Handout 3
Econ 502
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1 Finitely repeated games

Unsurprisingly, in repeated games we're analyzing a specific game (such as the prisoners' dilemma, battle of the sexes, etc) that is played over and over again in a supergame. At each stage of the repetition, let G denote a static game of complete information in which the players involved choose their corresponding actions simultaneously. The game G will then be called the **stage game** of the full game.

Definition 1.1. *Given a stage game G, let $G(T)$ denote the finitely repeated game in which G is played $T < \infty$ times, with the outcomes of all preceding plays observed before the next play begins. The payoffs of $G(T)$ are simply the sum of the payoffs for each of the T stages.*

Theorem 1.1. *If the stage game G has a unique Nash equilibrium, then for all finite T, the repeated game $G(T)$ has a unique subgame-perfect outcome where the unique NE is played at every stage.*

We have seen that if there are multiple equilibria at every stage, then there might be subgame-perfect NEs that do not involve playing "stage-game equilibrium strategies". Remember the example from last class where this occurred with the two-stage game.

2 Infinitely repeated games

In finitely repeated games, there might be credible threats that influence current behavior if the stage game has multiple equilibria. A stronger result can be stated for the infinitely repeated case: even if the stage game has a unique NE, there might be subgame-perfect NEs of the infinitely repeated game in which no choice of actions in any stage is a NE of G.

Definition 2.1. *Given a stage game G, let $G(\infty, \delta)$ denote the infinitely repeated game in which G is repeated indefinitely and players share the discount factor δ. For each t, the outcomes the the $t-1$ preceding plays are observed before the stage begins. The payoffs of each player are the present values of the sequence of payoffs from the stage games.*

2.1 Payoffs

In contrast to simply summing up payoffs like in the finitely repeated games, we're now discounting future payoffs to make the near future more salient and to keep things tractable. Given a discount factor δ and a sequence of payoffs $\{\pi_t^i\}_{t=1}^{\infty}$ to player i, the total payoff is $\pi_1^i + \delta\pi_2^i + \delta^2\pi_3^i + \ldots = \sum \delta^{t-1}\pi_t$. Occasionally this is rescaled by $1-\delta$ so we get rid of this factor when stage game payoffs are identical across all periods.

2.2 Folk theorems

Definition 2.2. *In an n-player game, call the vector $(x_1, \ldots, x_n)$ a feasible vector in stage game G if each x_i is a convex combination of the pure-strategy payoffs of G.*

Theorem 2.1. *(Friedman 1971) Let G be a finite, static game of complete information with n players. Let $(e_1, \ldots, e_n)$ be the payoffs from a Nash equilibrium of G, and let $(x_1, \ldots, x_n)$ be any other feasible payoffs from G, If $x_1 > e_i$ for each player i and δ is sufficiently close to one, then there exists a subgame-perfect Nash equilibrium of the infinitely repeated game $G(\infty, \delta)$ that achieves $(x_1, \ldots, x_n)$ as payoffs.*