4. (a) Prove that the following formula is unsatisfiable using the Nelson-Oppen procedure, where the variables are interpreted over the integers:

$$g(f(x_1 - 2)) = x_1 + 2 \ \wedge \ g(f(x_2)) = x_2 - 2 \ \wedge \ x_2 + 1 = x_1 - 1. \quad [15]$$

(b) A simple improvement to the Nelson-Oppen procedure for convex theories is to restrict the equality propagation step as follows:

We call a variable *local* when it appears only in the literals of a single theory. If an equality $x = y$ is implied by the literals F_i of a theory T_i and not by the literals F_j of a theory T_j, then we propagate it to F_j only if both x and y are not local to F_i.

Explain why this improvement is correct. [20]