Statistical Learning Theory, Exercise 1

Michael Hirsch

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Coin Flipping Inference

1. How many times should we flip the coin in order to achieve a precision of $\epsilon = 10^{-2}$ and error probability of $\alpha = 1/20$?

   Proof. We will solve for t in the equation $1/4t\epsilon^2 = \alpha$ by plugging in the values for ϵ and α.

$$\frac{1}{4t10^{-4}} = \frac{1}{20}$$
$$\frac{2500}{t} = \frac{1}{20}$$
$$t = 500000$$

$\square$

2. If we flip the coint $t = 10^3$ times and want the error probability to be less than $\alpha = 1/20$, what precision level can we obtain?

   Proof. Again, we solve the same equation using different values, this time noting that our epsilon value will be an upper bound on the precision.

$$\frac{1}{4000\epsilon^2} = \frac{1}{20}$$
$$4000\epsilon^2 = 20$$
$$\epsilon^2 = \frac{20}{4000}$$
$$\epsilon^2 = \frac{1}{200}$$
$$\epsilon = \sqrt{\frac{1}{200}}$$
$$\epsilon = \frac{1}{\sqrt{200}}$$
$$\epsilon = \frac{1}{10\sqrt{2}}$$

$\square$

3. If we flip the coin $t = 10^3$ times, what is the probability that the empirical frequency of heads deviates from the probability by more that $\epsilon = 10^{-2}$?

   Proof.

$$Pr\{|\frac{S_t}{t} - \mu| > 1/100\} \leq \frac{0.25/1000}{10^{-4}} = 2.5$$

I suppose that this means we are guaranteed to deviate at least ϵ from the mean? $\square$