7. The theme of Question 7 is the untyped λ-calculus. That is, the calculus of anonymous functions such as $\lambda n \;.\; (+1)\; n$ which form the logical foundation of functional programming.

   (a) Give a hand coded evaluation for the lambda expression, $(+)\;((\lambda n \;.\; (+)\; n\; 2)\; 3)\;((\lambda n \;.\; (*)\; 3\;((-)\; 6\; n))\; 4)$. [12]

   (b) The so-called *Y combinator* is defined in the original notation of untyped $\lambda-$calculus by, $Y \;=\; \lambda f \;.\; (\lambda x \;.\; f(x(x)))\;(\lambda x \;.\; f(x(x)))$. For any function g prove that $Y(g) \rightsquigarrow g(Y(g))$. That is, $Y(g)$ reduces to $g(Y(g))$. [8]