

$$[\text{HCO}_3^-] \approx \text{ZDIC} - \text{ALK}$$

$$[\text{CO}_3^{2-}] \approx \text{ALK} - \text{DIC}$$

$$p\text{CO}_2 = \frac{[\text{CO}_2]}{K_0} \quad \leftarrow \text{replace}$$

$$p\text{CO}_2 = \frac{1}{K_0} \cdot \frac{[\text{H}^+][\text{HCO}_3^-]}{K_1} \quad \leftarrow \text{replace}$$

$$p\text{CO}_2 = \frac{1}{K_0} \cdot \frac{[\text{HCO}_3^-]}{K_1} \cdot \frac{K_2 [\text{HCO}_3^-]}{[\text{CO}_3^{2-}]}$$

$$p\text{CO}_2 = \frac{K_2 [\text{HCO}_3^-]^2}{K_0 K_1 [\text{CO}_3^{2-}]}$$

Use approximations:

$$p\text{CO}_2 = \frac{K_2}{K_0 K_1} \cdot \frac{(\text{ZDIC} - \text{ALK})^2}{\text{ALK} - \text{DIC}}$$

quotient rule and chain rule
 $\frac{u'}{v} = \frac{u'v - uv'}{v^2}$

$$\frac{dp\text{CO}_2}{d\text{DIC}} = \frac{K_2}{K_0 K_1} \cdot \frac{2(\text{ZDIC} - \text{ALK}) \cdot 2 \cdot (\text{ALK} - \text{DIC}) - (\text{ZDIC} - \text{ALK})^2 \cdot (-1)}{(\text{ALK} - \text{DIC})^2}$$

$$\frac{dp\text{CO}_2}{d\text{DIC}} = \frac{K_2}{K_0 K_1} \cdot \frac{4(\text{ZDIC} - \text{ALK}) \cdot (\text{ALK} - \text{DIC}) + (\text{ZDIC} - \text{ALK})^2}{(\text{ALK} - \text{DIC})^2}$$

$$\frac{dp\text{CO}_2}{d\text{DIC}} = \frac{K_2}{K_0 K_1} \cdot \frac{4[\text{HCO}_3^-][\text{CO}_3^{2-}] + [\text{HCO}_3^-]^2}{[\text{CO}_3^{2-}]^2}$$

\swarrow really small compared to \downarrow

$$dp\text{CO}_2 \cdot K_0 = d[\text{CO}_2]$$

$$\frac{d[\text{CO}_2]}{d\text{DIC}} \approx \frac{K_2}{K_1} \cdot \frac{[\text{HCO}_3^-]^2}{[\text{CO}_3^{2-}]^2}$$

$$\text{recall } p\text{CO}_2 = \frac{K_2 [\text{HCO}_3^-]^2}{K_0 K_1 [\text{CO}_3^{2-}]}$$

$$K_0 p\text{CO}_2 = [\text{CO}_2] = \frac{K_2}{K_1} \cdot \frac{[\text{HCO}_3^-]^2}{[\text{CO}_3^{2-}]}$$

plug in

$$\frac{d[\text{CO}_2]}{d\text{DIC}} \approx \frac{[\text{CO}_2]}{[\text{CO}_3^{2-}]} \approx \frac{1}{20}$$

$$K_0 = \frac{[\text{CO}_2]}{p\text{CO}_2}$$

$$K_1 = \frac{[\text{H}^+][\text{HCO}_3^-]}{[\text{CO}_2]}$$

$$K_2 = \frac{[\text{H}^+][\text{CO}_3^{2-}]}{[\text{HCO}_3^-]}$$

Take partial derivative
of $p\text{CO}_2$ w/ respect to
DIC: $\frac{dp\text{CO}_2}{d\text{DIC}}$

$\swarrow [\text{CO}_3^{2-}] \quad \downarrow [\text{HCO}_3^-]$