

# Quantifying the relationship between water depth and carbonate facies

Blake Dyer<sup>a, b, \*</sup>, Adam C. Maloof<sup>b</sup>, Sam J. Purkis<sup>c</sup>, Paul M. (Mitch) Harris<sup>c</sup>

<sup>a</sup> Lamont-Doherty Earth Observatory, Columbia University, 61 Route 9W, Palisades, NY 10964, USA

<sup>b</sup> Department of Geosciences, Princeton University, Guyot Hall, Washington Road, Princeton, NJ 08544, USA

<sup>c</sup> CSL–Center for Carbonate Research, Department of Marine Geosciences, Rosenstiel School of Marine and Atmospheric Science, University of Miami, Miami, FL 33149, USA

## ARTICLE INFO

### Article history:

Received 9 January 2018

Received in revised form 18 May 2018

Accepted 19 May 2018

Available online 23 May 2018

Editor: Dr. B. Jones

### Keywords:

Great Bahama Bank

Carbonates

Water depth

Hidden Markov models

## ABSTRACT

Carbonate facies often are used to define meter-scale parasequence (or cyclic) structure in ancient sedimentary basins. These parasequences constitute the fundamental stratigraphic unit in many studies of ancient climate and life on Earth. Of interest is the uncertainty associated with the assumptions that underpin the parasequence definition and interpretation. This work presents a method that uses modern maps of bathymetry and the geographic distribution of facies atop the Great Bahama Bank to extract a signal of water depth change from facies transitions in vertically stacked carbonate strata. This probabilistic approach incorporates the observed complexity in the water depth distribution of immediately adjacent modern carbonate environments, and results in an impartial and explicit interpretation of stratigraphic data with quantified uncertainties. Specifically, this analytical tool can distinguish sequences of facies that are likely to record information about changing water depths from sequences of facies that do not. Additionally, the quantitative signal extracted from sequences of discrete, or qualitative data, can be used for the correlation of stratigraphic sequences. By quantifying geologic observations through a lens of modern data, the reproducibility and accuracy of sedimentary interpretations can be improved to build a more authentic picture of Earth history.

© 2018 Published by Elsevier B.V.

## 1. Introduction

At the most fundamental level, stratigraphy is the practice of extracting past environmental information from a sequence of layered sedimentary rocks. Much of our understanding of early Earth history is derived from sedimentary rocks that formed as a shallow shelf or platform, and a vast majority of these sediments are carbonates (Ronov, 1982; Walker et al., 2002; Peters and Husson, 2017). The grain, fossil, and chemical composition of these sediments records paleoenvironmental information. To accurately decode the environmental history of a sequence of sedimentary strata, the natural processes involved in the deposition and degradation of such properties over millions of years must be fully considered. This paper principally is concerned with the sedimentary process that controls the grain size distribution (or more generally the facies) of platform carbonate sediments. Specifically, we are interested in how the characteristics of carbonate sediments may record information

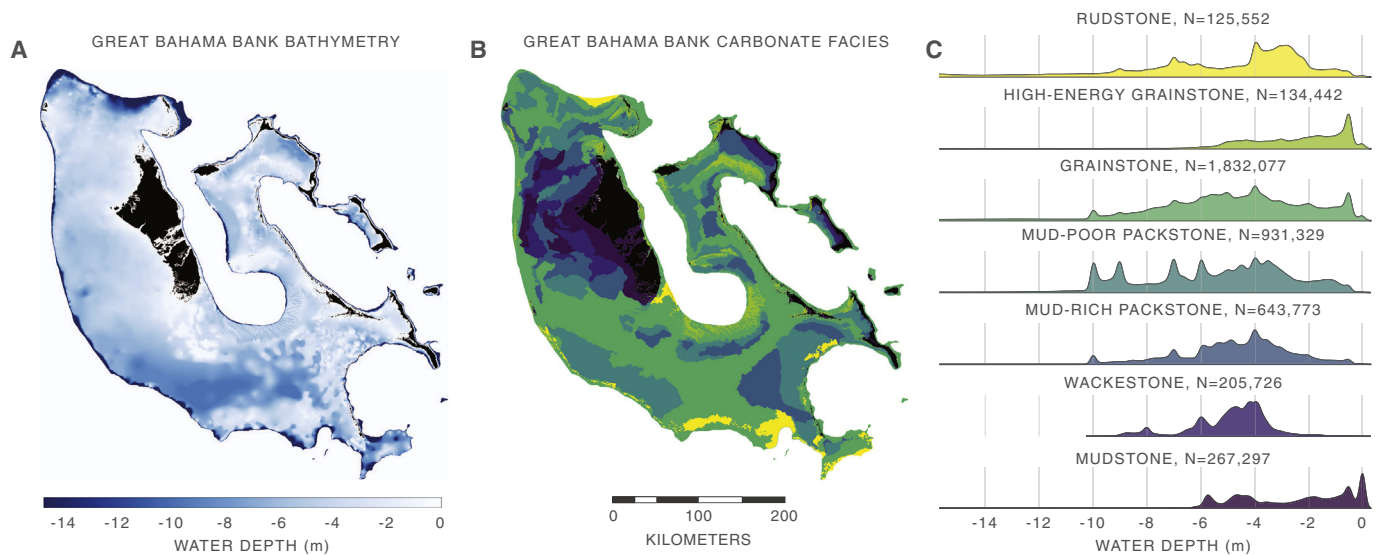
about water depth. Reconstructions of global sea level historically have underpinned much of our understanding of the long term evolution of climate and tectonics on Earth (Miller et al., 2005; Haq and Schutter, 2008), and we can study modern depositional environments to improve and quantify the accuracy of such water depth interpretations.

Over a century ago, Johannes Walther promoted this approach to sedimentology by suggesting that a sequence of paleoenvironments in a continuous stratigraphic column is related to the lateral distribution of those environments on the surface at the time of deposition (Walther, 1894). This scientific approach of comparing recent and ancient environments has been the foundation of many developments in sedimentology over the last century (Ginsburg, 1974). High resolution maps of depositional environments and bathymetry from modern carbonate platforms such as the Great Bahama Bank serve as a natural laboratory to observe directly the relationship between carbonate deposition and water depth, and statistical models can extend knowledge gained from those observations to stratigraphic interpretation (Fig. 1; Bosence, 2008; Reijmer et al., 2009; Purkis et al., 2015; Harris et al., 2015; Purkis and Harris, 2016).

Most platform carbonate sediment is generated in shallow water from the breakdown of skeletal components of an organism or by direct precipitation from the water column, and the accumulation

\* Corresponding author at: Lamont-Doherty Earth Observatory, Columbia University, 61 Route 9W, Palisades, NY 10964, USA.

E-mail address: [bdyer@ldeo.columbia.edu](mailto:bdyer@ldeo.columbia.edu) (B. Dyer).



**Fig. 1.** Geospatial dataset of carbonate facies on the Great Bahama Bank from Harris et al. (2015). A. Bathymetric map of the Great Bahama Bank, clipped below 25 m depth (white) and above mean sea level (black). B. Carbonate facies map generated from a combination of ground truthing and satellite images. C. The water depth distribution of each carbonate facies type across the Great Bahama Bank. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

of that sediment often is close to the production region (Schlager, 1981). On shallow platforms, the dominant control on the physical properties of the accumulating sediment is the energy of the coincident water column. Sometimes wave and current energy on carbonate platforms are completely controlled by water depth, resulting in patterns of carbonate sedimentation where water depth is very closely correlated with depositional facies. While this view is presented in classic texts (e.g., Tucker and Wright, 2009), studies of many shallow water carbonate platforms reveal a more complex mosaic of depositional environments (Rankey, 2004; Purkis et al., 2012, 2015; Rowlands et al., 2014; Harris et al., 2015; Gischler et al., 2017). For example, the distributions of facies by water depth across the entire Great Bahama Bank suggests that water depth alone is a poor predictor of sedimentary environment (Harris et al., 2015 and Fig. 1). However, at the local scale of immediately adjacent environments, the relationship between facies and water depth may be more in line with the classic view. Importantly, the observation that carbonate facies are an imperfect or noisy medium for recording water depth often is overlooked in studies of ancient stratigraphy and can lead to erroneous reconstructions of sea level change.

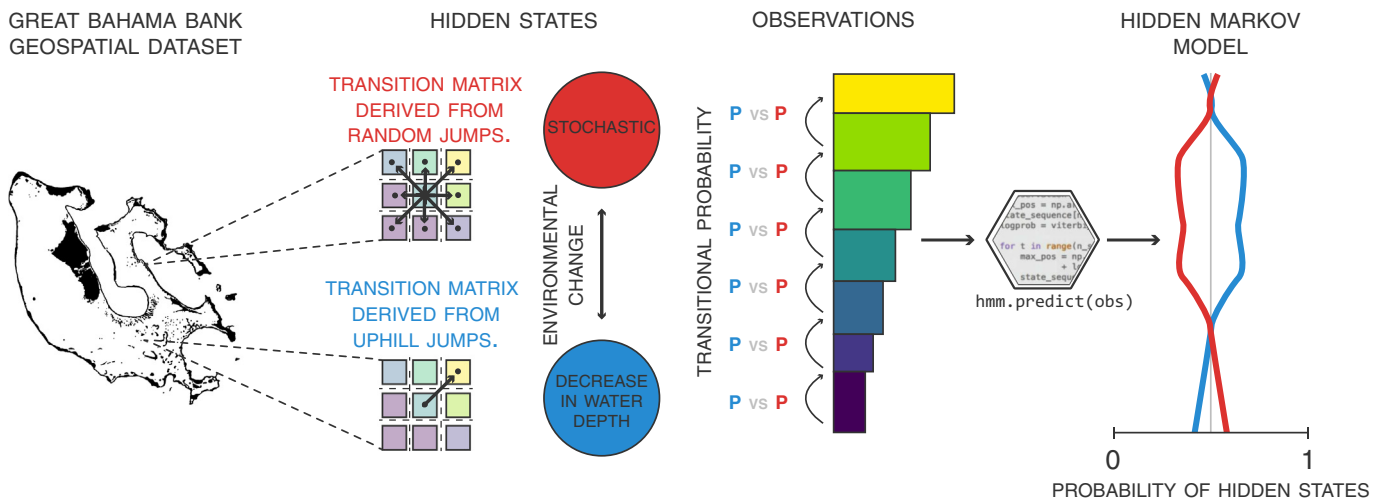
Shallow water carbonates commonly are organized vertically in meter-scale upward-shallowing packages that are bounded by flooding surfaces (e.g., parasequences; Drummond and Wilkinson, 1993; Lehrmann and Goldhammer, 1999; Lehrmann and Rankey, 1999; Rankey, 2004; Burgess, 2006). To understand the paleo-environmental implications of such records, there is a need to differentiate between parasequences that form as a result of a periodic external forcing (allogenic; Fisher, 1964; Goldhammer et al., 1987), such as Pleistocene style glacial-interglacial sea level variability, from parasequences that arise from the normal sedimentary process (autogenic; Ginsburg, 1971; Drummond and Wilkinson, 1993; Burgess, 2001). Variations in the orbital properties of Earth are responsible for periodic variations in continental ice volume and global mean sea level (Milankovitch, 1930; Hays et al., 1976; Imbrie and Imbrie, 1986). Assuming that such variations in sea level are responsible for meter-scale upward-shallowing parasequences, the depositional age of parasequence boundaries can be tuned to an astrochronologic timescale (Fisher, 1964; Goldhammer et al., 1987; Hinnov, 2000). This astrochronologic approach has the potential to

be an extremely precise measuring stick for calibrating rates that Earth systems respond to changes (Sadler, 1994), but the interpretation of shallow water parasequences suffers from the ambiguous relationship between water depth and facies.

We present a method that uses the spatial distribution of modern sediments mapped atop the vast Great Bahama Bank to extract water depth change from facies transitions in vertically stacked carbonate strata. This probabilistic approach incorporates the observed complexity in the water depth distribution of immediately adjacent carbonate environments to create quantitative signals from discrete stratigraphic data. These quantitative signals can be used to improve correlation of stratigraphic sections based on the aggregate transitions between facies, and provides quantitative insight into the absolute magnitude and variability of water depth change associated with sequences of carbonate facies. Moreover, the approach is a conservative one that integrates data from a meaningful range of water depths, depositional rates, local subsidence, and biological productivity, ensuring that the inferences derived from the approach are rooted in the processes important across all of these boundary conditions. Therefore, the uncertainty in the model interpretations encompasses some of the expected changes in the mosaic of sedimentary environments under large changes in sea level and different chemical and biological conditions that existed in early Earth history.

## 2. Methods

The stratigraphic thickness and vertical facies sequence contain information about the paleoenvironment. The observation of a vertical transition from one facies to another may indicate some change in the local sedimentary system, such as a change in water depth, or any number of other processes. To interpret a sequence of facies in a continuous stratigraphic column, we assume that the probability of that facies transition occurring is determined by the probability of the same two facies lying directly adjacent to one another on a map of the modern facies distribution (Walther, 1894). This broad assumption provides a statistical link between modern lateral facies data and ancient vertical facies transitions through Markov chains. Markov chains describe a sequence of possible states where



**Fig. 2.** Schematic overview of the hidden Markov model approach to quantifying discrete facies sequences in the context of water depth change. The geospatial dataset of bathymetry and carbonate facies on the Great Bahama Bank is the source of the transitional probabilities for stochastic and water depth driven facies changes. The probability that a lateral facies transition is related to a decrease in water depth is equal to the probability that a randomly selected location on the Great Bahama Bank is mapped as the starting facies and one of the immediate 8 neighbors is at a shallower water depth and mapped as the second facies. The hidden Markov model is a statistical framework to merge these transitional probabilities and discrete observations (facies) to estimate the underlying processes responsible for the sequence of observations.

the probability of each future state depends only on the immediately preceding state. In this specific case of carbonate sediments, facies observations are considered discrete states in such a Markov chain. The hidden Markov model is a statistical framework to estimate the underlying process responsible for generating a sequence of observations (Baum and Petrie, 1966; Rabiner, 1989). The hidden Markov model differs from simpler Markov chains only in that the state of the system is not directly observable. In this case, facies are observable features and the transitional properties between these observations record information about a hidden state. A sequence of facies may have a different probability of being generated when water depth is decreasing through time (one hidden state) than when water depth is constant (a second hidden state). For example, if mudstones are more likely to become grainstones when water depth is decreasing than when water depth is constant, then the observation of a mudstone to grainstone transition is more likely to represent the hidden state where water depth is decreasing through time.

To place ancient stratigraphic datasets of carbonate facies in a hidden Markov model framework, we consider the facies observations to be outputs of one of two depositional settings (hidden states) that represent either a decrease in water depth through time or constant or increasing water depth through time (referred to hereafter as stochastic noise; Figs. 2, 3). Thus, the model structure consists of two hidden states that can emit observations of the same facies. However, the probabilities of facies transitions for each hidden state are different. These probabilities were derived from a geospatial dataset that consists of water depth and carbonate facies mapped at 150 m<sup>2</sup> resolution across the 100,000 km<sup>2</sup> platform top of the Great Bahama Bank, an isolated carbonate platform where two thirds of the submerged area is less than 5 m deep. This grid size likely underestimates transitions into and out of facies that cover very narrow geographic areas, but should be representative of larger facies patches. Probabilities of lateral facies transitions were determined by iterating through each of the 150 m<sup>2</sup> grid spaces in the dataset and



**Fig. 3.** Facies transitions matrices for carbonates on the Great Bahama Bank. Colors along axes match facies colors in Fig. 1 and grey represents land or exposure. A. Facies transitional probability when moving to shallower water. B. Facies transitional probability when moving in a random direction. C. Difference between A and B. Blue colors highlight transitions that are more likely moving to shallower water (A) than moving in a random direction (B), and red colors match transitions that occur more often when moving in a random direction. The larger the negative value in the diagonal, the more that specific facies is water depth stratified at the local scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

inspecting the facies of immediately neighboring locations (similar to approaches used in Purkis et al., 2005; Riegl and Purkis, 2012). The dataset is a two dimensional regular four sided grid, so the immediate neighbors consist of the eight grid cells surrounding a central cell. In this model, one hidden state is described by lateral facies transitions that only occur over a decrease in water depth (Fig. 3A). The other hidden state is described by probability of lateral facies transitions regardless of the change in water depth (Fig. 3B). The probability of transitioning from one hidden state to another can be tuned to a length scale that fits the specific inference problem. For a length scale of  $N$ , the probability of switching from one hidden state to another,  $P$ , can be calculated as

$$P = 1 - e^{-\frac{1}{N}} \quad (1)$$

Practically, this tuneable parameter describes the shape of a continuous moving gaussian window over which observations inform on the likelihood of the inferred hidden state. Given the model described above, the probability that a set of facies observations was generated by each hidden state can be computed with the Viterbi algorithm (Viterbi, 1967). This algorithm computes the likelihood of all pathways through the hidden states that could generate the set of observations and returns the maximum likelihood pathway and the corresponding probability for each hidden state. In other words, this output indicates whether a sequence of vertical facies observations, such as from core or outcrop, are indicative of changing water depth.

### 3. Results: model exploration

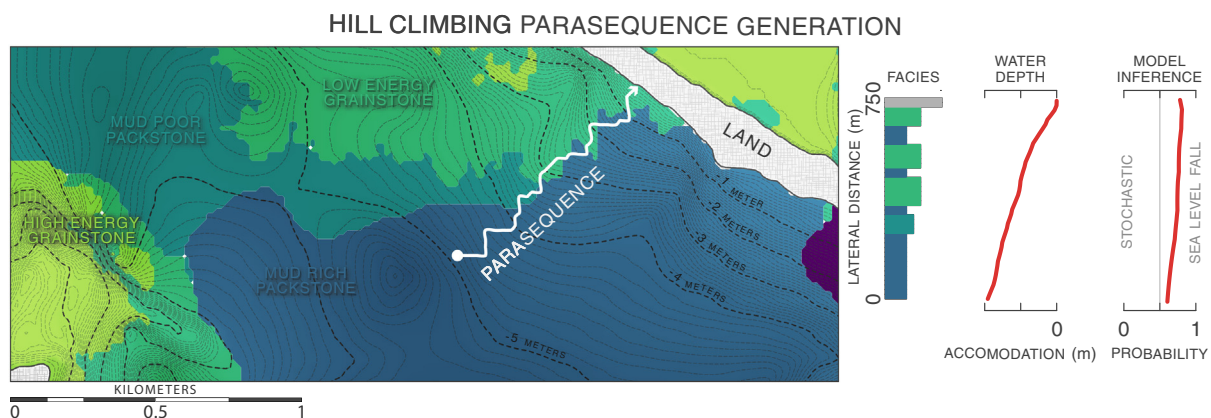
#### 3.1. Monte Carlo generated facies transitions

How does the difference in the two facies transition matrices (panel C in Fig. 3) translate into inferences about the facies record of changing water depth? If a specific transition only occurred when water depth is decreasing, then, at the moment that this transition occurs, the hidden state is completely determined (100% probability). However, on the Great Bahama Bank, each possible facies transition is observed in both of the hidden states (water depth change versus stochastic), although with different transitional probabilities. To explore these differences, we randomly generated sequences of facies directly from the each of the transitional matrices (A and B in Fig. 3). A sequence was generated by selecting a uniform random starting facies and transitioning to a new facies at a probability given by the transitional probabilities of the current facies in the matrix. When a facies change occurs, the next observation is generated by the probabilities that correspond to the new facies. 200,000

facies sequences were generated from each transition matrix (A and B in Fig. 3). Any transition to land represented the end of that sequence. The probability that each matrix could generate the sequence was determined with the Viterbi algorithm. In this scenario, the true answer to the inference problem is known, and the successes and failures offer some insight into the overall reliability of facies at recording changing water depths (and the uncertainty of such an interpretation). If facies were perfect recorders of water depth change, then the model always would predict the correct matrix that was used to generate the sequence. Fig. 5 shows two histograms that summarize the model interpretation of each randomly generated sequence. 52% of the total distribution under both curves is shared and represents the baseline uncertainty in differentiating whether or not a sequence of facies corresponds to changing water depths or other processes (Fig. 5). Facies sequences that correspond to regions where the two distributions overlap are difficult to interpret, but the sequences that correspond to regions where the distributions diverge contain the strongest inferences about the water depth change. When interpretations are guided by an explicit statistical model, the uncertainty can be quantified, and by focusing on the facies sequences that are the most likely to record changing water depths, the interpretations will be more accurate.

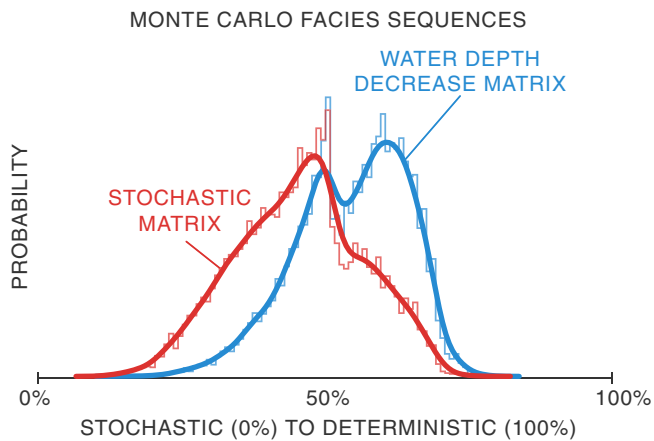
#### 3.2. Hill climbing generated parasequences

The relationship between water depth and facies on the Great Bahama Bank varies laterally (Fig. 1). To gain insight into the spatial variability in the organization of facies by water depth, and to evaluate the efficacy of the hidden Markov model at identifying changes in water depth from discrete facies sequences, we present a numerical experiment with synthetically generated parasequences. The ideal parasequence dataset would consist of down-core stratigraphic observations and known depositional water depths from every location on the Great Bahama Bank. Data from sediment and rock coring on the Great Bahama Bank is spatially limited, with only a few select areas having been cored in detail (Beach and Ginsburg, 1980; Manfrino and Ginsburg, 2001). Alternatively, and the approach conveyed in this work, a synthetic dataset of pseudo-parasequences can be generated from the map data. A parasequences will form in a location as accumulation outpaces accommodation and nearby shallower environments prograde over the location. For each location on the Great Bahama Bank we identify a path that goes through shallower cartesian neighbors ( $n = 8$ ) sequentially until there are no shallower neighbors, or land is reached (Fig. 4). We define the facies along that shallowing path as a synthetic parasequence, which may represent what a local progradational parasequence would look



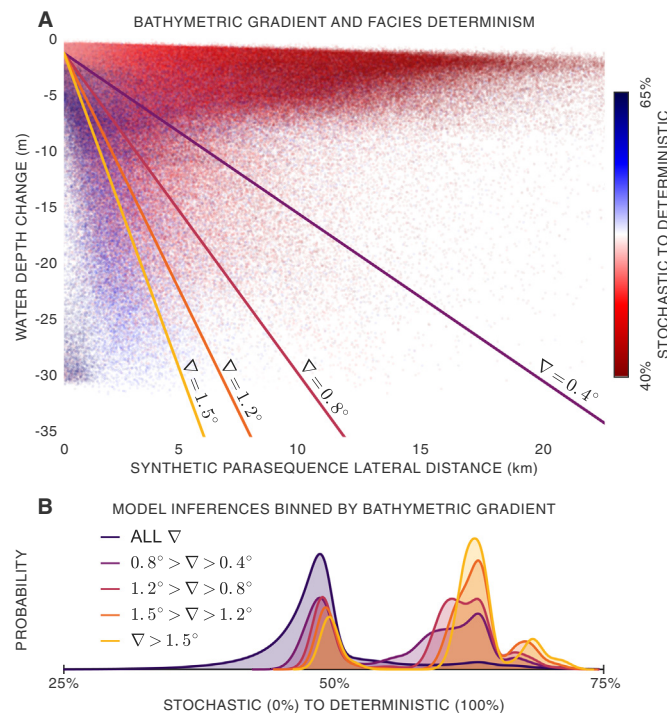
**Fig. 4.** Starting with any location on the Great Bahamas Bank, a synthetic parasequence is generated by moving to the shallowest of the cartesian neighbors ( $n = 8$ ) sequentially until reaching a local high point, or land. The sequence of facies observations is interpreted by the inference model without any information about the local bathymetry, and the results are compared to the known values of water depth.





**Fig. 5.** Distribution of modeled interpretations for 200,000 facies sequences of length 1 to 100 that are randomly generated by iterating through each transition matrix (A and B in Fig. 3). If facies distributions were completely determined by water depth there would be no overlap in the two distributions.

like. Ideally, the hidden Markov inference model should be able to identify that such a sequence of facies transitions resulted from a hidden state where water depth is decreasing. We generated 1 million such synthetic parasequences by selecting (uniform) random starting locations in the Bahamian dataset. The facies sequences were decoded with the Viterbi algorithm to quantify the probability of the sequence resulting from decreasing water depth (Figs. 4 and 6).



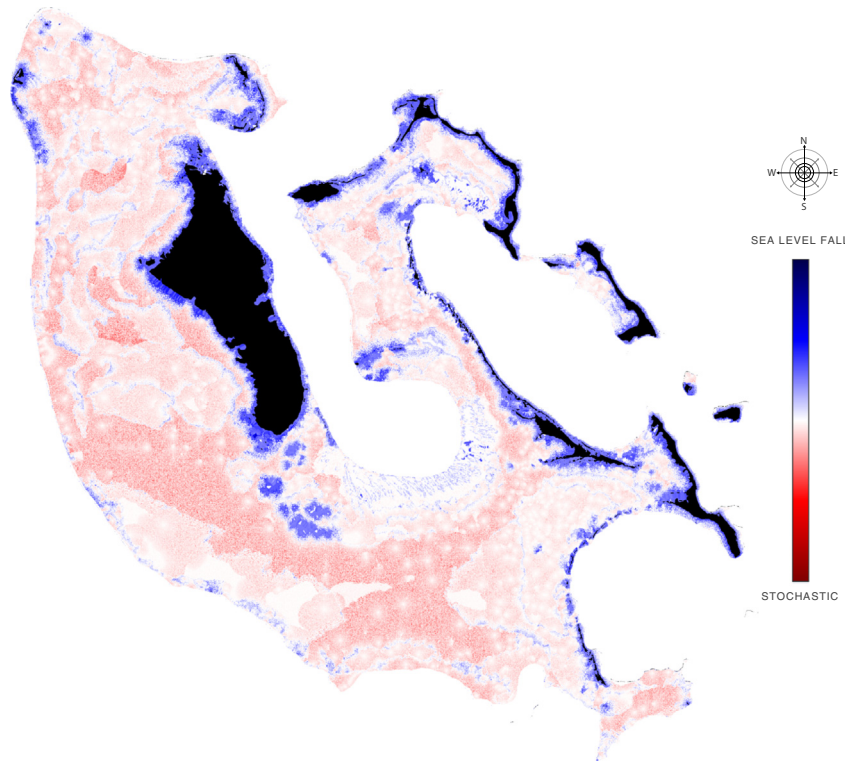
**Fig. 6.** 1 million synthetic parasequences are generated by selecting (uniform) random starting locations across the Great Bahama Bank and selecting sequentially shallower neighbors. The average probability that a particular sequence records a decrease in water depth is quantified by the inference model. A. Each dot records a single facies sequence from a random location on Great Bahama Bank. The dot is colored by the probability that the sequence records a decrease in water depth. Lines show the bathymetric bins for the distributions illustrated in B. B. Distributions of the modeled probabilities of recording water depth change from the 1 million synthetic parasequences illustrated in A. The distributions are illustrated for the entire dataset and four increasing bathymetric gradients. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The discrete sequence of facies observations are interpreted by the model without any information about the local bathymetry, so the results can be compared to the known values of water depth change to evaluate the model. The greater the bathymetric gradient (water depth change over lateral distance covered) for a facies sequence, the more likely the model correctly identifies water depth decrease as the hidden state driving the facies transitions (Fig. 6). Each synthetic parasequence corresponds to a location on the Great Bahama Bank, and inspection of the spatial patterns of model interpretations offers insight into the regions where facies are controlled by water depth and regions where stochastic variability dominates (Fig. 7).

#### 4. Discussion

The synthetic parasequence analysis demonstrates that local facies patterns in regions near islands are more determined by water depth (blue on Fig. 7) than the open areas of the platform (red). One possible reason for this relationship is that accumulation rates are highest near the islands (Purkis and Harris, 2016), perhaps amplifying the water depth signal recorded by the facies. Additionally, the bathymetric gradients on the platform top are highest in these near to land regions, and the higher gradients may aid in locally distributing facies by water depth better than low gradient regions where winds and currents can play a larger role in sedimentation. Alternatively, the accumulation of mud on Great Bahama Bank may be tightly coupled to the whitening phenomenon (Robbins et al., 1997), which does not appear to be a function of water depth (stochastic in the presented hidden Markov model). If whittings are direct responses to incursions of off bank currents (as suggested by Purkis et al., 2017), the shallow water margins of the islands on Great Bahama Bank are shielded from this effect, which could lead to the sediments in these regions more faithfully recording water depth. Roughly half of the synthetic parasequences are interpreted as a decrease in water depth, and the facies progressions in two nearby parasequences can have very different likelihoods of recording water depth change, even if both parasequences formed under decreasing water depth. This result is substantiated by the similarity of the two hidden states suggested from the overlapping distributions in Fig. 5. Moreover, a long vertical core through the platform may be expected to contain stacked parasequences from regions mapped as red (stochastic) as well as blue (deterministic). Incorporating laterally extensive data from parallel stratigraphic sections or cores and taking the mean of the stacked model inferences is a practical approach to overcome some of this uncertainty by boosting the signal to noise ratio.

The synthetic parasequences can be grouped into three broad archetypes: classic, grainstone, and mud capped. The three archetypes cover roughly the same span of water depth changes and seafloor gradients. Within each archetype, the facies sequences that were generated over greater magnitudes of water depth change typically have higher estimated probabilities of recording water depth change (Fig. 8). The classic parasequence pattern is the most similar to the textbook view of shallowing-upwards parasequences, with a gradational facies progression from high mud contents at the bottom to grainstone at the top. The grainstone parasequences lack high mud content facies such as mudstone and wackestones. These sequences often are entirely grainstone. Some sequences have many alternations between high-energy grainstone and low-energy grainstone (a grain size or grain type distinction), and for these sequences the model estimates a higher probability that changing water depth is responsible for the facies. Mud capped parasequences often match the general facies pattern from the grainstone or classic archetypes, but are capped with mudstone. These mud capped parasequences are the least likely to be interpreted by the model as stochastic signals, but differentiating the mud capped variant from



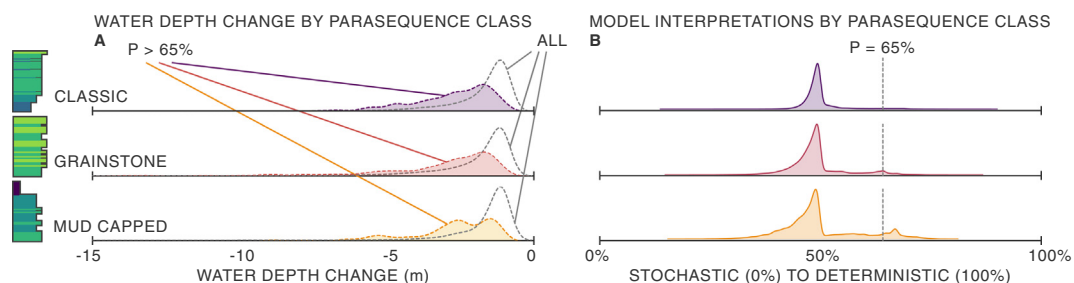
**Fig. 7.** Each pixel on the map of Great Bahama Bank is colored by the probability that the local lateral facies transitions are determined by water depth. For each location, a sequence was generated that corresponds to a series of continuously shallower neighbors. The probability that a particular series records a decrease in water depth was quantified by the inference model. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the other two may be difficult in practice (Fig. 8). In studies of ancient strata, a parasequence boundary often will be placed where a grainstone transitions upwards into mudstone, assuming an increase in water depth (flooding). However, this work emphasizes that to correctly identify parasequence bounding surfaces, it is crucial to identify evidence of exposure and flooding in addition to a facies transition, such as desiccation cracks or meteoric diagenesis that disappear upwards (Dyer and Maloof, 2015).

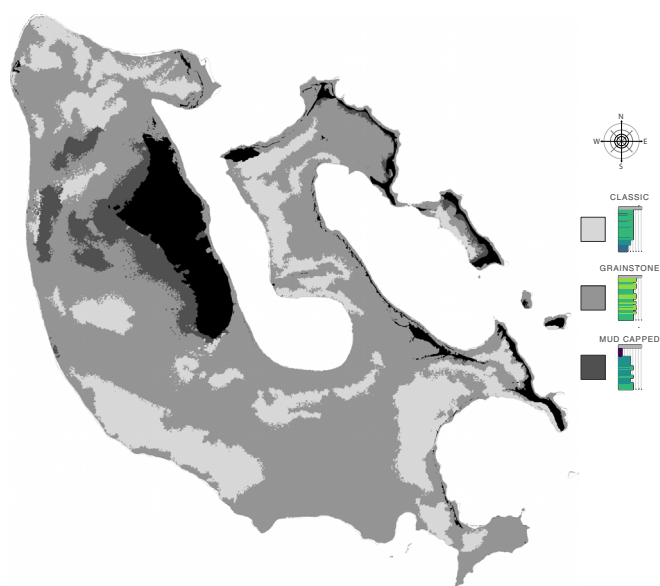
The spatial distribution of parasequence types seems closely related to the position of islands on the platform (Fig. 9). The winds in the Bahamas primarily come from an easterly direction (varying through the year from northeast to southeast) and the leeward sides of the islands accumulate more mud than the open platform (panel B in Fig. 1). This effect is the most apparent in the lee of Andros island, which is the largest island atop the Great Bahama Bank and has the largest width with respect to the direction of prevailing winds. Synthetic parasequences in these leeward regions frequently

are capped in mudstone prior to exposure, and may completely lack grainstone. Away from the islands the synthetic parasequence pattern is similar to the classical textbook parasequence with a gradual decrease mud content upwards. These classic parasequences frequently correspond to deeper water or under-filled regions (Figs. 1 and 9). However, most of the open platform lack mud-rich facies entirely, and the parasequences generated in those regions consist entirely of grainstone with alternating grain size and grain types. Minor decreases in platform sea level would drastically redistribute the locations of exposed portions of the platform, so long vertically successions of stacked parasequences should be expected to have all three of these parasequence archetypes expressed.

To illustrate how this hidden Markov model may be used to interpret ancient strata, we analyzed a well-studied stratigraphic dataset, the late Paleozoic outcrops along the San Juan River in Utah. These data comprise stacked carbonate parasequences that have been hypothesized to record cyclic changes in water depth that were



**Fig. 8.** The synthetic parasequences that are mapped in Fig. 7 can be grouped into three broad archetypes. Classic parasequences have a predictable and mostly complete facies progression from high mud contents to grainstone. Grainstone sequences lack mud but exhibit a wide range of grain sizes. Mud capped sequences can begin with a pattern that matches grainstone or classic sequences, but often is capped with pure mud. A. The distributions indicate the water depth change associated with each parasequence pattern, where the shaded distributions are for only the sequences that have a high probability of recording a decrease in water depth ( $P > 65\%$ ). B. Distributions of the model estimates for each parasequence class.



**Fig. 9.** The lateral distribution of parasequence archetypes is mapped across Great Bahama Bank. Mud capped sequences are closely associated with the leeward sides of islands, and are most abundant behind Andros island. Classic and grainstone dominated sequences cover much of the open platform, where classic sequences more commonly originate in the relatively deeper water or under-filled regions.

driven by glacio-eustasy (Weller, 1930; Wanless and Shepard, 1936; Goldhammer et al., 1991). During the Late Paleozoic Ice Age, the continental south pole (Gondwana) was variably glaciated from about the middle Carboniferous (330 Ma) to the early Permian (290 Ma; Veevers and Powell, 1987). Paleo-tropical sedimentary basins from this period are well known for their apparent cyclicity, which often is attributed to periodic glacio-eustatic variability. There remains significant debate, however, over the timing and pacing of such sedimentary cycles. Geochronology suggests that there is a significant 400 k.y. and 100 k.y. beat (Davydov et al., 2010; van den Belt et al., 2015). However, the higher frequency 100 k.y. cycles frequently contain more than the expected 5 precession driven meter-scale parasequences (mean 21 k.y.) (Eros et al., 2012; van den Belt et al., 2015). This inconsistency suggests that some number of identified parasequences are either autocyclic or that the interpreted parasequence boundaries must be better constrained. We can gain insight into the uncertainty involved in such an analysis by quantifying the interpretation of facies and parasequence boundaries through a lens of modern data with the hidden Markov model described above.

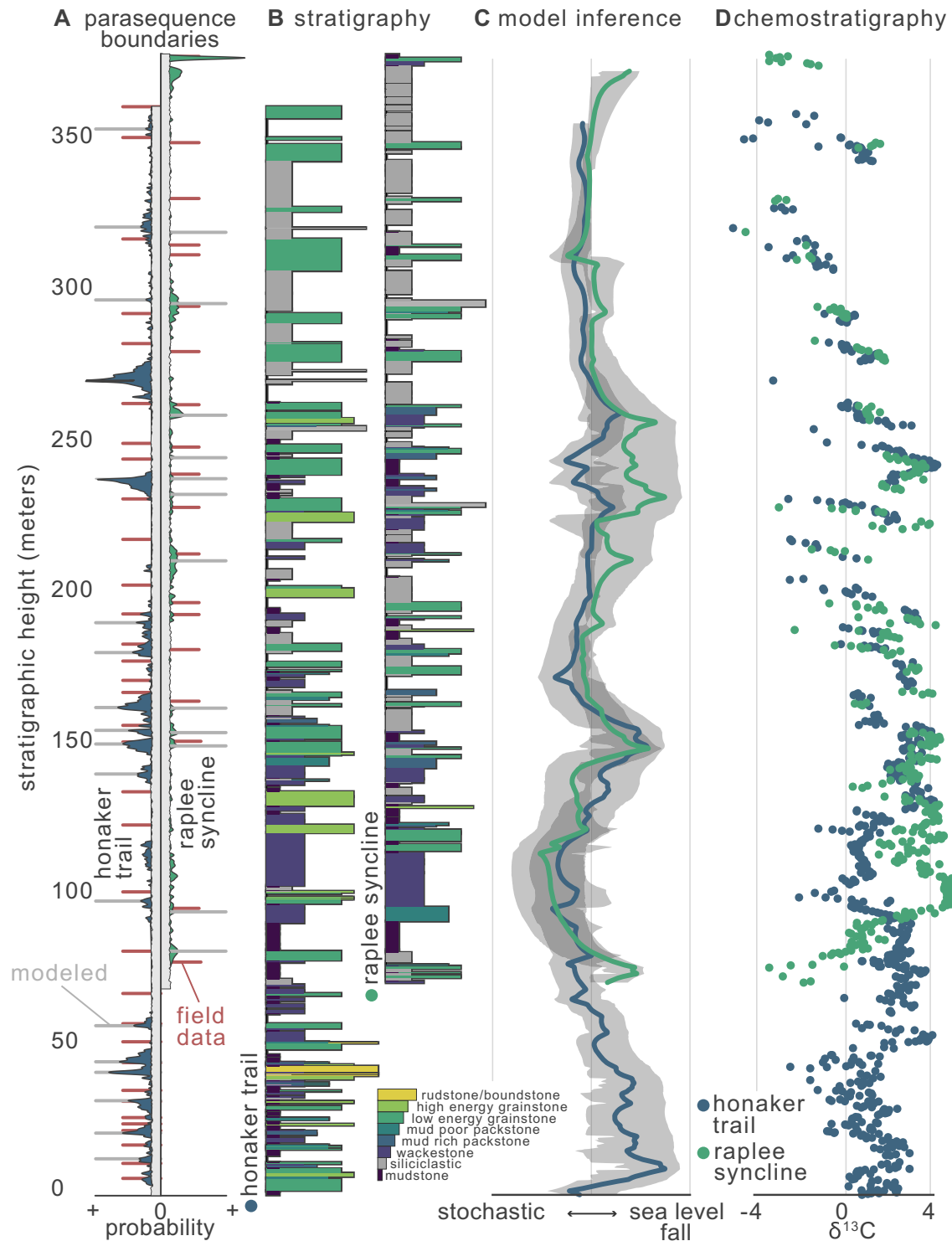
Near Mexican Hat, Utah, 350 vertical meters of the stacked carbonate parasequences outcrop along the San Juan River canyon walls (Peterson and Hite, 1969). The strata are organized into many sets of recessive mudstone to resistive grainstone parasequences (Wengert, 1962). Goldhammer et al. (1991) shows that the thicknesses of the parasequences fit a hierarchical pattern where roughly every five parasequences form a larger sequence. This 5:1 ratio is argued to represent a glacioeustatic climate signal arising from the modulation of precession and short term eccentricity. However, Lehmann and Goldhammer (1999) inspected the strata for statistical cyclicity or order and suggests that the sequence of facies has a distinctly random stacking pattern. Detailed lateral studies of facies in this region indicate a large degree of variability within a parasequence, but the vertical pattern of parasequence boundaries is relatively consistent (Grammer et al., 1996). While some of the identified parasequence boundaries contain evidence of relative sea level fall and exposure from meteoric diagenesis, the  $\delta^{13}\text{C}$  chemostratigraphy suggests that these parasequences formed under little to no change in sea level (Dyer and Maloof, 2015). Moreover, the Monte Carlo and

the synthetic parasequence analysis presented in this work highlight that there is significant uncertainty in identifying parasequence boundaries solely by water depth interpretations of facies.

To process the sequence of ancient strata with the previously described Hidden Markov model, two minor additions were necessary. First, the model must be able to explicitly handle observations that do not exist in the Bahamas dataset, such as siliciclastic rocks or covered intervals where a facies determination is not possible. Such observations were added to the model in a way so that they are equally likely to occur in both hidden states. Therefore, the occurrence of these observations in a sequence shifts the inference towards the middle (50%). Second, the complete stratigraphic section was split into many parasequences that were individually analyzed by the model, and then the extracted signals for each parasequence were recombined in stratigraphic order. By splitting the stratigraphic column into parasequences, the signal from one parasequence does not bleed over into the next sequence, and more importantly the upwards transition across a parasequence boundary is not considered. In the case where additional field or geochemical data suggest very clear parasequence boundaries, the splitting of the facies data at those locations is straightforward. However, a more general approach may be used to estimate the location of parasequence boundaries with the hidden Markov model. Carbonate parasequences form as sediments accumulate to fill some or all of the available accommodation space (upward-shallowing). The top of the parasequence is a period of non-deposition. When there is relative sea level rise (e.g., subsidence) during non-deposition, subsequent deposition will start at a deeper water depth and a new upward-shallowing parasequence can form. By simulating every possible combination of parasequence boundary locations in a stratigraphic dataset, it is possible to estimate the boundary positions that maximize this upward-shallowing hypothesis. To estimate these locations, parasequence count and boundary locations are (uniform) randomly generated for the entire sequence over many realizations. The probability that the complete stratigraphic sequence records decreasing water depth will depend on the randomly generated parasequence boundary locations. The full distribution of this signal over all realizations indicates the uncertainty associated with poorly constrained parasequence boundaries. Moreover, the best estimate of true parasequence boundary locations comes from the subset of realizations that maximize the probability that all other facies transitions are upward-shallowing (Fig. 10A).

The quantified signal extracted from facies data with a hidden Markov Model can serve as a metric to correlate similar aged strata. For context, a typical facies-based correlation may involve selecting marker beds or surfaces to stitch together the strata of a basin with the assumption that these features represent a single timeline. Another widely-used correlation technique involves alignment of a semi-continuous measurement, such as magnetic susceptibility or carbon isotopes, and an assumption that the measured property is laterally uniform in the paleo-environment. The quantified signal of aggregate facies locations, thicknesses, and transitional properties is a new correlation approach that hidden Markov Models provide. The stratigraphic columns in Fig. 10 have been correlated by vertically shifting the two records to minimize the difference between the two signals of water depth change (panel B). The sections are roughly 7 km apart and the correlation illustrated assumes no dilation due to differences in the long term development of accommodation space. The alignment of carbon isotopes illustrated in panel C support the effectiveness of this correlation approach. Moreover, regions where the carbon isotopes differ greatly between sections may suggest ancient lateral gradients in carbon isotopes, or regional diagenetic signals, that might have remained unidentified if the strata were aligned by their carbon isotopic values.

The power of hidden Markov models for correlation can be more generally applied by parameterizing alternative hidden states. For



**Fig. 10.** A. The distribution of modeled parasequence boundary locations that maximize the inferred signal of water depth change. These locations are based on the assumption that each parasequence should record a decrease in water depth (upward-shallowing). B. Measured stratigraphic sections of the Honaker Trail and Paradox Basin formations. The two sections are about 7 km apart, and the locations and stratigraphic interpretation is described in [Dyer and Maloof \(2015\)](#). C. The modeled estimates that the sequence of facies from each section records a decrease in water depth. The envelope is generated by analyzing 100,000 realizations of randomly generated parasequence boundary locations. The alignment of the two stratigraphic sections minimizes the difference between each modeled signal, without vertical stretching. D. The carbonate stable carbon isotopic composition of outcrop samples from each stratigraphic section.

example, the transitional matrices that underpin the model inference could be derived from vertical transitions in specific formations or intervals of interest. Then, the model output would indicate the probability that a new set of facies observations come from that

formation or interval of interest. In this general case, the modeled signal would indicate the probability that the aggregate facies thicknesses and transitional properties match the interval of interest. While robust correlation testing and robust depth to time calibration



often requires high precision geochronology, this approach provides an explicit method to develop baseline basin correlations.

The composite signal of water depth change in the two Paradox Basin stratigraphic sections suggests that some of the parasequences record a decrease in water depth. The facies in the lower portion of Honaker Trail are more likely to be upward-shallowing than recorders of stochastic processes. The signal in both sections over the span of +65 m to +125 m on Fig. 10 shift towards a stochastic origin. This shift may correspond to a change in the regional or global forcing of sea level at the parasequence time-scale. Alternatively, this change may indicate that the depositional locus recorded by these stratigraphic sections has shifted to a region where the vertical distribution of facies is more chaotic (moving from a blue region to a red region in Fig. 7). Differentiating between these two possibilities may be possible with more stratigraphic sections that cover a larger area of this basin.

Regions that correspond to stochastic signals tend to have fewer facies transitions per vertical meter. This observation is corroborated by the transitional matrices for each hidden state in the hidden Markov model (Fig. 3). Facies transitions to new facies types are more common in the water depth decrease matrix than the stochastic matrix. In other words, all facies on the Great Bahama Bank are more likely to transition to another facies when moving in a direction where water depth is decreasing than when moving in a random direction. The magnitude of this difference (identity diagonal in Fig. 3C) is largest in mudstones and low energy grainstones, indicating that these facies are the most water depth dependent at the local scale.

## 5. Conclusions

Discrete observations of facies can be converted into meaningful quantitative signals of past environmental change with hidden Markov models. This flexible statistical framework offers a tool for relating patterns in the transitional properties of facies sequences to one another. In this paper, we show how to extract information about the expected transitional properties of facies from maps of facies and water depth. This information forms parameters in a hidden Markov model that can estimate the probability that a sequence of facies records a change in water depth. This estimation is a function of modern distributions of immediately adjacent carbonate environments and ultimately offers an explicit and reproducible approach to interpreting shallow water carbonate parasequences. We anticipate that expanding this analysis to a greater variety of modern carbonate depositional environments and coupling this endeavor with vertical stratigraphic observations directly from those environments will lead to greater insight into the relationship between water depth and facies.

## Acknowledgments

B.D. and A.C.M. were supported by the Scott Vertebrate Fund, Princeton University, and NSF grant EAR-1410317. S.J.P. and P.M.H. thank the Comparative Sedimentology Laboratory of the University of Miami for support and lively discussions. We also are grateful to the reviewers and the Editor for their comments and time.

## References

- Baum, L.E., Petrie, T., 1966. Statistical inference for probabilistic functions of finite state Markov chains. *The Annals of Mathematical Statistics* 37, 1554–1563.
- Beach, D.K., Ginsburg, R.N., 1980. Facies succession of Pliocene-Pleistocene carbonates, northwestern Great Bahama Bank. *American Association of Petroleum Geologists Bulletin* 64, 1634–1642.
- Bosence, D., 2008. Randomness or order in the occurrence and preservation of shallow-marine carbonate facies? Holocene, South Florida. *Palaeogeography, Palaeoclimatology, Palaeoecology* 270, 339–348.
- Burgess, P.M., 2001. Modeling carbonate sequence development without relative sea-level oscillations. *Geology* 29, 1127–1130.
- Burgess, P.M., 2006. The signal and the noise: forward modeling of allocyclic and autocyclic processes influencing peritidal carbonate stacking patterns. *Journal of Sedimentary Research* 76, 962–977.
- Davydov, V.I., Crowley, J.L., Schmitz, M.D., Poletaev, V.I., 2010. High-precision U-Pb zircon age calibration of the global Carboniferous time scale and Milankovitch band cyclicity in the Donets Basin, eastern Ukraine. *Geochemistry, Geophysics, Geosystems* 11 (2).
- Drummond, C., Wilkinson, B., 1993. Carbonate cycle stacking patterns and hierarchies of orbitally forced eustatic sealevel change. *Journal of Sedimentary Research* 63, 369–377.
- Dyer, B., Maloof, A.C., 2015. Physical and chemical stratigraphy suggest small or absent glacioeustatic variation during formation of the Paradox Basin cyclothems. *Earth and Planetary Science Letters* 419, 63–70.
- Eros, J., Montañez, I., Osleger, D., Davydov, V., Nemyrovska, T., Poletaev, V., Zhykalyak, M., 2012. Sequence stratigraphy and onlap history of the Donets Basin, Ukraine: insight into Carboniferous icehouse dynamics. *Palaeogeography, Palaeoclimatology, Palaeoecology* 313–314, 1–25.
- Fisher, A., 1964. The Lofer cyclothems of the alpine Triassic. *Bulletin Kansas Geological Survey* 169, 107–149.
- Ginsburg, R., 1974. Introduction to comparative sedimentology of carbonates. *American Association of Petroleum Geologists Bulletin* 58, 781–786.
- Ginsburg, R.N., 1971. Landward movement of carbonate mud: new model for regressive cycles in carbonates: Abstract. *American Association of Petroleum Geologists Bulletin* 55, 340–340.
- Gischler, E., Isaack, A., Hudson, J.H., 2017. Sediments of the Dry Tortugas, south Florida, USA: facies distribution on a ramp-like isolated carbonate platform. *Sedimentary Geology* 351, 48–65.
- Goldhammer, R., Oswald, E., Dunn, P., 1991. Hierarchy of stratigraphic forcing; example from Middle Pennsylvanian shelf carbonates of the Paradox Basin. *Kansas Geological Survey Bulletin* 233, 361–413.
- Goldhammer, R.K., Dunn, P.A., Hardie, L., 1987. High frequency glacio-eustatic sealevel oscillations with Milankovitch characteristics recorded in Middle Triassic platform carbonates in northern Italy. *American Journal of Science* 287, 853–892.
- Grammer, G., Eberli, G., Bucher, F.V., Stevenson, G., Homewood, P., 1996. Application of high-resolution sequence stratigraphy to evaluate lateral variability in outcrop and subsurface—Desert Creek and Ismay intervals, Paradox basin. *Paleozoic systems of the Rocky Mountain region: Rocky Mountain Section Society for Sedimentary Geology* 235–266.
- Haq, B., Schutter, S., 2008. A chronology of Paleozoic sea-level changes. *Science* 322, 64.
- Harris, P.M., Purkis, S.J., Ellis, J., Swart, P.K., Reijmer, J.J., 2015. Mapping bathymetry and depositional facies on Great Bahama Bank. *Sedimentology* 62, 566–589.
- Hays, J.D., Imbrie, J., Shackleton, N.J., 1976. Variations in the Earth's orbit: pacemaker of the ice ages. *Science* 194, 1121–1132.
- Hinnov, L.A., 2000. New perspectives on orbitally forced stratigraphy. *Annual Review of Earth and Planetary Sciences* 28, 419–475.
- Imbrie, J., Imbrie, K.P., 1986. *Ice Ages: Solving the Mystery*. Harvard University Press.
- Lehrmann, D.J., Goldhammer, R.K., 1999. Secular variation in parasequence and facies stacking patterns of platform carbonates: a guide to application of stacking-patterns analysis in strata of diverse ages and settings. In: Harris, P.M., Saller, A.H., Simo, J.A. (Eds.), *Recent Advances in Carbonate Sequence Stratigraphy: Applications to Reservoirs, Outcrops and Models*. 62. Society for Sedimentary Geology, Special Publication., pp. 187–226.
- Lehrmann, D.J., Rankey, E.C., 1999. Do meter scale cycles exist? A statistical evaluation from vertical (1-D) and lateral (2-D) patterns in shallow-marine carbonate-siliciclastics of the “Fall In” strata of the Capitan Reef, Seven Rivers Formation, Slaughter Canyon, New Mexico. In: Saller, A.H., Harris, P.M., Krikland, B.L., Mazzullo, S.J. (Eds.), *Geologic Framework of the Capitan Reef*. 65. Society for Sedimentary Geology Special Publication., pp. 51–62.
- Manfrino, C., Ginsburg, R.N., 2001. Pliocene to Pleistocene deposition history of the upper platform margin. In: Ginsburg, R. (Ed.), *Subsurface Geology of a Prograding Carbonate Platform Margin, Great Bahama Bank*. 70. Results of the Bahamas Drilling Project, Society for Sedimentary Geology Special Publication., pp. 17–39.
- Milankovitch, M., 1930. *Mathematische Klimalehre und astronomische theorie der klimaschwankungen*. Handbuch der Klimatologie 1.
- Miller, K.G., Kominz, M.A., Browning, J.V., Wright, J.D., Mountain, G.S., Katz, M.E., Sugarman, P.J., Cramer, B.S., Christie-Blick, N., Pekar, S.F., 2005. The Phanerozoic record of global sea-level change. *Science* 310, 1293–1298.
- Peters, S.E., Husson, J.M., 2017. Sediment cycling on continental and oceanic crust. *Geology* 45, 323–326.
- Peterson, J., Hite, R., 1969. Pennsylvanian evaporite-carbonate cycles and their relation to petroleum occurrence, southern Rocky Mountains. *American Association of Petroleum Geologists Bulletin* 53, 884–908.
- Purkis, S., Cavalcante, G., Rohtla, L., Oehlert, A.M., Harris, P.M., Swart, P.K., 2017. Hydrodynamic control of whittings on Great Bahama Bank. *Geology* 45, 1–4.
- Purkis, S.J., Harris, P.M., 2016. The extent and patterns of sediment filling of accommodation space on Great Bahama Bank. *Journal of Sedimentary Research* 86, 294–310.
- Purkis, S.J., Harris, P.M., Ellis, J., 2012. Patterns of sedimentation in the contemporary Red Sea as an analog for ancient carbonates in rift settings. *Journal of Sedimentary Research* 82, 859–870.
- Purkis, S.J., Riegl, B.M., Andreouet, S., 2005. Remote sensing of geomorphology and facies patterns on a modern carbonate ramp (Arabian Gulf, Dubai, UAE). *Journal of Sedimentary Research* 75, 861–876.

- Purkis, S.J., Rowlands, G.P., Kerr, J.M., 2015. Unravelling the influence of water depth and wave energy on the facies diversity of shelf carbonates. *Sedimentology* 62, 541–565.
- Rabiner, L.R., 1989. A tutorial on hidden Markov models and selected applications in speech recognition. *Proceedings of the Institute for Electrical and Electronic Engineers* 77, 257–286.
- Rankey, E.C., 2004. On the interpretation of shallow shelf carbonate facies and habitats: how much does water depth matter? *Journal of Sedimentary Research* 74, 2–6.
- Reijmer, J.J., Swart, P.K., Bauch, T., Otto, R., Reuning, L., Roth, S., Zechel, S., 2009. A re-evaluation of facies on Great Bahama Bank I: new facies maps of western Great Bahama Bank. *Perspectives in Carbonate Geology: A Tribute to the Career of Robert Nathan Ginsburg* (Special Publication 41 of the International Association of Sedimentologists), vol. 98, pp. 29–46.
- Riegl, B.M., Purkis, S.J., 2012. Markov models for linking environments and facies in space and time (Recent Arabian Gulf, Miocene Paratethys). *Perspectives in Carbonate Geology: A Tribute to the Career of Robert Nathan Ginsburg* (Special Publication 41 of the International Association of Sedimentologists), 98, pp. 337–360.
- Robbins, L., Tao, Y., Evans, C., 1997. Temporal and spatial distribution of whittings on Great Bahama Bank and a new lime mud budget. *Geology* 25, 947–950.
- Ronov, A., 1982. The Earth's sedimentary shell (quantitative patterns of its structure, compositions, and evolution). *International Geology Review* 24, 1313–1365.
- Rowlands, G., Purkis, S., Bruckner, A., 2014. Diversity in the geomorphology of shallow-water carbonate depositional systems in the Saudi Arabian Red Sea. *Geomorphology* 222, 3–13.
- Sadler, P.M., 1994. The expected duration of upward-shallowing peritidal carbonate cycles and their terminal hiatuses. *Geological Society of America Bulletin* 106, 791–802.
- Schlager, W., 1981. The paradox of drowned reefs and carbonate platforms. *Geological Society of America Bulletin* 92, 197–211.
- Tucker, M.E., Wright, V.P., 2009. *Carbonate Sedimentology*. John Wiley & Sons.
- van den Belt, F.J., van Hoof, T.B., Pagnier, H.J., 2015. Revealing the hidden Milankovitch record from Pennsylvanian cyclothem successions and implications regarding late Paleozoic chronology and terrestrial-carbon (coal) storage. *Geosphere* 11, 1062–1076.
- Veevers, J.T., Powell, C.M., 1987. Late Paleozoic glacial episodes in Gondwanaland reflected in transgressive-regressive depositional sequences in Euramerica. *Geological Society of America Bulletin* 98, 475–487.
- Viterbi, A., 1967. Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *Institute for Electrical and Electronic Engineers transactions on Information Theory* 13, 260–269.
- Walker, L.J., Wilkinson, B.H., Ivany, L.C., 2002. Continental drift and Phanerozoic carbonate accumulation in shallow-shelf and deep-marine settings. *The Journal of Geology* 110, 75–87.
- Walther, J., 1894. *Einleitung in die Geologie als historische Wissenschaft*. Jena. Verlag von Gustav Fisher, Germany.
- Wanless, H., Shepard, F., 1936. Sea level and climatic changes related to late Paleozoic cycles. *Geological Society of America Bulletin* 47, 1177–1206.
- Weller, J.M., 1930. Cyclical sedimentation of the Pennsylvanian period and its significance. *The Journal of Geology* 38, 97–135.
- Wengert, S.A., 1962. Pennsylvanian sedimentation in Paradox basin, Four Corners region. In: Branson, C.C. (Ed.), *Pennsylvanian System in the United States - A Symposium*. American Association of Petroleum Geologists, Tulsa, Oklahoma, pp. 264–330.