Statistical Learning Theory, Exercise 2

Michael Hirsch

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Membership Processes

Consider the membership process defined by the set of all intervals $[a, b] \subseteq \mathbb{R}$

1. Compute $N(t)$ for $t = 1, 2, 3, 4$.

   Without loss of generality, we will assume that our questions are ordered.

   $$N(1) = 2$$
   $$N(2) = 4$$
   $$N(3) = 7$$
   $$N(4) = 11$$

   □

2. Find a general formula for $N(t)$.

   After some mathematical manipulation, the formula for $N(t)$ appears to be:

   $$N(t) = \frac{t^2}{2} + \frac{t}{2} + 1$$

   □

3. Compare $N(t)$ to $\Phi(2, t)$.

   $$\Phi(2, t) = \binom{t}{0} + \binom{t}{1} + \binom{t}{2} = \frac{1}{2}(t^2 + t + 2)$$

   This is equal to the $N(t)$ that was computed above.

   □