UiO : Department of Physics
University of Oslo

Oslo University Hospital

Interaction theory – Photons

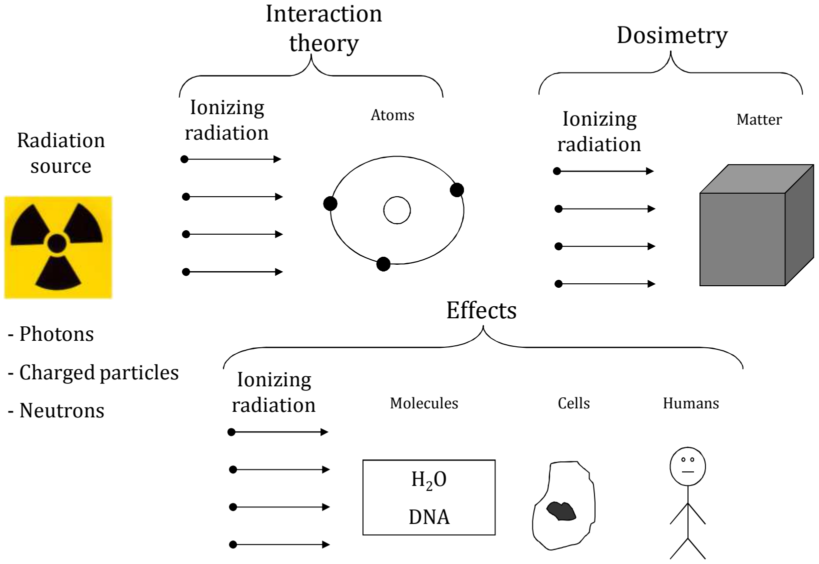
Eirik Malinen

INTRODUCTION TO RADIOLOGICAL PHYSICS AND RADIATION DOSIMETRY

FRANK HERBERT ATTIX
Professor of Medical Physics
University of Wisconsin Medical School
Madison, Wisconsin

ISBN-13: 978-0-471-01146-0
ISBN-10: 0-471-01146-0

Introduction



UiO : Department of Physics
University of Oslo

Oslo University Hospital

Objectives

- To understand primary effects of ionizing radiation
- How radiation doses are calculated and measured
- To appreciate applications of ionizing radiation

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Contents FYSKJM4710

- Interactions between ionizing radiation and matter
- Radioactive and non-radioactive sources
- Calculations and measurement of absorbed doses (dosimetry)

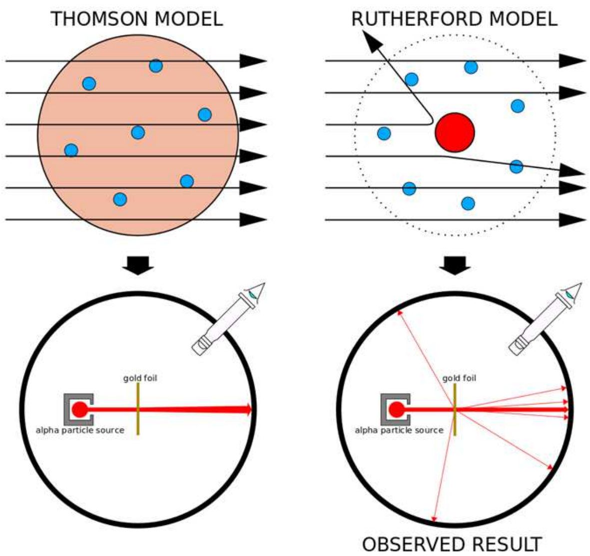
Relevant issues

- X-ray and CT investigations
- Radiotherapy
- Positron emission tomography
- Radiation protection
- Radiation Biology

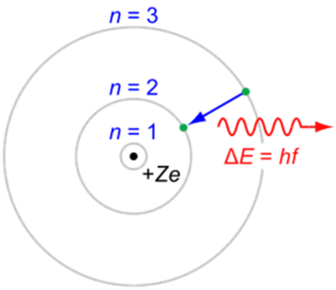
UiO : Department of Physics
University of Oslo
Oslo University Hospital

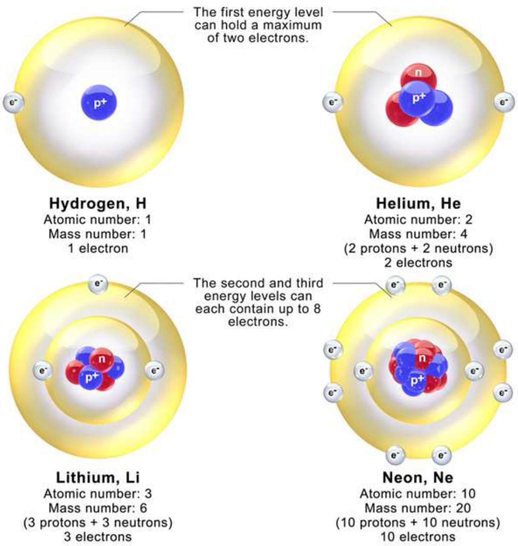
- X-ray contrast: only a matter of differences in density?

Basic theory of the atom



Bohr model

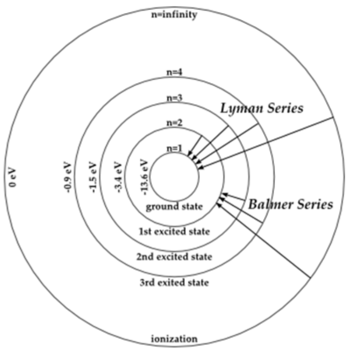


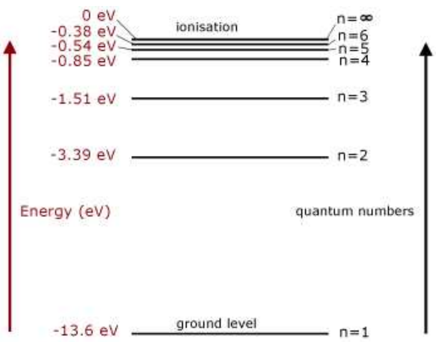


UiO : Department of Physics
University of Oslo

Oslo University Hospital

Hydrogen



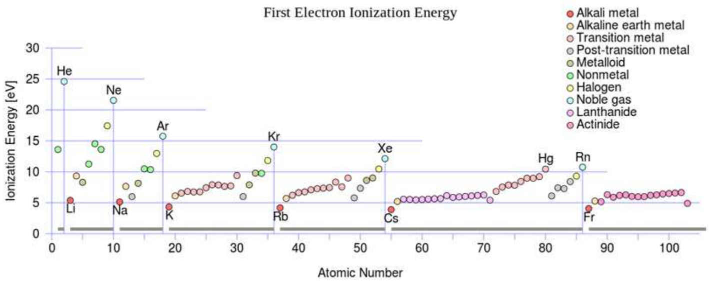


UiO : Department of Physics
University of Oslo

Oslo University Hospital

Ionization

- Liberation of electron from atom
- Requires energy transfer ~4-25 eV



- A lethal whole-body dose of radiation (5 J/kg) results in a temperature increase of 0.001 °C

Ionizing radiation

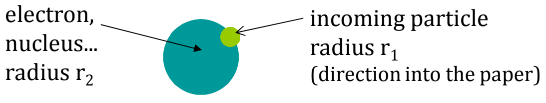
Directly ionizing radiation: Fast charged particles, which deliver their energy to matter directly, through many small Coulomb-force interactions

Indirectly ionizing radiation: Photons (γ or X-rays) or neutrons, which transfer their energy to charged particles in the matter

UiO : Department of Physics
University of Oslo
Oslo University Hospital

Cross section 1

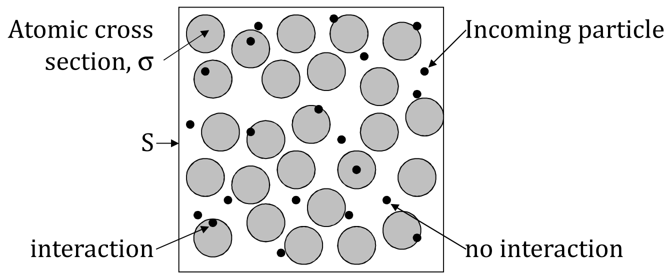
- Cross section s: “target area”, effective target covering a certain area
- Proportional to the interaction strength between an incoming particle and the target particle
- Consider two discs, one target and one incoming:



- s is the total area: $\pi(r_1^2 + r_2^2)$

Cross section 2

- N particles move towards an area S with n atoms



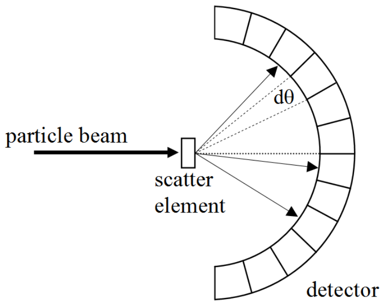
- Probability of interaction: p = nσ/S
- Number of interacting particles: Np= Nnσ/S

Cross section 3

- Separate between *electronic* and *atomic* cross section
- The cross section depends on:
  - Type of target (nucleus, electron, ..)
  - Type of and energy of incoming particle (photon, electron…)
- Cross section calculated with quantum mechanics
  - here visualized in a classical window

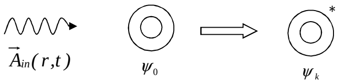
Cross section 4

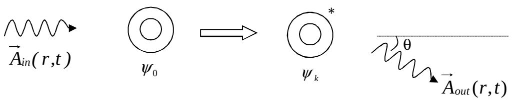
- *Differential cross section* with respect to scattering angle



$$\frac{d\sigma}{d\Omega} = \frac{number\ of\ \ particles\ scattered\ into\ d\Omega}{number\ of\ \ particles\ \ per\ unit\ area} \frac{1}{d\Omega}$$

Photon interactions

- Photon represented by a plane wave $\vec{A}_{in}(r,t) \sim e^{i(\vec{p}_{in} \cdot \vec{r} - \omega_{in} t)}$ in quantum mechanical calculations
- In principle, two different processes:
  - Absorption 

  - Scattering 

- Scattering: coherent (elastic) og incoherent (inelastic)

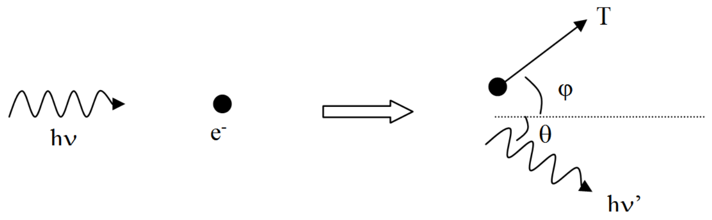
Coherent (Rayleigh) scattering

- Scattering without loss of energy: hν=hν'
- Photon is absorbed by atom, thereby emitted at a small deflection angle
- Depends on atomic structure and photon energy
- Atomic cross section:

$$\sigma_R \propto \left(\frac{Z}{h\nu} \right)^2$$

Incoherent (Compton) scattering

- Scattering with loss of energy: hν'<hν
- Photon-electron scattering; electron may be assumed free (i.e. unbound)



- Thomson scattering: low energy limit, hν→0

Compton scattering – kinematics

- Conservation of energy and momentum:

$$h\nu = h\nu' + T$$

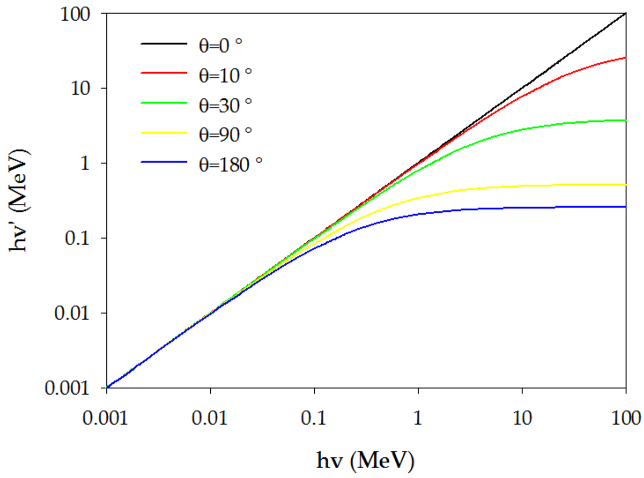
$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p\cos\varphi \quad , \quad \frac{h\nu'}{c}\sin\theta = p\sin\varphi$$

$$(pc)^2 = T^2 + 2Tm_ec^2$$

→

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_ec^2}(1-\cos\theta)} \quad , \quad \cot\varphi = \left(1 + \frac{h\nu}{m_ec^2}\right)\tan\left(\frac{\theta}{2}\right)$$

Compton scattering – kinematics



UiO : Department of Physics
University of Oslo

Oslo University Hospital

Compton scattering – example

- An X-ray unit is to be installed, with the beam direction towards the ground. Employees in the floor above the unit are worried. Maximum X-ray energy is 250 keV. What is the maximum energy of the backscattered photons?

$$\theta = 180° \Rightarrow h\nu' = \frac{h\nu}{1+\frac{h\nu}{m_e c^2}(1-\cos\theta)} = \frac{h\nu}{1+\frac{2h\nu}{m_e c^2}}$$

$$h\nu = 250\ keV \Rightarrow h\nu' = \frac{250}{1+\frac{2\times 250}{511}} = \underline{\underline{126\ keV}}$$

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Compton scattering – cross section 1

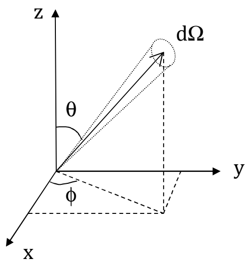
- Klein and Nishina derived the cross section for Compton scattering, assuming free electron
- Differential cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)=\frac{r_0^2}{2}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu'}{\nu}+\frac{\nu}{\nu'}-\sin^2\theta\right)$$

$$d\Omega=\sin\theta d\theta d\phi$$

r_0: classical electron radius

incoming photon along z-axis



UiO : Department of Physics
University of Oslo

Oslo University Hospital

Compton scattering – cross section 2

- Cylinder symmetry results in:

$$\left(\frac{d\sigma}{d\theta}\right)=\pi r_0^2\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu'}{\nu}+\frac{\nu}{\nu'}-\sin^2\theta\right)\sin\theta$$

- ~ probability of finding a scattered photon in the interval [θ, θ+dθ]
- Total electronic cross section:

$${}_e\sigma=\int_0^\pi \pi r_0^2\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu'}{\nu}+\frac{\nu}{\nu'}-\sin^2\theta\right)\sin\theta d\theta$$

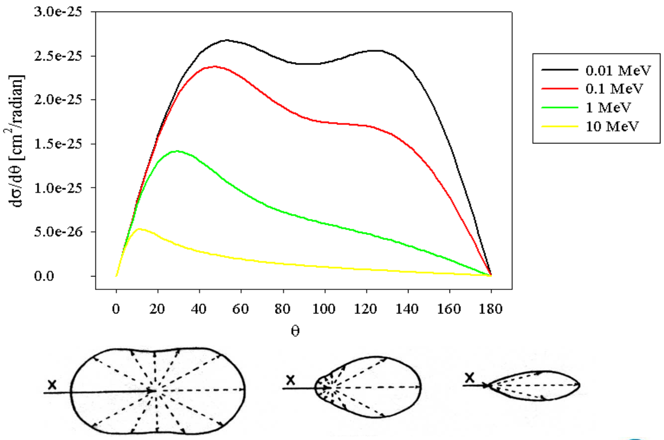
- Atomic cross section: ${}_a\sigma=Z{}_e\sigma$

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Compton scattering – cross section 3

- Scattered photons are more forwardly directed with increasing photon energy:



10 keV 200 keV 2 MeV

Compton scattering – cross section 3

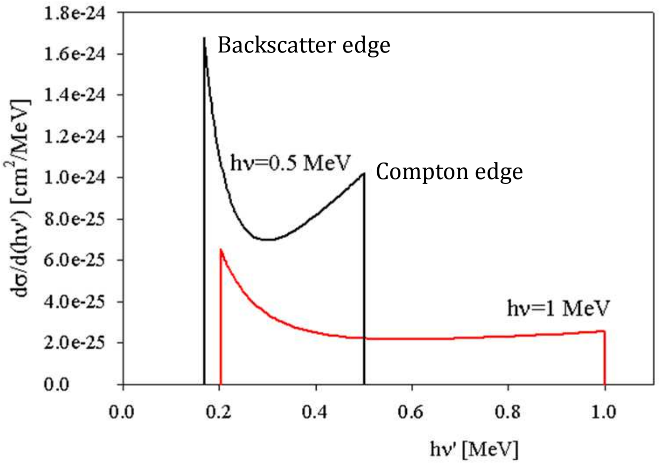
- Cross section may be modified with respect to energy:

$$\frac{d\sigma}{d(h\nu')} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d(h\nu')} = \frac{d\sigma}{d\Omega} 2\pi \sin\theta \frac{d\theta}{d(h\nu')}$$

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_e c^2}(1-\cos\theta)}$$

$$\Rightarrow \frac{d\sigma}{d(h\nu')} = \frac{\pi r_0^2 m_e c^2}{(h\nu)^2}\left[\frac{h\nu'}{h\nu} + \frac{h\nu}{h\nu'} - 1 + \left(1 - \left(\frac{h\nu}{h\nu'} - 1\right)\frac{m_e c^2}{h\nu}\right)^2\right]$$

Compton scattering – cross section 4

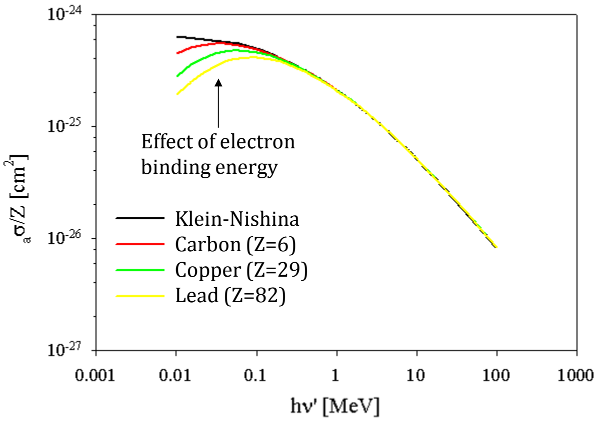


UiO : Department of Physics
University of Oslo

Oslo University Hospital

Compton scattering – cross section 5

- Correct atomic cross section:



UiO : Department of Physics
University of Oslo

Oslo University Hospital

Compton scattering – transferred energy 1

- The energy transferred to an electron in a Compton process:

$$T = h\nu - h\nu'$$

- The cross section for energy transfer:

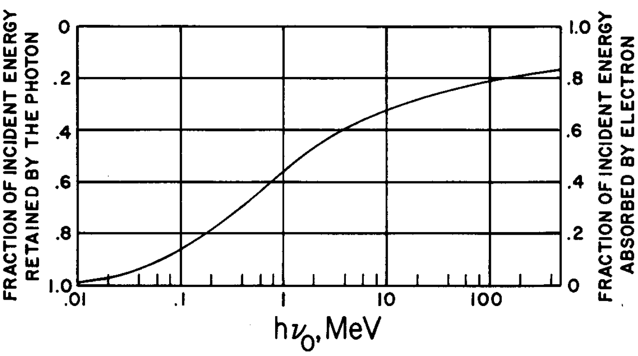
$$\frac{d\sigma_{tr}}{d\Omega} = \frac{d\sigma}{d\Omega}\frac{T}{h\nu} = \frac{d\sigma}{d\Omega}\frac{h\nu - h\nu'}{h\nu}$$

- Mean energy transferred:

$$\overline{T} = \frac{\int T \frac{d\sigma}{d\Omega} d\Omega}{\int \frac{d\sigma}{d\Omega} d\Omega} = \frac{\int \frac{h\nu - h\nu'}{h\nu}\frac{d\sigma}{d\Omega} d\Omega}{\sigma} = \frac{\sigma_{tr}}{\sigma} \times h\nu$$

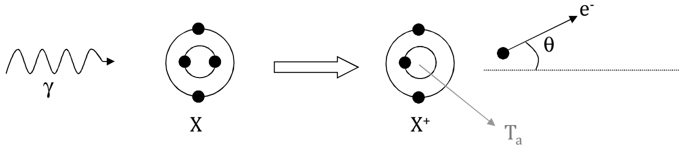
Compton scattering – transferred energy 2

- The fraction of incident energy transferred:

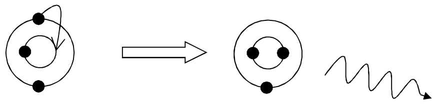


Photoelectric effect 1

- Photon is absorbed by atom/molecule; the result is an excitation or ionization



- Atom may deexcite and emit characteristic radiation:



Photoelectric effect 2

- In the kinematics, the binding energy of the ejected electron should be taken into account:

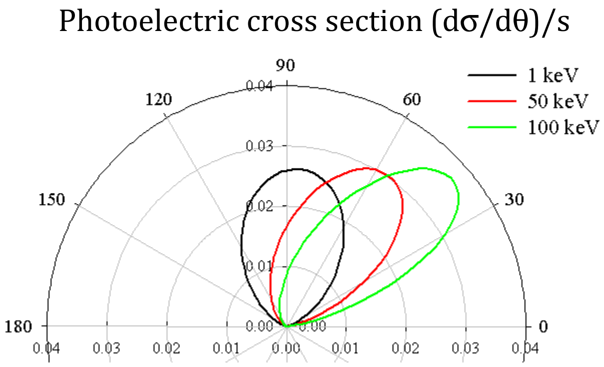
$$T = h\nu - E_b - T_a \approx h\nu - E_b$$

- Assuming E_b=0, the atomic cross section is:

$$\frac{d\tau}{d_e W} = 2\sqrt{2} r_0^2 \alpha^4 Z^5 \left(\frac{m_e c^2}{h\nu}\right)^{7/2} \sin^2\theta \left(1 + 4\sqrt{\frac{2h\nu}{m_e c^2}} \cos\theta\right)$$

α: The fine-structure constant
Solid angle $_e\Omega$ gives the direction of the ejected electron

Photoelectric effect 2



UiO : Department of Physics
University of Oslo

Oslo University Hospital

Characteristic radiation

- Energy of characteristic radiation depends on elektronic structure and transition probabilities
- ”K- and L-shell” vacancies ↔ $h\nu_K$ and $h\nu_L$
- Isotropic emission
- Fraction of photoelectric interactions:
  $P_K\ [h\nu > (E_b)_K]$ and $P_L\ [(E_b)_L < h\nu < (E_b)_K]$
- Probability for emission: Y_K og Y_L (*flourescence yield*)
- Energy emitted from the atom:
  $P_K Y_K h\nu_K + (1-P_K) P_L Y_L h\nu_L$

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Auger effect

- Energy release by ejection of losely bound electron
- Energy of emitted electron equal to deexcitation energy
- Low Z: Auger dominates
- High Z: characteristic radiation dominates

Photoelectric cross section

- General formula:

$$\tau \propto \frac{Z^n}{(h\nu)^m} \ , 4<n<5 \ , 1<m<3$$

- Fraction of energy transferred to photoelectron:

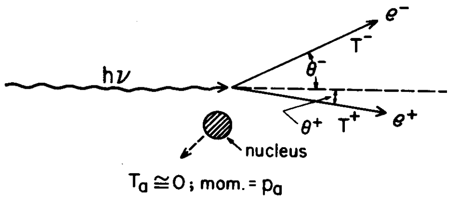
$$\frac{T}{h\nu} = \frac{h\nu - E_b}{h\nu}$$

- However: don’t forget Auger electron(s)
- Cross section for energy transfer to photoelectron:

$$\tau_{tr} = \tau \frac{\left(h\nu - P_K Y_K h\nu_K \text{-}(1\text{-}P_K)P_L Y_L h\nu_L\right)}{h\nu}$$

Pair production 1

- Photon absorption in the nuclear electromagnetic field where an electron-positron pair is created



- Triplet production: in the electromagnetic field of an electron

Pair production 2

- Conservation of energy:

$$h\nu = 2m_ec^2 + T^+ + T^-$$

- Average kinetic energy after absorption:

$$\overline{T} = \frac{h\nu - 2m_ec^2}{2}$$

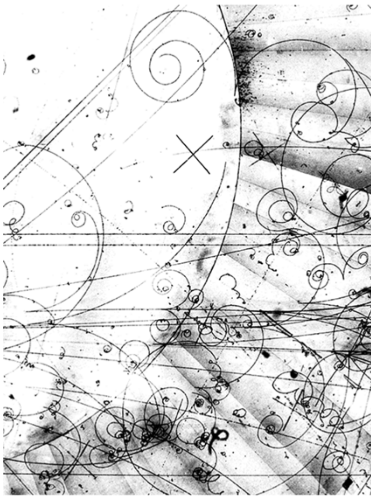
- Estimated electron/positron scattering angle:

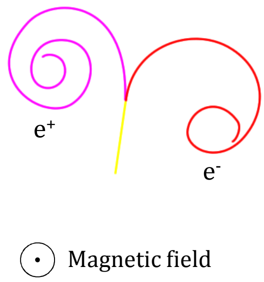
$$\overline{\theta} \approx \frac{m_ec^2}{\overline{T}}$$

- Total cross section:

$$\kappa \approx \alpha r_0^2 Z^2 \overline{P}$$

Discovery of pair production





Triplet production

- In the electromagnetic field from an electron, an electron-positron pair is created
- Energy conservation:

$$h\nu = 2m_ec^2 + T^+ + T_1^- + T_2^-$$

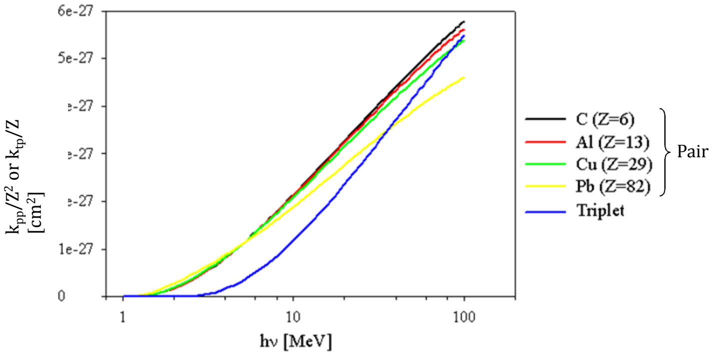
- Average kinetic energy:

$$\overline{T} = \frac{h\nu - 2m_ec^2}{3}$$

- Primary electron is also given energy
- Threshold: $4m_0c^2$

Pair- and triplet production

- Pair production dominates:



UiO : Department of Physics
University of Oslo

Oslo University Hospital

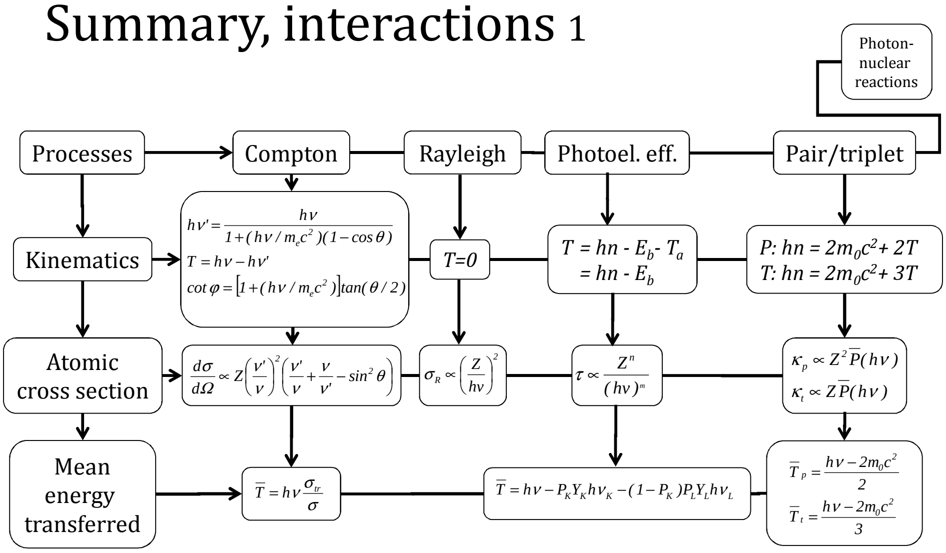
Photonuclear reactions

- Photon (energy above a few MeV) excites a nucleus
- Proton or neutron is emitted
- (γ, n) interactions may have consequences for radiation protection
- Example: Tungsten W (γ, n)

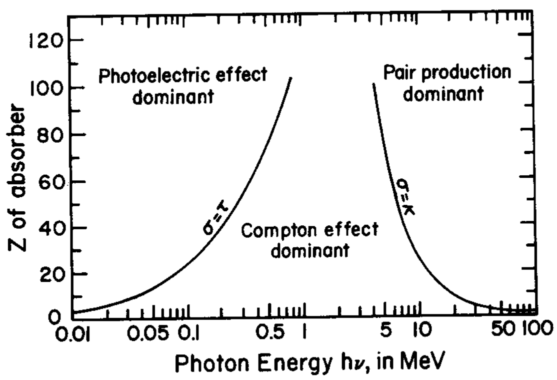
UiO : Department of Physics
University of Oslo

Oslo University Hospital

Summary, interactions 1

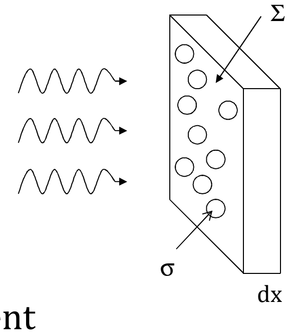


Summary, interactions 2



Attenuation coefficients 1

- n_V atoms per volume = $\rho(N_A/A)$
- Number of atoms:
  $n=n_V V=n_V \Sigma dx$
- Interaction probability
  $p=n\sigma/\Sigma=n_V\sigma dx$
- Probability per unit length:
  $\mu=p/dx=n_V\sigma=\rho(N_A/A)\sigma$
  μ: linear attenuation coefficient



UiO : Department of Physics
University of Oslo

Oslo University Hospital

Attenuation coefficients 2

- N_A : Avogadro's constant; 6.022×10^{23} mole^{-1}
- A: number of grams per mole
- N_A/A: number of atoms per gram
- $N_A Z/A$: number of electrons per gram
- Number of atoms per volume: $r(N_A/A)$
- Etc.

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Attenuation coefficients 3

- Total *mass* attenuation coeffecient:

$$\frac{\mu}{\rho} = \frac{\tau}{\rho} + \frac{\sigma}{\rho} + \frac{\kappa}{\rho} + \frac{\sigma_R}{\rho}$$

- Coefficient for energy transfer:

$$\frac{\mu_{tr}}{\rho} = \frac{\mu}{\rho} \frac{\overline{T}}{h\nu}$$

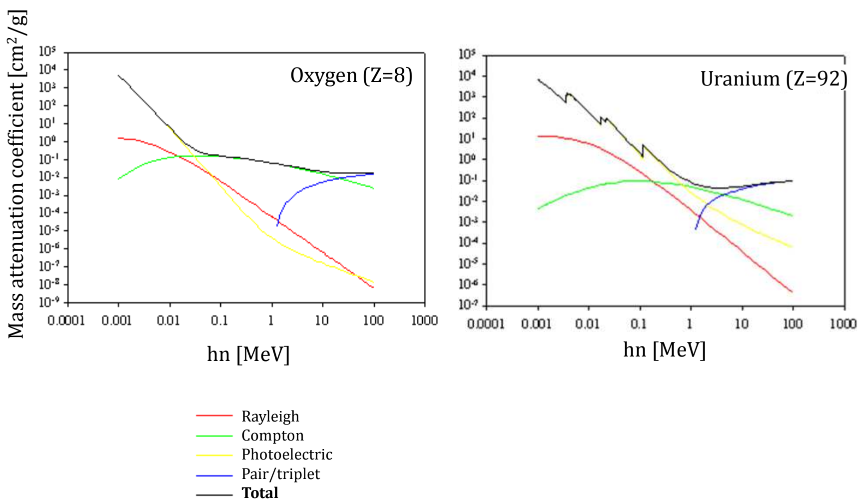
- Braggs rule for mixture of atoms:

$$\left(\frac{\mu}{\rho}\right)_{mix} = \sum_{i=1}^{n} f_i \left(\frac{\mu}{\rho}\right)_i \quad , \quad f_i = \frac{m_i}{\sum_{i=1}^{n} m_i}$$

UiO : **Department of Physics**
University of Oslo

Oslo University Hospital

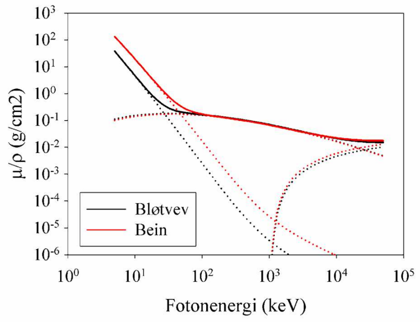
Attenuation coefficients 4

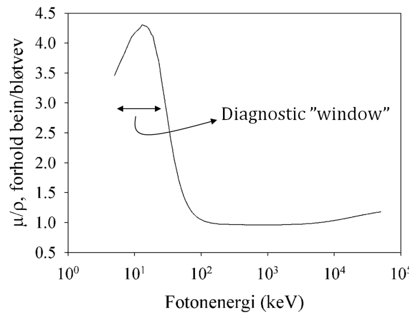


UiO : **Department of Physics**
University of Oslo

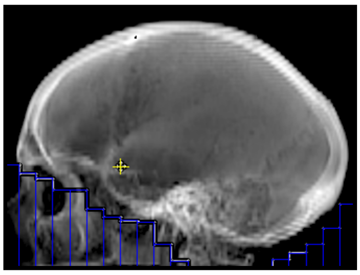
Oslo University Hospital

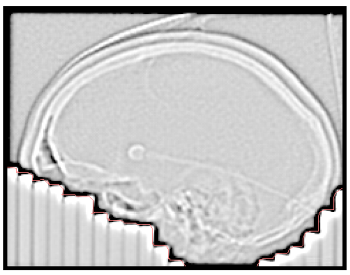
Attenuation coefficients 5





X-ray images at different energies



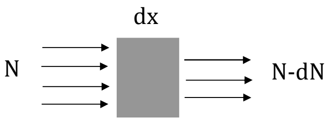


Conventional X-rays (120 kV)

Linear accelerator (5 MV)

Attenuation 1

- Beam with *N* photons impinge absorber with thickness *dx*:



- Probability for interaction: μdx
- Number of photons interacting: $N\mu dx$

$$dN = N\mu dx \quad \Rightarrow \quad \int \frac{dN}{N} = \int \mu dx$$

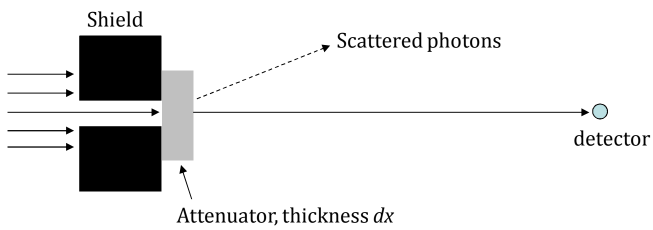
$$\Rightarrow \underline{\underline{N = N_0 e^{-\mu x}}}$$

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Attenuation 2

- Note that μ is the *interaction probability* per unit lenght – *not* the *absorption* probability
- $e^{-\mu x}$ corresponds to a narrow beam measurement geometry:



UiO : Department of Physics
University of Oslo

Oslo University Hospital

Attenuation 3

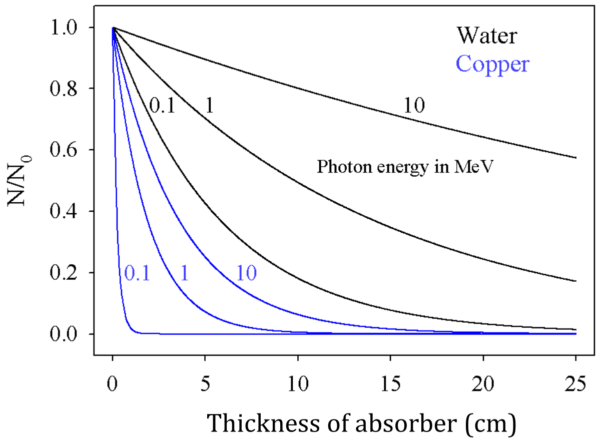
- 'Probability' for photon not interacting: $e^{-\mu x}$
- Normalized probability

$$p_{ni} = Ce^{-\mu x} \; , \int_0^\infty p_{ni}\,dx \overset{!}{=} 1 \; , \; \Rightarrow p_{ni} = \mu e^{-\mu x}$$

- Mean free path:

$$\langle x \rangle = \int_0^\infty x p_{ni}\,dx = \int_0^\infty x \mu e^{-\mu x} dx = \frac{1}{\mu}$$

Attenuation 4



Attenuation 4

- 'Probability' for photon not interacting: ~$e^{-\mu x}$
- Normalized probability

$$p_{ni} = Ce^{-\mu x} \; , \; \int_0^\infty p_{ni}\, dx \overset{!}{=} 1 \; , \; \Rightarrow p_{ni} = \mu e^{-\mu x}$$

- Mean free path:

$$\langle x \rangle = \int_0^\infty x p_{ni}\, dx = \int_0^\infty x \mu e^{-\mu x} dx = \frac{1}{\mu}$$

- A distance of 3 MFP reduces the beam intensity to 5%

UiO : Department of Physics
University of Oslo

Oslo University Hospital

Attenuation - example

- 2 MeV photons

Pb: $\mu = 0{,}516\ \text{cm}^{-1}$

H_2O: $\mu = 0{,}049\ \text{cm}^{-1}$

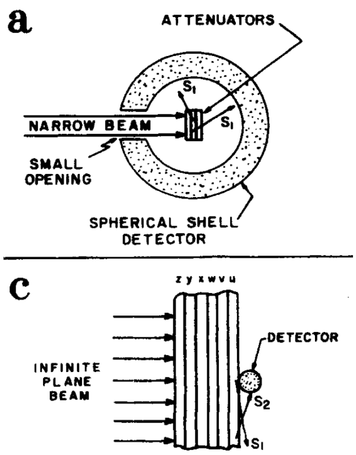
$$e^{-\mu_{H_2O} x_{H_2O}} = e^{-\mu_{Pb} x_{Pb}}$$

$$\Rightarrow \frac{x_{H_2O}}{x_{Pb}} = \frac{\mu_{Pb}}{\mu_{H_2O}}$$

- 10 times as much water necessary

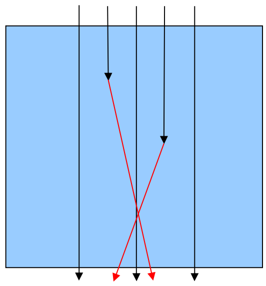
Broad beam attenuation

- Broad-beam geometry: every scattered or secondary uncharged particle strikes the detector



Scattered photons

- $e^{-\mu x}$: number of primary photons at a given depth
- What about the scattered photons?

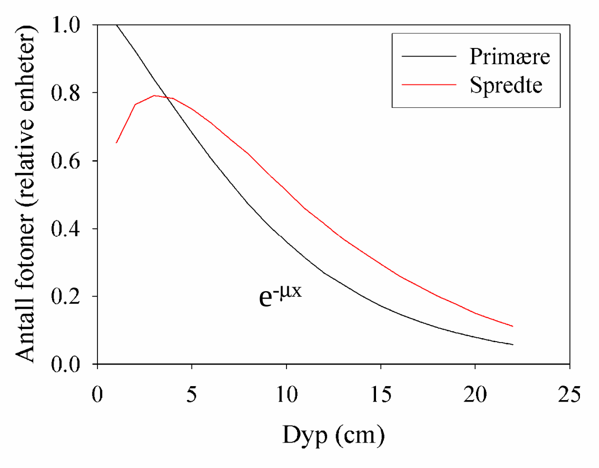


- Monte Carlo simulations

UiO : Department of Physics
University of Oslo

Oslo University Hospital

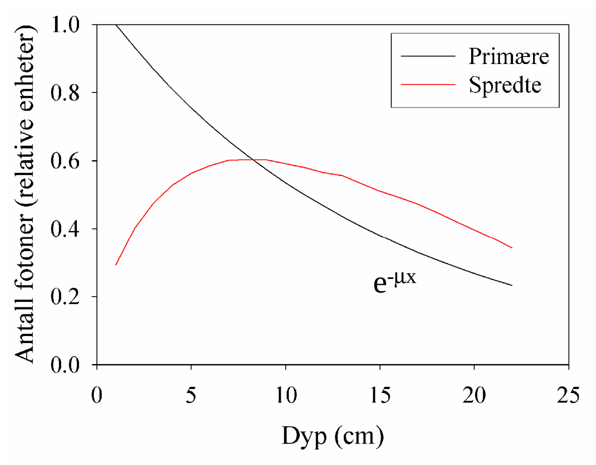
Primary and scattered photons, 100 keV



UiO : Department of Physics
University of Oslo

Oslo University Hospital

Primary and scattered photons, 1 MeV



UiO : Department of Physics
University of Oslo

Oslo University Hospital