11. (a) Evaluate [8]

$$\text{(i)} \quad \int_0^{\frac{1}{4}\pi} \cos^2 x \quad dx, \qquad\qquad \text{(ii)} \quad \int_0^{\frac{1}{4}\pi} x\cos^2 x \quad dx.$$

Tip: Make use of the trigonometric functions:

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos^2 x + \sin^2 x = 1$

(b) The interval $0 \leq x \leq 1$ is divided into n equal subintervals by points x_r $(r = 0, 1, \ldots, n)$ such that [9]

$$0 = x_0 < x_1 < x_2 < \cdots < x_r < \ldots < x_n = 1.$$

By considering

$$\sum_{r=1}^{n} \mathrm{f}(x_r)(x_r - x_{r-1}),$$

where $\mathrm{f}(x) = 1/(1+x^2)$, show that

$$\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \cdots + \frac{n}{n^2+n^2}\right) = \frac{1}{4}\pi.$$