- It is possible also to use the `int` command by typing the expression to be integrated as a string without having the variables in the expression first created as symbolic objects. However, the variables in the integrated expression do not exist as independent symbolic objects.
- Integration can sometimes be a difficult task. A closed-form answer may not exist, or if it exists, MATLAB might not be able to find it. When that happens MATLAB returns `int(S)` and the message `Explicit integral could not be found`.

11.6 *SOLVING AN ORDINARY DIFFERENTIAL EQUATION*

An ordinary differential equation (ODE) can be solved symbolically with the `dsolve` command. The command can be used to solve a single equation or a system of equations. Only single equations are addressed here. Chapter 10 discusses using MATLAB to solve first-order ODEs numerically. The reader's familiarity with the subject of differential equations is assumed. The purpose of this section is to show how to use MATLAB for solving such equations.

A first-order ODE is an equation that contains the derivative of the dependent variable. If t is the independent variable and y is the dependent variable, the equation can be written in the form

$$\frac{dy}{dt} = f(t, y)$$

A second-order ODE contains the second derivative of the dependent variable (it can also contain the first derivative). Its general form is:

$$\frac{d^2y}{dt^2} = f\left(t, y, \frac{dy}{dt}\right)$$

A solution is a function $y = f(t)$ that satisfies the equation. The solution can be general or particular. A general solution contains constants. In a particular solution the constants are determined to have specific numerical values such that the solution satisfies specific initial or boundary conditions.

The command `dsolve` can be used for obtaining a general solution or, when the initial or boundary conditions are specified, for obtaining a particular solution.

General solution:

For obtaining a general solution, the `dsolve` command has the form:

`dsolve('eq')` or `dsolve('eq','var')`

- `eq` is the equation to be solved. It has to be typed as a string (even if the variables are symbolic objects).
- The variables in the equation don't have to first be created as symbolic objects. (If they have not been created, then, in the solution the variables will not be sym-

bolic objects.)

- Any letter (lowercase or uppercase), except `D` can be used for the dependent variable.
- In the `dsolve('eq')` command the independent variable is assumed by MATLAB to be `t` (default).
- In the `dsolve('eq','var')` command the user defines the independent variable by typing it for `var` (as a string).
- In specifying the equation the letter `D` denotes differentiation. If y is the dependent variable and t is the independent variable, `Dy` stands for $\frac{dy}{dt}$. For example, the equation $\frac{dy}{dt} + 3y = 100$ is typed in as `'Dy + 3*y = 100'`.
- A second derivative is typed as `D2`, third derivative as `D3`, and so on. For example, the equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 5y = \sin(t)$ is typed in as: `'D2y + 3*Dy + 5*y = sin(t)'`.
- The variables in the ODE equation that is typed in the `dsolve` command do not have to be previously created symbolic variables.
- In the solution MATLAB uses `C1`, `C2`, `C3`, and so on, for the constants of integration.

For example, a general solution of the first-order ODE $\frac{dy}{dt} = 4t + 2y$ is obtained by:

```
>> dsolve('Dy=4*t+2*y')
ans =
C1*exp(2*t) - 2*t - 1
```

The answer $y = C_1e^{2t} - 2t - 1$ is displayed.

A general solution of the second-order ODE $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$ is obtained by:

```
>> dsolve('D2x+2*Dx+x=0')
ans =
C1/exp(t)+(C2*t)/exp(t)
```

The answer $x = C_1e^{-t} + C_2te^{-t}$ is displayed.

The following examples illustrate the solution of differential equations that contain symbolic variables in addition to the independent and dependent variables.

```
>> dsolve('Ds=a*x^2')
ans =
a*t*x^2 + C1
```

The independent variable is t (default). MATLAB solves the equation $\frac{ds}{dt} = ax^2$.

The solution $s = ax^2t + C_1$ is displayed.

```
>> dsolve('Ds=a*x^2','x')

ans =
(a*x^3)/3 + C1

>> dsolve('Ds=a*x^2','a')

ans =
(a^2*x^2)/2 + C2
```

The independent variable is defined to be x. MATLAB solves the equation $\frac{ds}{dx} = ax^2$.

The solution $s = \frac{1}{3}ax^3 + C_1$ is displayed.

The independent variable is defined to be a. MATLAB solves the equation $\frac{ds}{da} = ax^2$.

The solution $s = \frac{1}{2}a^2x^2 + C_1$ is displayed.

Particular solution:

A particular solution of an ODE can be obtained if boundary (or initial) conditions are specified. A first-order equation requires one condition, a second-order equation requires two conditions, and so on. For obtaining a particular solution, the `dsolve` command has the form

First-order ODE: `dsolve('eq','cond1','var')`

Higher-order ODE: `dsolve('eq','cond1','cond2',....,'var')`

- For solving equations of higher order, additional boundary conditions have to be entered in the command. If the number of conditions is less than the order of the equation, MATLAB returns a solution that includes constants of integration (`C1`, `C2`, `C3`, and so on).
- The boundary conditions are typed in as strings in the following:

Math form	**MATLAB form**
$y(a) = A$	`'y(a)=A'`
$y'(a) = A$	`'Dy(a)=A'`
$y''(a) = A$	`'D2y(a)=A'`

- The argument `'var'` is optional and is used to define the independent variable in the equation. If none is entered, the default is t.

For example, the first-order ODE $\frac{dy}{dt} + 4y = 60$, with the initial condition $y(0) = 5$ is solved with MATLAB by:

```
>> dsolve('Dy+4*y=60','y(0)=5')
ans =
15 - 10/exp(4*t)
```

The answer $y = 15 - 10/e^{4t}$ is displayed.