Install AMPL IDE and/or python link. URL will be sent!
Reading: Vanderbei, Chapter 1 (Intro)
Extra credit for helping others

Each set $H = \{x \in \mathbb{R}^n : a_i^T x = b_i\}$ is a <u>hyperplane</u>.

row of $A$

generalization of plane in $\mathbb{R}^3$
flat level-set of dim. $n-1$

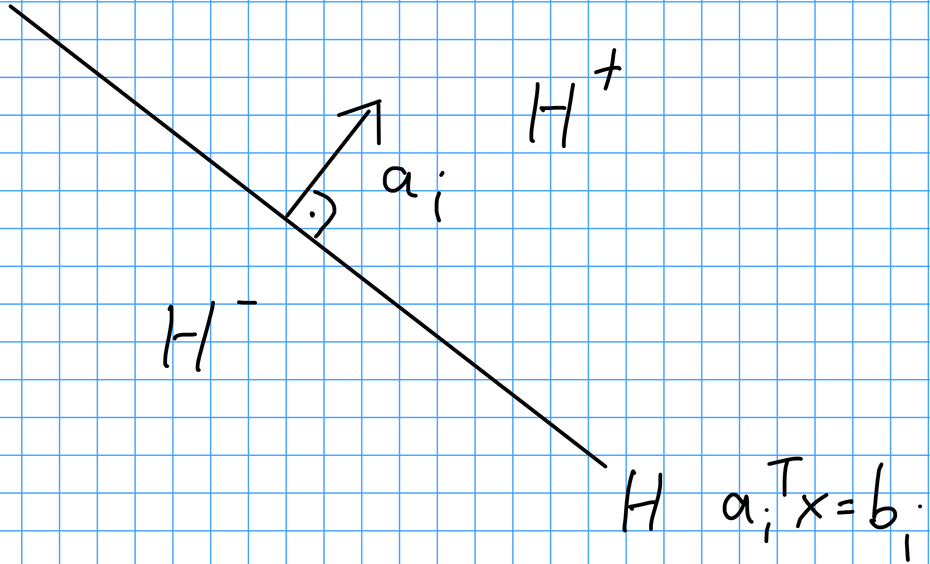
A hyperplane induces two <u>halfspaces</u>

$H^+ = \{x \in \mathbb{R}^n : a_i^T x \geq b_i\}$ and $H^- = \{x \in \mathbb{R}^n : a_i^T x \leq b_i\}$

positive halfspace    negative halfspace

The <u>normal vector</u> $a_i \in \mathbb{R}^n$ points orthogonally into $H^+$.

$H$ can be considered a level se w.r.t. $a_i^T x$.

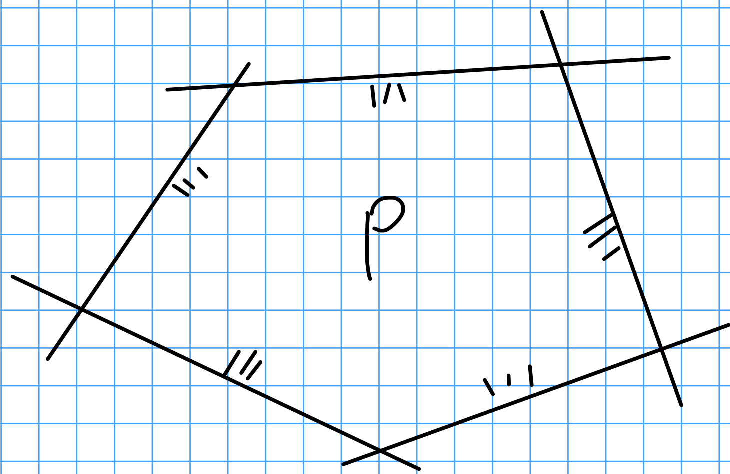



The feasible region of an LP (in canonical form $Ax \le b$) is the intersection of finitely many halfspaces.

**Polyhedral sets** are finite intersections of halfspaces

→ polyhedron, polytope

bounded polyhedron

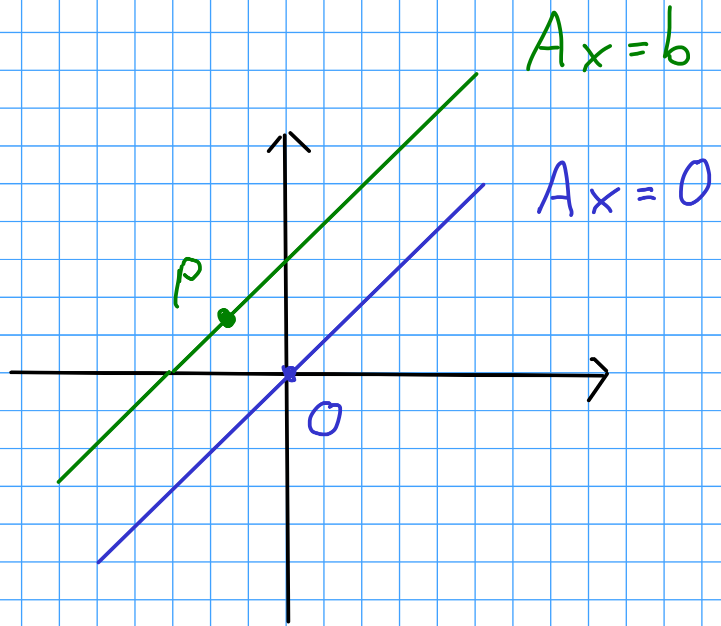


The feasible sets of linear programming are called polyhedra.

The set $S = \{x \in \mathbb{R}^n : Ax = b\}$ is an <u>affine</u> subspace.

$S_0 = \{x \in \mathbb{R}^n : Ax = 0\}$ is a <u>linear</u> subspace.

$S$ and $S_0$ are parallel to each other. $S$ runs through a <u>particular solution</u> $p$ instead of the origin




Special case: hyperplanes are $n-1$-dim. affine subspaces

The set $C = \{x \in \mathbb{R}^n : x \geq 0\}$ is a special <u>convex cone</u>.

the first orthant

- A set $C$ is a <u>cone</u> if $\alpha \cdot c \in C$ whenever $c \in C$ and $\alpha \geq 0$.
- A set $C$ is <u>convex</u> if $(1-\alpha)c + \alpha \cdot d \in C$ whenever $c, d \in C$ and $0 \leq \alpha \leq 1$.

The feasible region of an LP in standard form $Ax = b, x \geq 0$ is the intersection of an affine subspace with a convex cone/the first orthant.

# Visualization

## Example (*)

Geometrically characterize / visualize the feasible set

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ x_1 + x_2 + x_3 + x_4 &= 4 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

This describes a two-dimensional object in $\mathbb{R}^4$!

$\Rightarrow$ Can eliminate two variables for a visualization without loss of information!

we here eliminate $x_1$ and $x_4$

$$x_1 = 6 - 2x_2 - 3x_3 \geq 0$$

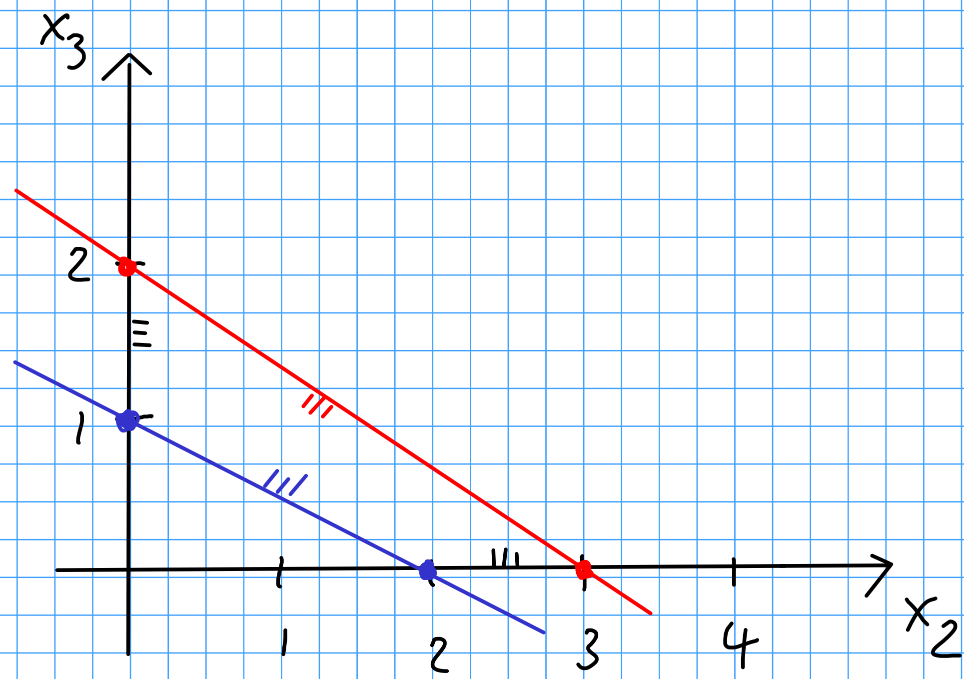
$$x_4 = 4 - x_1 - x_2 - x_3 = 4 - (6 - 2x_2 - 3x_3) - x_2 - x_3 =$$

$$= x_2 + 2x_3 - 2 \geq 0$$

$$\Rightarrow \quad 2x_2 + 3x_3 \leq 6$$

$$x_2 + 2x_3 \geq 2$$

$$x_2, \; x_3 \geq 0$$




HW2: Pick another combination of variables and show this process (algebra & sketch) for your choice!

## Vertices

$x$ is a vertex (or extreme point) of a polyhedron $P$ if

$x = y = z$ whenever $x = \alpha y + (1-\alpha) z$ for $y, z \in P$ and $0 \leq \alpha \leq 1$.

$x$ is not the convex combination of any other points

## The Fundamental Theorem of Linear Programming

If there is an optimal solution to an LP,

then there is an optimal vertex.

Note: There could be no opt. solution / unboundedness in direction of $c$ or even no feasible point.