CSCE 222 Discrete Structures for Computing – Fall 2021
Hyunyoung Lee

**Problem Set 3**

**Due dates:** Electronic submission of *yourLastName-yourFirstName-hw3.tex* and *yourLastName-yourFirstName-hw3.pdf* files of this homework is due on **Friday, 9/24/2021 11:59 p.m.** on `https://canvas.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing or unreadable, you will receive zero points for this homework.**

**Name: Blake Lun** **UIN: 131001178**

**Resources.** (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

**Electronic signature: Blake Lun**

Total 100 points.

The problems are from the lecture notes posted on Perusall.

**Problem 1.** (15 points) Section 2.9, Exercise 2.35

**Solution.** If $m$ and $n$ are consecutive integers, then we can say that $n = m+1$. We can substitute n in the equation $m + n$ to make it $m + (m + 1)$ or $2m + 1$. Since $2m$ will always be an even integer no matter what the value of m, adding 1 to $2m$ makes it an odd integer, which proves the claim.

**Problem 2.** (15 points) Section 2.9, Exercise 2.36

**Solution.** If n is an odd integer, then we can assume $n = 2m + 1$ where m is any integer. For this question, instead we'll use $n = 4m + 1$ which is still an odd integer. This is so we have enough to factor out an 8 later to prove that it is divisible. $n^2 - 1$ can be factored out to $n^2 - 1 = (n + 1)(n - 1)$. Now we substitute $n$ for $4m+1$. The expression then becomes $((4m+1)+1)((4m+1)-1)$ which further simplifies to $(4m+2)(4m)$. From here, we can factor out a 2 from $(4m + 2)$ and a 4 from $(4m)$ which gives us $2(2m + 1) * 4(m)$. The constants before the parenthesis can be grouped together at the front to give $8(2m+1)(m)$, which proves the claim.

**Problem 3.** (20 points) Section 2.9, Exercise 2.41

**Solution.** To do proof by contraposition, we are trying to prove $\neg B \to \neg A$. For this question, we we negate both sides, the problems comes to be if $m \leqslant 40$ and $n \leqslant 60$, then $m+n \leqslant 100$. In this case, if we were to give m and n their greatest possible values, 40 and 60 respectively, that would make both statements true since m and n would stay within their integer restriction and $40 + 60 \leqslant 100$ is a true statement. This proves the claim.

**Problem 4.** (20 points) Section 2.9, Exercise 2.45

**Solution.** To do a proof by contradiction, we are trying to prove that $\neg A$ is false which in turn makes $A$ true. For this problem, we're looking at the claim $42m + 70n = 1000$ DOES NOT have an integer solution. For proof by contradiction, we'll look at the claim $42m + 70n = 1000$ DOES have an integer solution. We can begin by factoring out a 14 on the left side of the equation giving us $14(3m + 5n) = 1000$. We then divide both sides by 14 giving us $3m + 5n = 1000/14$. If we are trying to prove that there is an integer solution, then both m and n must be integers, which also moves the entirety of the left side ends up being an integer. If the left side of the equation is an integer, that means the right side must also be an integer. However, if we look at the right side, 1000/14 or 500/7 cannot be reduced any further to an integer. This is the simplest fraction that it can be which means that there is no integer solution, which proves the claim.

**Problem 5.** (5 points $\times$ 2 = 10 points) Section 3.1, Exercise 3.4 (a) and (c)

**Solution.** a. $\{n \epsilon Z | n^2\}$, $P(n) = n^2$
c. $\{n \epsilon Z | 2^n\}$, $P(n) = 2^n$

**Problem 6.** (20 points) Section 3.1, Exercise 3.5 [Hint: Use the definition of $\subseteq$]

**Solution.** Since $A \subseteq B$, $x \epsilon A$ implies $x \epsilon B$ which means $A = B$. Since $B \subseteq C$, $x \epsilon B$ implies $x \epsilon C$ which means $B = C$. Since $A = B$ and $B = C$, we can substitute C for B in the first equation and get $A = C$ which proves the claim $A \subseteq C$.

**Checklist:**
☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
☐ Did you sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit the .tex and .pdf files of your homework to the correct link on Canvas?