Shining Light on Dark Matter, One Photon at a Time

by

Brandon Leigh Allen

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

Doctorate of Science in Physics

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2019

Author . . . .
Department of Physics
May 18, 2019

Certified by . . . .
Christoph E.M. Paus
Professor
Thesis Supervisor

Accepted by . . . .
Nergis Mavalvala
Associate Department Head for Education

Shining Light on Dark Matter, One Photon at a Time

by

Brandon Leigh Allen

Submitted to the Department of Physics
on May 18, 2019, in partial fulfillment of the
requirements for the degree of
Doctorate of Science in Physics

Abstract

A search is conducted for new physics in final states containing a photon and missing transverse momentum in proton-proton collisions at $\sqrt{s} = 13$ TeV. The data collected by the CMS experiment at the CERN LHC correspond to an integrated luminosity of 35.9 inverse femtobarns. No deviations from the predictions of the standard model are observed. The results are interpreted in the context of dark matter production and limits on new physics parameters are calculated at 95% confidence level. For the two simplified dark matter production models considered, the observed (expected) lower limits on the mediator masses are both 950 (1150) GeV for 1 GeV dark matter mass.

Thesis Supervisor: Christoph E.M. Paus
Title: Professor

Acknowledgments

This is the acknowledgements section. You should replace this with your own acknowledgements.

Contents

Chapter 1

The Standard Model

The Standard Model (SM) of particle physics describes the physical properties and dynamics of fermions, the fundamental constituents of matter, and their interactions in the language of a Lorentz-invariant quantum field theory (QFT). The discussion of the SM in this chapter is heavily indebted to the book *Modern Particle Physics* by Mark Thomson [1], in both organization and content. Additional references are included where necessary.

The Standard Model consists of a set of fermion fields, shown in Table 1.1, and the local gauge symmetry group that acts on them

$$G_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y, \tag{1.1}$$

which is composed of the subgroups

$$\begin{aligned} G_{\mathrm{QCD}} &= \mathrm{SU}(3)_C \qquad \text{and} \\ G_{\mathrm{EWK}} &= \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y, \end{aligned} \tag{1.2}$$

corresponding to the strong and electroweak interactions, respectively. Each fermion field exists in a unique representation of G_{SM}, also summarized in Table 1.1. The possible represenations of $\mathrm{SU}(3)_C$ are triplet, conjugate, and singlet, denoted by $\mathbf{3}$, $\bar{\mathbf{3}}$, and $\mathbf{1}$, respectively, while the possible representations of $\mathrm{SU}(2)_L$ are doublet and singlet, denoted by **2** and **1**, respectively. All fermions exist in the singlet representation of $\mathrm{U}(1)_Y$, only distinguished by differing values of the weak hypercharge Y. Conversely, all fermions in non-singlet representations of $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ have the same interaction strength, a feature known as universality.

Name	Symbol	Y	$\mathrm{SU}(2)_L$ rep.	$\mathrm{SU}(3)_C$ rep.
Left-handed quark	q_L	$^1/_6$	**2**	**3**
Right-handed up-type quark	u_R	$^2/_3$	**1**	**3**
Right-handed down-type quark	d_R	$-^1/_3$	**1**	**3**
Left-handed lepton	ℓ_L	$-^1/_2$	**2**	**1**
Right-handed charged lepton	e_R	-1	**1**	**1**
Right-handed neutrino	ν_R	$^1/_6$	**1**	**1**

Table 1.1: The categories of SM fermions and the action of the SM local gauge symmetry group G_{SM}. Each category contains three members, one for each generation of the Standard Model. A corresponding table exists for the charge conjugated fields representing the anti-fermions. The subscripts L and R denote whether the field is left- or right-handed.

For each category of fermion listed in Table 1.1, there exist three generations or copies in the Standard Model, identical except for differing masses. The lepton electroweak doublets contain the left-handed charged leptons and neutrinos

$$\ell_L = \begin{pmatrix} \nu_e \\ e_L^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu_L^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau_L^- \end{pmatrix}, \tag{1.3}$$

and the right-handed lepton singlets contain the right-handed projections of the same leptons and neutrinos. The quark electroweak doublets contain the left-handed up-type and down-type quarks

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \tag{1.4}$$

and the right-handed quark singlets contain the right-handed projections of the same quarks. Quarks also exist in a strong triplet, which will be denoted with a superscript c as necessary.

1.1 Strong Interactions

The strong interactions of quarks and gluons are described by quantum chromodynamics (QCD) [2, 3], with the Lagrangian

$$\mathcal{L}_{\text{QCD}} = i\bar{q}_f^a \not{D}^{ab} q_f^b + m_f \bar{q}_f^a q_f^a - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu} + \theta \frac{g_s^2}{72\pi^2} \epsilon_{\mu\nu\rho\sigma} G^{c,\mu\nu} G^{c,\rho\sigma}, \tag{1.5}$$

where repeated indices are contracted. The q_f^a are the quark-field Dirac spinors of flavor $f \in \{u, d, c, s, t, b\}$, color $a \in \{r, g, b\}$ the basis elements of the triplet representation of $\mathrm{SU}(3)_C$, and mass m_f. The first term in Equation 1.5 contains the QCD covariant derivative

$$D_\mu^{ab} = \delta^{ab} \partial_\mu - i g_s \sum_c t_c^{ab} G_{c,\mu}, \tag{1.6}$$

where g_s is the strong interaction coupling strength with associated coupling constant $\alpha_s = g_s^2/(4\pi)$, the t_c are the eight 3×3 Hermitian traceless matrices that serve as the generators of the triplet representation of $\mathrm{SU}(3)_C$, and the G_c are the corresponding eight gluon fields. The third term in Equation 1.5 contains the gluon field strength tensors

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^C, \tag{1.7}$$

where f^{abc} are the structure constants of $\mathrm{SU}(3)_C$. The non-Abelian structure of the $\mathrm{SU}(3)_C$ group allows for 3-gluon and 4-gluon interactions in addition to the quark-antiquark-gluon interactions.

The last term in Equation 1.5 violates CP conservation and produces a non-zero electric dipole moment (EDM) for the neutron. Experimental limits on the neutron EDM constrain the observed QCD vaccum angle θ to be smaller than 10^{-10} [4]. The Peccei-Quinn theory [5, 6] provides a possible explanation for this contradiction by introducing the hypothetical axion particle a with the following Lagrangian

$$\mathcal{L}_a = \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{a}{f_a} \frac{g_s^2}{72\pi^2} \epsilon_{\mu\nu\rho\sigma} G^{c,\mu\nu} G^{c,\rho\sigma}, \tag{1.8}$$

where f_a is axion decay constant that determines its characteristic scale. The second term in Equation 1.8 cancels the last term in Equation 1.5 when the axion field dynamically assumes its vacuum expectation value $\langle a \rangle = -f_a \theta$. The axion is a potential dark matter candidate.

1.2 Renormalization and Hadrons

Due to higher-order corrections to propagators in a QFT, physical quantities such as coupling constants and masses acquire a scale-dependence, where the value of the quantity changes as a function of the probed energy scale q^2. The process of recovering scale-invariance is called renormalization and ensures that any divergent terms from the higher-order corrections cancel out in the physical values. Given the value of an arbitrary coupling constant α at some known scale μ^2, the value of α at arbitrary scale q^2 is

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \alpha(\mu^2)\left[\Pi(q^2) - \Pi(\mu^2)\right]}, \tag{1.9}$$

where $\Pi(q^2)$ and $\Pi(\mu^2)$ are the self-energy correction of the propagator at scales q^2 and μ^2. While these individual terms are separately divergent, their difference is finite and calculable.

For values of q^2 and μ^2 larger than the QCD confinement scale $\Lambda_{\mathrm{QCD}} = 218\,\mathrm{MeV}$, the difference between the gluon self-energy corrections to one-loop order is given by

$$\Pi_s(q^2) - \Pi_s(\mu^2) \approx -\frac{\beta_{\mathrm{QCD}}}{4\pi} \ln\left(\frac{q^2}{\mu^2}\right) \tag{1.10}$$

where β_{QCD} depends on the number of quark and gluon loops. For N_c colors and N_f quark flavors with mass below $|q|$,

$$\beta_{\mathrm{QCD}} = \frac{11N_c - 2N_f}{12\pi}. \tag{1.11}$$

In the Standard Model, $N_c = 3$ and $N_f \leq 6$ regardless of energy, thus β_{QCD} is always positive. Combining Equations 1.9 and 1.10, the evolution of α_s is given by

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \beta_{\text{QCD}}\alpha_s(\mu^2)\ln\left(\frac{q^2}{\mu^2}\right)} \approx \frac{1}{\beta \ln\left(\frac{q^2}{\Lambda^2_{\text{QCD}}}\right)} \tag{1.12}$$

for a sufficiently large energy scale $q^2 \gg \Lambda^2_{\text{QCD}}$. Through electron-positron collisions, the value of α_s at the Z-pole has been measured to be $\alpha_s(m_Z^2) = 0.1181 \pm 0.0011$ [4].

From Equation 1.12, we see that α_s decreases with increasing q^2. At $|q| \sim 1\,\text{GeV}$, the value of α_s is of $\mathcal{O}(1)$ confining quarks and gluons to hadrons in a strongly-bound non-perturbative state. However, $|q| \gtrsim 100\,\text{GeV}$, we have $\alpha_s \approx 0.1$ which is small enough that perturbation theory can be used and quarks can be treated as quasi-free particles. This property of QCD is known as asymptotic freedom [7].

Below the confinement scale Λ_{QCD}, colored objects are always confined to color singlet states and no objects with non-zero color charge propagate as free particles. This low-energy non-pertubative phenomenon is known as color confinement. Thus, free quarks and gluons are not observed in nature, only in colorless bound states called hadrons [8]. The most common states consist of a quark-antiquark pair or three quarks, called mesons and baryons, respectively. Rarer pentaquark states have recently been found by the LHCb collaboration [9].

1.3 Electroweak Interactions

The electroweak interactions of fermions are described by the $\text{SU}(2)_L \times \text{U}(1)_Y$ gauge group [10–12], with the Lagrangian

$$\mathcal{L}_{\text{EWK}} = i\bar{\psi}_i \not{D} \psi_i - \frac{1}{4}\vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{1.13}$$

where repeated indices are contracted and $\psi \supseteq \{q_L, u_R, d_R, \ell_L, e_R, \nu_R\}$ is the set of SM fermions, and the gauge field tensors are given by

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \qquad \text{and} \\ \vec{W}_{\mu\nu} &= \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + g\vec{W}^\mu \times \vec{W}^\nu, \end{aligned} \tag{1.14}$$

where $\vec{W}_\mu$ and B_μ are the gauge fields for $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$, respectively, and g is the coupling strength for $\mathrm{SU}(2)_L$. The first term in Equation 1.13 contains the EWK covariant derivative

$$D_\mu = \partial_\mu - ig\vec{T} \cdot \vec{W}_\mu - ig'YB_\mu, \tag{1.15}$$

where g' is the coupling strength for $\mathrm{U}(1)_Y$, Y is the $\mathrm{U}(1)_Y$ hypercharge of the fermion field, and $\vec{T}$ are the generators of the doublet representation of $\mathrm{SU}(2)_L$. The generators can be written in terms of the Pauli spin matrices $\vec{T} = \vec{\sigma}/2$ and only have non-zero action on left-handed particles. The values of the hypercharge Y shown in Table 1.1 are chosen such that the physical electric charge of each fermion is given by $Q = T_3 + Y$.

However, this theory of the electroweak interactions is not sufficient to explain the observed behavior of the weak force. Equation 1.13 contains three massless gauge bosons for the weak charge and one massless gauge boson for hypercharge, but experimentally three massive weak gauge bosons and one massless photon have been observed. Introducing explicit mass terms of the form $-m_W^2 W_\mu W^\mu$ to the Lagrangian breaks the $\mathrm{SU}(2)_L$ gauge invariance as well as making the theory non-renormalizable. Spontaneous breaking of the $\mathrm{SU}(2)_L$ gauge invariance provides the mechanism we need to provide mass to the weak gauge bosons while maintaining the underlying symmetries and gauge invariance of Equation 1.13.

1.4 Electroweak Symmetry Breaking

The development of a dynamical photon mass in the Bardeen-Cooper-Schrieffer theory of superconductivity [13] provided the template for spontaneous symmetry breaking in the electroweak sector. Such spontaneous symmetry breaking occurs when the vacuum is degenerate with none of the possible ground states exhibiting the symmetry of the underlying theory [14, 15]. However, as a consequence of the Nambu-Goldstone theorem [16, 17], massless spin-0 bosons appear after spontaneous symmetry breaking but no such particles are observed in nature. Fortunately, Brout, Englert, and Higgs as well as Guralnik, Hagen, and Kibble discovered that when an additional field is used to break a gauge symmetry, the gauge bosons acquire a nonzero mass by absorbing the Nambu-Goldstone bosons [18–23]. Building upon these ideas, Glashow, Weinberg, and Salam developed a theory of electroweak unification [10–12] that explained the observed massive weak bosons in terms of the massless bosons from Equation 1.13. Finally, t'Hooft and Veltman proved that this model is renormalizable [24]. We shall walk through the key points of these developments now.

The $\mathrm{SU}(2)_L$ symmetry is broken by introducing a left-handed complex scalar doublet ϕ with $Y_\phi = 1/2$ to the Lagrangian in the following manner

$$\mathcal{L}_{\mathrm{EWK}} \mapsto \mathcal{L}_{\mathrm{EWK}} + |D_\mu \phi|^2 + \mu^2 \phi^2 - \lambda |\phi|^4 . \tag{1.16}$$

We choose to write this complex doublet, known as the complex Higgs field, in terms of four real-valued fields so that

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} . \tag{1.17}$$

Fortunately, the two self-interaction terms create a Higgs potential with a degenerate global minimum at the vacuum expectation value (vev)

$$v \equiv \langle |\phi| \rangle = \sqrt{\frac{\mu^2}{\lambda}}, \tag{1.18}$$

and through gauge rotations we set $\langle \phi_{1,2,4} \rangle = 0$, removing three degrees of freedom and producing three massless Nambu-Goldstone bosons. The remaining degree of freedom is the real Higgs field H which expresses small peturbations around the vev in the third component of the complex Higgs field $\phi_3 = v + H$.

The kinetic term in Equation 1.16 couples the complex Higgs field to the EWK gauge bosons as follows at the vev

$$|D_\mu\phi|^2 = \frac{v^2}{8}\left[\left(gW_\mu^1\right)^2 + \left(gW_\mu^2\right)^2 + \left(g'B_\mu - gW_\mu^3\right)^2\right]. \tag{1.19}$$

Diagonalizing this term gives rise to the three massive weak bosons and the massless photon that we observe in nature:

$$\begin{array}{l|l} W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp W_\mu^2\right) & m_W = \frac{1}{2}vg \\ Z_\mu \equiv \cos\theta_\mathrm{W} W_\mu^3 - \sin\theta_\mathrm{W} B_\mu & m_Z = \frac{1}{2}v\sqrt{g^2+(g')^2} \\ A_\mu \equiv \sin\theta_\mathrm{W} W_\mu^3 + \cos\theta_\mathrm{W} B_\mu & m_A = 0, \end{array} \tag{1.20}$$

where $\tan\theta_\mathrm{W} = g'/g$ and $\cos\theta_\mathrm{W} = m_W/m_Z$. With this, we rewrite Equation 1.13 in terms of the observed electromagnetic (EM), charged weaked (CC), and neutral weak (NC) currents as follows:

$$\begin{aligned} \mathcal{L}_\mathrm{EWK} = \bar{\psi}_i\left(i\partial\!\!\!/ - eQA\!\!\!/\right)\psi_i - \frac{g}{2\sqrt{2}}\bar{\psi}_i\left(T^+W\!\!\!\!/^+ + T^-W\!\!\!\!/^-\right)\psi_i - \frac{1}{2}m_\mathrm{W}^2 W_\mu^+ W^{-\mu} \\ - \frac{g}{2\cos\theta_\mathrm{W}}\bar{\psi}_i(g_V - g_A\gamma^5)Z\!\!\!/\psi_i - \frac{1}{2}m_Z^2 Z_\mu Z^\mu, \end{aligned} \tag{1.21}$$

where $e = g'\cos\theta_\mathrm{W}$ is the charge of the electron with associated coupling constant $\alpha = e^2/(4\pi)$, $T^\pm = (T_1 \mp iT_2)/\sqrt{2}$ are the weak isospin raising and lowering operators, and $g_V = T_3$ and $g_A = T_3 - 2Q\sin^2\theta_\mathrm{W}$ are the vector and axial-vector couplings for the neutral weak current. We can also expand Equation 1.16 about the vev giving us the following Higgs Lagrangian

$$\begin{aligned} \mathcal{L}_H = \frac{1}{2}\partial_\mu H\partial^\mu H - \frac{1}{2}m_H^2 H^2 + \frac{m_H^2}{2v}H^3 + \frac{2m_W^2}{v}W_\mu^+W^{-\mu}H + \frac{m_Z^2}{v}Z_\mu Z^\mu H \\ + \frac{m_H^2}{8v^2}H^4 + \frac{m_W^2}{v^2}W_\mu^+W^{-\mu}H^2 + \frac{m_Z^2}{2v^2}Z_\mu Z^\mu H^2, \end{aligned} \tag{1.22}$$

where $m_H = \mu\sqrt{2}$. Thus, we see that the real Higgs field H has trilinear and quartic couplings to itself and the weak gauge bosons with coupling strengths proportional to the mass squared of the appropriate boson.

1.5 Fermion Masses

Notice that Equation 1.21 does not contain a Dirac mass term like that found in Equation 1.5. This is because the term

$$m\bar{\psi}\psi = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right) \tag{1.23}$$

mixes the left-handed and right-handed fermions leading to a Lagrangian that is no longer invariant under $\mathrm{SU}(2)_L$. As the observed fermions are not massless, the Lagrangian given in Equation 1.13 is incomplete. Thankfully, introducing Yukawa couplings between the complex Higgs field ϕ and the SM fermion fields provides an economical way to add mass terms for the fermions.

First, we start with the terms for charged leptons,

$$\mathcal{L}_{\mathrm{Y}}^{\mathrm{leptons}} = -\bar{\ell}_L Y_e \phi e_R - \bar{e}_R Y_e^\dagger \phi^\dagger \ell_L, \tag{1.24}$$

where Y_e is the Yukawa matrix for the charged leptons. In general, Yukawa matrices and thus mass matrices are non-diagonal and hence we need to convert from the electroweak eigenstates $f_{L,R}$ to the mass eigenstates $\tilde{f}_{L,R} = U^f_{L,R} f_{L,R}$ where $U^f_{L,R}$ is a unitary matrix. With this we rewrite Equation 1.24 in terms of the mass eigenstates

$$\begin{aligned}
\mathcal{L}_{\mathrm{Y}}^{\mathrm{leptons}} &= -\bar{\tilde{\ell}}_L U_L^e Y_e \phi U_R^{e\dagger} \tilde{e}_R - \bar{\tilde{e}}_R U_R^e Y_e^\dagger \phi^\dagger U_L^{e\dagger} \tilde{\ell}_L \\
&= -\bar{\tilde{\ell}}_L \tilde{Y}_e \phi \tilde{e}_R - \bar{\tilde{e}}_R \tilde{Y}_e^\dagger \phi^\dagger \tilde{\ell}_L,
\end{aligned} \tag{1.25}$$

where $\tilde{Y}_e = U_L^e Y_e U_R^{e\dagger}$ is the diagonalized Yukawa matrix for the charged leptons. After electroweak symmetry breaking, these terms become

$$\begin{aligned}
\mathcal{L}_{\mathrm{Y}}^{\mathrm{leptons}} &= -\frac{v+H}{\sqrt{2}}\left(\bar{\tilde{e}}_L \tilde{Y}_e \tilde{e}_R + \bar{\tilde{e}}_R \tilde{Y}_e^\dagger \tilde{e}_L\right) \\
&= -\left(1 + \frac{H}{v}\right)\left(\bar{\tilde{e}}_L \tilde{M}_e \tilde{e}_R + \bar{\tilde{e}}_R \tilde{M}_e^\dagger \tilde{e}_L\right) \\
&= -\tilde{M}_e \bar{e} e - \frac{\tilde{M}_e}{v} \bar{e} e H,
\end{aligned} \tag{1.26}$$

where $\tilde{M}_e = v\tilde{Y}_e/\sqrt{2}$ is the diagonalized mass matrix for the charged leptons and e is the set of massive Dirac spinors for the charged leptons.

From Equation 1.26, we see that the Yukawa couplings between the complex Higgs field ϕ and the charged leptons result in a Dirac mass term and a coupling to the real Higgs field H that is proportional to the mass of the charged leptons and the vev. The same procedure is used to introduce mass terms for the down-type quarks whereas for the neutrinos and up-type quarks we must use the conjugate doublet $\phi_c = -i\sigma_2\phi^*$ in place of ϕ to obtain the same result.

1.6 Flavor Mixing

For the charged leptons and up-type quarks, it is possible to define a basis of simultaneous electroweak and mass eigenstates, so in practice $\tilde{Y}_{e,u} = Y_{e,u}$ as $U_L^{e,u} = U_R^{e,u} = \mathbf{I}$. However, it is not possible to do this for the neutrinos at the same time as the charged leptons or for the down-type quarks at the same time as the up-type quarks.

In Equation 1.21, the charged current term involves interactions between the up-type and down-type quarks and is not preserved under the transform $f \to \tilde{f}$. Writing this in terms of the mass eigenstates we have

$$\begin{aligned}\mathcal{L}_{\text{CC}} &= -\frac{g}{2\sqrt{2}}\left(\bar{u}_L T^+ \not{W}^+ d_L + \bar{d}_L T^- \not{W}^- u_L\right)\\ &= -\frac{g}{2\sqrt{2}}\left(\bar{u}_L T^+ \not{W}^+ V_{\text{CKM}}\tilde{d}_L + \bar{\tilde{d}}_L T^- \not{W}^- V_{\text{CKM}}^\dagger u_L\right),\end{aligned} \tag{1.27}$$

where $V_{\text{CKM}} = U_L^{u\dagger} U_L^d$ is the Cabibbo-Kaboyshi-Maskawa matrix [25, 26] and $u_L = \tilde{u}_L$ by construction. The CKM matrix is unitary with four free parameters, the mixing angles between quark generations ϕ_{12}, ϕ_{23}, and ϕ_{13} as well as a CP-violating phase δ. In terms of these parameters, the CKM matrix is

$$V_{\text{CKM}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}, \tag{1.28}$$

where $s_{ij} = \sin\phi_{ij}$ and $c_{ij} = \cos\phi_{ij}$. It has been experimentally determined that the CKM is mostly diagonal with $s_{13} \ll s_{23} \ll s_{12} \ll 1$.

The equivalent mixing matrix for the neutrinos is the Pontecorvo-Maki-Nakagawa-Sakata matrix U_{PMNS} [27–29], which converts from the mass eigenstates ν_1, ν_2, and ν_3 to the electroweak eigenstates ν_e, ν_μ, ν_τ. Unlike the CKM matrix, the PMNS matrix is non-diagonal resulting in stronger mixing in the neutrino sector. The values of the mixing angles θ_{12}, θ_{23}, and θ_{13} have been measured in neutrino oscillation experiments while the CP-violating phase δ' has not yet been directly measured. From cosmological measurements of the large-scale structure of the universe, it is known that the sum of the neutrino masses is less than one eV.

1.7 Summary

Parameter	Description	Best Fit Value
ϕ_{12}	CKM 12-mixing anlge	13.1°
ϕ_{23}	CKM 23-mixing angle	2.4°
ϕ_{13}	CKM 13-mixing angle	0.4°
δ	CKM CP-violating phase	0.995
$\sin^2\theta_{12}$	PMNS 12-mixing anlge	0.297
$\sin^2\theta_{23}$	PMNS 23-mixing angle	0.437
$\sin^2\theta_{13}$	PMNS 13-mixing angle	0.0214
δ'	PMNS CP-violating phase	1.35
α_s	QCD coupling constant	0.1181
α	EM coupling constant	$1/137.036$
θ	QCD vacuum angle	$< 10^{-10}$

Table 1.2: The free parameters of the Standard Model, not including masses [4]

.

The Standard Model has a total of 26 free parameters and 17 physical particles. The parameters are the twelve Yukawa couplings for the fermions, the four parameters of the CKM matrix, the four parameters of the PMNS matrix, the two coupling constants α_s and α, the masses of the weak gauge bosons m_W and m_Z, the mass of the Higgs boson m_H, and the QCD vacuum angle θ. The best fit values of the SM parameters, excluding masses, are summarized in Table 1.2.

Name	Symbol	Spin	Charge	Mass
up quark	u	½	⅔	2.2 MeV
down quark	d	½	-⅓	4.7 MeV
charm quark	c	½	⅔	1.28 GeV
strange quark	s	½	-⅓	95 MeV
top quark	t	½	⅔	173 GeV
bottom quark	b	½	-⅓	4.18 GeV
electron neutrino	ν_e	½	0	-
electron	e	½	-1	511 keV
muon neutrino	ν_μ	½	0	-
muon	μ	½	-1	105 MeV
tau neutrino	ν_τ	½	0	-
tau	τ	½	-1	1.78 GeV
gluon	g	1	0	0
photon	γ	1	0	0
Z boson	Z	1	0	91.2 GeV
W boson	$W^\pm$	1	±1	80.4 GeV
Higgs boson	H	0	0	125 GeV

Table 1.3: The physical particles of the Standard Model [4].

The physical particles are the single-particle states of the various mass eigenfields and their properties are summarized in Table 1.3. Each of the fermion fields has a corresponding anti-particle with the electromagnetic and color charges inverted. Most of these single-particle states have finite lifetimes and decay to lower energy configurations. The only particles whose decays have not been observed are the photon, the electron, the neutrinos, and the proton (a baryon of flavor content uud). Additionally, stable bound states of protons and neutrons (a baryon of flavor content udd) exist in the form of atomic nuclei.

Bibliography

[1]M. Thomson, *Modern particle physics* (Cambridge University Press, 2013).

[2]D. J. Gross and F. Wilczek, “Ultraviolet behavior of non-abelian gauge theories”, Phys. Rev. Lett. **30**, 1343–1346 (1973).

[3]H. D. Politzer, “Reliable perturbative results for strong interactions?”, Phys. Rev. Lett. **30**, 1346–1349 (1973).

[4]M. T. et al., “Review of particle physics”, Phys. Rev. D **98**, 030001 (2018).

[5]R. D. Peccei and H. R. Quinn, “CP conservation in the presence of pseudoparticles”, Phys. Rev. Lett. **38**, 1440–1443 (1977).

[6]R. D. Peccei, “The strong cp problem and axions”, in *Axions: theory, cosmology, and experimental searches* (Springer Berlin Heidelberg, Berlin, Heidelberg, 2008), pp. 3–17.

[7]D. J. Gross and F. Wilczek, “Asymptotically free gauge theories. i”, Phys. Rev. D **8**, 3633–3652 (1973).

[8]M. Gell-Mann, “A schematic model of baryons and mesons”, Physics Letters **8**, 214–215 (1964).

[9]R. A. et al., “Observation of $j/\psi p$ resonances consistent with pentaquark states in $\Lambda b0 \to j/\psi K^- p$ decays”, Phys. Rev. Lett. **115**, 072001 (2015).

[10]S. L. Glashow, “Partial-symmetries of weak interactions”, Nuclear Physics **22**, 579–588 (1961).

[11]S. Weinberg, “A model of leptons”, Phys. Rev. Lett. **19**, 1264–1266 (1967).

[12]A. Salam, “Weak and Electromagnetic Interactions”, Conf. Proc. **C680519**, 367–377 (1968).

[13]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, “Theory of superconductivity”, Phys. Rev. **108**, 1175–1204 (1957).

[14]Y. Nambu, “Quasi-particles and gauge invariance in the theory of superconductivity”, Phys. Rev. **117**, 648–663 (1960).

[15]P. W. Anderson, “Plasmons, gauge invariance, and mass”, Phys. Rev. **130**, 439–442 (1963).

[16]J. Goldstone, “Field theories with « superconductor » solutions”, Il Nuovo Cimento (1955-1965) **19**, 154–164 (1961).

[17]J. Goldstone, A. Salam, and S. Weinberg, “Broken symmetries”, Phys. Rev. **127**, 965–970 (1962).

[18]F. Englert and R. Brout, “Broken symmetry and the mass of gauge vector mesons”, Phys. Rev. Lett. **13**, 321–323 (1964).

[19]P. W. Higgs, “Broken symmetries, massless particles and gauge fields”, Physics Letters **12**, 132–133 (1964).

[20]P. W. Higgs, “Broken symmetries and the masses of gauge bosons”, Phys. Rev. Lett. **13**, 508–509 (1964).

[21]P. W. Higgs, “Spontaneous symmetry breakdown without massless bosons”, Phys. Rev. **145**, 1156–1163 (1966).

[22]G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, “Global conservation laws and massless particles”, Phys. Rev. Lett. **13**, 585–587 (1964).

[23]T. W. B. Kibble, “Symmetry breaking in non-abelian gauge theories”, Phys. Rev. **155**, 1554–1561 (1967).

[24]G. t’Hooft and M. Veltman, “Regularization and renormalization of gauge fields”, Nuclear Physics B **44**, 189–213 (1972).

[25]N. Cabibbo, “Unitary symmetry and leptonic decays”, Phys. Rev. Lett. **10**, 531–533 (1963).

[26]M. Kobayashi and T. Maskawa, “CP-Violation in the Renormalizable Theory of Weak Interaction”, Progress of Theoretical Physics **49**, 652–657 (1973).

[27]B. Pontecorvo, “Mesonium and anti-mesonium”, Sov. Phys. JETP **6**, [Zh. Eksp. Teor. Fiz.33,549(1957)], 429 (1957).

[28]B. Pontecorvo, “Neutrino Experiments and the Problem of Conservation of Leptonic Charge”, Sov. Phys. JETP **26**, [Zh. Eksp. Teor. Fiz.53,1717(1967)], 984–988 (1968).

[29]M. Nakagawa, S. Sakata, and Z. Maki, “Remarks on the Unified Model of Elementary Particles”, Progress of Theoretical Physics **28**, 870–880 (1962).