Blake Van Dyken

Assignment 5

Methods Used:

Governing Equation: $\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2}$

Initial Conditions: $u(x\epsilon(0, L), t = 0) = f(x) = 20℃$

Boundary Conditions: $u(x = 0, t) = 60℃$

$u(x = L, t) = 20℃$

Derived 1D Heat Equation:

Delta_X = Length / N - 1

Determine the stencil for $\frac{\partial^2 u}{\partial x^2}$, where $u_i'' = \frac{\partial^2 u_i}{\partial x^2}$

$$u_{i+1} = u_i + \frac{u_i'}{1!}\cancel{\Delta x} + \frac{u_i''}{2!}\Delta x^2 + \cancel{\frac{u_i'''}{3!}\Delta x^3} + O(\Delta x^4)$$

$$u_{i-1} = u_i + \frac{u_i'}{1!}\cancel{(-\Delta x)} + \frac{u_i''}{2!}(-\Delta x)^2 + \frac{u_i'''}{3!}\cancel{(-\Delta x)^3} + O(\Delta x^4)$$

$$u_{i-1} = 2u_i + u_i''\Delta x^2 + O(\Delta x^4) \Rightarrow -u_i''\frac{\Delta x^2}{\Delta x^2} = \frac{2u_i + O(\Delta x^4)}{\Delta x^2}$$

$$u_i'' = \frac{\partial^2 u_i}{\partial t^2} = u_i'' = \frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta x^2} + \frac{O(\Delta x^4)}{\Delta x^2}$$

t_0 t_1 t_n

$$\frac{\Delta u}{\Delta t} = \frac{u_i(\Delta t + t) - u(t)}{\cancel{t} + \Delta t - \cancel{t}} = a\cdot\frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta x^2}$$

$$= u_i(t) + a\frac{\Delta t}{\Delta x^2}\cdot(u_{i-1} - 2u_i + u_{i+1})$$

For $\Delta t = 1s$

Expand the following equations

$u_1(t + \Delta t) = u_1(t) = 25°C$

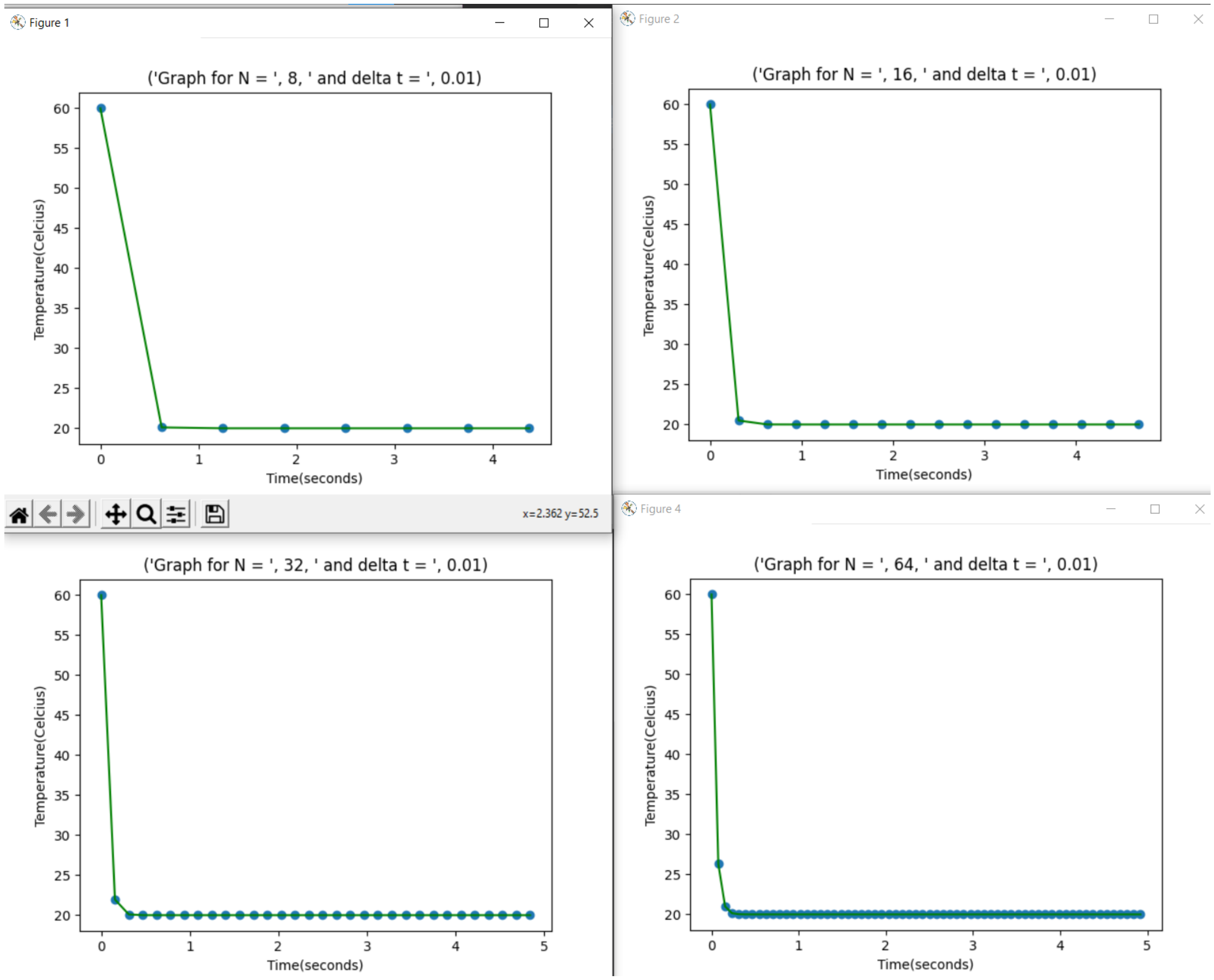
$u_2(t + \Delta t) = a\cdot\Delta t\cdot\frac{u_1 - 2u_2 + u_3}{\Delta x^2} + u_2 = 75 + \frac{.01\cdot 1}{1^2}(25 - 2\cdot 75 + 75) = 74.5$

$u_3(t + \Delta t) = u_3 + \frac{a\Delta t}{\Delta x^2}(u_2 - 2u_3 + u_4) = 75 + \frac{.01\cdot 1}{1^2}(75 - 2\cdot 75 + 75) = 75$

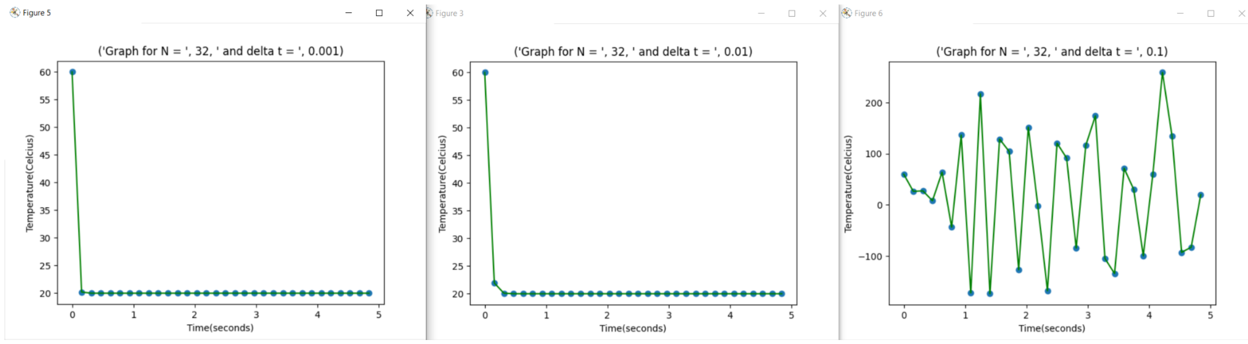
$u_4(t + \Delta t) = u_4 + \frac{a\Delta t}{\Delta x^2}(u_3 - 2u_4 + u_5) = 75 + \frac{.01}{1^2}(75 - 2\cdot 75 + 25) = 74.5$

$u_5(t + \Delta t) = u_5(t) = 25°C$

Results and Figures:



In these first four graphs, our delta t is the same, 0.01, but our N is changing. As N increases, we get more and more measurements, thus we get more accurate measurements as we can see in the first graph we have fewer measurements taken across the five-second interval. However, as we go on to the following graphs up to N = 64, we have many more measurements that will be more accurate.



In these three graphs, delta t is changing, while N stays at 32. Thus we are taking the same amount of measurements at each timestep, however, we are measuring at fewer intervals of time. Looking at when t = 0.001, we have accurate results here because there is less of a gap between each time the measurements are taken, whereas looking at the last chart when delta t = 0.1, we get very inaccurate results and the graph just becomes wrong because we wait too long between measurements to get accurate results.

Sources:

All my code is submitted with the project and on GitHub here:

https://github.com/iPupkin/Theory-3200