

9. (a) Give the definition of the following concepts: [6]
- i. The *span* of a set of vectors V
 - ii. A subspace S of \mathbb{R}^n
 - iii. A basis of a subspace S
- (b) Prove that the following sets each form a basis for \mathbb{R}^4 : [4]
- i. $V = \{(1, 1, 1, 0), (1, 1, 0, 1), (1, 0, 1, 1), (0, 1, 1, 1)\}$
 - ii. $W = \{(1, 1, 1, 1), (1, 1, -1, -1), (1, -1, 0, 0), (0, 0, 1, -1)\}$
- (c) An orthogonal basis is one where every distinct pair of vectors in the basis is perpendicular. For example, the standard basis is orthogonal. Let M be the square matrix formed with the vectors of a basis as columns. Prove that $M^T M$ is diagonal if the corresponding basis is orthogonal. Is either of V or W an orthogonal basis? [7]