISION*

The division operation is also associated with the rules of linear algebra. This operation is more complex, and only a brief explanation is given below. A full explanation can be found in books on linear algebra.

The division operation can be explained with the help of the identity matrix and the inverse operation.

Identity matrix:

The identity matrix is a square matrix in which the diagonal elements are 1s and the rest of the elements are 0s. As was shown in Section 2.2.1, an identity matrix can be created in MATLAB with the eye command. When the identity matrix multiplies another matrix (or vector), that matrix (or vector) is unchanged (the multiplication has to be done according to the rules of linear algebra). This is equivalent to multiplying a scalar by 1. For example:

$$\begin{bmatrix} 7 & 3 & 8 \\ 4 & 11 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 8 \\ 4 & 11 & 5 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ 2 \\ 15 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 15 \end{bmatrix} \quad \text{or}$$

$$\begin{bmatrix} 6 & 2 & 9 \\ 1 & 8 & 3 \\ 7 & 5 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 9 \\ 1 & 8 & 3 \\ 7 & 5 & 4 \end{bmatrix}$$

If a matrix A is square, it can be multiplied by the identity matrix, I, from the left or from the right:

$$AI = IA = A$$

Inverse of a matrix:

The matrix B is the inverse of the matrix A if, when the two matrices are multiplied, the product is the identity matrix. Both matrices must be square, and the multiplication order can be BA or AB.

$$BA = AB = I$$

Obviously B is the inverse of A, and A is the inverse of B. For example:

$$\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 8 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 5.5 & -3.5 & 2 \\ 2 & -1 & 0 \\ -3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5.5 & -3.5 & 2 \\ 2 & -1 & 0 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 8 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The inverse of a matrix A is typically written as A^{-1}. In MATLAB the inverse of a matrix can be obtained either by raising A to the power of –1, `A^-1`, or with the `inv(A)` function. Multiplying the matrices above with MATLAB is shown below.

```
>> A=[2 1 4; 4 1 8; 2 -1 3]                Creating the matrix A.
A =
     2     1     4
     4     1     8
     2    -1     3
>> B=inv(A)                                Use the inv function to find the
                                           inverse of A and assign it to B.
B =
    5.5000   -3.5000    2.0000
    2.0000   -1.0000         0
   -3.0000    2.0000   -1.0000
>> A*B              Multiplication of A and B gives the identity matrix.
ans =
     1     0     0
     0     1     0
     0     0     1
```

```
>> A*A^-1
ans =
     1     0     0
     0     1     0
     0     0     1
```

Use the power –1 to find the inverse of `A`. Multiplying it by `A` gives the identity matrix.

Not every matrix has an inverse. A matrix has an inverse only if it is square and its determinant is not equal to zero.

Determinants:

A determinant is a function associated with square matrices. A short review on determinants is given below. For a more detailed coverage refer to books on linear algebra.

The determinant is a function that associates with each square matrix A a number, called the determinant of the matrix. The determinant is typically denoted by det(A) or $|A|$. The determinant is calculated according to specific rules. For a second-order 2×2 matrix, the rule is:

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}\text{, for example, } \begin{vmatrix} 6 & 5 \\ 3 & 9 \end{vmatrix} = 6 \cdot 9 - 5 \cdot 3 = 39$$

The determinant of a square matrix can be calculated with the `det` command (see Table 3-1).

Array division:

MATLAB has two types of array division, right division and left division.

Left division, \ :

Left division is used to solve the matrix equation $AX = B$. In this equation X and B are column vectors. This equation can be solved by multiplying, on the left, both sides by the inverse of A:

$$A^{-1}AX = A^{-1}B$$

The left-hand side of this equation is X, since

$$A^{-1}AX = IX = X$$

So the solution of $AX = B$ is:

$$X = A^{-1}B$$

In MATLAB the last equation can be written by using the left division character:

$$X = A \backslash B$$

It should be pointed out here that although the last two operations appear to give the same result, the method by which MATLAB calculates X is different. In the first, MATLAB calculates A^{-1} and then uses it to multiply B. In the second (left division), the solution X is obtained numerically using a method that is based on Gauss elimination. The left division method is recommended for solving a set of linear equations, because the calculation of the inverse may be less accurate than the Gauss elimination method when large matrices are involved.

Right division, / :

The right division is used to solve the matrix equation $XC = D$. In this equation X and D are row vectors. This equation can be solved by multiplying, on the right, both sides by the inverse of C:

$$XCC^{-1} = DC^{-1}$$

which gives

$$X = DC^{-1}$$

In MATLAB the last equation can be written using the right division character:

$$X = D/C$$

The following example demonstrates the use of the left and right division, and the `inv` function to solve a set of linear equations.

Sample Problem 3-1: Solving three linear equations (array division)

Use matrix operations to solve the following system of linear equations.

$$4x - 2y + 6z = 8$$
$$2x + 8y + 2z = 4$$
$$6x + 10y + 3z = 0$$

Solution

Using the rules of linear algebra demonstrated earlier, the above system of equations can be written in the matrix form $AX=B$ or in the form $XC=D$:

$$\begin{bmatrix} 4 & -2 & 6 \\ 2 & 8 & 2 \\ 6 & 10 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 4 & 2 & 6 \\ -2 & 8 & 10 \\ 6 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 4 & 0 \end{bmatrix}$$

Solutions for both forms are shown below:

```
>> A=[4 -2 6; 2 8 2; 6 10 3];
>> B=[8; 4; 0];
>> X=A\B
X =
   -1.8049
    0.2927
    2.6341
>> Xb=inv(A)*B
Xb =
   -1.8049
    0.2927
    2.6341
```

Solving the form $AX = B$.

Solving by using left division: $X = A \backslash B$.

Solving by using the inverse of A: $X=A^{-1}B$.