



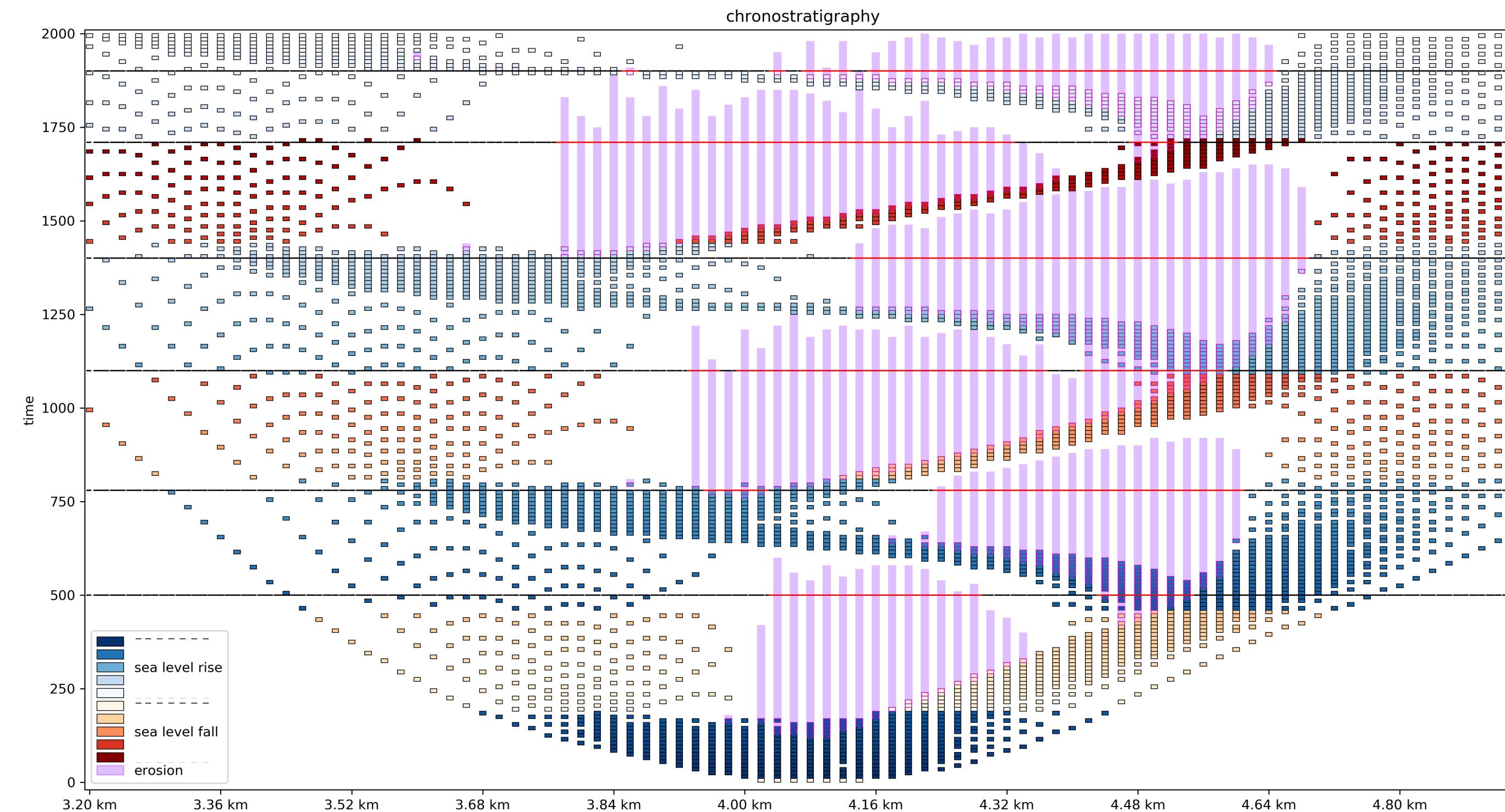
Lectures 11-12: Cycles

1. Quantitative correlation
2. An example from our model
3. Enter cyclostratigraphy
4. How to identify a cycle
5. Fourier Transforms

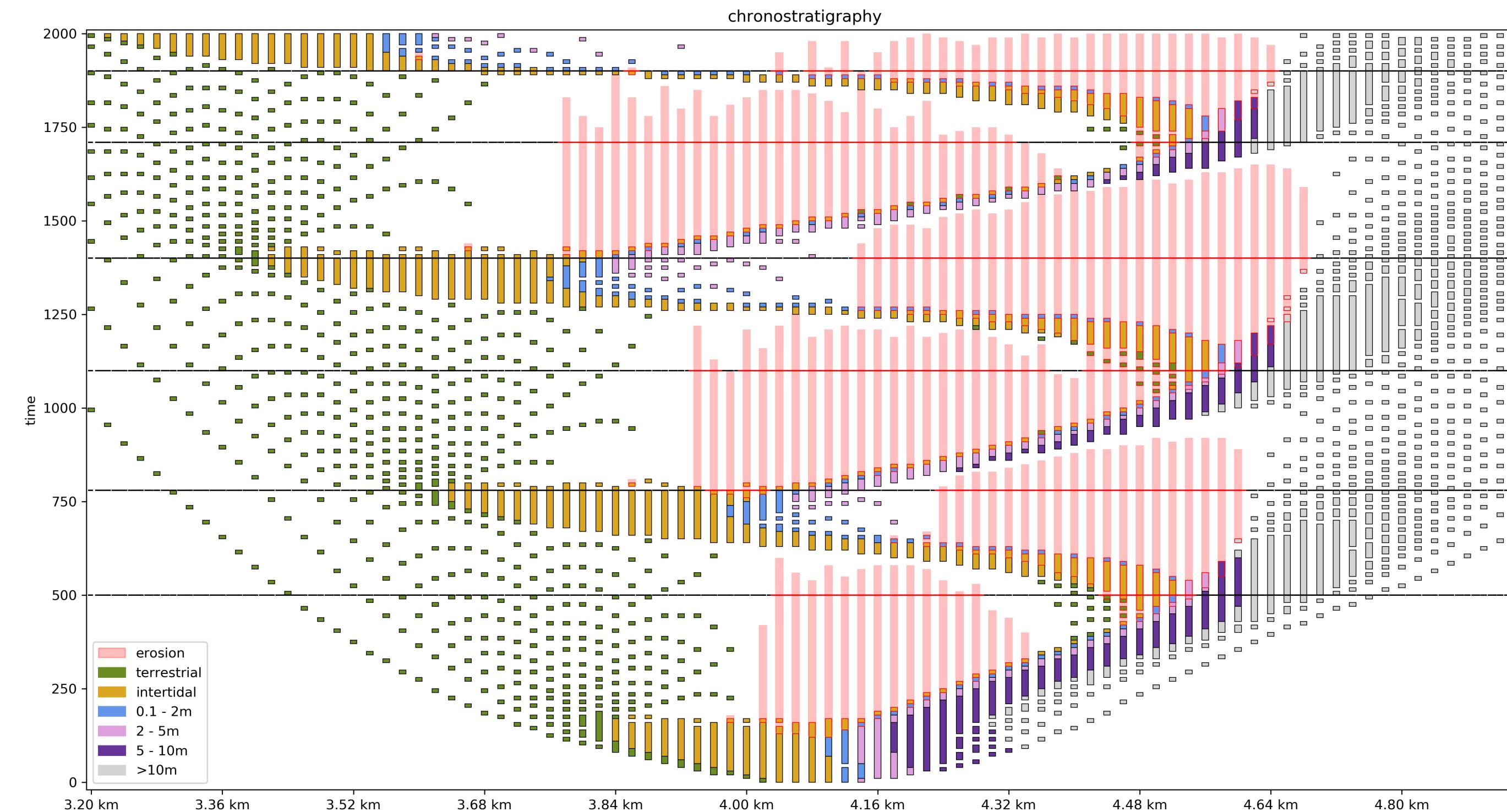
We acknowledge and respect the *lək'ʷəŋən* peoples on whose traditional territory the university stands and the Songhees, Esquimalt and *WSÁNEĆ* peoples whose historical relationships with the land continue to this day.



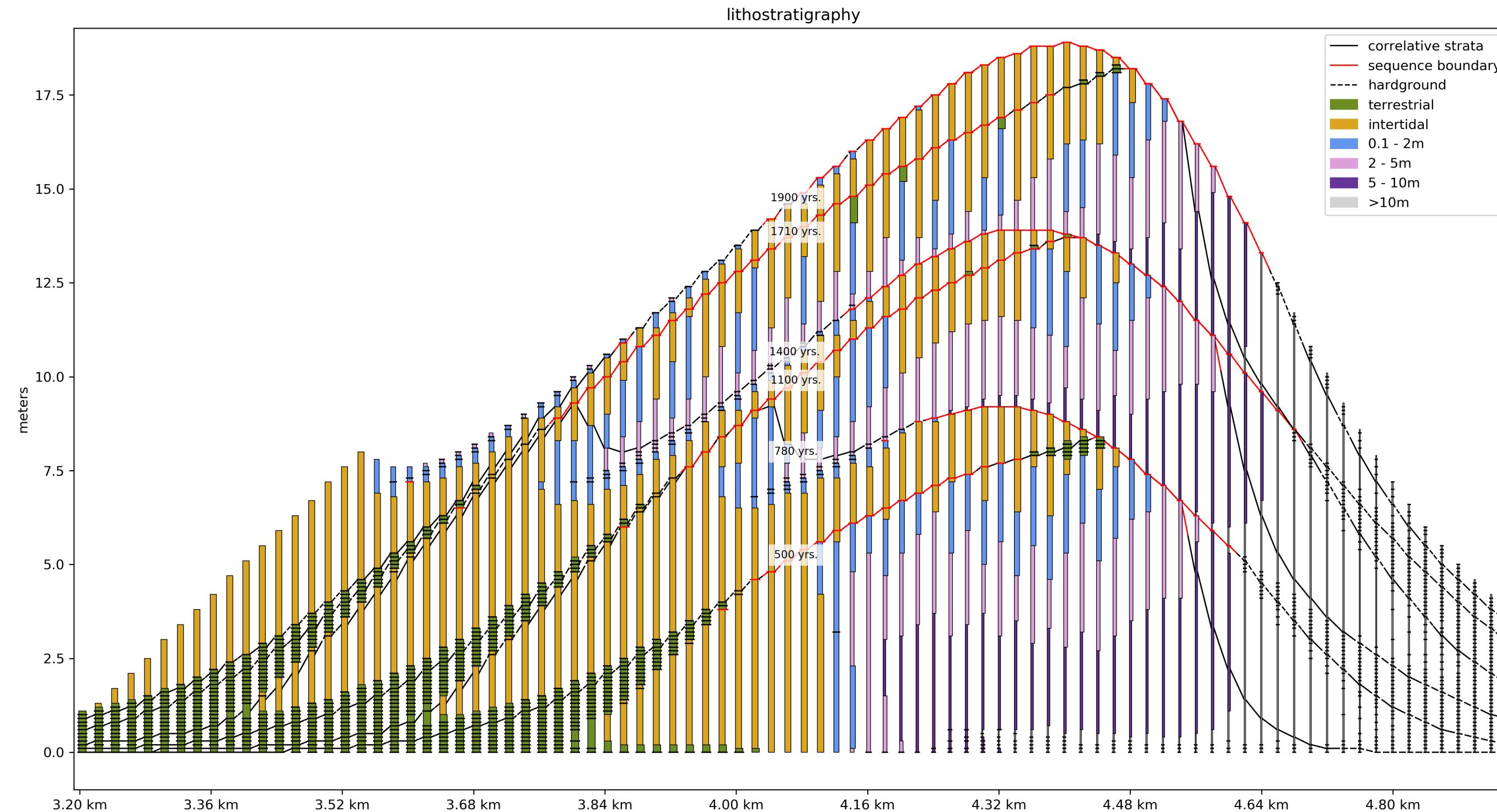
Brief return to Wheeler diagrams



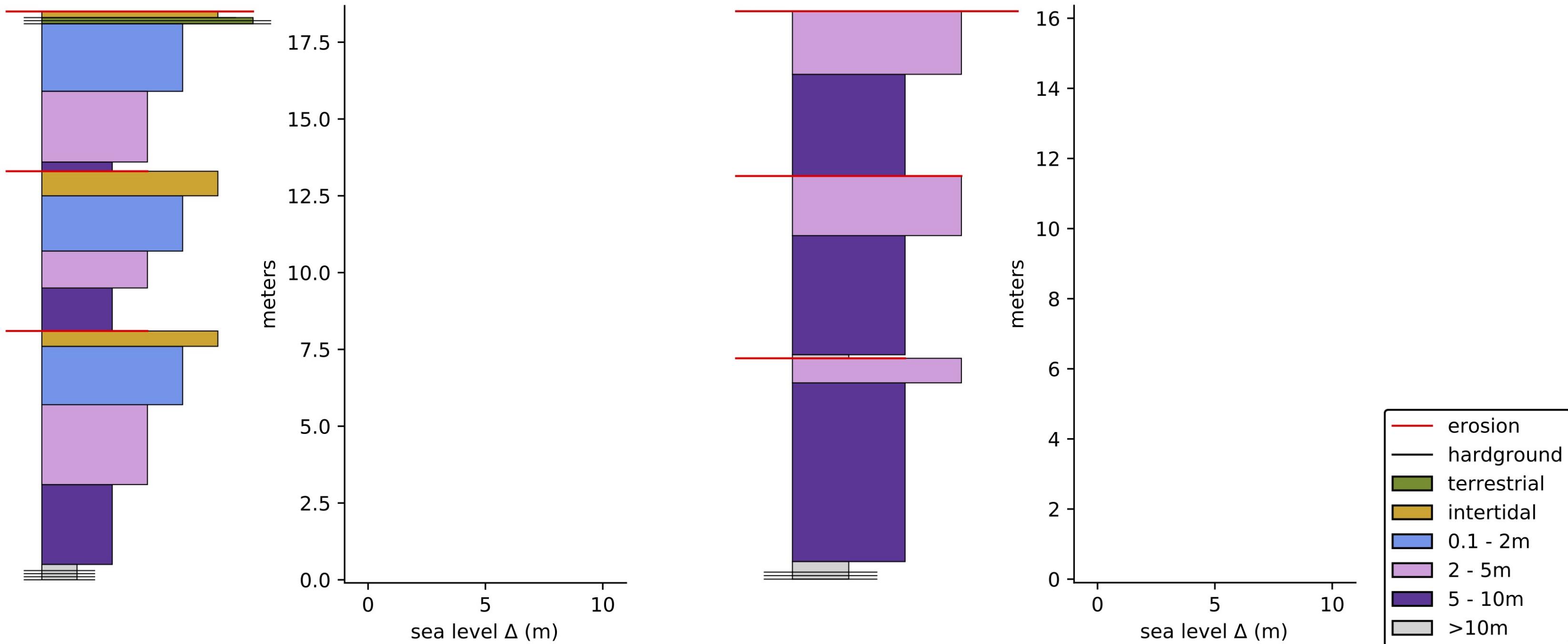
Brief return to Wheeler diagrams



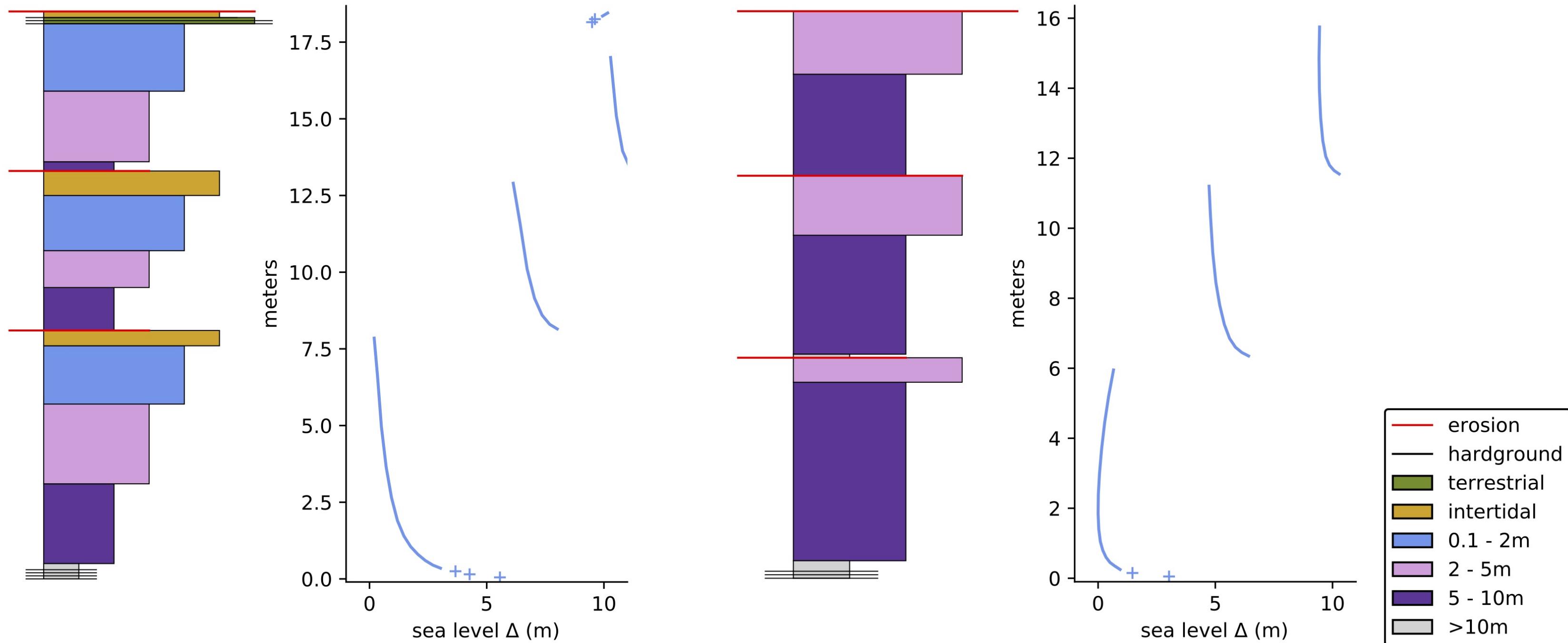
Brief return to Wheeler diagrams



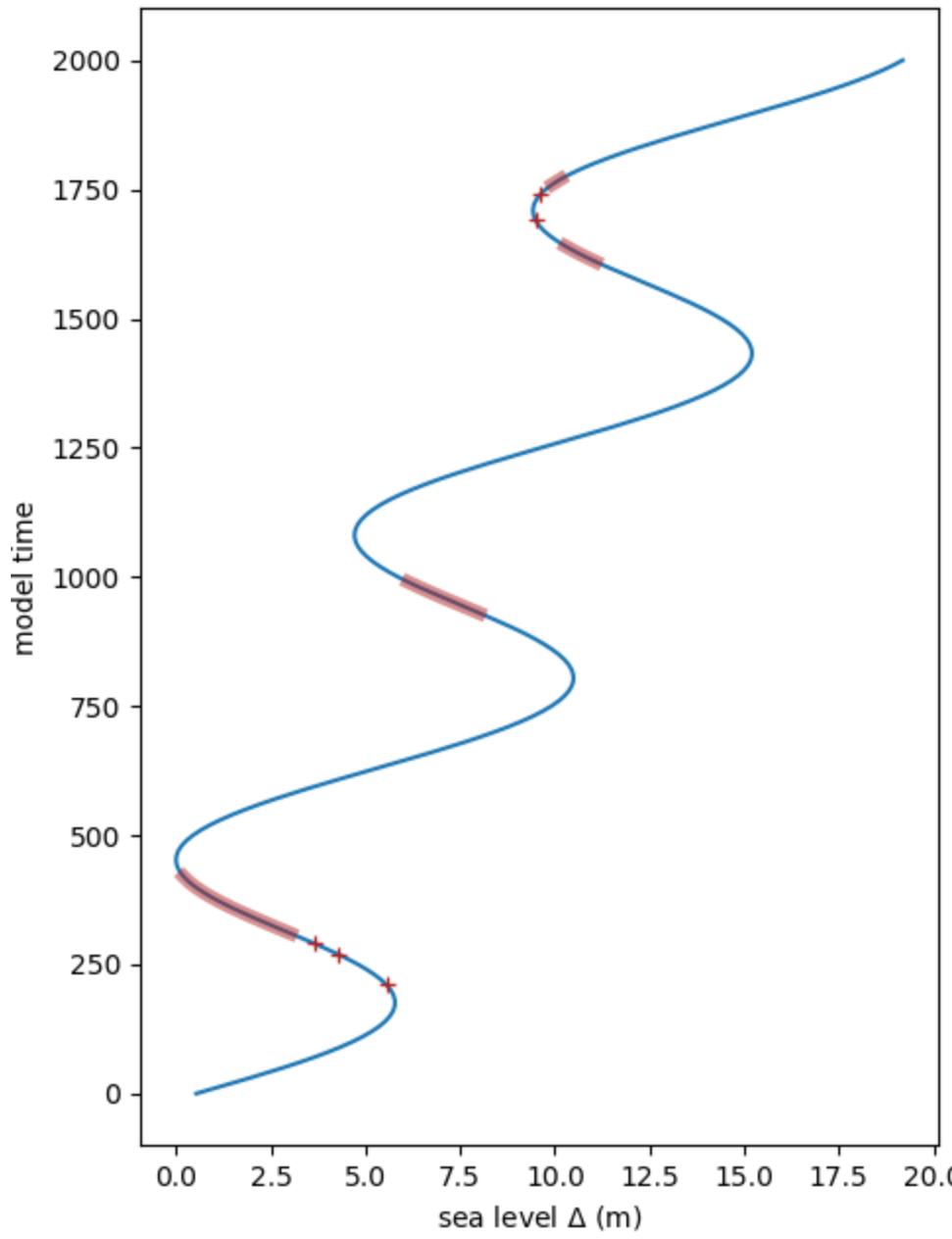
Correlating sequences: an example from our model



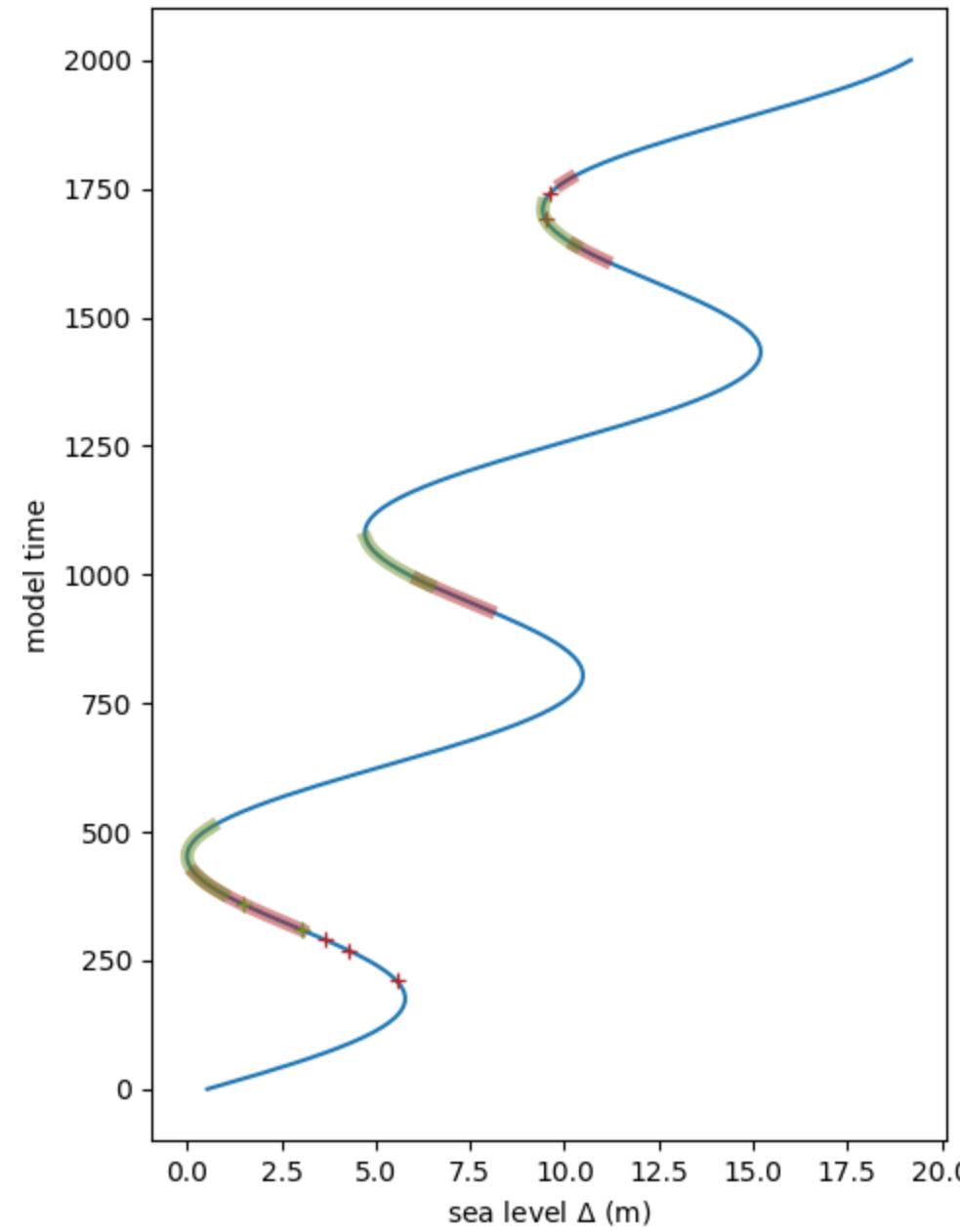
Correlating sequences: an example from our model



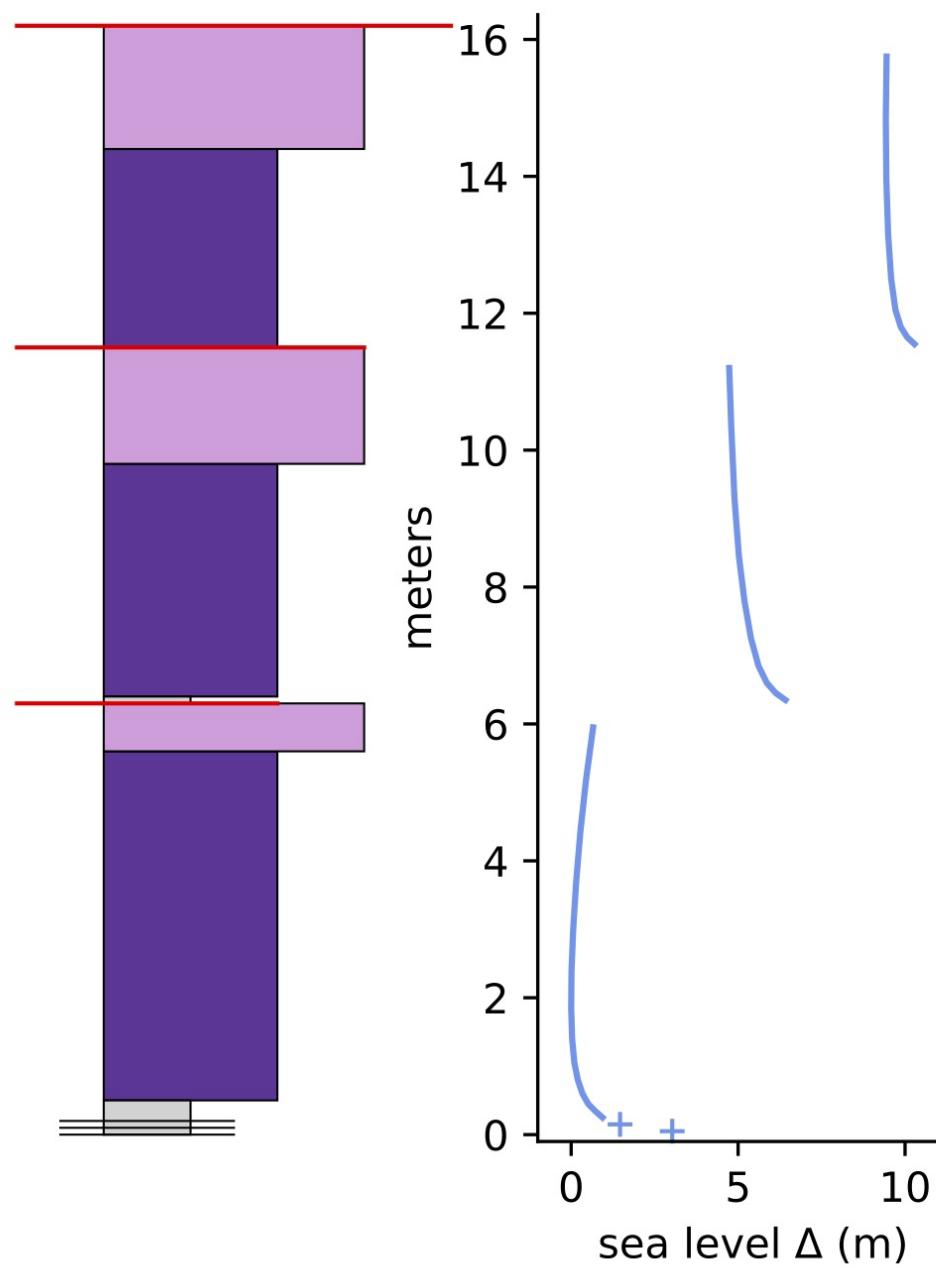
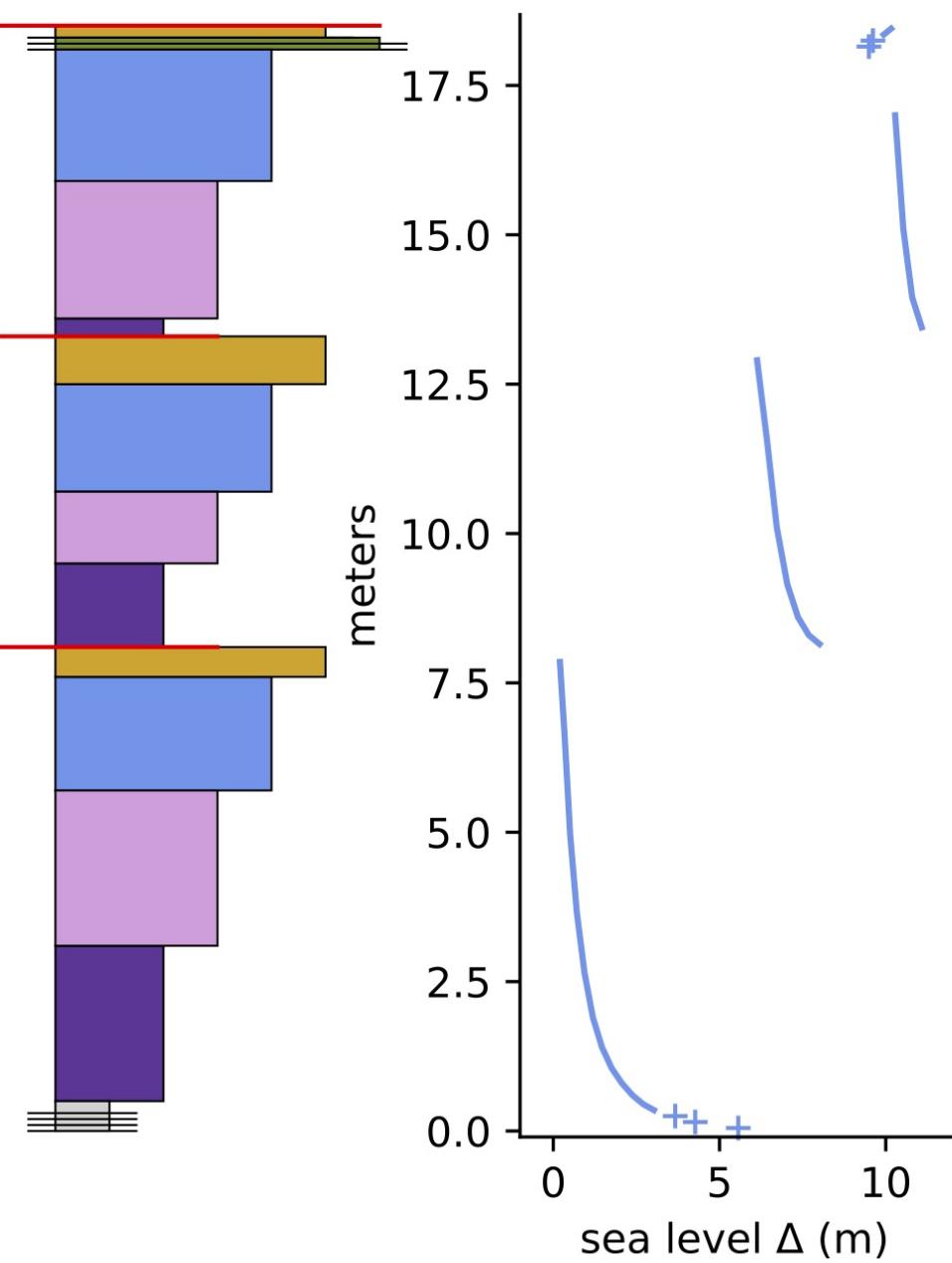
Correlating sequences: an example from our model



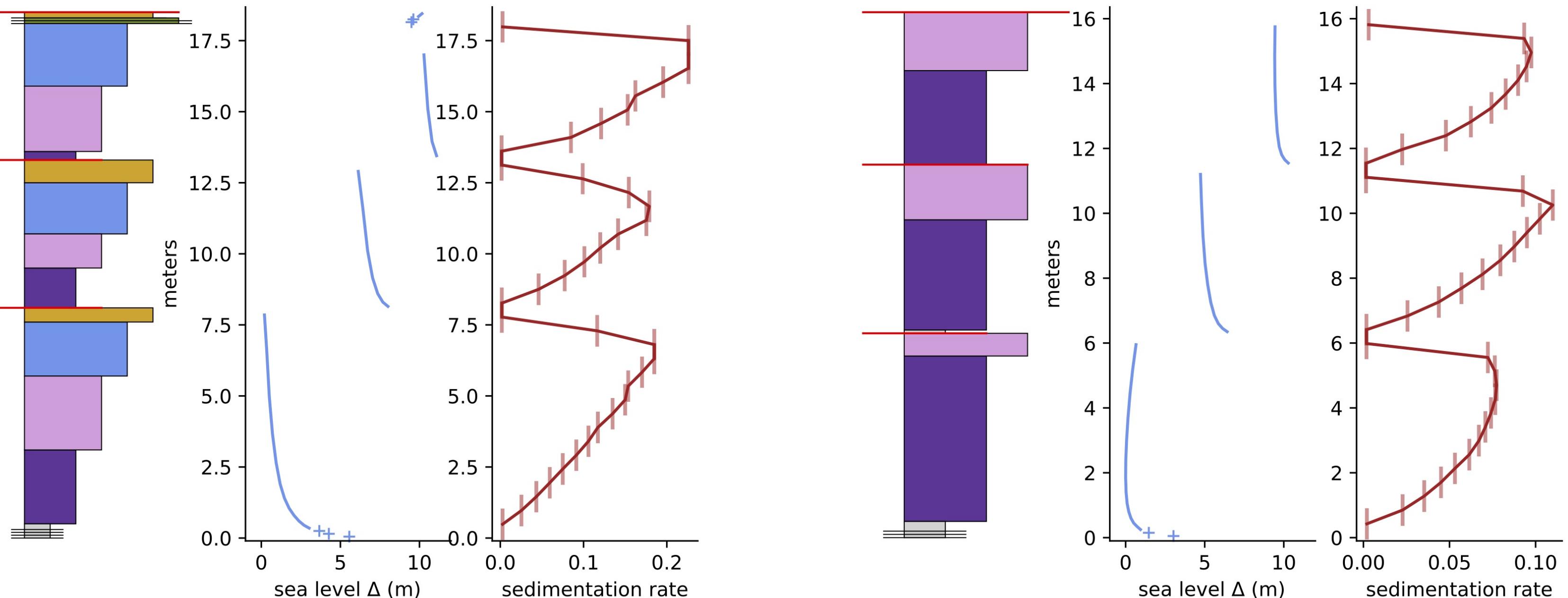
Correlating sequences: an example from our model



Why do the sea-level cycles look funny?

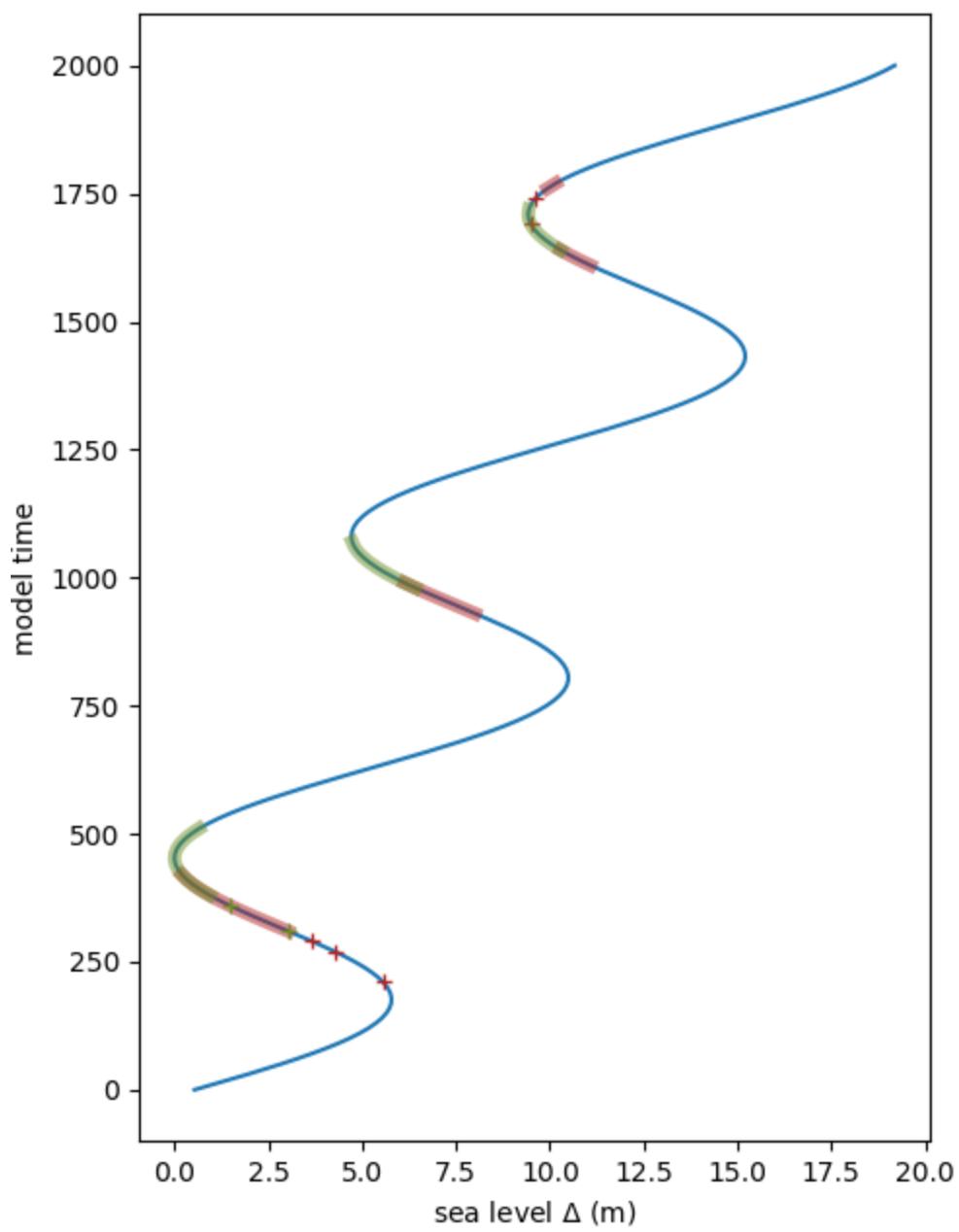


Why do the sea-level cycles look funny?



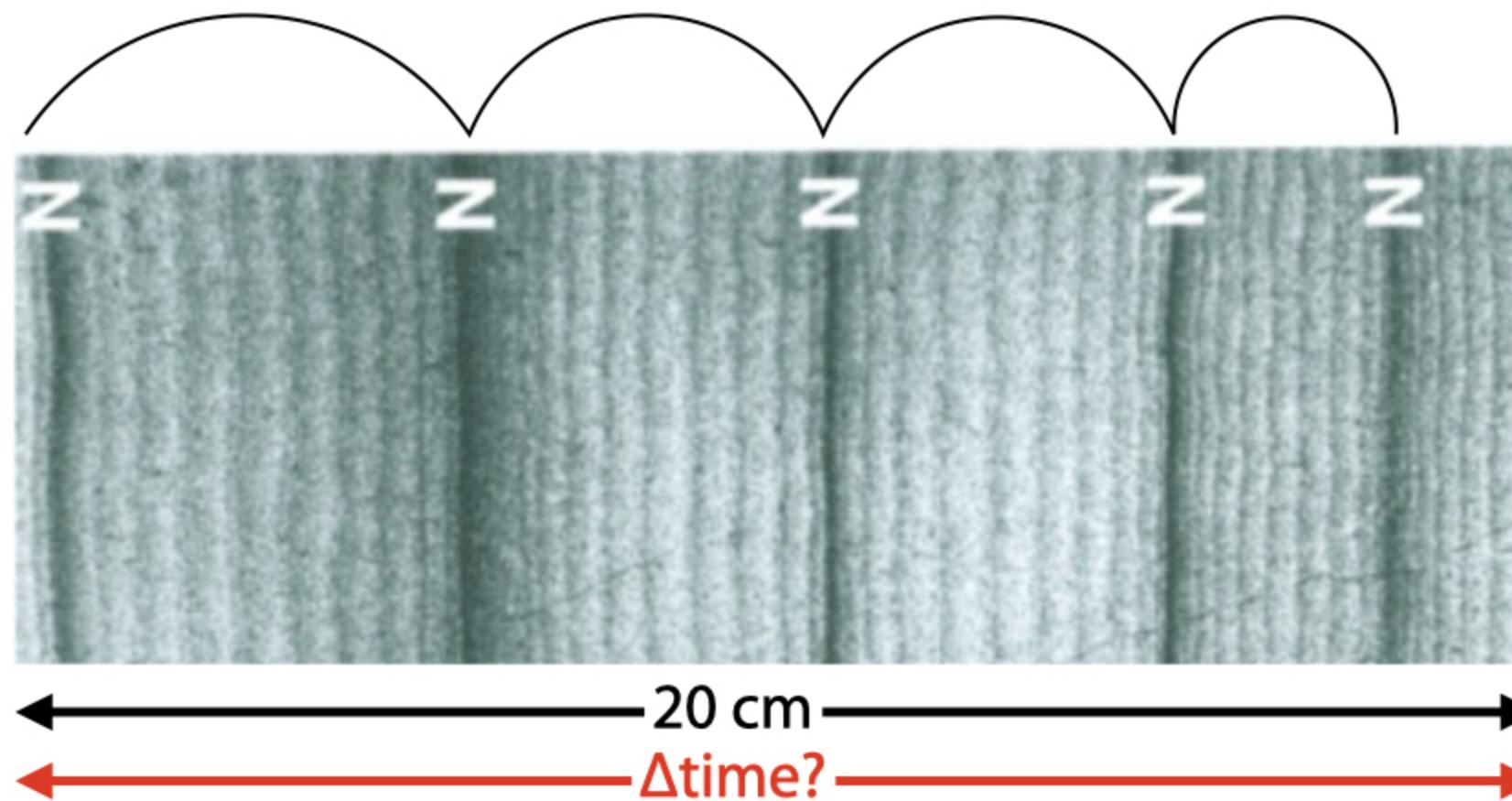
Correlating sequences

- an amazing key would be detecting this sea level signal
- how could this be done?



Cyclostratigraphy

- Cyclostratigraphy is a sub-discipline of stratigraphy that seeks to identify, characterize and interpret cyclic variations in the stratigraphic record
 - identify cycles → interpret timing of cycles → age models and correlations
 - fundamentally, it is about explaining the processes *behind* a record



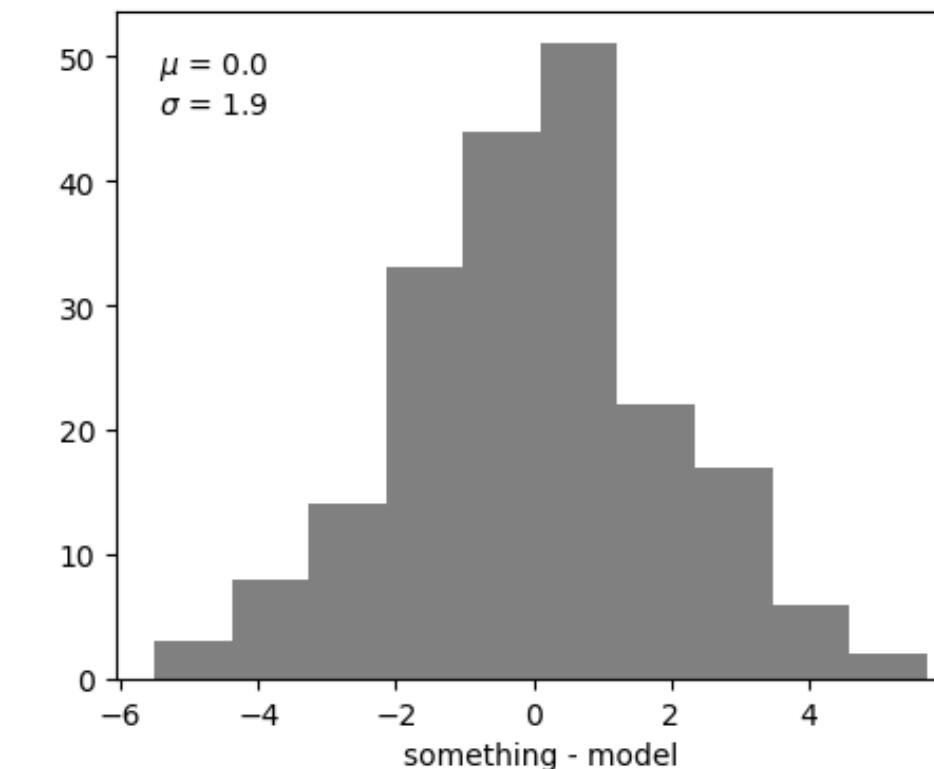
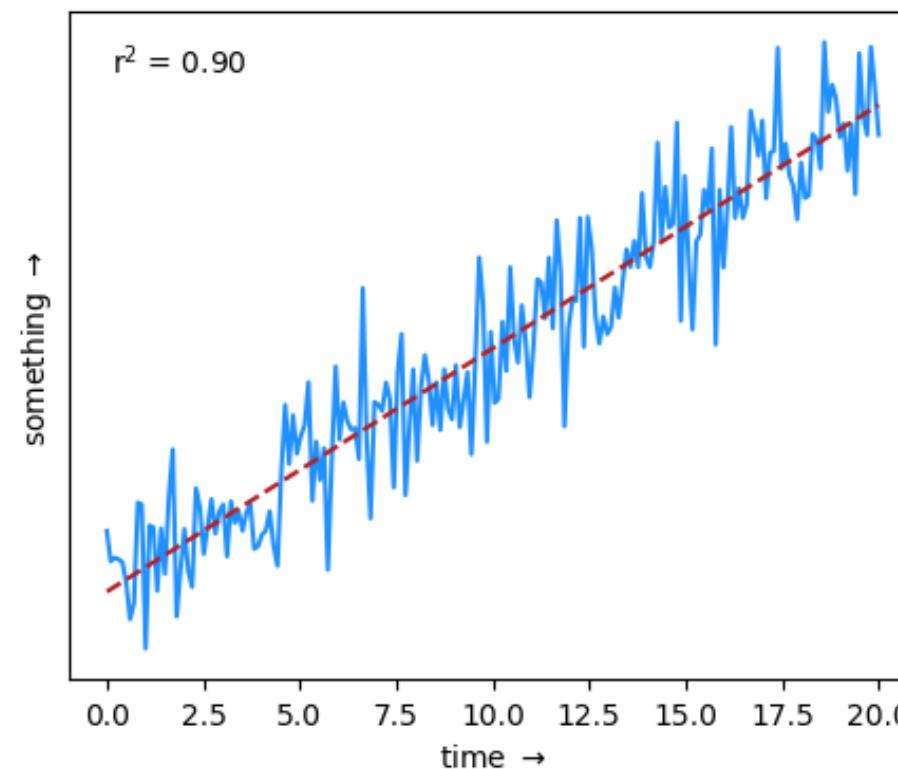
Identifying cycles

```
In [5]: t = 20 # total time series  
N=200 # sample spacing  
x = np.linspace(0.0, t, N) #noise amplitude  
a=2  
noise=np.random.normal(0,a,N) #total signal  
y=x+noise # plot_data_res(x,y)
```



Identifying cycles

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Identifying cycles

In [10]:

```
#add a sine wave
y=y+8*np.sin(0.5*2*np.pi*x)

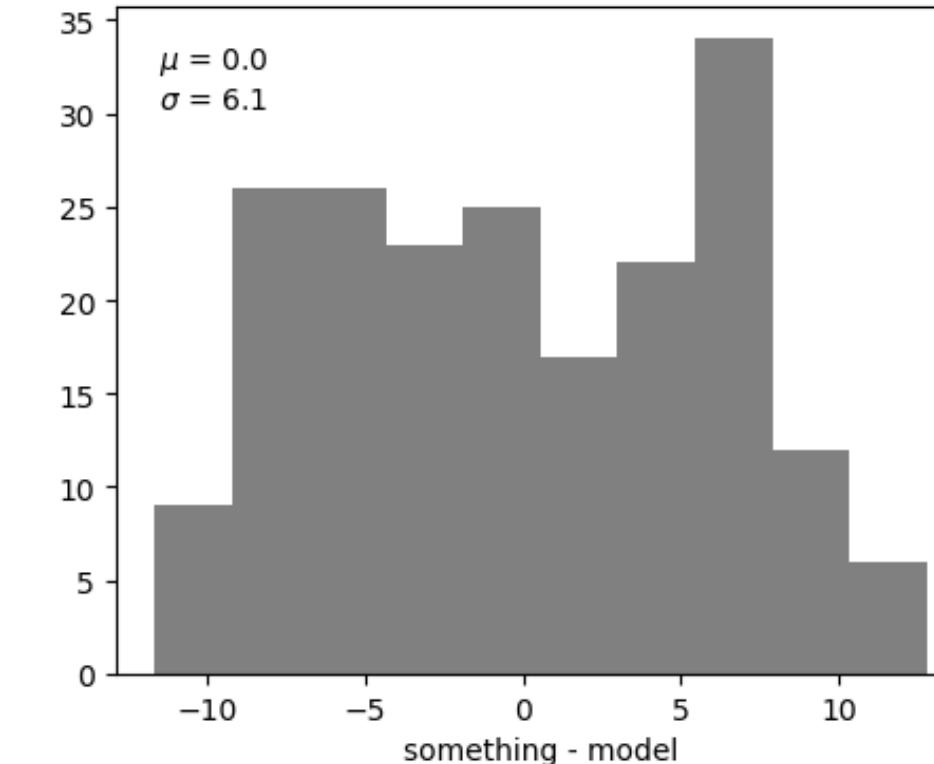
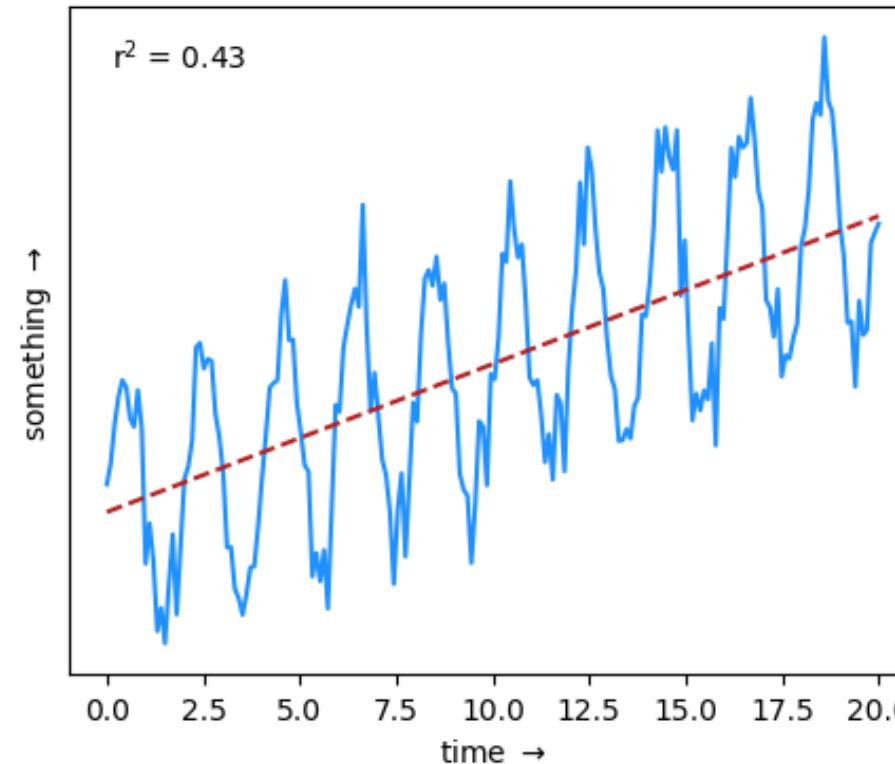
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Identifying cycles

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Identifying cycles

In [6]:

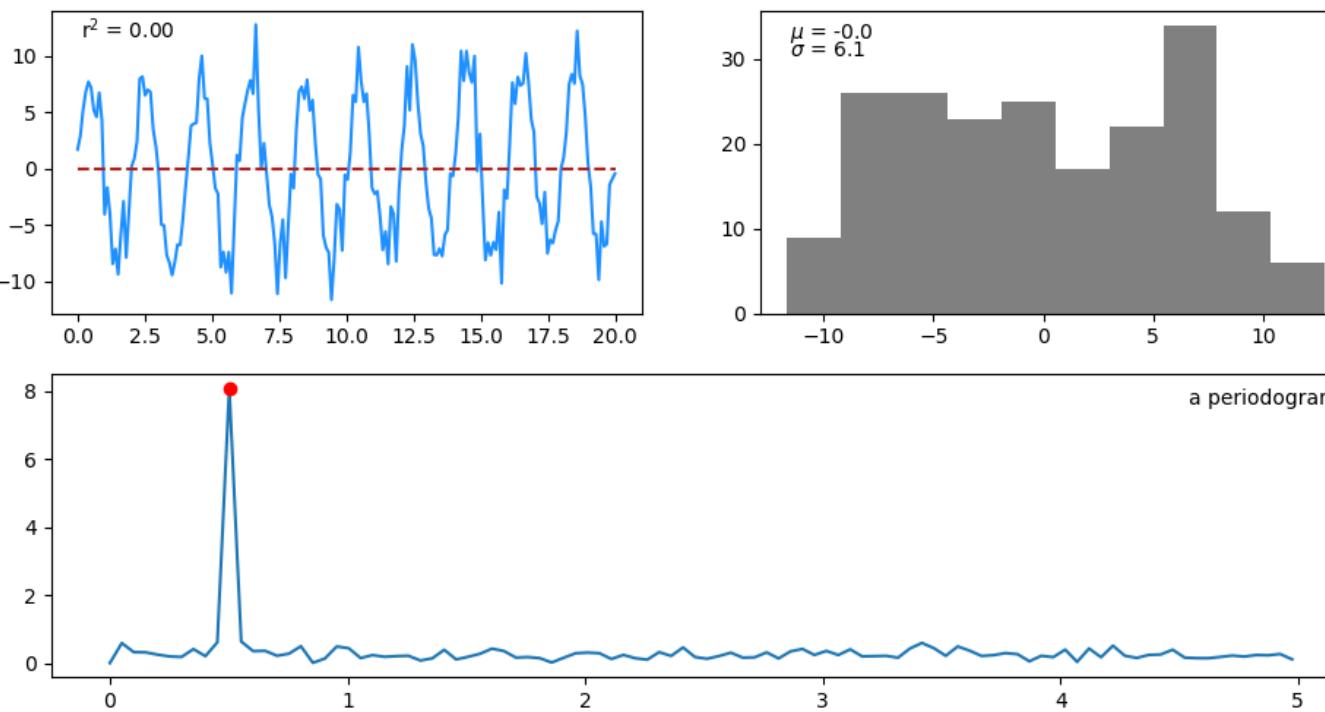
```
#remove the linear rise
y=signal.detrend(y)
#define sampling interval for frequency axis
dt=np.round(np.diff(x),10)[0]
#perform Fast Fourier transform
yf = fft(y)
xf = np.linspace(0.0, 1.0/(2.0*dt), N//2)
#plot results
# axes,max_power,freq=plot_data_res_fft(x,y,yf,xf)
```



Identifying cycles

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Identifying cycles

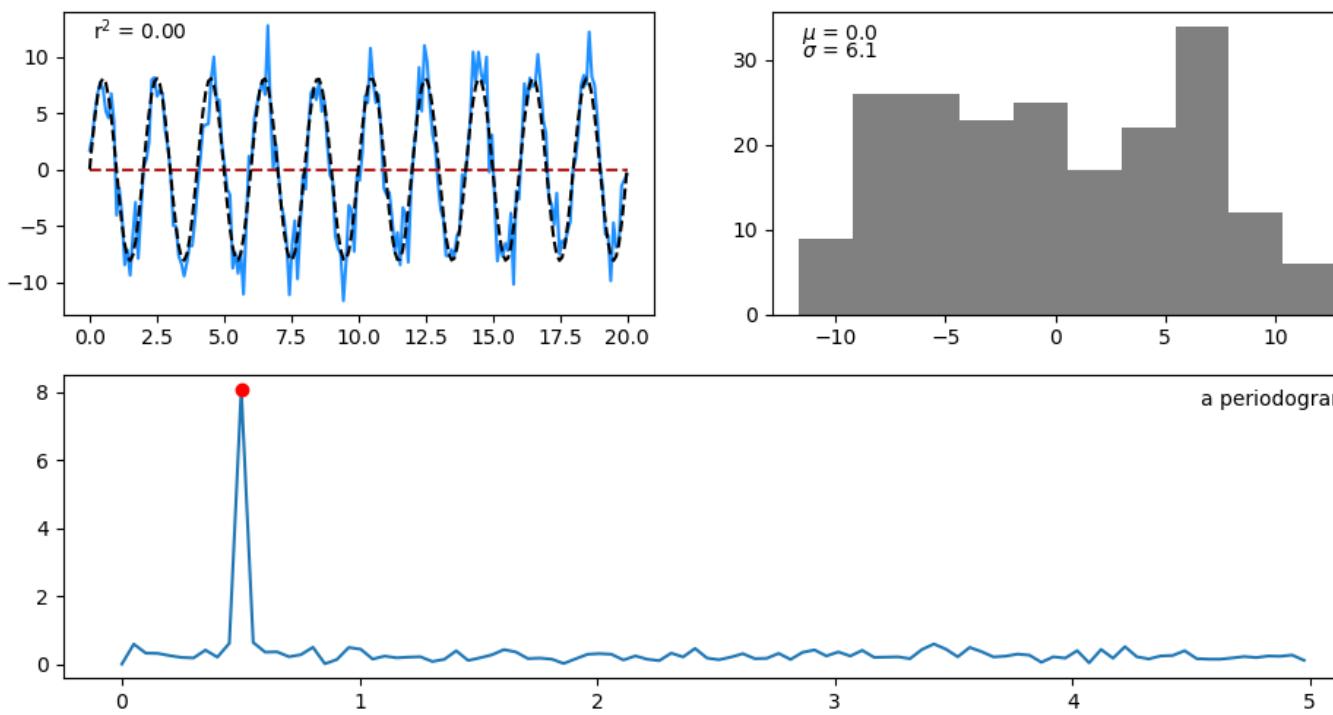
```
In [8]: #create and plot the recovered sinusoid  
amp=max_power  
ang_freq=freq * 2*np.pi  
phase=0 * np.pi/180  
cycle=amp*np.sin(ang_freq*x + phase)  
#axes,max_power,freq=plot_data_fft(x,y,yf,xf)  
#axes[0].plot(x,cycle,'k--')
```



Identifying cycles

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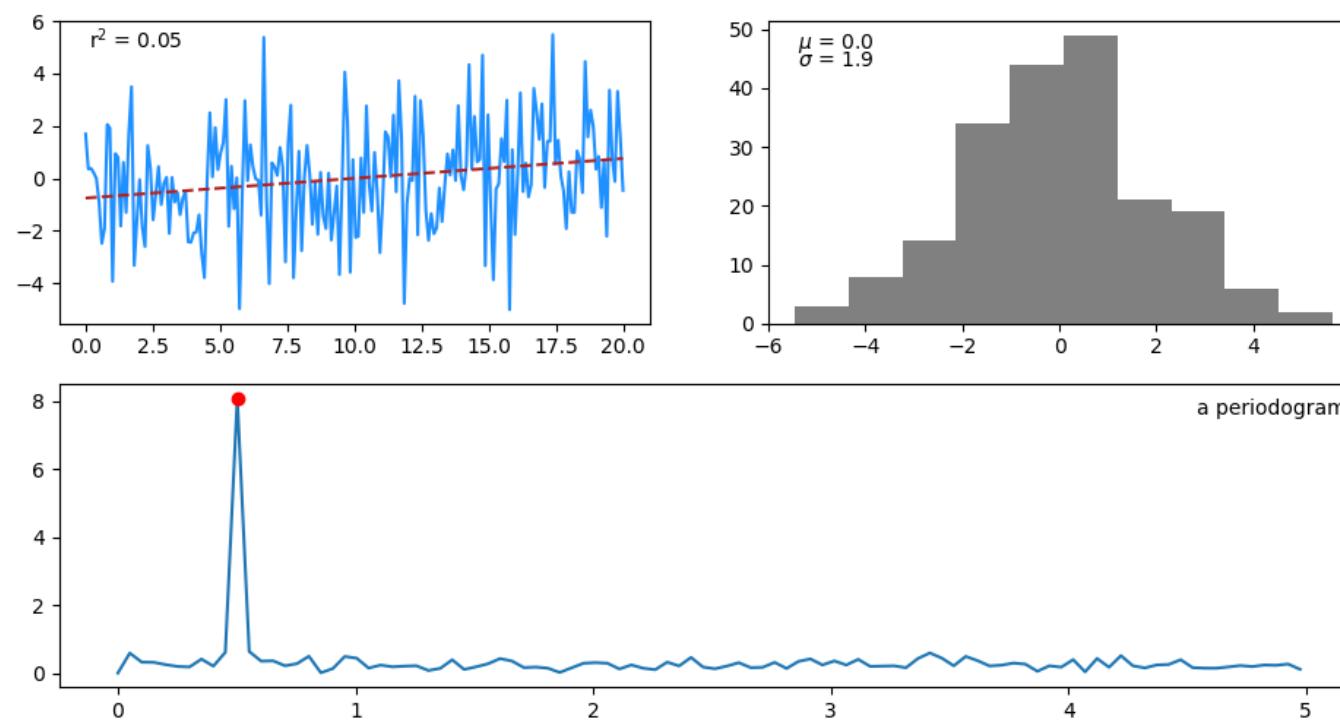
Identifying cycles

```
In [18]: ##recalculate residuals  
#axes=plot_data_res_fft(x,y-cycle,yf,xf)
```

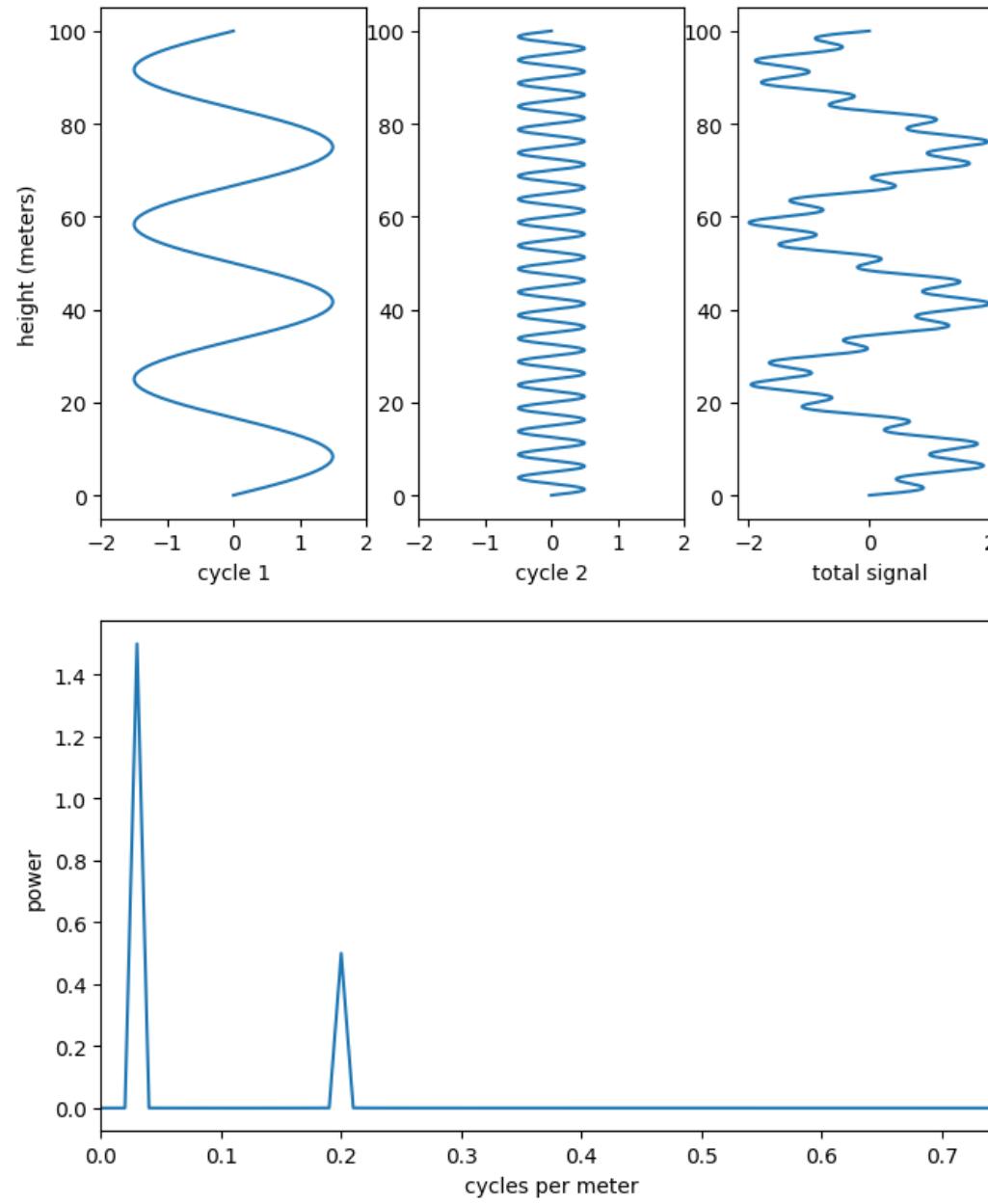


Identifying cycles

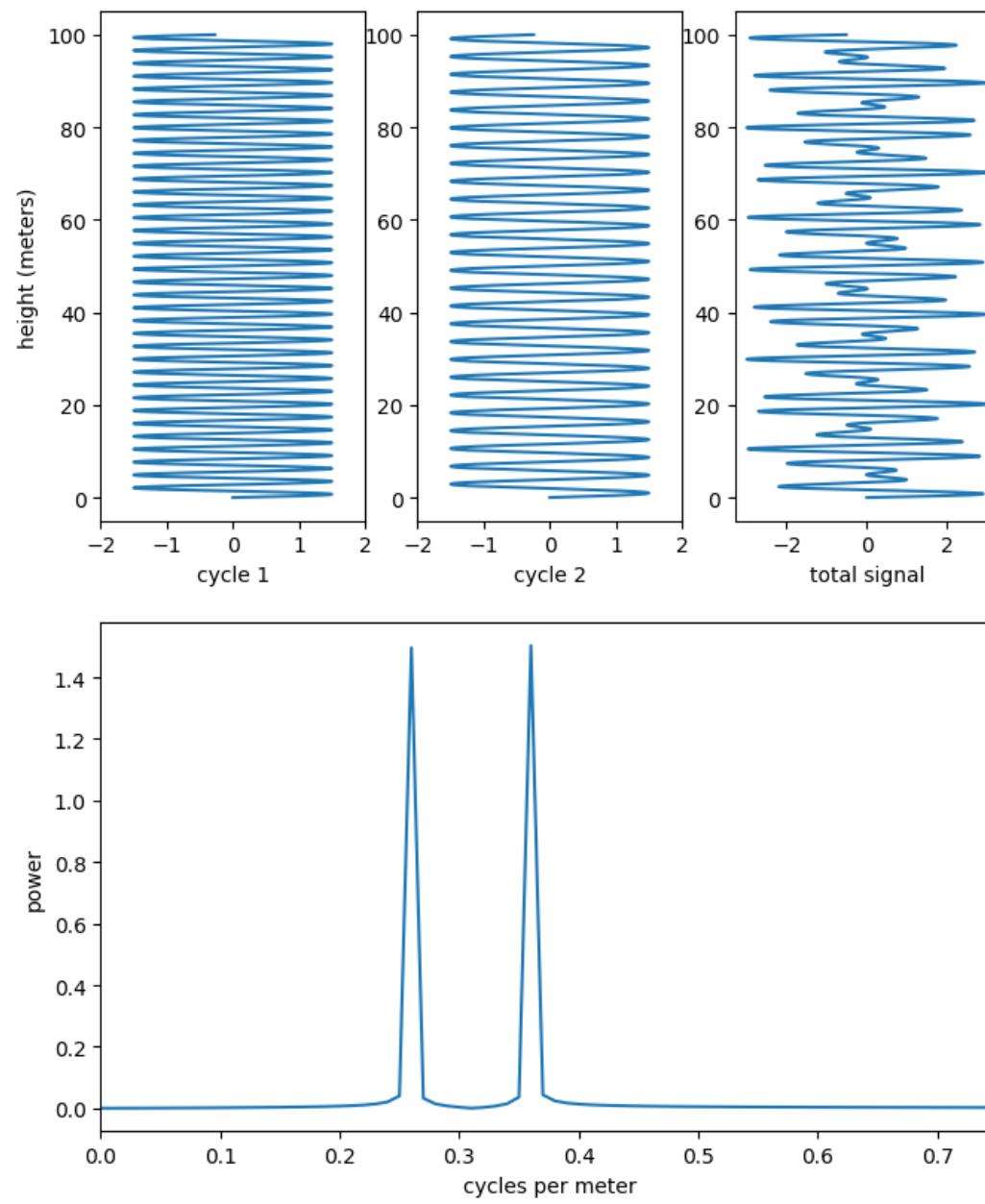
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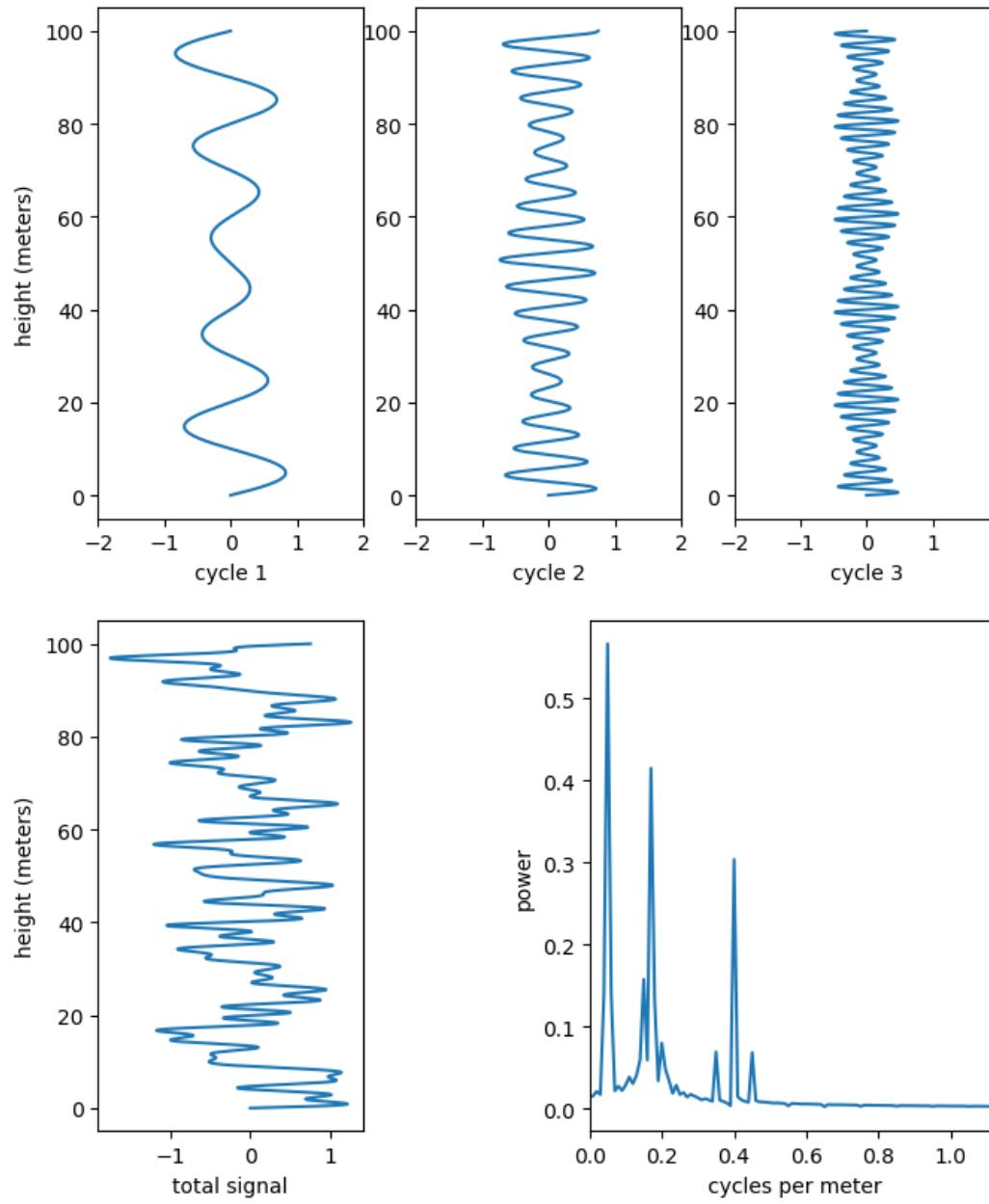
quickly we cannot rely on our eyes ...



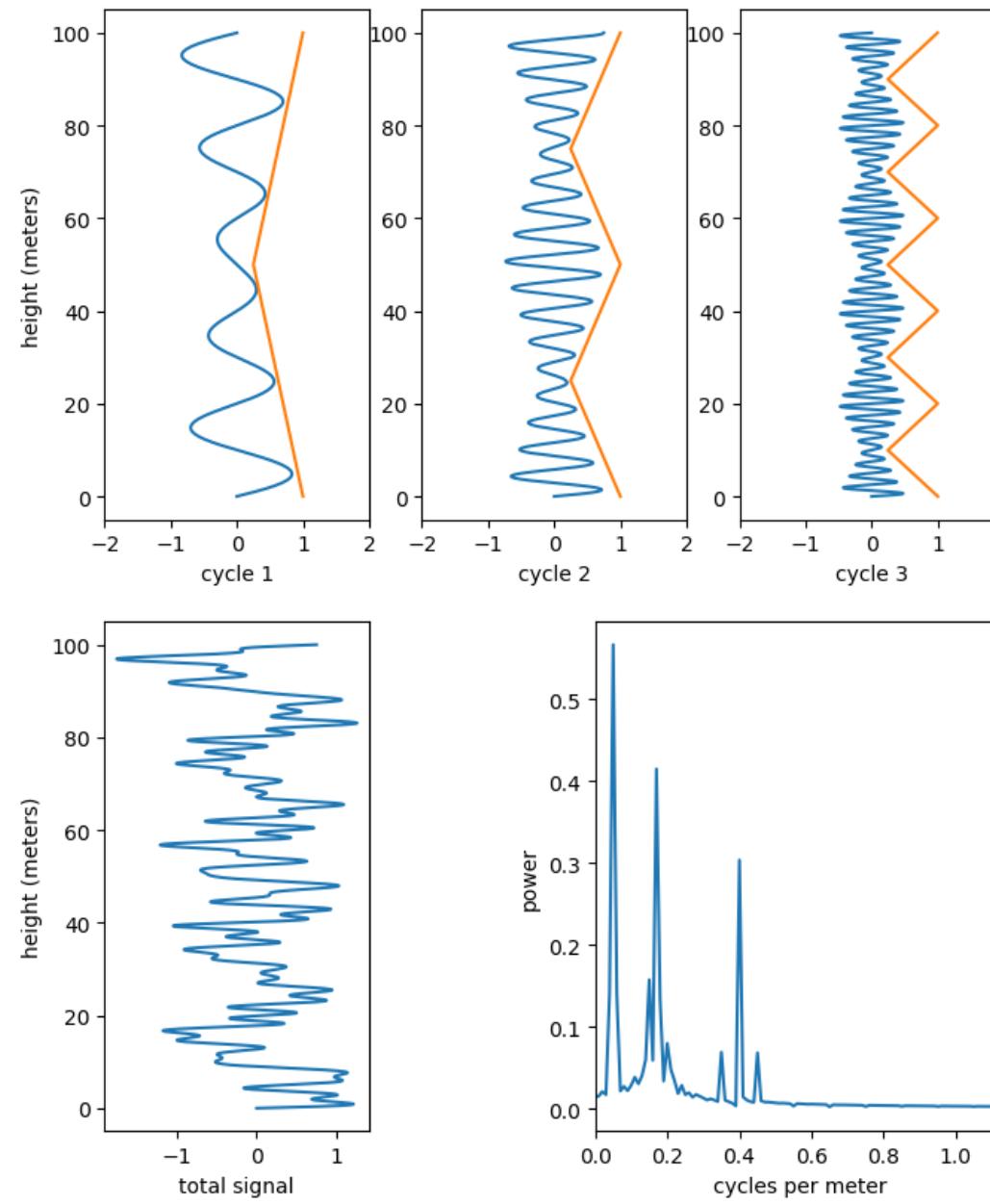
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quickly we cannot rely on our eyes ...

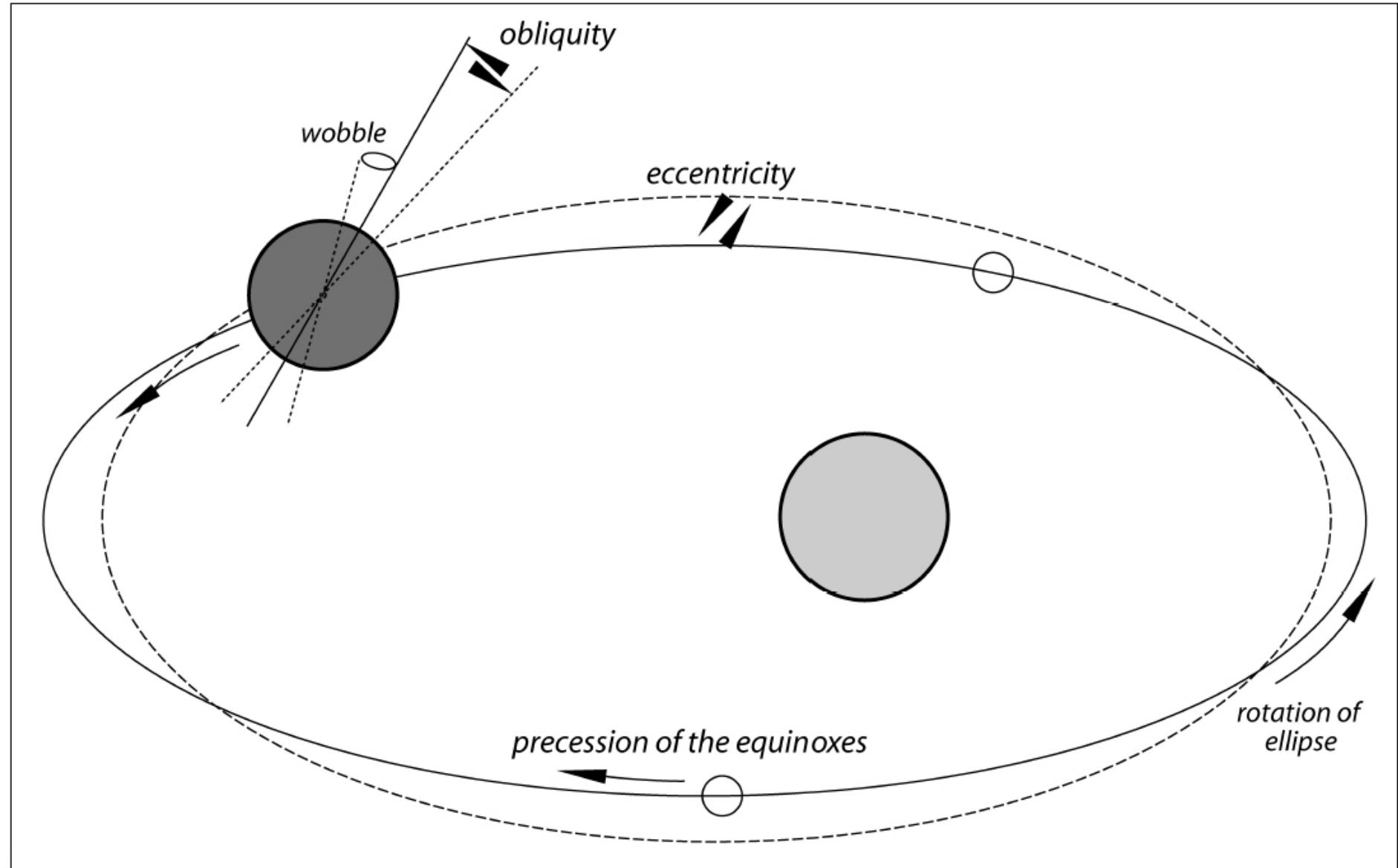


quickly we cannot rely on our eyes ...



(Draw cycles in sediments)





Axial Precession (Wobble)

26,000-year cycles

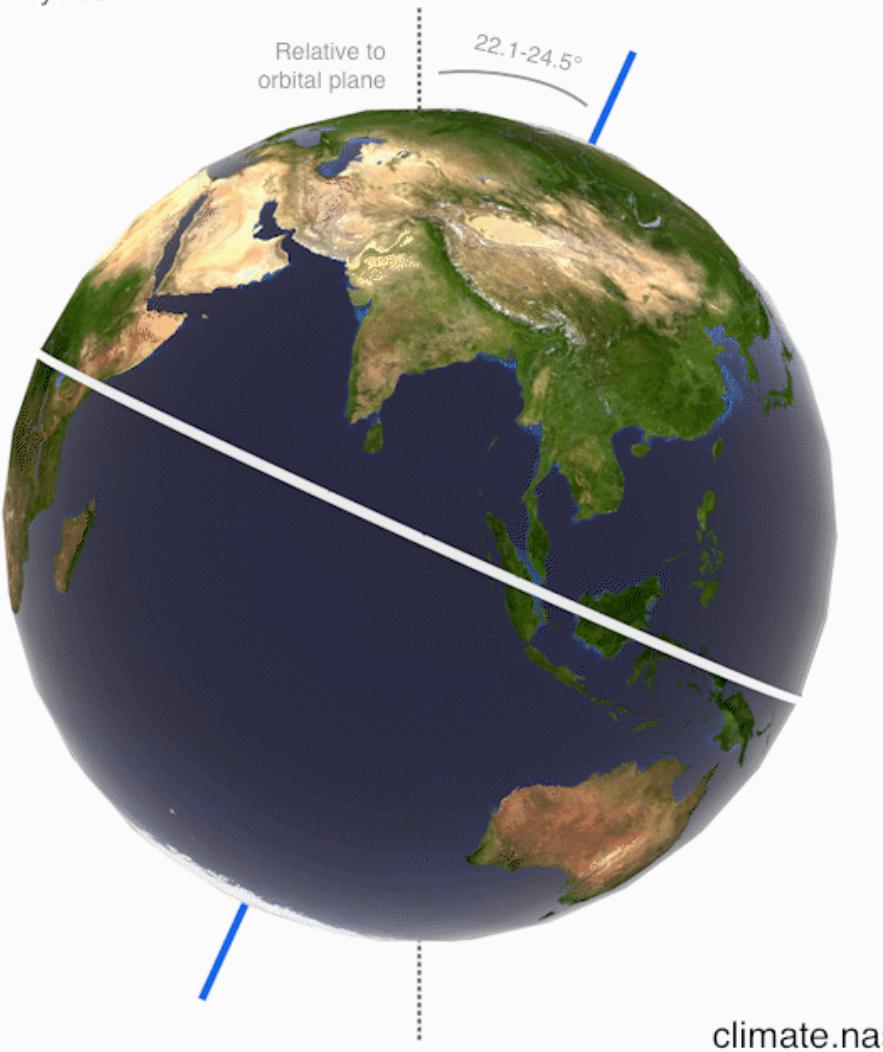


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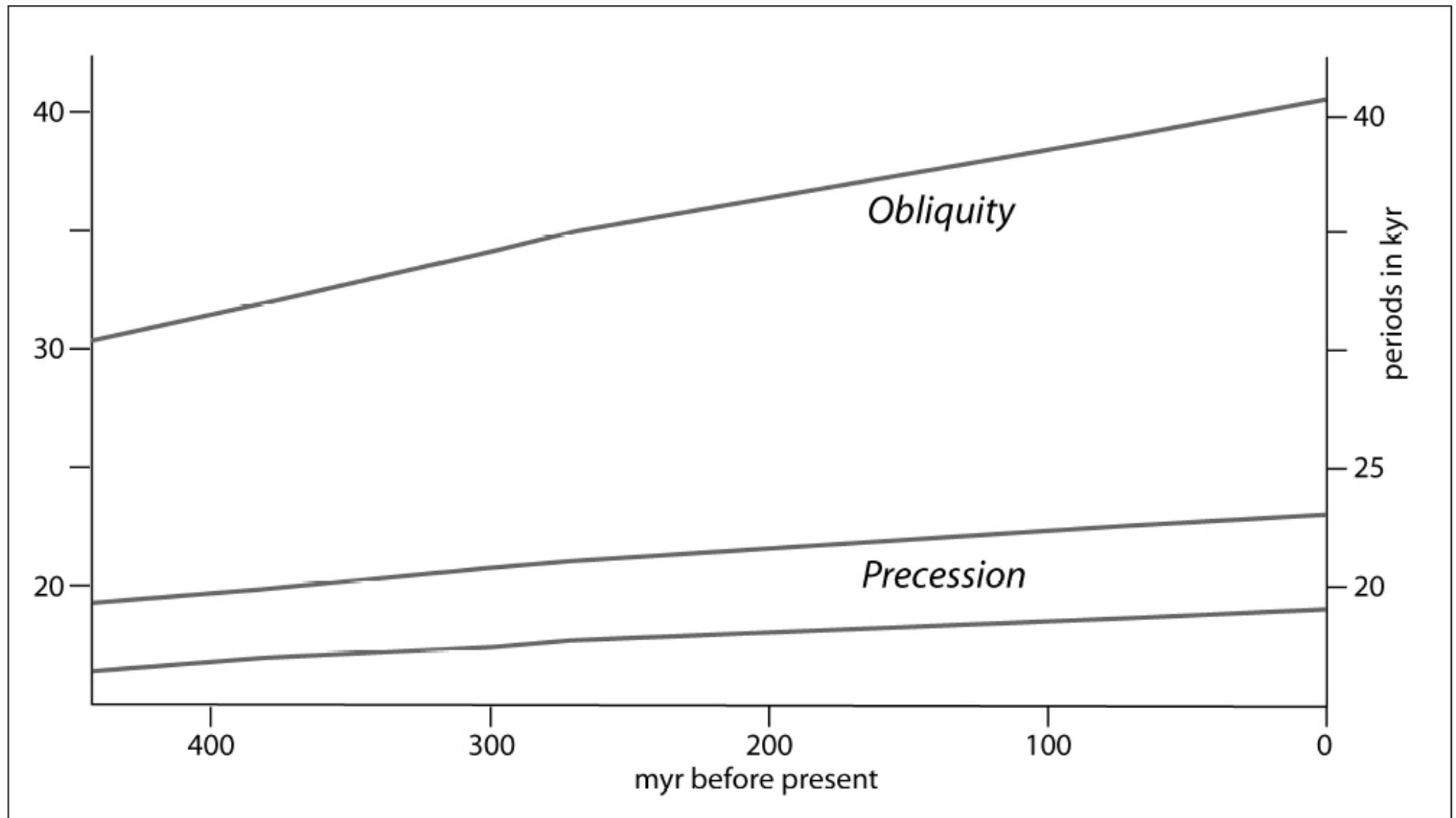
Changes in Obliquity (Tilt)

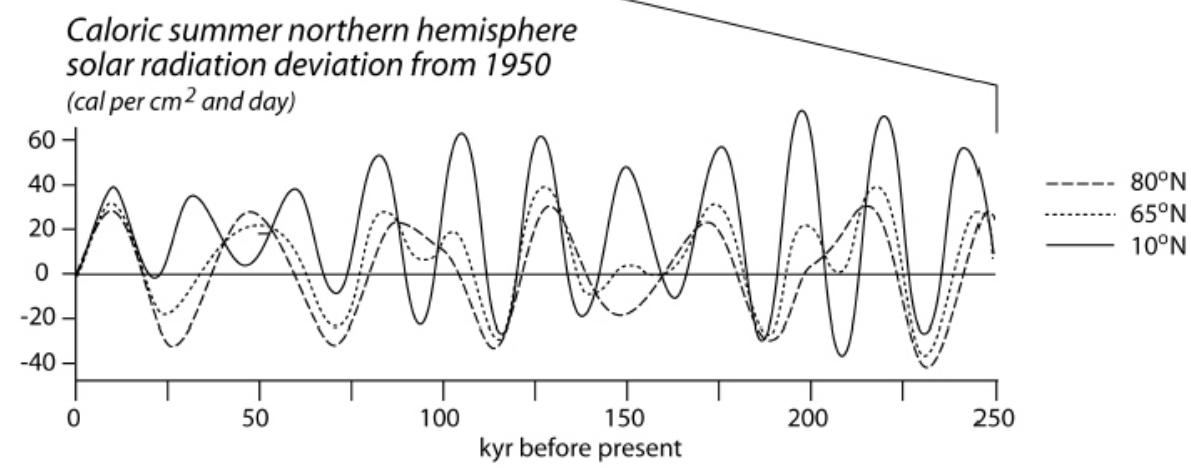
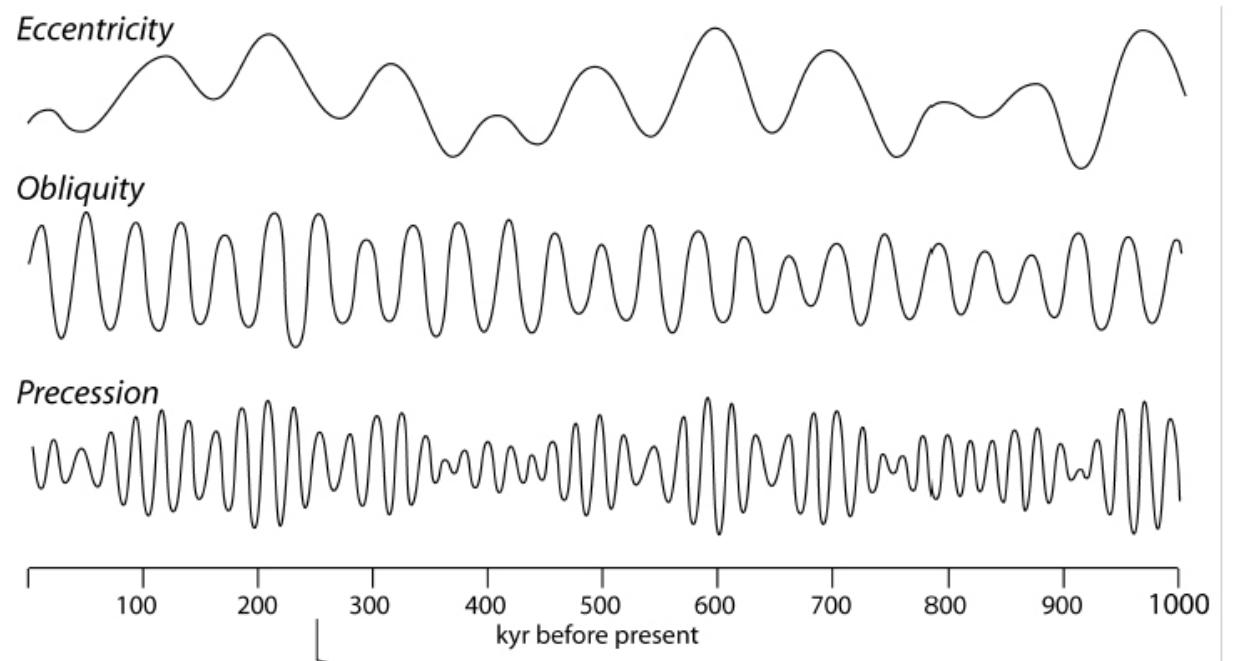
41,000-year cycles

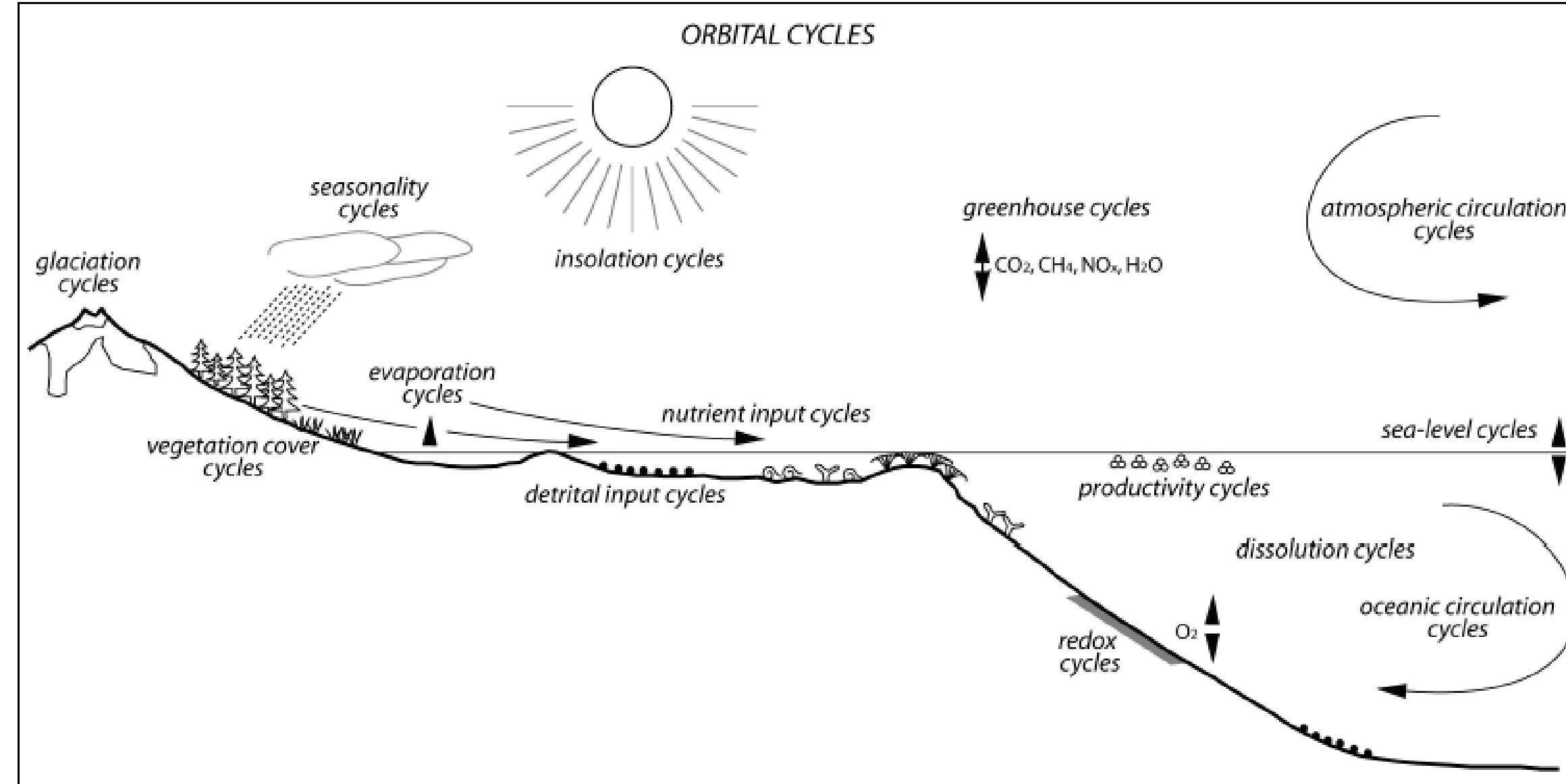


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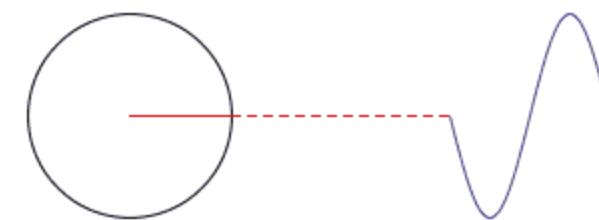




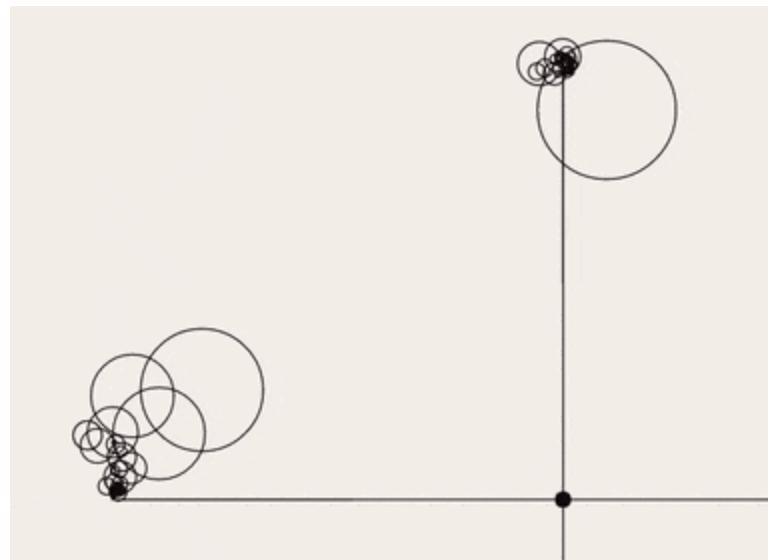
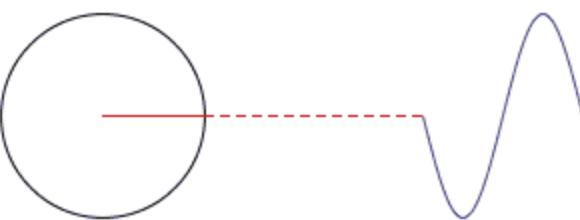
Information can be described as an infinite series of sin and cos waves of differing frequencies and magnitudes.



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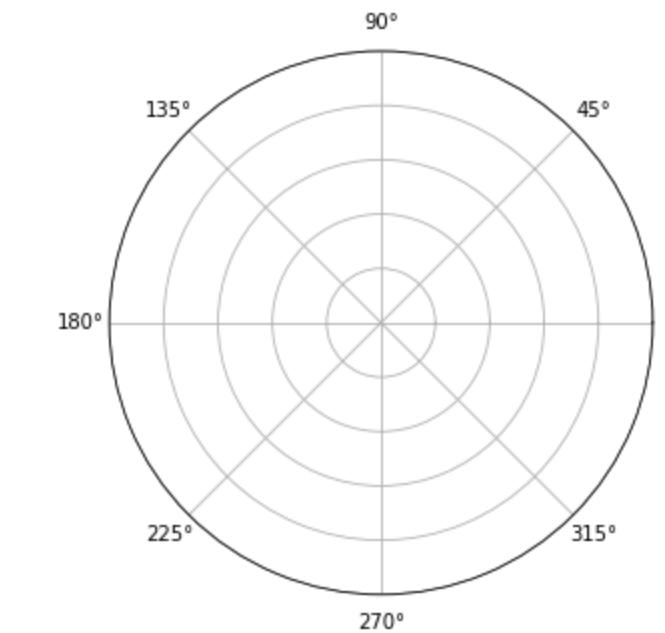
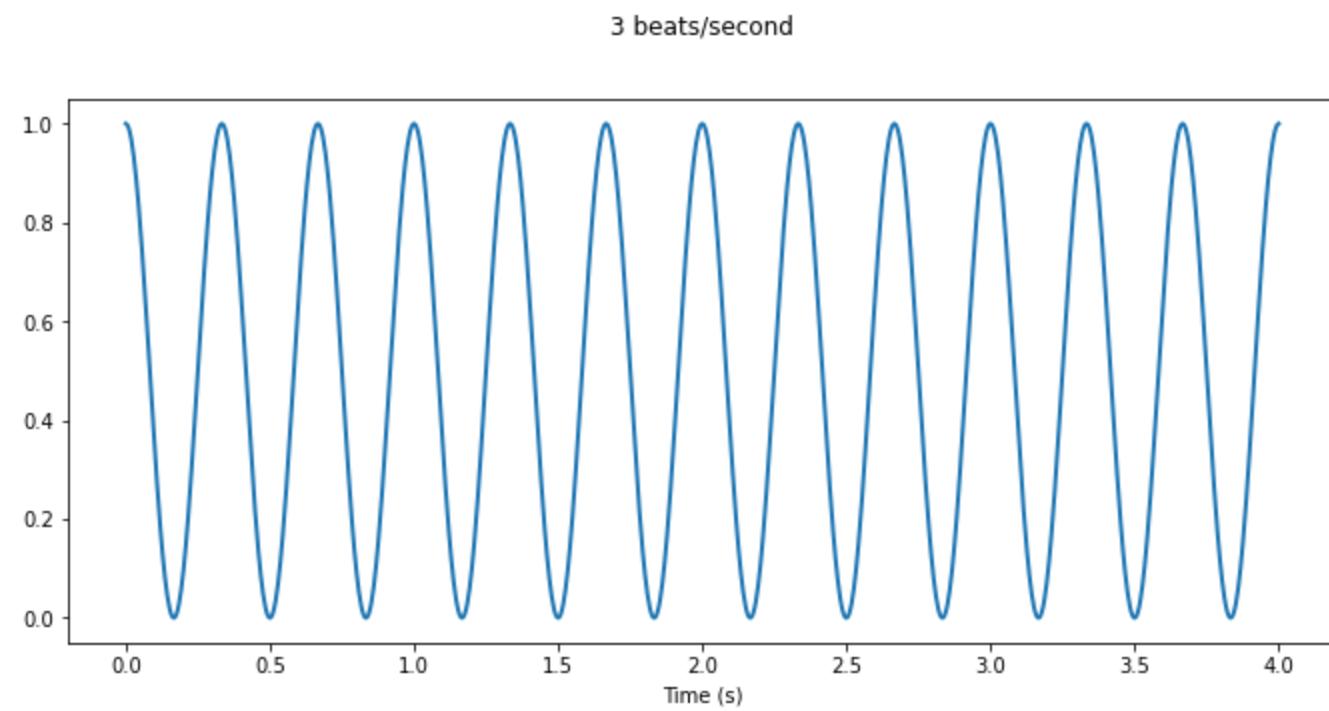


Information can be described as an infinite series of sin and cos waves of differing frequencies and magnitudes.



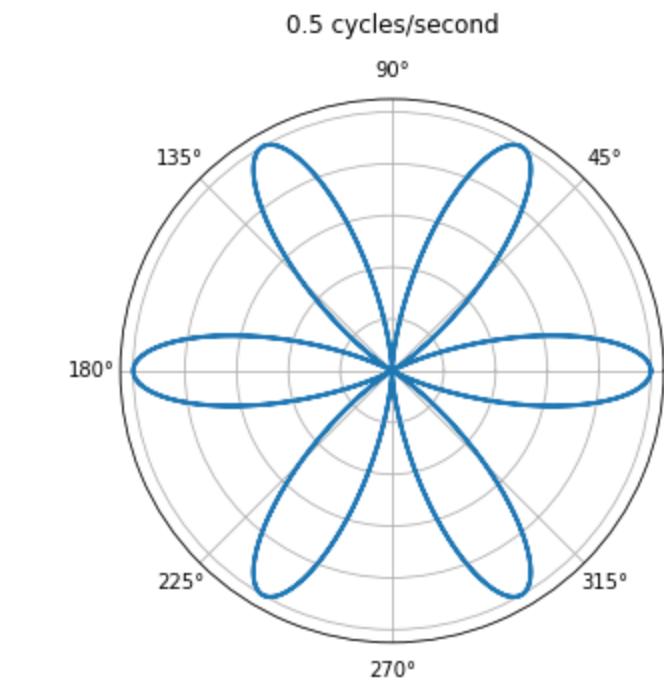
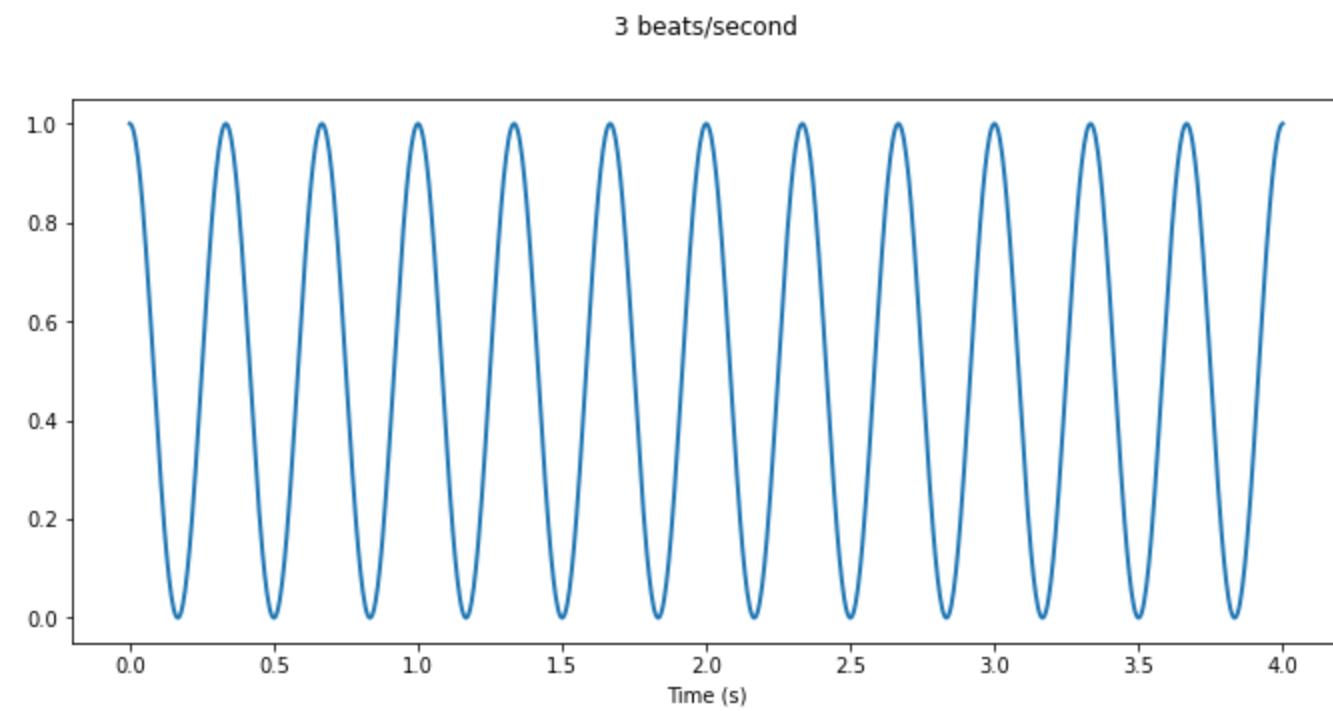
In [6]:

```
plt.figure(figsize=(25,5))
plt.subplot(1,2,1)
xt = np.linspace(0,4,1000)
y = (np.cos(xt*3*2*np.pi)+1)/2
plt.plot(xt,y,lw=2)
plt.gca().set_title('3 beats/second',y=1.1)
plt.gca().set_xlabel('Time (s)')
plt.subplot(1,2,2, polar=True)
_=plt.gca().set_yticklabels([])
```



In [7]:

```
plt.figure(figsize=(25,5))
plt.subplot(1,2,1)
xt = np.linspace(0,4,1000)
y = (np.cos(xt*3*2*np.pi)+1)/2
plt.plot(xt,y,lw=2)
plt.gca().set_title('3 beats/second',y=1.1)
plt.gca().set_xlabel('Time (s)')
plt.subplot(1,2,2, polar=True)
plt.plot(xt*.5*2*np.pi,y,alpha=1,lw=2)
plt.gca().set_yticklabels([])
_=plt.gca().set_title('0.5 cycles/second',y=1.1)
```



Some set up to cycle through different frequencies:

```
In [10]: %%capture
import time
import pylab as pl
from IPython import display
xt = np.linspace(0,4,1000)
y = (np.cos(xt*3*2*np.pi)+1)/2
y = (np.cos(xt*3*2*np.pi)+1)/4+(np.cos(xt*2*2*np.pi)+1)/4
plt.gca().set_title('3 beats/second',y=1.1)
plt.gca().set_xlabel('Time (s)')
```



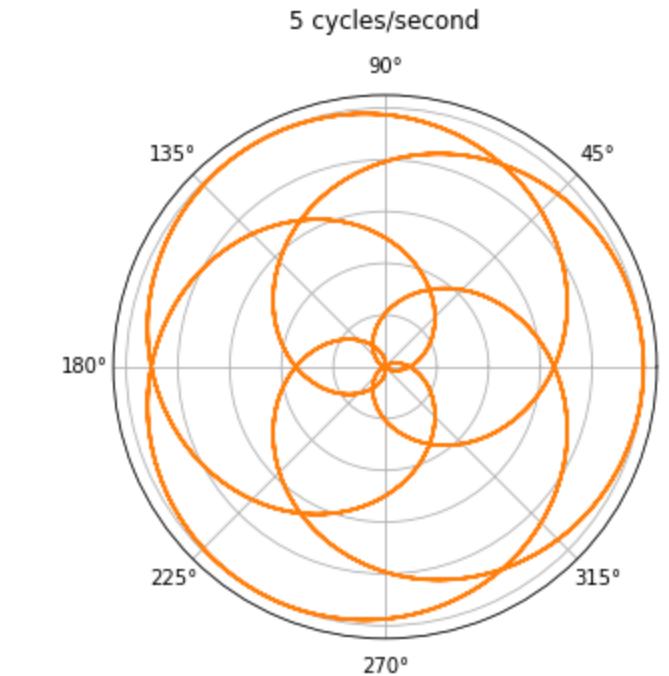
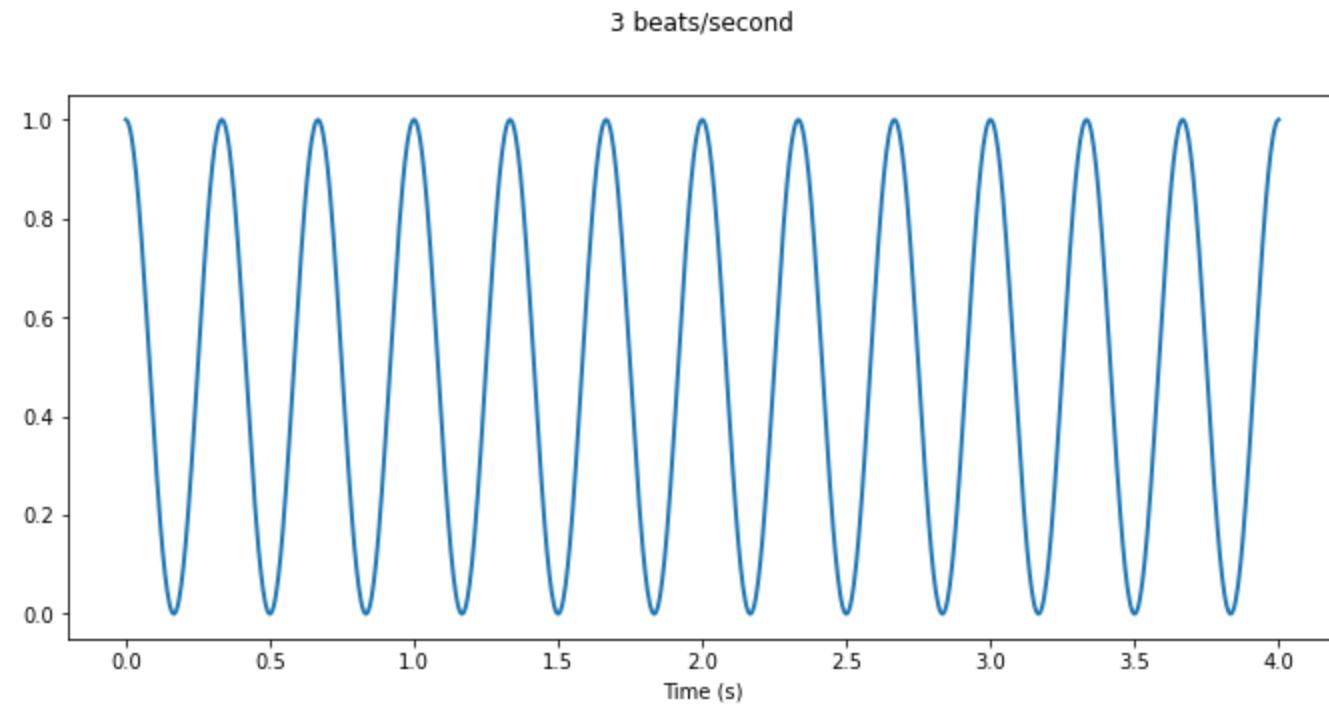
Wrapping the signal (left) around the unit circle (right) at different frequencies:

```
In [ ]: for i in np.linspace(.1,6,100):
    fig=plt.figure(figsize=(15,5))
    plt.subplot(1,2,1)
    plt.plot(xt,y,lw=2)
    plt.subplot(1,2,2, polar=True)
    display.clear_output(wait=True)
    plt.plot(xt*i*2*np.pi,y,alpha=1,lw=2)
    plt.gca().set_yticklabels([])
    _=plt.gca().set_title(f'{i:.2f} cycles/second',y=1.1)
    display.display(pl.gcf())
    time.sleep(.25)
```



```
In [12]: plt.figure(figsize=(25,5))
plt.subplot(1,2,1)
xt = np.linspace(0,4,1000)
y = (np.cos(xt*3*2*np.pi)+1)/2
plt.plot(xt,y,lw=2)
plt.gca().set_title('3 beats/second',y=1.1)
plt.gca().set_xlabel('Time (s)')
plt.subplot(1,2,2, polar=True)
plt.plot(xt*.5*2*np.pi,y,alpha=0)
plt.plot(xt*5*2*np.pi,y)
plt.gca().set_yticklabels([])
plt.gca().set_title('5 cycles/second',y=1.1)
```

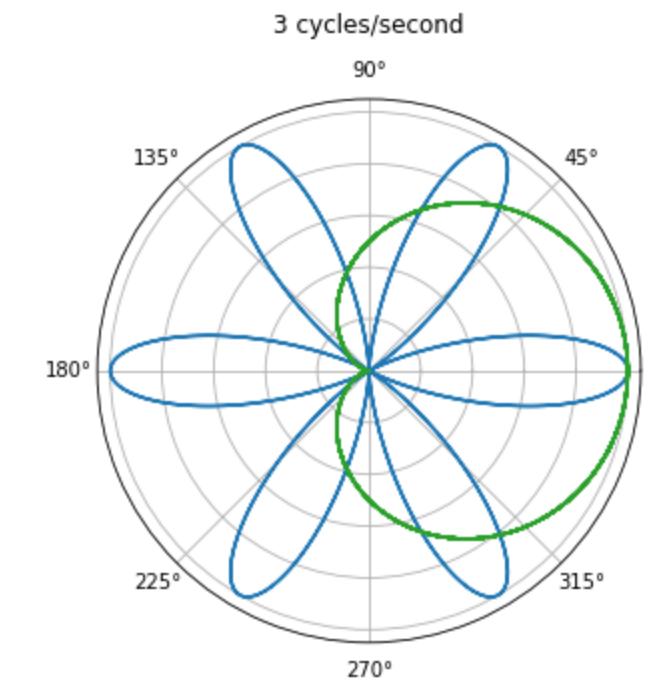
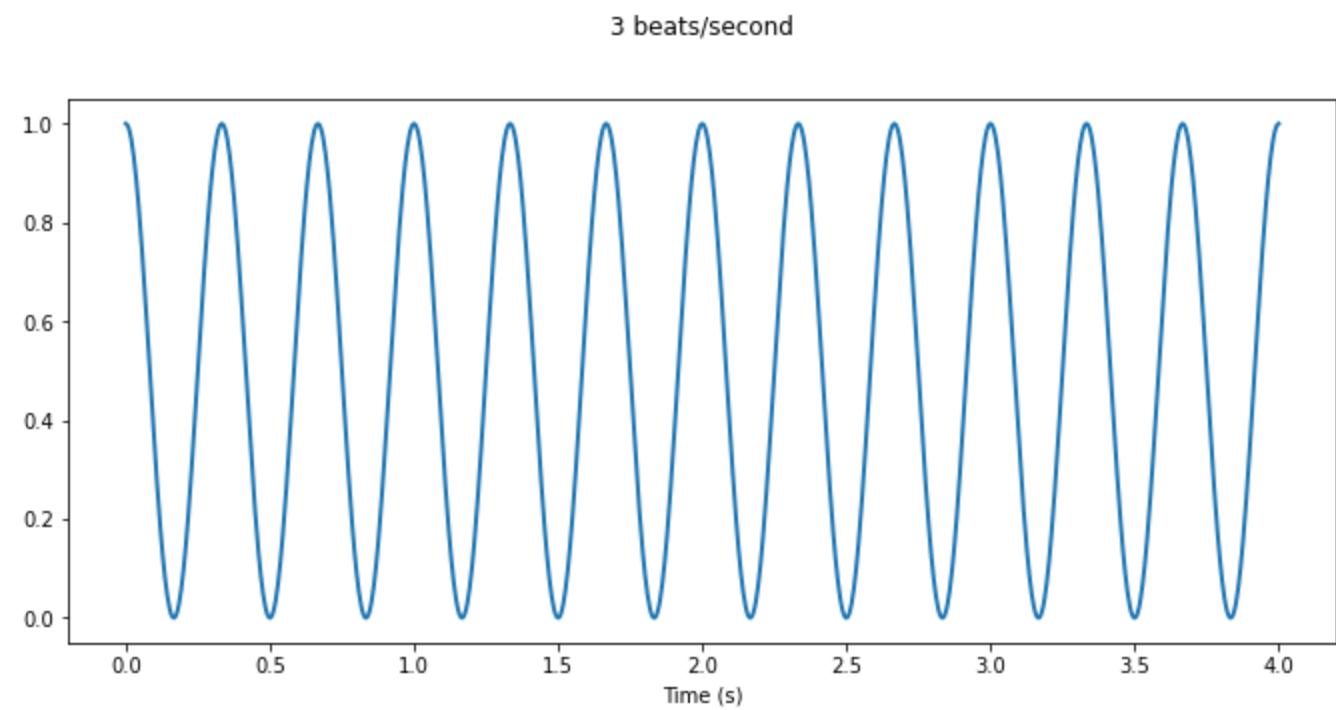
Out[12]: Text(0.5, 1.1, '5 cycles/second')



In [13]:

```
plt.figure(figsize=(25,5))
plt.subplot(1,2,1)
xt = np.linspace(0,4,1000)
y = (np.cos(xt*3*2*np.pi)+1)/2
plt.plot(xt,y,lw=2)
plt.gca().set_title('3 beats/second',y=1.1)
plt.gca().set_xlabel('Time (s)')
plt.subplot(1,2,2, polar=True)
plt.plot(xt*.5*2*np.pi,y,alpha=1) #add back
plt.plot(xt*5*2*np.pi,y,alpha=0)
plt.plot(xt*3*2*np.pi,y)
plt.gca().set_yticklabels([])
plt.gca().set_title('3 cycles/second',y=1.1)
```

Out[13]: Text(0.5, 1.1, '3 cycles/second')



Euler's formula

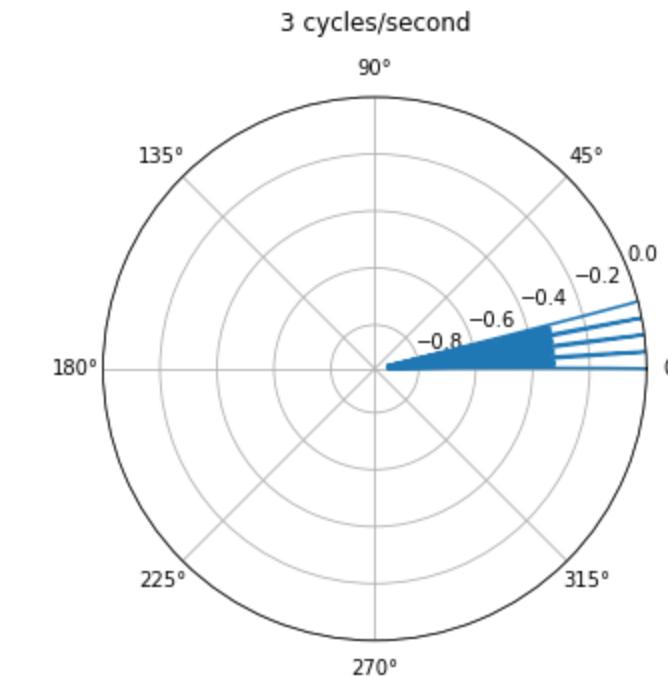
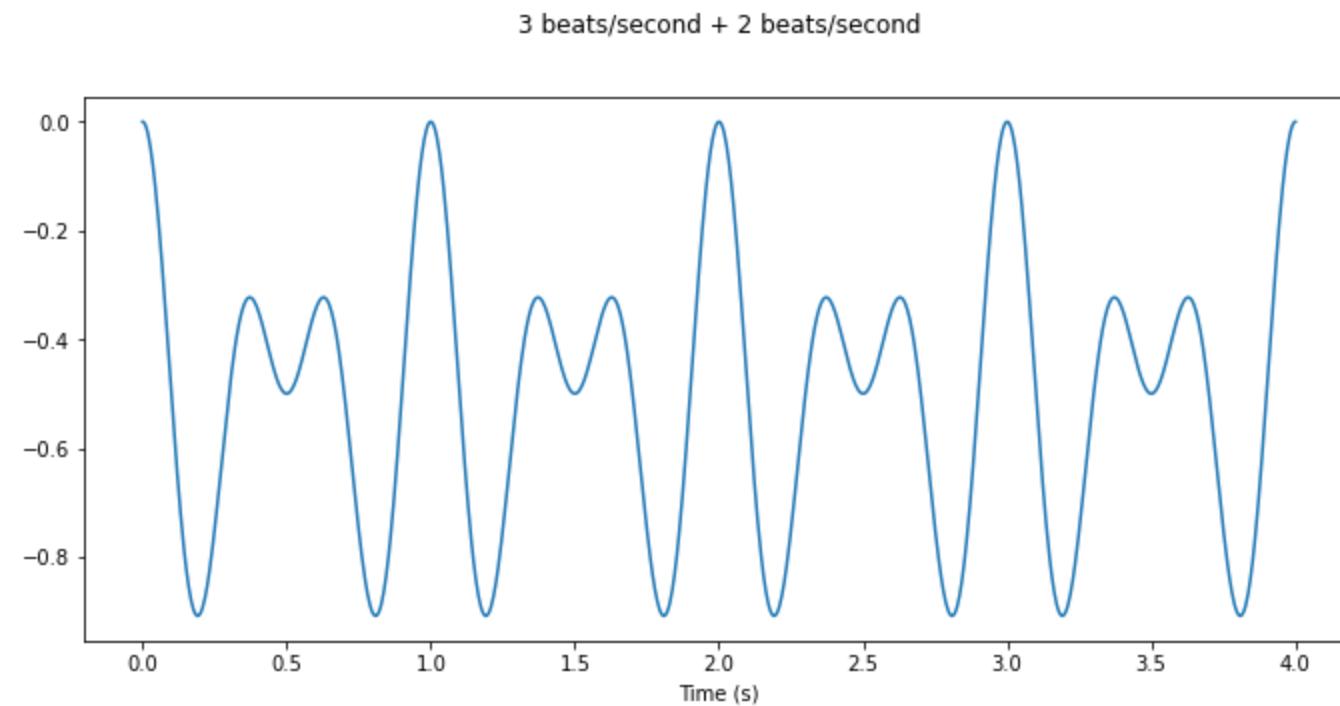
$$e^{ix} = \cos x + i \sin x$$



In [20]:

```
plt.figure(figsize=(25,5))
plt.subplot(1,2,1)
xt = np.linspace(0,4,1000)
y = -1 + (np.cos(xt*3*2*np.pi)+1)/4+(np.cos(xt*2*2*np.pi)+1)/4
plt.plot(xt,y)
plt.gca().set_title('3 beats/second + 2 beats/second',y=1.1)
plt.gca().set_xlabel('Time (s)')
plt.subplot(1,2,2, polar=True)
# plt.polar(xt*.5*2*np.pi,y,label='0.5 c/s')
# plt.polar(xt*2*2*np.pi,y,label='2 c/s')
plt.plot(xt*.01*2*np.pi,y,label='3 c/s')
# plt.gca().set_yticklabels([])
# plt.legend(loc='lower right', fontsize=10, bbox_to_anchor=(1.3, -.1))
plt.gca().set_title('3 cycles/second',y=1.1)
```

Out[20]: Text(0.5, 1.1, '3 cycles/second')



In [17]:

```
plt.figure(figsize=(25, 5))
plt.subplot(1, 2, 1)
xt = np.linspace(0, 4, 1000)
y = (np.cos(xt * 3 * 2 * np.pi) + 1) / 4 + (np.cos(xt * 2 * 2 * np.pi) + 1) / 4 - .5
complex_y = y * np.exp(-2 * np.pi * 1j * 3 * xt) #using imaginary j and exp to convert to polar
plt.plot(complex_y.real, complex_y.imag)
plt.gca().set_aspect(1); plt.gca().set_title("3 cycles/second"); plt.subplot(1, 2, 2)
x_COM = []
freqs = np.linspace(0.1, 10, 1000)
# freqs = np.arange(0,1000-1)
for i in freqs:
    complex_y = y * np.exp(-2 * np.pi * 1j * i * xt)
    x_COM.append((np.sum(complex_y.real)**2+np.sum(complex_y.imag)**2)**(1/2))

plt.plot(freqs, x_COM, alpha=1, zorder=2); _ = plt.gca().set_xticks(range(0, 11)); plt.gca().set_ylabel("'sum'");
plt.gca().set_xlim([0,10])
```

Out[17]: (0.0, 10.0)

