10. (a) If x is real show that the maximum value of [8]

$$y = \frac{x^2 - x + 1}{x^2 + x + 1}$$

is 3, and the minimum is $\frac{1}{3}$. Draw the graph of y as a function of x.

(b) i. Find the general solution to the 2nd order homogeneous linear recurrence [9]

$$2u_{n+1} + 3u_n - 2u_{n-1} = 0.$$

ii. Give a necessary and sufficient condition on u_0 and u_1 such that the sequence defined by the recurrence converges.

iii. Find the general solution to the 2nd order linear recurrence:

$$2v_{n+1} + 3v_n - 2v_{n-1} = n - 3.$$

iv. If $u_0 = u_1 = 0$, give a formula for u_n.