Handout 5
Econ 502
February 22, 2012
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1 Mechanism Design with Money

Last week we saw how to bypass the Gibbard-Satterthwaite theorem by restricting preferences to be single-peaked. Another setting with positive results is when agents have quasi-linear preferences over transfers. The ability of the mechanism designer to charge or compensate agents opens up new possibilities for incentives besides changing the direct outcome. With new possibilities comes new problems, though. The designer must to be careful agents aren't charged so much they're no longer interested in participating. The mechanism might also run a surplus or deficit if transfers between agents don't balance out.

Definition 1.1. *Agent i has* ***quasi-linear preferences*** *over transfers iff the agent's utility function is representable as $u_i(\theta_i, a, t_i) = v_i(\theta_i, a) + t_i$ for some function $v_i(\theta_i, a)$.*

For now, we are concerned with implementing an efficient choice. Later we will consider alternative objectives like maximizing the revenue of the designer.

Definition 1.2. *A choice $a^* \in A$ is* ***efficient*** *in state θ iff $a^* \in \text{argmax}_{a \in A} \sum_i v_i(\theta_i, a)$.*

Definition 1.3. *The* ***efficiency correspondence*** *$\bar{F} : \Theta \twoheadrightarrow A$ assigns to each state the efficient choices. Similarly, an* ***efficiency decision rule*** *is a function that assigns some efficient choice.*

Definition 1.4. *In a setting with tranfers and n agents, a social choice function $f : \Theta \to A \times \mathbb{R}^n$ is a pair $(d(\theta), t(\theta))$, where d is a decision rule which selects some collective option from A and t defines a vector of transfers to agents (with negative transfers being payments by agents).*

We'll typically assume the designer can't run a deficit, so the feasible mechanisms are self-financing ones.

Definition 1.5. *A transfer function t is* ***feasible*** *if $\sum_i t_i(\theta) \leq 0$ for all $\theta \in \Theta$. A transfer function is* ***ex-post budget balanced*** *if $\sum_i t_i(\theta) = 0$ for all θ.*

We don't want to assume agents are forced to participate, so we have the notion of ex-post individual rationality.

Definition 1.6. *A social choice function (d, t) is* ***ex-post individually rational*** *iff $v_i(\theta_i, F(\theta)) + t_i(\theta) \geq 0$ for all i and θ_i.*

The *ex-post* label refers to these properties holding for all possible realizations of types. Alternately, we can consider *ex-ante* or *interim* properties that hold in expectation over all type profiles or in expectation conditional on one agent's type, respectively. More on these in later classes.

2 Vickrey-Clarke-Groves Mechanisms

The basic VCG mechanism is a rule for selecting the efficient choice along with specific monetary transfers. Here "basic" refers to the fact that an entire class of mechanisms have the same properties.

Definition 2.1. *Given an efficient decision rule d and reports θ^* of the n agents, the* ***basic VCG*** *mechanism selects $d(\theta^*) \in A$ and allocates transfers*

$$t_i^{VCG}(\theta^*) = \sum_{j \neq i} v_j(\theta_j^*, d(\theta^*))$$

Definition 2.2. *Given an efficient decision rule d, a mechanism is in the **VCG family** or is a **Groves mechanism** if for reports θ^* of the n agents*

1. *The mechanism selects $d(\theta^*) \in A$.*
2. *The transfer has the form $t_i(\theta^*) = t_i^{VCG}(\theta^*) + h_i(\theta_{-i})$, for any set of n functions $h_i : \Theta_{-i} \to \mathbb{R}$.*

This additional term $h_i(\theta_{-i})$ is sometimes called an individualized tax. On an exam, if you are asked to define the family of VCG mechanisms for a given problem, you have to adjust this definition and find the objective and transfers up to some arbitrary functions.

Notice that with a VCG mechanism, each agent has the social welfare criterion as his own objective, plus something that is unchanged by his report. The individuals' incentives are now aligned with the group, so honest reporting becomes (weakly) dominant. Surprisingly, the converse is also true.

Theorem 2.3. *(Groves 1973) The family of VCG mechanisms is dominant strategy incentive compatible (strategy-proof).*

Theorem 2.4. *(Green and Laffont 1977) Conversely, if d is an efficient decision rule, the social choice function (d, t) is strategy-proof, and the type spaces are complete in the sense that each $a \in A$ is selected by d for some type profile, then the mechanism is in the VCG family.*

Even weakening dominant-strategy IC to Bayesian IC doesn't change the class of efficient mechanisms:

Theorem 2.5. *(Williams 1999) Every efficient and Bayesian incentive compatible mechanism with transfers gives the same interim payoffs as some Groves mechanism, assuming type spaces are connected, open subsets of Euclidean space and interim valuations are continuously differentiable.*

We still have an open question about the "best" h_i's. If we let h_i be zero as in the basic mechanism, then each player receives large transfers, violating feasibility. Intuitively, we would prefer something like a Pigouvian tax, where transfers reflect the externality of adding a person to society.

Definition 2.6. *The **Clarke pivotal mechanism** for an efficient decision rule d is a VCG mechanism where*

$$t_i(\theta) = \sum_{j \neq i} v_j(\theta_j, d(\theta)) - \max_{a \in A} \sum_{j \neq i} v_j(\theta_j, a)$$

Theorem 2.7. *If $v_i(\theta_i, a) \geq 0$ for all i, the pivotal mechanism always makes negative transfers, is feasible, and is ex-post individually rational.*

Sometimes the pivotal mechanism is referred to as *the* VCG mechanism, while any other choice of h_i's is *a* VCG mechanism or a Groves mechanism.

3 Virtues and Vices of the VCG

The work leading up to the VCG was a theoretical *tour-de-force*. The characterization of how to achieve perfect efficiency in dominant strategies is elegant and appealing. However, while shedding insight into the workings of mechanisms, it's not something you'd actually want to use. "Thirteen reasons why the VCG process is not practical" (Rothkopf 2007) documents these clearly:

1. the nonexistence of dominant-strategy equilibria in models with reasonable bid preparation costs;
2. problems associated with the disclosure of valuable confidential information;
3. the exponential growth of effort related to bid preparation and bid communication;
4. the NP-completeness of the winner determination problem;

5. the dominant strategy equilibrium is a weak equilibrium and there may exist alternative weak equilibria;
6. problems related to capital-limited bidders;
7. problems associated with various kinds of cheating including:
   (a) false bids by the bid taker,
   (b) conspiracies by competing bidders,
   (c) conspiracies in two-sided markets between bidders offering to sell and those offering to buy,
   (d) and the use of false-name bids by single bidders;
8. the fact that strategies in sequences of strategy-proof auctions may not be strategy-proof;
9. the fact that the process can be revenue deficient.

The first three problems are common to most direct mechanisms in combinatorial settings. Direct mechanisms are very useful theoretically, but communication costs (from transmission, preparation, or privacy-violations) break the revelation principle.

Notice that the efficiency of the VCG applies only to the allocation/decision, assuming negative transfers aren't lost from the economy. The pivotal mechanism actually turns out to maximize total payments among feasible, ex-post IR mechanisms, which can be a good thing when payments are seen as revenue. However, if there isn't someone external to absorb the payments, the benefit of efficiency in decisions is partially reduced.

Every cloud has a silver lining though. Each deficiency with the VCG is a new opportunity for budding market designers. For instance, redistribution mechanisms [1] return some of the transfers back to participants relative to the pivotal mechanism while maintaining feasibility. Clippel et al. (2012), "Destroy to save" shows that allowing a tiny bit of inefficiency in the allocation can substantially reduce the amount of surplus taken from participants, improving overall participant welfare.

4 Exercises

Exercise 4.1. (Inspired by Fishburn (1973), "Summation Social Choice Functions") Assume agents' utility functions over the choice sets X_i are taken from the set of positive-valued, continuous, strictly concave, and strictly monotone real-valued functions, and agents are heterogeneous. Assume the feasible vectors of choices make up a compact set, denoted by $Y \subset \prod^n X_i$. Define the social choice function as a mapping that maximizes the sum of all individual utilities over the feasible vectors. Setup the SCF and check: Is it well-defined? Is this scf monotone? Does it satisfy NVP?

Exercise 4.2. Given a set of alternatives A and n players, let $S(\theta) \subset \mathbb{R}^n$ be the image of A under players' utility functions, describing the profiles of feasible utilities. Given some status-quo alternative $\underline{a}$, suppose $S(\theta) \cap \{s : s_i \geq u_i(\underline{a}, \theta_i)\}$ is compact and convex. Define the Nash bargaining correspondence as

$$f^N(\theta) = \underset{\substack{a\in A: \\ u_i(a,\theta_i)\geq u_i(\underline{a},\theta_i)}}{\operatorname{argmax}} \prod_{i=1}^{n} \left(u_i(a,\theta_i) - u_i(\underline{a},\theta_i)\right)$$

Is f^N well-defined, monotone, or satisfy NVP?

Exercise 4.3. (Dutta and Sen (2009), "Nash Implementation with Partially Honest Individuals") Assume at least one individual in the population is partially honest, i.e. the agent lexicographically prefers the honest or helpful strategy to deceptive ones. Show NVP is sufficient for Nash implementation.

[1] See Ruggerio (2006), "Optimal decision-making with minimal waste", Guo and Conitzer (2010), "Optimal-in-expectation redistribution mechanisms", and Ruggerio (2012) "Fairness and welfare through redistribution when utility is transferable", among others.

Exercise 4.4. Describe the VCG family and the pivotal mechanism for the following applications:

1. Living arrangements: Three grad students, Abel, Baker, and Charlie, are renting a three-bedroom apartment together. The rent is \$600 and they've committed to paying equal shares. Now they have to decide who gets the big room, the small room with a view, and the small dark room. Abel values the three at an additional \$200, \$100, and \$0 respectively. Baker's valuations are \$100, \$60, and \$0. Charlie's valuations are \$25, \$0, and \$50. Now suppose Charlie's valuations are \$150, \$25, and \$0.

2. Combinatorial auction: Later, during finals season when no one has time to go shopping, the roommates discover the only things left in their refrigerator are a slice of ham, a carrot, and a can of PBR. Abel values the slice of ham and the PBR jointly at \$6, but would rather get nothing than any single item. Baker is a vegetarian and values the carrot and the PBR at \$4, the carrot at \$1, and the PBR at \$2. Charlie is only after the PBR, which he values at \$4.

3. Multi-unit auction: K units of an identical good are up for sale and each buyer has a valuation v_i for a single item.

4. Multi-unit auction 2: K units of an identical good are up for sale and each buyer has a valuation v_i^k for the k-th unit they receive. For instance, if player i receives two units, they have utility $v_i^1 + v_i^2$.

5. Buying a path in a network: Consider a series of roads between towns modeled as a weighted directed graph $G = (V, E)$, where each road $e \in E$ is owned by a local Baron and has a cost $c_e \geq 0$ if the road is used by the King's Army. Suppose we want to procure a path for soldiers between two towns $s, t \in V$. How should the King's economists compensate the Barons for routing the army through their fiefdoms?