CSCE 222 Discrete Structures for Computing – Fall 2021
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Problem Set 9

Due dates: Electronic submission of *yourLastName-yourFirstName-hw9.tex* and *yourLastName-yourFirstName-hw9.pdf* files of this homework is due on **Monday, 11/22/2021 11:59 p.m.** on `https://canvas.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing or unreadable, you will receive zero points for this homework.**

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Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic signature: **(type your full name here)**

Total 100 points. (The extra credit points for Problem 6 will be recorded separately.)

The problems except Problem 5 are from the lecture notes posted on Perusall. **Explain** everything carefully *in your own words*!

Problem 1. $(6+6+8 = 20$ points) Section 12.2, Exercise 12.6 [Grading rubric: (a) & (b) Explanation 4 points + correct final answer 2 points. (c) Explanation 6 points + correct final answer 2 points. In the explanation, the student should justify why combinations or permutations were chosen.]

Solution. a. For question a, the order of the boxes does not matter which makes this combination. There are 6 boxes as options and we're choosing 3 so our combination is C(6, 3). This translates to $\frac{n!}{k!(n-k)!} = \frac{6!}{3!(6-3)!} = \frac{6*5*4}{3!} = \frac{120}{6} = 20$. This mean there are 20 different ways he can load 3 boxes into his car.

b. This question again is combination since the order of the boxes does not matter. But for this one, John is making two trips instead. So we need to account for two combinations. C(6, 3) and C(3, 3) since for the second trip the amount of boxes lessens. This makes our combinations come out to be $\frac{6!}{3!(6-3)!} * \frac{3!}{3!(3-3)!}$. The first one is C(6, 3) and the second one is C(3, 3). The numbers come out to be 20 * 1 which is 20 different ways.

c. Permutations occur when the order of the choices matter. For question c, John is making 3 stacks of different sizes so the order matters since after choosing a box it can't be used for the next stacks. For the first stack, the notation is P(6, 3) meaning 6 options choose 3 of them. For the second stack, it's P(3, 2) meaning 3 options choose 2 of them. For the third stack, it's P(1, 1) since there will be only one box left. The formula for P(n, k) is $\frac{n!}{(n-k)!}$. For the first stack, this is $\frac{6!}{(6-3)!}$ which simplifies to $6 * 5 * 4 = 120$. For the second stack, it's $\frac{3!}{(3-2)!}$ which simplifies to $3 * 2 = 6$. For the third stack, it's $\frac{1!}{(1-1)!}$ which simplifies to $1 * 1 = 1$. Multiply these 3 permutations together to get $120 * 6 * 1 = 720$ different ways.

Problem 2. (10 + 10 = 20 points) Section 12.3, Exercise 12.11. For (a), use the formula involving the factorials. For (b), use the hint given in the problem statement, and explain carefully your double counting (combinatorial) proof in your own words.

Solution. a. $\binom{n}{k}k$, the $\binom{n}{k}$ portion translates to $\frac{n^k}{k!}$. In the original formula, this is multiplied by k which cancels out a k from the bottom factorial. This makes the equation become $\frac{n^k}{(k-1)!}$. If we solve backward from what we're trying to solve, n$\binom{n-1}{k-1}$ can be changed to n$\frac{(n-1)^{k-1}}{(k-1)!}$. If we go back to our other equation of $\frac{n^k}{(k-1)!}$, we can take out an n from the numerator which makes it come out to n$\frac{(n-1)^{k-1}}{(k-1)!}$. Which matches what we changed the end result to showing that they're equal to each other.

Problem 3. (20 points) Section 12.6, Exercise 12.31. Explain carefully in your own words and show your work step-by-step.

Solution. Let $S = \{1, 2, \ldots, 500\}$. We denote by S_1 the subset of pages that have a 1 in the ones place of S, by S_2 the subset of pages that have a 1 in the tens place of S, and by S_3 the subset of pages that have a 1 in the hundreds place. Then $|S_1| = |\{1, 11, \ldots, 481, 491\}| = 50$ since there's 10 numbers have a 1 in the ones place every 100 numbers so $10 * 5$ gives us the cardinality, $|S_2| = |\{10, 11, \ldots, 410, 419\}| = 50$ since there are again 10 numbers that have a 1 in the tens place every 100 numbers, $|S_3|\{100, 101, \ldots, 198, 199\}| = 100$ since the numbers that have a 1 in the hundred place range from $100 - 199$. Furthermore,
$|S_1 \cap S_2| = 5$, $|S_1 \cap S_3| = 10$, $|S_2 \cap S_3| = 10$, and $|S_1 \cap S_2 \cap S_3| = 1$.

The inclusion-exclusion formula yields
$|S \backslash (S_1 \cup S_2 \cup S_3)|$
$= |S| - |S_1| - |S_2| - |S_3| + |S_1 \cap S_2| + |S_1 \cap S_3| + |S_2 \cap S_3| - |S_1 \cap S_2 \cap S_3|$
$= 500 - 50 - 50 - 100 + 5 + 10 + 10 - 1$
$= 324$ pages that don't have a 1 in the page number
To find the number of pages that do have a 1 just subtract 324 from 500 to get 176 pages that have a 1 in them.

Problem 4. (20 points) Section 12.7, Exercise 12.33. Explain your reasoning carefully.

Solution.

Problem 5. (20 points) What is the smallest number of ordered pairs of integers (x, y) that are needed to guarantee that there are three ordered pairs $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) such that $x_1 \textbf{ mod } 5 = x_2 \textbf{ mod } 5 = x_3 \textbf{ mod } 5$ and $y_1 \textbf{ mod } 4 = y_2 \textbf{ mod } 4 = y_3 \textbf{ mod } 4$? Explain your reasoning carefully.

Solution.

Problem 6. (**Extra Credit** 30 points) Section 12.7, Exercise 12.36 (a). Explain your reasoning carefully.

Solution.

Checklist:
☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
☐ Did you sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit the .tex and .pdf files of your homework to the correct link on Canvas?