

Project 2: Simplified PageRank Algorithm

This document outlines the design, computational complexity, and reflections on the implementation of the Simplified PageRank algorithm.

1. Graph Data Structure and Justification

The web graph is implemented using an Adjacency List data structure, comprising:

- **`std::map<std::string, int> url_to_id`**: Maps URLs to unique integer IDs.
- **`std::vector<std::string> id_to_url`**: Maps integer IDs back to URL strings.
- **`std::vector<std::vector<int>> adj`**: The core adjacency list, where `adj[i]` contains IDs of pages that page `i` links to.
- **`std::vector<int> out_degree`**: Stores the count of outgoing links for each page.

Justification: Adjacency Lists are ideal for sparse web graphs, saving significant memory compared to adjacent matrices. They enable efficient iteration over a page's outgoing links during rank calculation and support dynamic sizing as new URLs are discovered.

2. Computational Complexity of Each Method

Let V be the number of unique webpages (vertices), E be the number of directed links (edges), and p be the number of power iterations.

- **`AdjacencyList()` (Constructor)**: $O(1)$. Initializes member variables.
- **`getOrCreateId(const std::string& url)`**: $O(\log V)$. Dominated by `std::map` lookup/insertion. `std::vector::push_back` is amortized $O(1)$, and `std::vector::resize` (if triggered) is amortized $O(1)$ over the total V insertions.
- **`addEdge(const std::string& from_url, const std::string& to_url)`**: $O(\log V)$. Dominated by two `getOrCreateId` calls.
- **`PageRank(int powerIterations)`**: $O(p \cdot E + V \log V)$.
 - **PageRank Iterations**: The main loop runs p times. Each iteration involves traversing all edges and vertices to distribute ranks based on incoming links according to the simplified formula. This results in $O(V+E)$, simplifying to $O(E)$. Total: $O(p \cdot E)$.

- Preparing Results: Populating the `std::map` for final sorted output involves V insertions, each $O(\log V)$. Total: $O(V \log V)$.
- Overall: The sum of these dominant phases.

3. Computational Complexity of the Main Method (runProgram)

Let N be the number of lines of input (equivalent to E , the number of edges).

- **runProgram():** $O(N \log V + p \cdot E + V \log V)$.
 - Graph Construction: Reading N input lines and calling `addEdge` for each takes $O(N \log V)$.
 - PageRank Calculation: Calling `AdjacencyList::PageRank(p)` takes $O(p \cdot E + V \log V)$.
 - Overall: The total complexity is the sum of these two phases.

4. What I Learned and What I Would Do Differently

What I Learned:

- **Core PageRank Mechanics:** Gained a clear understanding of iterative rank distribution, including the nuances of the *simplified* formula and how it differs from standard PageRank.
- **Adjacency List Practicality:** Solidified practical graph representation in C++ using adjacency lists.
- **Algorithm Efficiency & Data Structure Impact:** Enhanced my ability to analyze computational complexity and reinforced the importance of appropriate data structure selection for performance.
- **Output Management:** Learned the criticality of careful output aggregation (e.g., using `std::map` for unique, sorted results) to meet specific formatting requirements.

What I Would Do Differently if I Had to Start Over:

- **`std::unordered_map` for URL Mapping:** Use `std::unordered_map` for `url_to_id` to improve average-case lookup/insertion to $O(1)$, potentially speeding up graph construction.

- **Modular Design for PageRank Variants:** Design the PageRank method with more modularity (e.g., helper functions for iteration steps) for better readability and extensibility.
- **Enhanced Input Validation:** Add more robust checks for input data validity to improve program resilience in a production environment.
- **Precise Algorithm Implementation:** Emphasize upfront understanding of the exact mathematical PageRank formulation (as provided in the assignment) to avoid misinterpretations and ensure correct numerical results.