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Quantities and concepts in interaction theory and dosimetry

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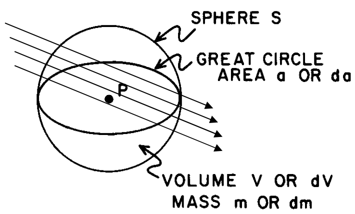
Ionizing radiation field

- Field of ionizing particles, where the particles may have a directional- and energy distribution
- Radiation field striking a small sphere:



- Number of particles N striking the sphere is proportional to dose

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Fluence

- Fluence *F*: number of particles *dN* striking the sphere per unit area *da:*

$$\Phi = \frac{dN}{da}$$ *(da* is the great circle area)

- The small sphere defines a point in space
- Fluence is as an expectation value; *N* is in reality a stochastic quantity
- For a radiation field through a medium, the fluence varies due to absorption, scattering and creation of new particles → $\Phi = \Phi(\vec{r})$

Fluence 2

- The fluence may vary in time – the fluence rate is defined as:

$$\Phi_t = \frac{d\Phi}{dt} = \frac{d^2N}{dtda}$$

- Thus

$$\Phi = \int_0^{t_0} \Phi_t dt$$

- For a time-independent field:

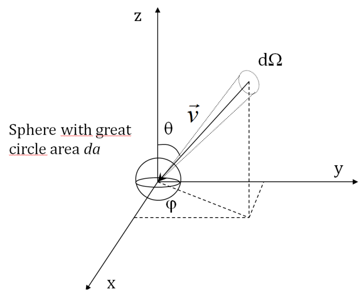
$$\Phi = \Phi_t \Delta t$$

Fluence 3

- The radiation field may have a an energy and directional dependence. The differential fluence is:

$$\Phi_T = \frac{d\Phi}{dT} \quad , \quad \Phi_\Omega = \frac{d\Phi}{d\Omega} \quad (d\Omega = \sin\theta d\theta d\varphi)$$

- Φ_T is the number of particles per energy and area in the energy interval [T, T+dT] striking the sphere



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Energy fluence

- How much energy 'strikes' the sphere?
- The energy fluence is defined as:

$$\Psi = \int_0^{T_{max}} T\Phi_T dT$$

- For a monoenergetic field:

$$\Psi = T\Phi = T\frac{dN}{da}$$

- Differentiated:

$$\Psi_T = \frac{d\Psi}{dT} = T\Phi_T \quad , \quad \Psi_\Omega = \frac{d\Psi}{d\Omega} = \int_0^{T_{max}} T\frac{d\Phi_T}{d\Omega} dT$$

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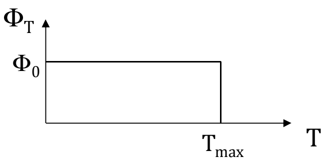
Fluence *vs* energy fluence

- Differential fluence with respect to energy is constant up to T_{max}:

$$\Phi_T = \Phi_0 \;\Rightarrow\; \Phi = \int_0^{T_{max}} \Phi_T \, dT$$

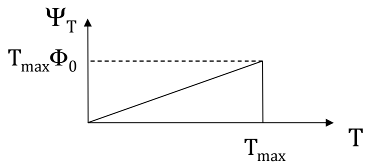
$$\Rightarrow \underline{\underline{\Phi = T_{max}\Phi_0}}$$



- Differential energy fluence is:

$$\Psi_T = T\Phi_T \;\Rightarrow\; \Psi = \int_0^{T_{max}} \Psi_T \, dT = \int_0^{T_{max}} T\Phi_T \, dT$$

$$\Rightarrow \underline{\underline{\Psi = \frac{1}{2} T_{max}^2 \Phi_0}}$$



Average particle energy in field

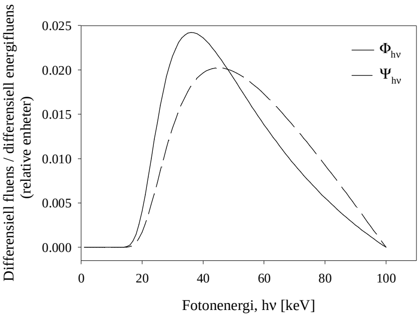
- Differential fluence and energy fluence are distribution functions
- Average energy defined as:

$$< T >_\Phi = \frac{\int_0^{T_{max}} T\Phi_T \, dT}{\int_0^{T_{max}} \Phi_T \, dT} = \frac{\Psi}{\Phi}$$

$$< T >_\Psi = \frac{\int_0^{T_{max}} T\Psi_T \, dT}{\int_0^{T_{max}} \Psi_T \, dT} = \frac{\int_0^{T_{max}} T^2\Phi_T \, dT}{\int_0^{T_{max}} \Psi_T \, dT} \neq < T >_\Phi$$

Fluence *vs* energy fluence 2

- X-ray spectrum is either differential fluence or differential energy fluence
- Problem: is often given as ”intensity”



Fluence *vs* energy fluence 3

- In our example:

$$< T >_{\Phi} \approx 48 \text{ keV}$$
$$< T >_{\Psi} \approx 54 \text{ keV}$$

- Always ask what the unit of the ordinate is in X-ray (or e.g. e^-) spectra!

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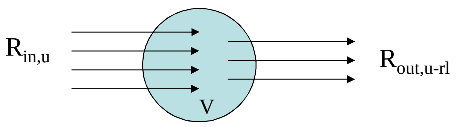
Indirectly ionizing radiation

- Indirectly ionizing radiation experience few interactions, but releases relatively large amounts of energy in each interaction
- Example: photons, neutrons
- Secondary charged particles (electrons most relevant) will deposit the transferred energy over a short distance
- How large are the energy transfers from e.g. photons to matter for a given volume element?
- The energy-mass budget is important!

Energy transferred, ε_{tr}

- A photon field with total energy $R_{in,u}$ enters a volume, while $R_{out,u\text{-}rl}$ is the energy leaving the volume:



- Energy transferred:

$$\varepsilon_{tr} = R_{in,u} - R_{out,u-rl} + \Sigma Q$$

- ε_{tr} is the total energy transferred from photons to electrons, and is the sum of all kinetic energy released

Energy transferred, ε_{tr} 2

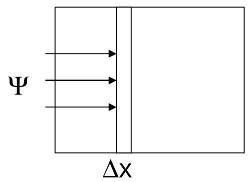
- u-rl: uncharged minus radiative losses; radiative losses by secondary electrons should not be included
- ε_{tr} is a stochastic quantity
- ΣQ: energy from conversion of rest mass or *vice versa*
- Example, pair production $\Sigma Q = -2m_e c^2$

KERMA

- Kinetic Energy Release per MAss:

$$K = \frac{d\varepsilon_{tr}}{dm} \quad \text{unit: [J/kg]}$$

- K is the expectation value of the energy transferred per unit mass in a point of interest
- Consider monoenergetic photons (quantum energy hν) passing a thin layer:



S: cross section of photon field

KERMA 2

- Probability per unit lenght for photon interaction multiplied with fraction of energy transferred: μ_{tr}
- Total energy transferred to electrons: $\varepsilon_{tr}=N(h\nu)\mu_{tr}\Delta x$
- Energy fluence for monoenergetic photons:

$$\Psi = (h\nu)\Phi = \frac{N(h\nu)}{S}$$

- KERMA becomes: $K = \frac{\varepsilon_{tr}}{m} = \frac{N(h\nu)\mu_{tr}\Delta x}{\rho V} = \frac{N(h\nu)\mu_{tr}\Delta x}{\rho S \Delta x}$

$$= \Psi \frac{\mu_{tr}}{\rho}$$

KERMA 3

- KERMA is determined by the energy fluence and the mass energy transfer coeffecient
- For a distribution of photons:

$$K = \int_0^{h\nu_{max}} \Psi_{h\nu} \frac{\mu_{tr}}{\rho} d(h\nu)$$

- Remember that μ_{tr}/ρ is dependent on the photon energy and atomic number of the absorber

Components of KERMA

- Kerma includes all kinetic energy given to secondary electrons, and this energy may be lost by:
  - Collisions
  - Radiative losses
- Kerma may be divided into two components:

  $K=K_c+K_r$
- K_c: collision Kerma; provides a measure of the energy loss per unit mass from photons resulting in collisional losses for secondary electrons!

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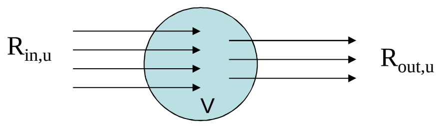
Net energy transferred ε_{tr}^{n}

- ε_{tr}^{n} is defined as:



$$\varepsilon_{tr}^{n} = R_{in,u} - R_{out,u} + \Sigma Q$$

- $R_{out,u}$ is all photon energy leaving the volume element (including brehmsstrahlung)
- ε_{tr}^{n} is thus the total kinetic energy of secondary electrons which is not lost as brehmsstrahlung

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Collision Kerma

- Is defined by:

$$K_c = \frac{d\varepsilon_{tr}^{n}}{dm}$$

- May take radiative losses into account by defining the qantity *g*; the fraction of kinetic energi lost as brehmsstrahlung

$$K_c = K(1-g) = \Psi \frac{\mu_{tr}}{\rho}(1-g)$$

- Definition: $\frac{\mu_{en}}{\rho} \equiv \frac{\mu_{tr}}{\rho}(1-g)$
- μ_{en}/ρ: mass energy absorption coeffecient

Collision Kerma 2

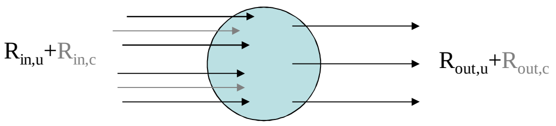
- K_c is thus:

$$K_c = \Psi \frac{\mu_{en}}{\rho}$$

- Generally: $K_c < K$
- Special case: Low energy photons releases low energy electrons in an absorber of low atomic number Z. Radiative losses are insignificant, and $g \approx 0$ and $K_c \approx K$

Energy imparted and absorbed dose

- Look at all energy transport (both charged and uncharged particles) through the volume of interest:



$$\varepsilon = R_{in,u} + R_{in,c} - R_{out,u} - R_{out,c} + \Sigma Q$$

- Absorbed dose is (at last) defined as:

$$\boxed{D = \frac{d\varepsilon}{dm}}$$ unit: [Gy] = [J/kg]

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Absorbed dose

- The absorbed dose is all energy imparted to the volume per mass
- May not be directly related to photon interaction coefficients
- However, in some cases the dose may be approximated by K_c

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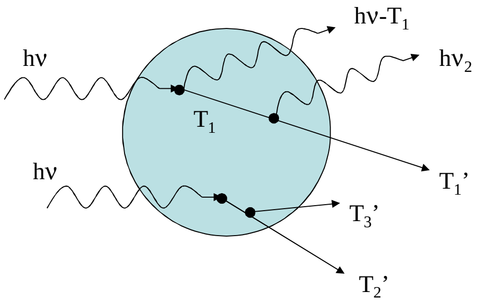
ε_{tr} , ε_{tr}^{n} , ε : example

- Two photons interacts in a volume of interest ($\Sigma Q=0$):



ε_{tr} , ε_{tr}^{n} , ε : example 2

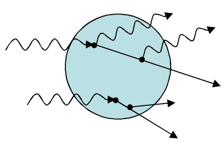
- Photon 1:

  $\varepsilon_{tr} = R_{in,u} - R_{out,u\text{-}rl} = h\nu - (h\nu - T_1) = T_1$

  $\varepsilon_{tr}^{n} = R_{in,u} - R_{out,u} = h\nu - (h\nu - T_1) - h\nu_2 = T_1 - h\nu_2$

  $\varepsilon = R_{in,u} + R_{in,c} - R_{out,u} - R_{out,c}$

  $= h\nu + 0 - (h\nu - T_1) - h\nu_2 - T_1' = T_1 - h\nu_2 - T_1'$



- Photon 2:

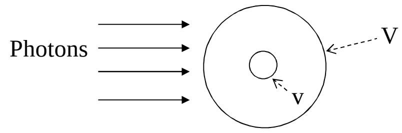
  $\varepsilon_{tr} = h\nu - 0 = h\nu$

  $\varepsilon_{tr}^{n} = h\nu - 0 = h\nu$

  $\varepsilon = h\nu + 0 - T_2 - T_3 = h\nu - T_2' - T_3'$

Charged particle equilibrium (CPE)

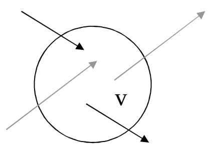
- Photons enter a volume V, which includes a smaller volume v:



- CPE: Number of charged particles of a given type and energy entering v is equal to the number of particles of the same type and energy leaving
- Certain conditions must be fullfilled:
  - V must be homogeneous
  - Photon attenuation must be negligible

CPE 2

CPE:



- If CPE is present, $R_{in,c} = R_{out,c}$
- Energy imparted:

$$\varepsilon = R_{in,u} + R_{in,c} - R_{out,u} - R_{out,c} = R_{in,u} - R_{out,u} = \varepsilon_{tr}^{n}$$

- In this case, absorbed dose equals collision Kerma:

$$\boxed{D = \frac{\varepsilon}{m} \overset{CPE}{=} \frac{\varepsilon_{tr}^{n}}{m} = K_c = \Psi \frac{\mu_{en}}{\rho}}$$

Absorbed doses under CPE

- K_c, and thus dose, is given by $\Psi\mu_{en}/\rho$, and is thus proportional to the interaction probability in a given absorber
- Two different absorbers A og B placed in the same point in a radiation field:

$$\frac{D_A}{D_B} = \frac{\Psi\left(\frac{\mu_{en}}{\rho}\right)_A}{\Psi\left(\frac{\mu_{en}}{\rho}\right)_B} = \frac{\left(\frac{\mu_{en}}{\rho}\right)_A}{\left(\frac{\mu_{en}}{\rho}\right)_B}$$

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CPE, example

- Two small volumes of water and air is placed in same point in a radiation field (1 MeV photons) where CPE exists. What is the dose ratio?
- Use tabulated values for μ_{en}/ρ (Attix):

μ_{en}/ρ(water) = 0.0309

μ_{en}/ρ(air) = 0.0278

$\rightarrow$ D(air) / D(water) = 0.90

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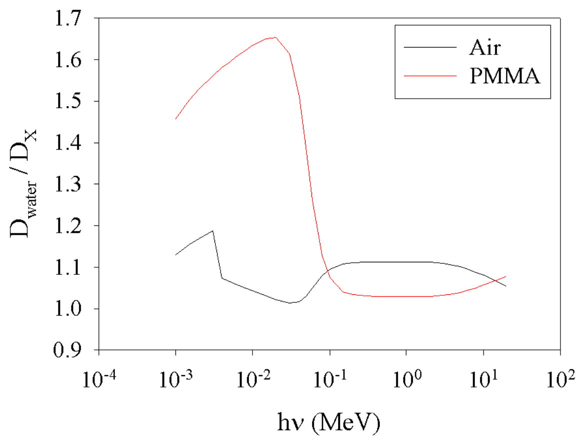
CPE, example 2



CPE, example

- A (real) patient was treated with 60 kV X-rays with the dose reported as 400 R x 3. Estimate the dose to tissue (water).

✓ R is the *Roentgen* unit, giving a measure of *exposure* (liberated charges in *air*; see later lectures on ionometry).

✓ 1 R = 0.00877 Gy dose to *air*

✓ 60 kV X-rays have mean X-ray energy of roughly 30 keV (strongly depends on filtration; see lectures on X-ray production)

CPE, example

- ✓ Assuming CPE, Dair/Dw is proportional to μ_{en}/ρ-ratio
- ✓ Use NIST tables -> μ_{en}/ρ(air) = 0.1537
  μ_{en}/ρ(water) = 0.1557
- ✓ Dose:

$$\frac{D_w}{D_{air}} = \left(\frac{\mu_{en}}{\rho}\right)_{air}^{w}$$

$$\Rightarrow D_w = D_{air}\left(\frac{\mu_{en}}{\rho}\right)_{air}^{w} = 3\times 400R\times 0.00877Gy/R\times\frac{0.1557}{0.1537}$$

$$\underline{\underline{D_w = 10.7Gy}}$$

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CPE, problems

- When the photon energy increases, the range of the secondary electrons increases more than the photon pathlenght

Photon energy(MeV)	Photon attenuation (%) in water within the range of a secondary electron
0.1	0
1	1
10	7
30	15

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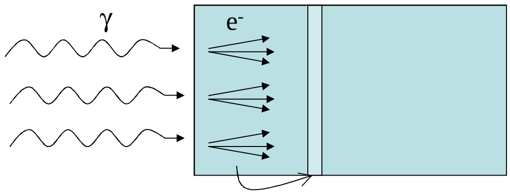
CPE, problems 2



e- with long range contributes to the dose at the layer. Photon beam significantly attenuated between the interaction point and the layer – fewer electrons are generated in the layer than what was generated upstream.

- Thus: $R_{in,c} > R_{out,c}$ and:

$$\Rightarrow \varepsilon = R_{in,u} + R_{in,c} - R_{out,u} - R_{out,c} > \varepsilon_{tr}^{n}$$

$$\Rightarrow D > K_c$$

- Most relevant for high photon energies

TCPE

- Transient Charged Particle Equilibrium: electrons originating from upstream contributes to the dose, while the photon contribution ($R_{in,u}$-$R_{out,u}$) is given by the collision Kerma
- Assumption: absorbed dose propotional to K_c

$$D^{TCPE} = K_c(1+f_{TCPE})$$

$$f_{TCPE} \geq 0$$