Handout 6
Econ 502
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1 Auction Theory

Remember that if $v \in \mathbb{R}^n$ *iid* $\sim F$, then $P(v_i > v_j, \forall j \neq i) = F(v_i)^{n-1}$, so the density of the largest order statistic of $n-1$ other agents is $(n-1)F(v_{(1)})^{n-2} f(v_{(1)})$.

1.1 Second-price Auction

Denote the second-price auction with reserve price r and entry fee c as SPA(r, c). We have the following result:

Theorem 1.1 (SPA$(r, 0)$ in dominant strategies)**.** *The following consitutes a weakly dominant strategy for player i in the* SPA$(r, 0)$*:*

$$b(v) = \begin{cases} \text{No} & \textit{if } v < r \\ v & \textit{if } v \geq r \end{cases} \tag{1}$$

where No *indicates no participation.*

Let $v_0(r, c)$ be the *marginalvalue*, the valuation for which a potential bidder is indifferent between participating and not. This is part of the equilibrium, and clearly depends on the distribution of values F.

Theorem 1.2 (SPA(r, c))**.** *Assume $0 \leq c \leq 1 - r \leq 1$. Then, a BNE of the* SPA(r, c) *with n bidders consists of each player using the following strategy:*

$$b(v) = \begin{cases} \text{No} & \textit{if } v < v_0(r, c) \\ v & \textit{if } v \geq v_0(r, c) \end{cases} \tag{2}$$

where $v_0(r, c)$ solves $(v_0 - r)F(v_0)^{n-1} = c$

1.2 First-price Auction

Denote the first-price auction with reserve price r and entry fee c as FPA(r, c).

Theorem 1.3 (Symmetric Equilibrium in FPA$(0, 0)$)**.** *Consider a* FPA$(0, 0)$ *with n bidders and iid private values with distribution F. THen the following is a symmetric BNE strategy:*

$$b(v) = \int_0^v t \frac{g(t)}{G(v)}\, dt = v - \int_0^v \frac{G(t)}{G(v)}\, dt = \mathrm{E}[v_{(1)} \mid v_{(1)} < v] \tag{3}$$

where $G(v) = F(v)^{n-1}$ and $v_{(1)}$ is the first order statistic among over $n-1$ agents. Moreover, b is strictly increasing on $[0, 1]$ and is differentiable.

2 Revenue Equivalence

Myerson (1981) asks which mechanism maximizes the revenue of the seller among all feasible mechanism (where feasible means implementable in Bayesian Nash equilibrium and individually rational). It turns out that just changing the format of the auction without changing who gets the item won't make much of a difference:

Theorem 2.1 (Revenue Equivalence)**.** *Suppose bidders are risk neutral and have iid private values. Then, all equilibria of any two auctions that generate the same allocation rule and the same conditional expected payoffs for each buyer with value* 0 *produce the same expected revenue for the seller.*

Revenue equivalence in auctions is a special case of the following theorem:

Theorem 2.2 (General Revenue Equivalence)**.** *Suppose agents are risk neutral and have independent private types distributed on connected supports. Then, all equilibria of any two mechanisms that implement the same social choice function and give the same conditional expected payoffs for some type of each agent produce the same expected revenue.*

Revenue equivalence has two important implications. First, we'll have to depart from efficiency or IR if we want to increase revenue. Second, since we don't observe empirical equivalence between first- and second-price auctions, we'll need to invoke risk-aversion, common-values, or non-expected-utility maximization to fully explain behavior in auctions.