7. (a) Work out the Bezier polynomials for a cubic spline. [5]

(b) Derive a Bezier matrix, $\mathbf{B}$, for a cubic spline satisfying the blending formula:

$$\mathbf{q}(u) = \mathbf{U}\mathbf{B}\mathbf{b}$$

making the definitions of $\mathbf{U}$ and $\mathbf{b}$ clear. [7]

(c) Derive the conditions under which 1st and 2nd order continuity can be achieved between successive knots of the spline designed above. [7]

(d) Given the forward-differing approximation:

$$\Delta x(u) = x(u + \delta) - x(u),$$

find an expression for $\Delta x(u)$ if

$$x(u) = a_3 x^3 + a_2 x^2 + a_1 x + a_0.$$

Explain why this result is helpful in reducing the calculations required to draw splines. How many operations are required to calculated one step, δ, forward when drawing a 2D cubic spline using forward-differencing? [6]