Statistical Learning Theory, Exercise 1

Michael Hirsch

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Coin Flipping Inference

1. How many times should we flip the coin in order to achieve a precision of $\epsilon = 10^{-2}$ and error probability of $\alpha = 1/20$?

   Proof. We will solve for t in the equation $1/4t\epsilon^2 = \alpha$ by plugging in the values for ϵ and α.

$$\frac{1}{4t10^{-4}} = \frac{1}{20}$$
$$\frac{2500}{t} = \frac{1}{20}$$
$$t = 500000$$

   $\square$

2. If we flip the coint $t = 10^3$ times and want the error probability to be less than $\alpha = 1/20$, what precision level can we obtain?

   Proof. Again, we solve the same equation using different values, this time noting that our epsilon value will be a bound on the precision.

$$\frac{1}{4000\epsilon^2} = \frac{1}{20}$$
$$4000\epsilon^2 = 20$$
$$\epsilon^2 = \frac{20}{4000}$$
$$\epsilon^2 = \frac{1}{200}$$
$$\epsilon = \sqrt{\frac{1}{200}}$$
$$\epsilon = \frac{1}{\sqrt{200}}$$
$$\epsilon = \frac{1}{10\sqrt{2}}$$

   $\square$

3. If we flip the coin $t = 10^3$ times, what is the probability that the empirical frequency of heads deviates from the probability by more that $\epsilon = 10^{-2}$?

   Proof.

$$Pr\{|\frac{S_t}{t} - \mu| > 1/100\} \leq \frac{0.25/1000}{10^{-4}} = 2.5$$

   $\square$

   It appears that in this example, the Chebyshev bound is not effective given the number of trials and our desired degree of precision. A probaility of at most 2.5 indicates that we are entirely in the dark when predicting the empirical frequency of the number of heads.