

Lecture 12: Model ages

1. Decay chains and half-lives

2. Model ages

A. Sm-Nd system

B. Isochrons

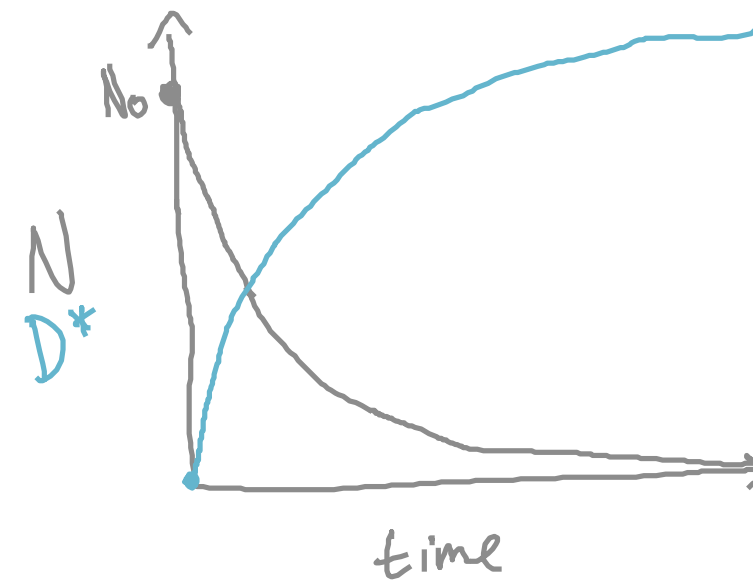
We acknowledge and respect the $lək'əŋən$ peoples on whose traditional territory the university stands and the Songhees, Esquimalt and $W̱SÁNEĆ$ peoples whose historical relationships with the land continue to this day.



The decay equation.

$$\begin{array}{c} \text{not} \\ \text{measurable} \end{array} \uparrow N_0 e^{-\lambda t} = N \uparrow \text{measurable}$$

measurable



N = parent isotope

D = descendant isotope

$$\begin{array}{l} N_0 e^{-\lambda t} = N \uparrow \\ N e^{\lambda t} = N_0 \downarrow \end{array} \text{ reversible}$$

$$D^* = N_0 - N \quad \text{Descendant created by the decay of } N$$

$$D^* = N e^{\lambda t} - N_0 e^{-\lambda t}$$

- λt * hard to measure absolute values, so ratio w/ stable isotope common

$$D^* = N (e^{\lambda t} - 1)$$

↑ measured

$$\frac{D}{x} = \frac{D_0}{x} + \frac{N}{x} (e^{\lambda t} - 1)$$

$$D = D_0 + D^*$$

↑ initial D ↑ generated D



Half life

Half life

how long does it take for half of N to decay?

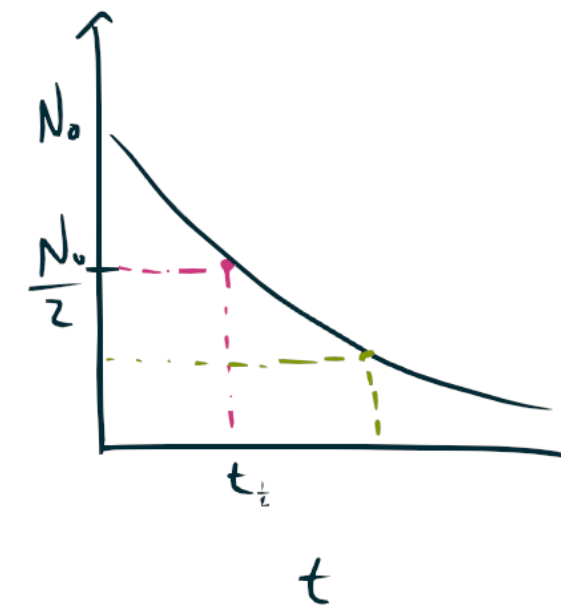
$$N = \frac{N_0}{2}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

$$\frac{1}{2} = e^{-\lambda t_{\frac{1}{2}}}$$

$$-\frac{\ln \frac{1}{2}}{\lambda} = t_{\frac{1}{2}}$$

large decay constant means short half life



Decay chains

Consider



$$\frac{dN}{dt} \propto N$$

$$\frac{d^{238}\text{U}}{dt} = -^{238}\lambda^{238}\text{U}$$

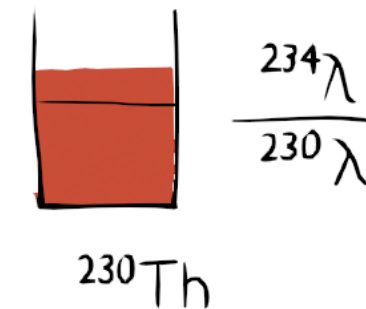
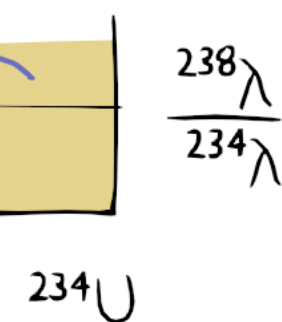
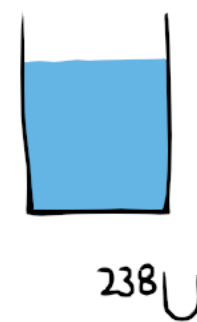
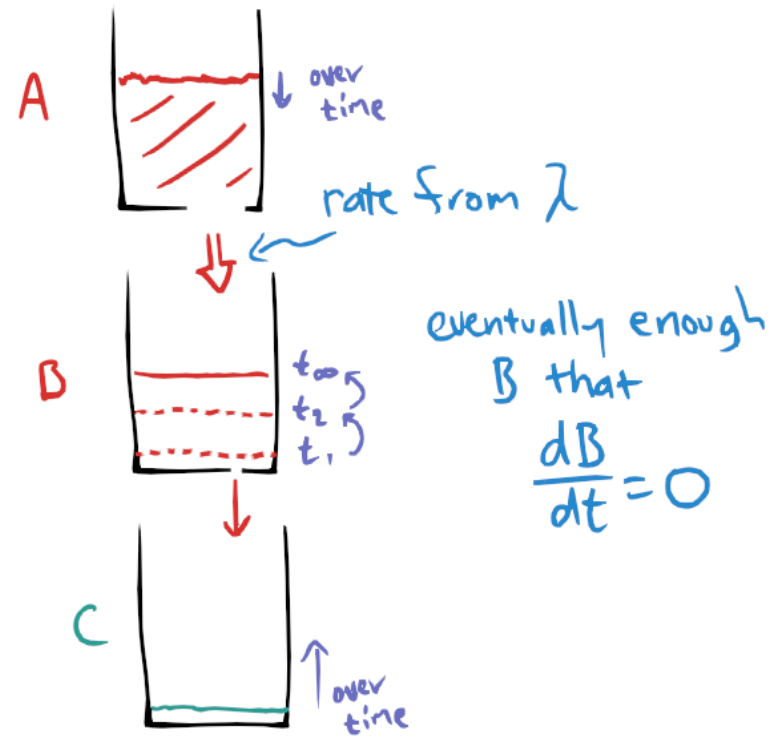
$$\frac{d^{234}\text{U}}{dt} = ^{238}\lambda^{238}\text{U} - ^{234}\lambda^{234}\text{U}$$

$$\frac{d^{230}\text{Th}}{dt} = ^{234}\lambda^{234}\text{U} - ^{230}\lambda^{230}\text{Th}$$

$$\frac{d^{234}\text{U}}{dt} = \frac{d^{230}\text{Th}}{dt} = 0 \quad \text{at steady state}$$

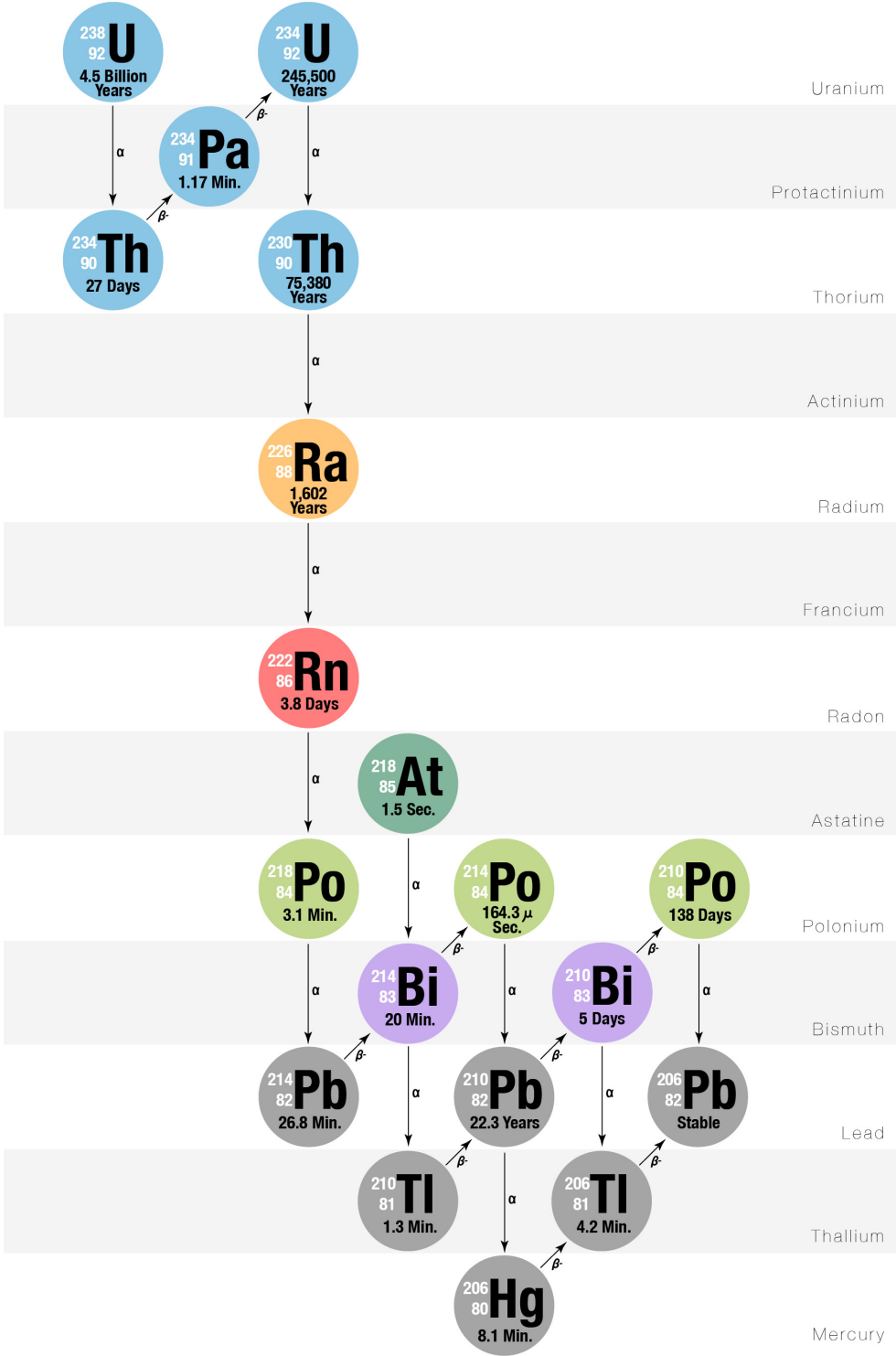
$$\frac{^{238}\lambda}{^{234}\lambda} = \frac{^{234}\text{U}}{^{238}\text{U}}$$

Modern seawater not in secular equilibrium



$$\frac{^{238}\lambda}{^{234}\lambda}$$

$$\frac{^{234}\lambda}{^{230}\lambda}$$







half life 105 billion years

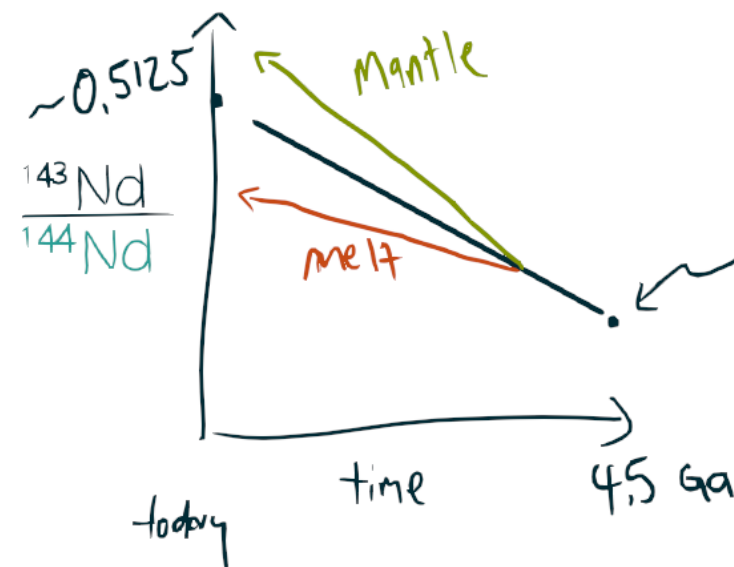
* isotope ratio more meaningful than absolute abundance, so we divide by a stable isotope

$$\left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}} \right)_{\text{today}} = \left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}} \right)_{\text{initial}} + \left(\frac{^{147}\text{Sm}}{^{144}\text{Nd}} \right)_{\text{today}} (e^{\lambda^{147}t} - 1)$$

Sm, Nd are RLE

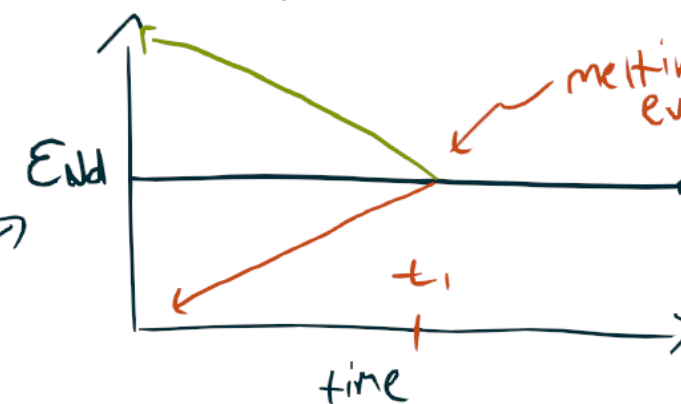
What happens when we melt the mantle? Sm, Nd incompatible

Nd more incompatible melt $\frac{\text{Sm}}{\text{Nd}}$ lower than solid



chondritic ratio evolution (~ Primitive Mantle)

lab 5



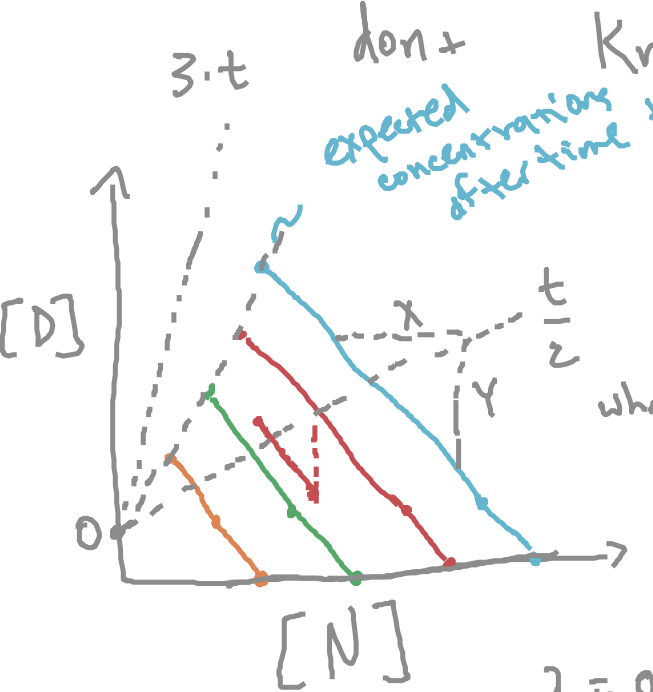
with model age t₁

Isochrons

how much Descendant isotope does our sample start with?

Partition coefficients for N
 $D_A^N = 4$
 $D_B^N = 3$
 $D_C^N = 2$
 $D_D^N = 1$
 $D_D^D = 0$
 $N \rightarrow D$

how do determine the age of a sample if we

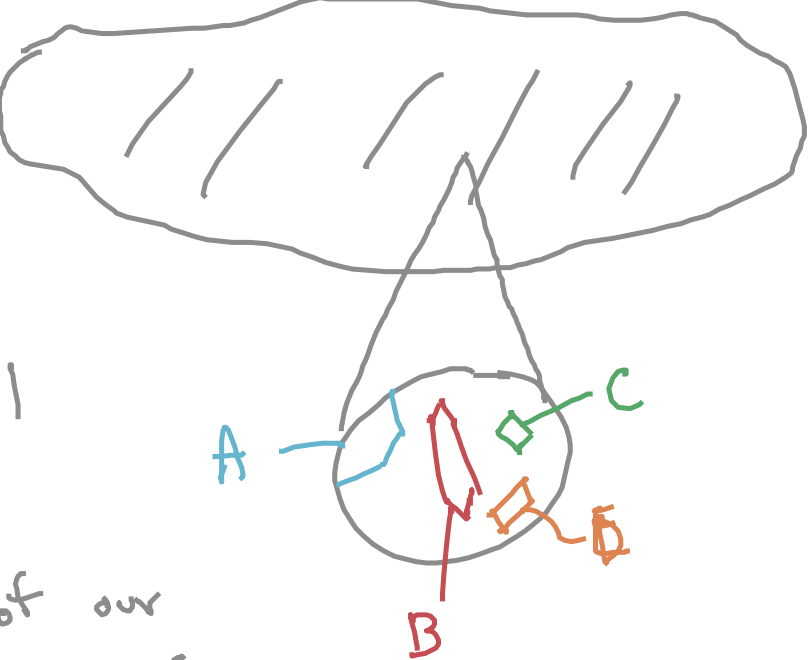


Know D_0 ?

$$D = D_0 + N(e^{\lambda t} - 1)$$

$Y = B + X M$

what is $\frac{X}{Y}$? slope of 1



isochrons:
the slope of our
sample observations
is a function of time

if $m=10$

$$10 = e^{\lambda t} - 1$$
$$11 = e^{\lambda t}$$
$$\ln 11 = \lambda t$$

$\ln e^x = x$

$\lambda = 0.01$

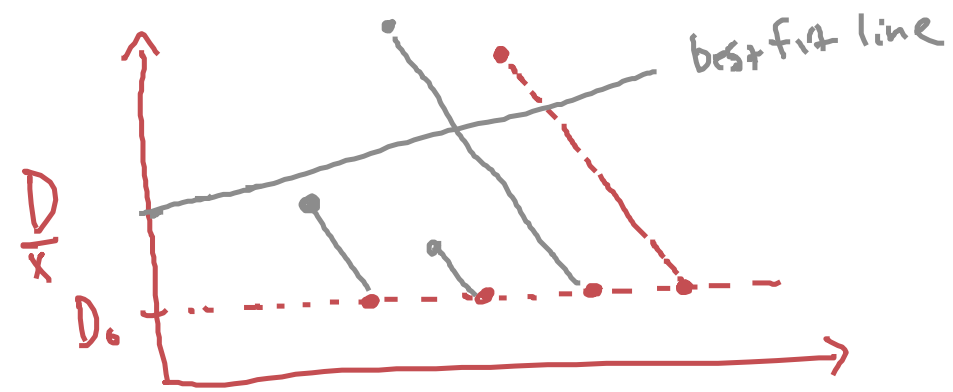


Isochrons

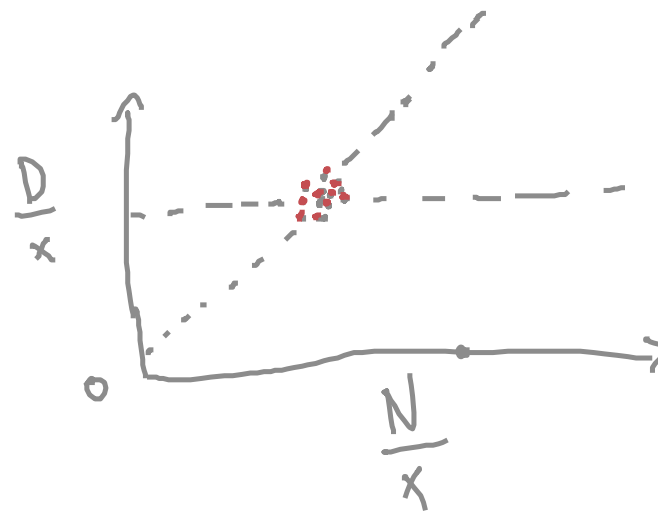
if a suite of samples forms a line in $\frac{D}{N}$, then the samples may have the same age and initial D_0 .

to use an isochron:

1. Samples have same age
2. Samples have the same initial Descendant isotope
3. D and N isotopes do not enter or leave the mineral or rock since formation (closed system)



$\frac{N}{X}$



it helps when the samples have very different N concentrations

