8. (a) i. Calculate (as a function of the variable x) the determinant of matrix [8]

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & (1+x) & -3 \\ -3 & 2 & (2-x) \end{bmatrix}$$

ii. Under what conditions on x is there an inverse for the matrix A?

iii. Find the adjoint, $\mathrm{adj}(A)$.

(b) An upper diagonal matrix is a square matrix of order n such that all entries below the main diagonal are 0. That is, any matrix B for which $B_{ij} = 0$ if $i > j$. Prove that $|B| = B_{11}B_{22}\ldots B_{nn}$. [6]

(c) Calculate the eigenvalues of the matrix [3]

$$C = \begin{bmatrix} 2 & 1 & \pi & \sqrt{2} \\ 0 & 0 & 3 & e \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$