

4. (a) Determine whether the following set of vectors are linearly independent in \mathbb{R}^3 : [2]

$$\{(2, 3, 4), (2, 1, -1), (-6, 1, 13)\}$$

(b) Determine whether the following transformations are linear transformations: [3]

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ defined by } T(x, y) = (2x - 3y, 3x - 2y)$$

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ defined by}$$

$$T(x, y) = \left(\frac{(x + y)^2 - (x - y)^2}{x}, (x + iy)(x - iy) - (y + ix)(y - ix), \frac{x^2 - y^2}{x + y} \right)$$

(c) Find the dimension, and a basis, of the subspace spanned by the following set of vectors: [3]

$$\{(2, 0, 1, -4), (-1, -1, 0, 2), (1, -3, 2, -2), (0, 0, -2, 0), (0, 4, -2, 0)\} \subseteq \mathbb{R}^3.$$