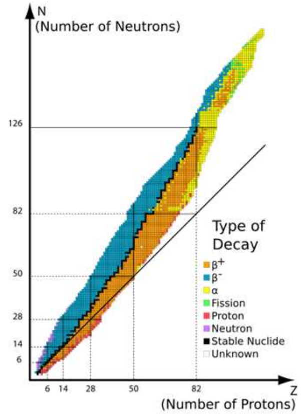
UiO : Department of Physics
University of Oslo

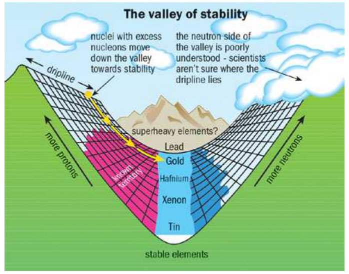
Oslo University Hospital

Radioactivity and dose in radioactive media

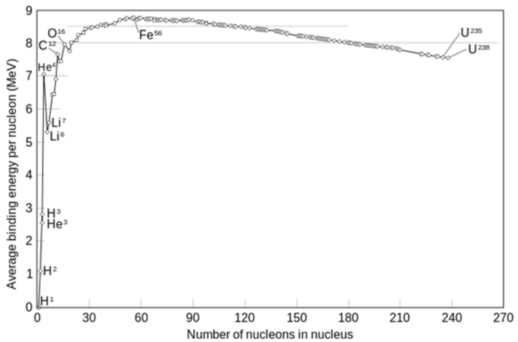
Eirik Malinen

The valley of stability





Nuclear binding energy



Radioactivity and radiation dose

- About 255 stable nuclei
- Nuclides in nature: 340
- Decay modes in radioactive nuclei
  - β^- : neutron rich
  - β^+ : neutron deficient
  - EC : neutron deficient near valley
  - α : heavy nuclei
  - Fission : very heavy nuclei

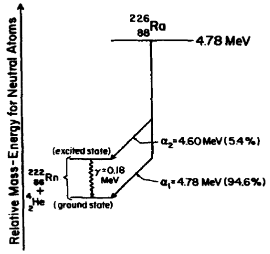
α-disintegration

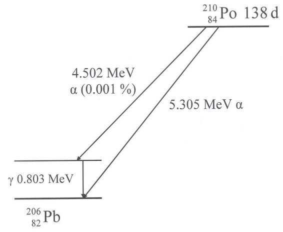
X ⟶ α + Y

X	α	Y
Z protons N neutrons A nucleons	2 protons 2 neutrons 4 nucleons	Z-2 protons N-2 neutrons A-4 nucleons

- α-particle typically have 2-7 MeV energy
- ~4 cm range in air; 50-80 μm in water
- Becomes He-atom after slowing down

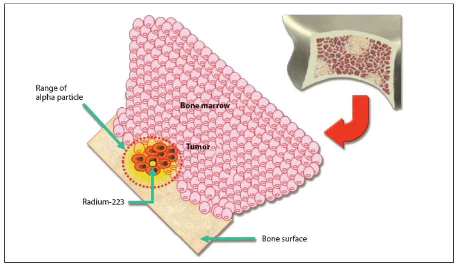
α- decay scheme





Ra-223 - therapy

- Radiopharmaceutical for bone cancer



- Ra-223 and daughters emit over 95% of the decay energy in the form of alpha radiation

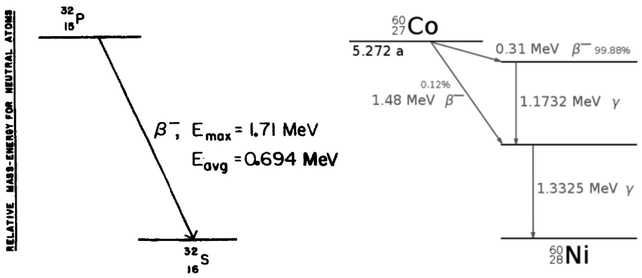
β^--disintegration

X ⟶ e^- + $\bar{\nu}_e$ + Y

X	e^-	$\bar{\nu}_e$	Y
Z protons N neutrons A nucleons	electron	anti- neutrino	Z+1 protons N-1 neutrons A nucleons

- Spectrum of electrons emitted: T=[0, $T_{\beta,max}$]
- Remember: $n \rightarrow p + e^- + \bar{\nu}_e$

β^--decay scheme



β^+-disintegration

X ⟶ e^+ + ν_e + Y

Z protons
N neutrons
A nucleons

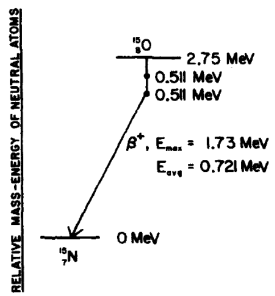
positron

neutrino

Z-1 protons
N+1 neutrons
A nucleons

- Spectrum of positrons emitted
- Here: p +1.022 MeV → n + e^+ + ν_e

β^+-decay scheme



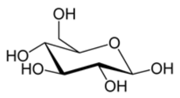
Fluorine-18

- Irradiation of stable oxygen-18 isotope with high energy protons (18 MeV)

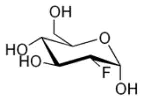
$$_8O^{18} + _1p^1 \rightarrow {}_9F^{18} + _0n^1$$

- The target is either normal or enriched water

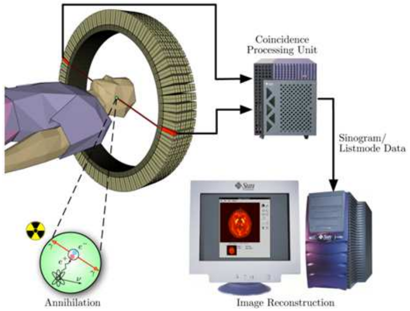
Fluorine-18 - PET



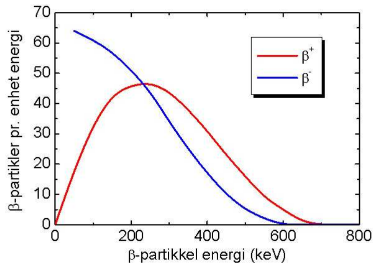
Glukose



Fluordeoksyglukose



β^-/β^+ decay spectrum



- β^- spectrum shifted due to electrostatic attraction with nucleus
- Rule of thumb: $\overline{T} = 1/3\ T_{max}$

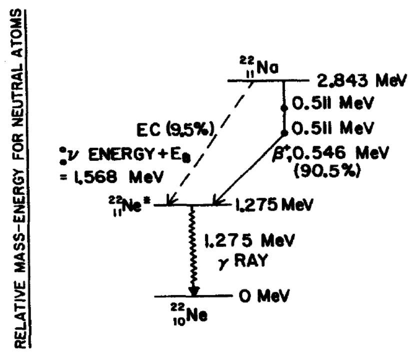
Disintegration by electron capture

X + e^- ⟶ ν_e + Y

X			Y
Z protons N neutrons A nucleons	electron	neutrino	Z-1 protons N+1 neutrons A nucleons

- Same daughter nuclei as β^+ disintegration

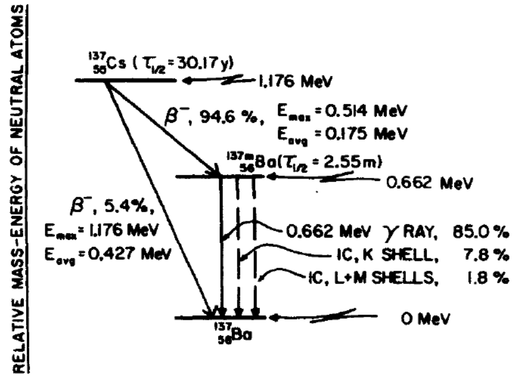
EC / β^+-decay scheme



Internal conversion

- γ-ray emission with energy $h\nu$ often accompanies decay
- Alternative: impart kinetic energy to atomic electron
- Energy: $T = h\nu - E_b$

Internal conversion cont'd



Radioactivity and radiation dose

- Radioactivity important in
  - Radiation protection – external and internal exposure
  - Brachytherapy
  - Nuclear medicine – PET / SPECT, Radionuclide / Radioimmunotherapy

Radioactivity

- Number of decay events *dN* occurring during *dt* is proportional to number of atoms N:

$$\frac{dN}{dt} \propto -N$$

- Decay constant λ introduced:

$$\frac{dN}{dt} = -\lambda N$$

- Simple differential equation with solution:

$$N = N_0 e^{-\lambda t}$$

Radioactivity

- λ is characteristic of each nuclide; decay probability per unit time per atom
- Rate of decay; activity is given by:

$$\left|\frac{dN}{dt}\right| = \left|\frac{d}{dt}\left(N_0 e^{-\lambda t}\right)\right| = \lambda N$$

- If nucleus has several modes of disintegration:

$$\lambda = \lambda_A + \lambda_B + \cdots = \sum_i \lambda_i$$

Radioactivity cont'd

- The total activity is then

$$\lambda N = N \sum_i \lambda_i$$

- Partial activity for i'th mode of disintegration:

$$\lambda_i N = \lambda_i N_0 e^{-\lambda t}$$

- Activity given in units of Becquerel [Bq] = s^{-1}
- Earlier: 1 Ci = 3.7×10^{10} Bq

Radioactivity cont'd

- Time needed to decay to 1/e of original number of nuclei:

$$\frac{N}{N_0} = \frac{1}{e} = e^{-\lambda\tau} \Rightarrow \tau = \frac{1}{\lambda}$$

- τ: mean life, average lifetime of a nucleus.
- Probability of nucleus not having decayed:

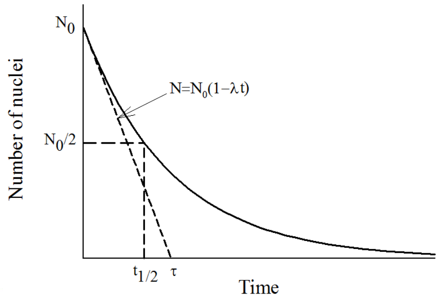
$$p(t) = C\exp^{-\lambda t} \Rightarrow \int_0^\infty p(t)dt \overset{!}{=} 1 \Rightarrow C = \frac{1}{\lambda}$$

- Mean lifetime: $\tau = \int_0^\infty t p(t)dt = \frac{1}{\lambda}\int_0^\infty t e^{-\lambda t}dt = \frac{1}{\lambda}$

Radioactivity cont'd

- Half life $t_{1/2}$ defined as

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda t_{1/2}} \Rightarrow t_{1/2} = \frac{\ln(2)}{\lambda}$$



Parent-daughter relationships

- Initial population of parent nuclei $N_{P,0}$
- Decays into a radioactive daughter by λ_P
- Daughter activity:

$$\frac{dN_D}{dt} = \lambda_P N_P - \lambda_D N_D = \lambda_P N_{P,0} e^{-\lambda t} - \lambda_D N_D$$

- It can be shown that:

$$N_D = N_{P,0} \frac{\lambda_P}{\lambda_D - \lambda_P} \left(e^{-\lambda_P t} - e^{-\lambda_D t}\right)$$

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Parent-daughter relationships cont'd

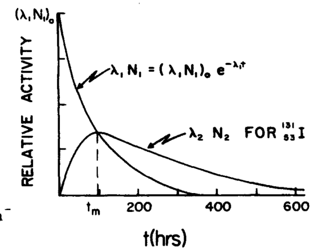
- Daughter will have zero activity at t=0 and t=∞. Maximum of N_D reached at:

$$\frac{dN_D}{dt} = 0 \;\Rightarrow\; -\lambda_P e^{-\lambda_P t_m} + \lambda_D e^{-\lambda_D t_m} = 0$$

$$\Rightarrow t_m = \frac{\ln(\lambda_D / \lambda_P)}{\lambda_D - \lambda_P}$$

$${}^{131m}_{52}\mathrm{Te} \xrightarrow[\tau_{1/2} = 30\,\mathrm{h}]{\beta^-} {}^{131}_{53}\mathrm{I} \xrightarrow[\tau_{1/2} = 193\,\mathrm{h}]{\beta^-} {}^{131}_{54}\mathrm{Xe}$$

$\lambda_1 = 2.31 \times 10^{-2}\ \mathrm{h}^{-1}$, $\lambda_2 = 3.59 \times 10^{-3}\ \mathrm{h}^{-}$



UiO : Department of Physics
University of Oslo

Oslo University Hospital

Parent-daughter relationships cont'd

- Ratio of activities:

$$\frac{\lambda_D N_D}{\lambda_P N_P} = \frac{\lambda_D N_{P,0} \frac{\lambda_P}{\lambda_D - \lambda_P}\left(e^{-\lambda_P t} - e^{-\lambda_D t}\right)}{\lambda_P N_{P,0} e^{-\lambda_P t}}$$

$$= \frac{\lambda_D}{\lambda_D - \lambda_P}\left(1 - e^{-(\lambda_D t - \lambda_P t)}\right)$$

- For $\lambda_D < \lambda_P$, the ratio increases with time
- For $\lambda_D > \lambda_P$ at t>> t_m, the ratio is $\lambda_D / (\lambda_D - \lambda_P)$

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Parent-daughter relationships cont'd

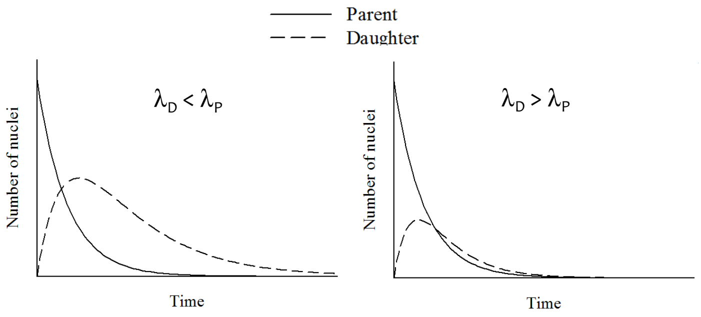
- For $\lambda_D >> \lambda_P$, the activity ratio is unity
- Example

$${}^{226}_{88}\mathrm{Ra} \xrightarrow[\substack{\tau_{1/2} = 1602\mathrm{y} \\ \lambda_1 = 1.1845 \times 10^{-6}\mathrm{d}^{-1}}]{\alpha,\gamma} {}^{222}_{86}\mathrm{Rn} \xrightarrow[\substack{\tau_{1/2} = 3.824\,\mathrm{d} \\ \lambda_2 = 0.18125\,\mathrm{d}^{-1}}]{\alpha} {}^{218}_{84}\mathrm{Po}$$

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Parent-daughter relationships cont'd



Absorbed dose in radioactive media

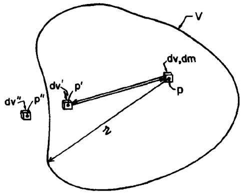
- Dose from α-particles: $D = n \times T$
- Dose from β's: $D \approx n \times \overline{T}$
- Dose from γ's; two limiting cases:

1. In a *small* radioactive object V, CPE is obtained at least a distance d=<t> (or a few MFPs) from the boundary of V. The dose is given by KERMA.
2. In a large radioactive object (>> $1/\mu$), the dose will be given by the sum of the kinetic energies emitted.

Absorbed dose in radioactive media

- At intermediate radioactive objects, we use the absorbed fraction

$$AF = \frac{\gamma - \text{ray energy absorbed}}{\gamma - \text{ray energy emitted}} = \frac{\varepsilon_{dv,V}}{R_{dv,V}}$$



- Assuming exponential attenuation over *r*

$$AF = \frac{1}{4\pi}\iint\left(1 - e^{-\mu r}\right)\sin\theta d\theta d\varphi$$

Absorbed fraction