

interest formula, Equation (1).

(c) Determine the number of years and months that correspond to t .

The problem is solved by writing the following program in a script file:

```
% Solution of Sample Problem 1-4
```

```
P=5000; r=0.085; ta=17; n=12;
```

```
B=P*(1+r)^ta
```

Step (a): Calculate B from Eq. (2).

```
t=log(B/P)/(n*log(1+r/n))
```

Step (b): Solve Eq. (1)
for t , and calculate t .

```
years=fix(t)
```

Step (c): Determine the number of years.

```
months=ceil((t-years)*12)
```

Determine the number of months.

When the script file is executed, the following (the values of the variables B, t , years, and months) is displayed in the Command Window:

```
>> format short g
```

```
B =
```

```
20011
```

```
t =
```

```
16.374
```

```
years =
```

```
16
```

```
months =
```

```
5
```

The values of the variables B, t , years, and months are displayed (since a semicolon was not typed at the end of any of the commands that calculate the values).

1.10 PROBLEMS

The following problems can be solved by writing commands in the Command Window or by writing a program in a script file and then executing the file.

1. Calculate:

$$(a) \left(5 - \frac{19}{7} + 2.5^3\right)^2$$

$$(b) 7 \times 3.1 + \frac{\sqrt{120}}{5} - 15^{5/3}$$

2. Calculate:

$$(a) \sqrt[3]{8 + \frac{80}{2.6}} + e^{3.5}$$

$$(b) \left(\frac{1}{\sqrt{75}} + \frac{73}{3.1^3}\right)^{1/4} + 55 \times 0.41$$

3. Calculate:

$$(a) \frac{23 + \sqrt[3]{45}}{16 \times 0.7} + \log_{10} 589006 \quad (b) (36.1 - 2.25\pi)(e^{2.3} + \sqrt{20})$$

4. Calculate:

$$(a) \frac{3.8^2}{2.75 - 41 \times 25} + \frac{5.2 + 1.8^5}{\sqrt{3.5}} \quad (b) \frac{2.1 \times 10^6 - 15.2 \times 10^5}{3 \cdot \sqrt[3]{6 \times 10^{11}}}$$

5. Calculate:

$$(a) \frac{\sin(0.2\pi)}{\cos(\pi/6)} + \tan 72^\circ \quad (b) (\tan 64^\circ \cos 15^\circ)^2 + \frac{\sin^2 37^\circ}{\cos^2 20^\circ}$$

6. Define the variable z as $z = 4.5$; then evaluate:

$$(a) 0.4z^4 + 3.1z^2 - 162.3z - 80.7 \quad (b) (z^3 - 23) / \left(\sqrt[3]{z^2 + 17.5} \right)$$

7. Define the variable t as $t = 3.2$; then evaluate:

$$(a) \frac{1}{2}e^{2t} - 3.81t^3 \quad (b) \frac{6t^2 + 6t - 2}{t^2 - 1}$$

8. Define the variables x and y as $x = 6.5$ and $y = 3.8$; then evaluate:

$$(a) (x^2 + y^2)^{2/3} + \frac{xy}{y-x} \quad (b) \frac{\sqrt{x+y}}{(x-y)^2} + 2x^2 - xy^2$$

9. Define the variables a , b , c , and d as:

$$c = 4.6, d = 1.7, a = cd^2, \text{ and } b = \frac{c+a}{c-d}; \text{ then evaluate:}$$

$$(a) e^{d-b} + \sqrt[3]{c+a} - (ca)^d \quad (b) \frac{d}{c} + \left(\frac{ct}{b} \right)^2 - c^d - \frac{a}{b}$$

10. Two trigonometric identities are given by:

$$(a) \cos^2 x - \sin^2 x = 1 - 2\sin^2 x \quad (b) \frac{\tan x}{\sin x - 2 \tan x} = \frac{1}{\cos x - 2}$$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting $x = \pi / 10$.

11. Two trigonometric identities are given by:

$$(a) (\sin x + \cos x)^2 = 1 + 2 \sin x \cos x \quad (b) \frac{1 - 2 \cos x - 3 \cos^2 x}{\sin^2 x} = \frac{1 - 3 \cos x}{1 - \cos x}$$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting $x = 20^\circ$.

12. Define two variables: $\alpha = \pi/8$, and $\beta = \pi/6$. Using these variables, show that the following trigonometric identity is correct by calculating the values of the left and right sides of the equation.

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

13. Given: $\int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2)\sin x$. Use MATLAB to calculate the following definite integral: $\int_{\pi/6}^{\pi/3} x^2 \cos x \, dx$.

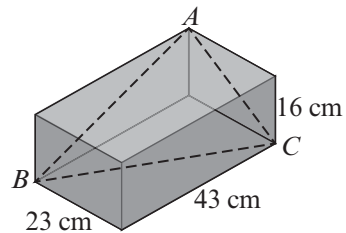
14. A rectangular box has the dimensions shown.
(a) Determine the angle BAC to the nearest degree.

- (b) Determine the area of the triangle ABC to the nearest tenth of a centimeter.

Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos \gamma$

Heron's formula for triangular area:

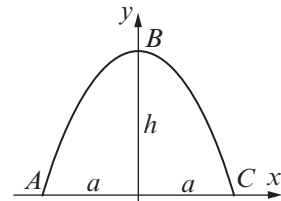
$$A = \sqrt{p(p-a)(p-b)(p-c)}, \text{ where } p = (a+b+c)/2.$$



15. The arc length of a segment of a parabola ABC is given by:

$$L_{ABC} = \sqrt{a^2 + 4h^2} + \frac{a^2}{2h} \ln \left(\frac{2h}{a} + \sqrt{\left(\frac{2h}{a} \right)^2 + 1} \right)$$

Determine L_{ABC} if $a=8$ in. and $h=13$ in.



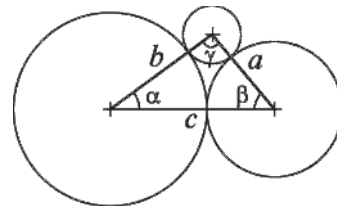
16. The three shown circles, with radius 15 in., 10.5 in., and 4.5 in., are tangent to each other.

- (a) Calculate the angle γ (in degrees) by using the law of cosines.

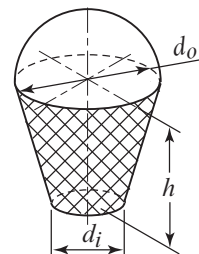
(Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos \gamma$)

- (b) Calculate the angles γ and α (in degrees) using the law of sines.

- (c) Check that the sum of the angles is 180° .



17. A frustum of cone is filled with ice cream such that the portion above the cone is a hemisphere. Define the variables $d_i=1.25$ in., $d_o=2.25$ in., $h=2$ in., and determine the volume of the ice cream.



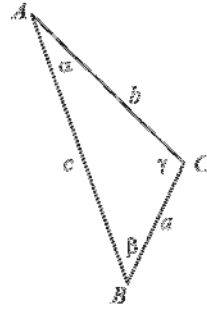
18. In the triangle shown $a = 27$ in., $b = 43$ in., and $c = 57$ in. Define a , b , and c as variables, and then:

(a) Calculate the angles α , β , and γ by substituting the variables in the law of cosines.

(Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos \gamma$)

(b) Verify the law of tangents by substituting the results into the right and left sides of:

$$\text{law of tangents: } \frac{b-c}{b+c} = \frac{\tan \left[\frac{1}{2}(\beta-\gamma) \right]}{\tan \left[\frac{1}{2}(\beta+\gamma) \right]}$$



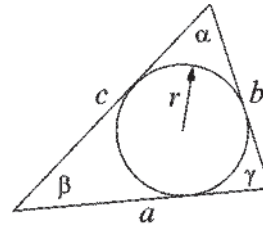
19. For the triangle shown, $\alpha = 72^\circ$, $\beta = 43^\circ$, and its perimeter is $p = 114$ mm. Define α , β , and p , as variables, and then:

(a) Calculate the triangle sides (Use the law of sines).

(b) Calculate the radius r of the circle inscribed in the triangle using the formula:

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

where $s = (a + b + c) / 2$.



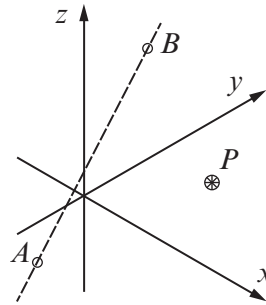
20. The distance d from a point $P(x_P, y_P, z_P)$ to the line that passes through the two points $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$ can be calculated by $d = 2S / r$ where r is the distance between the points A and B , given by

$r = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$ and S is the area of the triangle defined by the three points calculated by $S = \sqrt{s_1^2 + s_2^2 + s_3^2} / 2$ where

$$s_1 = x_P y_A + x_A y_B + x_B y_P - (y_P x_A + y_A x_B + y_B x_P)$$

$$s_2 = y_P z_A + y_A z_B + y_B z_P - (z_P y_A + z_A y_B + z_B y_P)$$

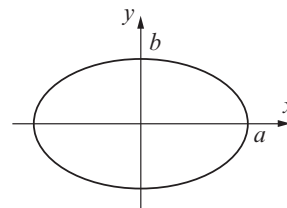
$s_3 = x_P z_A + x_A z_B + x_B z_P - (z_P x_A + z_A x_B + z_B x_P)$. Determine the distance of point $P(2, 6, -1)$ from the line that passes through point $A(-2, -1.5, -3)$ and point $B(-2.5, 6, 4)$. First define the variables x_P , y_P , z_P , x_A , y_A , z_A , x_B , y_B , and z_B , and then use the variable to calculate s_1 , s_2 , s_3 , and r . Finally calculate S and d .



21. The perimeter of an ellipse can be approximated by:

$$P = \pi(a+b) \left(3 - \frac{\sqrt{(3a+b)(a+3b)}}{a+b} \right)$$

Calculate the perimeter of an ellipse with $a = 18$ in. and $b = 7$ in.

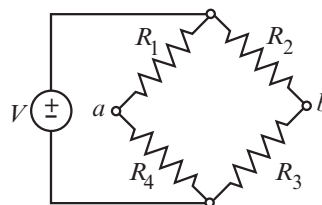


22. A total of 4217 eggs have to be packed in boxes that can hold 36 eggs each. By typing one line (command) in the Command Window, calculate how many eggs will remain unpacked if every box that is used has to be full. (Hint: Use MATLAB built-in function `fix`.)
23. A total of 777 people have to be transported using buses that have 46 seats and vans that have 12 seats. Calculate how many buses are needed if all the buses have to be full, and how many seats will remain empty in the vans if enough vans are used to transport all the people that did not fit into the buses. (Hint: Use MATLAB built-in functions `fix` and `ceil`.)
24. Change the display to `format long g`. Assign the number 7E8/13 to a variable, and then use the variable in a mathematical expression to calculate the following by typing one command:
- Round the number to the nearest tenth.
 - Round the number to the nearest million.

25. The voltage difference V_{ab} between points a and b in the Wheatstone bridge circuit is given by:

$$V_{ab} = V \left(\frac{c-d}{(c+1)(d+1)} \right)$$

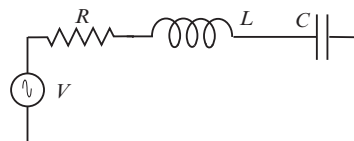
where $c = R_2 / R_1$ and $d = R_3 / R_4$. Calculate the V_{ab} if $V = 15$ V, $R_1 = 119.8 \, \Omega$, $R_2 = 120.5 \, \Omega$, $R_3 = 121.2 \, \Omega$ and $R_4 = 119.3 \, \Omega$



26. The current in a series RCL circuit is given by:

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}}$$

where $\omega = 2\pi f$. Calculate I for the circuit shown if the supply voltage is 80 V, $f = 50$ Hz, $R = 6 \, \Omega$, $L = 400 \times 10^{-3}$ H, and $C = 40 \times 10^{-6}$ F.



27. The monthly payment M of a mortgage P for n years with a fixed annual interest rate r can be calculated by the formula:

$$M = P \frac{\frac{r}{12} \left(1 + \frac{r}{12}\right)^{12n}}{\left(1 + \frac{r}{12}\right)^{12n} - 1}$$

Determine the monthly payment of a 30-year \$450,000 mortgage with interest rate of 4.2% ($r = 0.042$). Define the variables P , r , and n and then use them in the formula to calculate M .

28. The number of permutations ${}_nP_r$ of taking r objects out of n objects without repetition is given by:

$${}_nP_r = \frac{n!}{(n-r)!}$$

- (a) Determine how many six-letter passwords can be formed from the 26 letters in the English alphabet if a letter can only be used once.
 (b) How many passwords can be formed if the digits 0, 1, 2, ..., 9 can be used in addition to the letters.

29. The number of combinations $C_{n,r}$ of taking r objects out of n objects is given by:

$$C_{n,r} = \frac{n!}{r!(n-r)!}$$

In the Powerball lottery game the player chooses five numbers from 1 through 59, and then the Powerball number from 1 through 35.

Determine how many combinations are possible by calculating $C_{59,5} C_{35,1}$. (Use the built-in function `factorial`.)

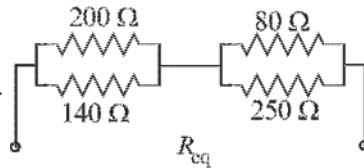
30. The equivalent resistance of two resistors R_1 and R_2 connected in parallel is given by

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

The equivalent resistance of two resistors R_1 and R_2 connected in series

is given by $R_{eq} = R_1 + R_2$. Determine the

equivalent resistance of the four resistors in the circuit shown in the figure.

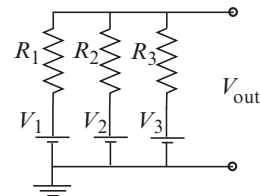


31. The output voltage V_{out} in the circuit shown is given by (Millman's theorem):

$$V_{out} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Calculate V_{out} given $V_1 = 36\text{ V}$, $V_2 = 28\text{ V}$, $V_3 = 24\text{ V}$,

$R_1 = 400\ \Omega$, $R_2 = 200\ \Omega$, $R_3 = 600\ \Omega$.



32. Radioactive decay of carbon-14 is used for estimating the age of organic material. The decay is modeled with the exponential function $f(t) = f(0)e^{kt}$, where t is time, $f(0)$ is the amount of material at $t = 0$, $f(t)$ is the amount of material at time t , and k is a constant. Carbon-14 has a half-life of approximately 5,730 years. A sample taken from the ancient footprints of Acahualinca in Nicaragua shows that 77.45% of the initial ($t = 0$) carbon-14 is present. Determine the estimated age of the footprint. Solve the problem by writing a program in a script file. The program first determines the constant k , then calculates t for $f(t) = 0.7745f(0)$, and finally rounds the answer to the nearest year.
33. The greatest common divisor is the largest positive integer that divides the numbers without a remainder. For example, the greatest common divisor of 8 and 12 is 4. Use the MATLAB Help Window to find a MATLAB built-in function that determines the greatest common divisor of two numbers. Then use the function to show that the greatest common divisor of:
- (a) 91 and 147 is 7.
 - (b) 555 and 962 is 37.

34. The amount of energy E (in joules) that is released by an earthquake is given by:

$$E = 1.74 \times 10^{19} \times 10^{1.44M}$$

where M is the magnitude of the earthquake on the Richter scale.

- (a) Determine the energy that was released from the Anchorage earthquake (1964, Alaska, USA), magnitude 9.2.
 - (b) The energy released in Lisbon earthquake (Portugal) in 1755 was one-half the energy released in the Anchorage earthquake. Determine the magnitude of the earthquake in Lisbon on the Richter scale.
35. According to the Doppler effect of light, the perceived wavelength λ_p of a light source with a wavelength of λ_s is given by:

$$\lambda_p = \lambda_s \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$

where c is the speed of light (about 300×10^6 m/s) and v is the speed the observer moves toward the light source. Calculate the speed the observer has to move in order to see a red light as green. Green wavelength is 530 nm and red wavelength is 630 nm.

36. Newton's law of cooling gives the temperature $T(t)$ of an object at time t in terms of T_0 , its temperature at $t = 0$, and T_s , the temperature of the surroundings.

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

A police officer arrives at a crime scene in a hotel room at 9:18 PM, where he finds a dead body. He immediately measures the body's temperature and finds it to be 79.5°F. Exactly one hour later he measures the temperature again and finds it to be 78.0°F. Determine the time of death, assuming that victim body temperature was normal (98.6°F) prior to death and that the room temperature was constant at 69°F.

37. The velocity v and the falling distance d as a function of time of a skydiver that experience the air resistance can be approximated by:

$$v(t) = \sqrt{\frac{mg}{k}} \tanh\left(\sqrt{\frac{kg}{m}} t\right) \quad \text{and} \quad d(t) = \frac{m}{k} \ln \left[\cosh\left(\sqrt{\frac{kg}{m}} t\right) \right]$$

where $k = 0.24 \text{ kg/m}$ is a constant, m is the skydiver mass, $g = 9.81 \text{ m/s}^2$ is the acceleration due to gravity, and t is the time in seconds since the skydiver starts falling. Determine the velocity and the falling distance at $t = 8 \text{ s}$ for a 95-kg skydiver

38. Use the Help Window to find a display format that displays the output as a ratio of integers. For example, the number 3.125 will be displayed as 25/8. Change the display to this format and execute the following operations:
 (a) $5/8 + 16/6$ (b) $1/3 - 11/13 + 2.7^2$

39. Gosper's approximation for factorials is given by:

$$n! = \sqrt{\left(2n + \frac{1}{3}\right)} \pi n^n e^{-n}$$

Use the formula for calculating 19!. Compare the result with the true value obtained with MATLAB's built-in function `factorial` by calculating the error ($\text{Error} = (\text{True Val} - \text{Approx Val}) / \text{True Val}$).

40. According to Newton's law of universal gravitation, the attraction force between two bodies is given by:

$$F = G \frac{m_1 m_2}{r^2}$$

where m_1 and m_2 are the masses of the bodies, r is the distance between the bodies, and $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ is the universal gravitational constant. Determine how many times the attraction force between the sun and the Earth is larger than the attraction force between the Earth and the moon. The distance between the sun and Earth is $149.6 \times 10^9 \text{ m}$, the distance between the moon and Earth is $384.4 \times 10^6 \text{ m}$, $m_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}$, $m_{\text{sun}} = 2.0 \times 10^{30} \text{ kg}$, and $m_{\text{moon}} = 7.36 \times 10^{22} \text{ kg}$.