HW 7: Exercise 4-5 from AMPL book.

/// End of Homework Collection 1

Submission Date extended to Sep. 23

Reading: Vanderbei 14.1 & 15.1-2

AMPL Chapter 15

Task Scheduling and the Critical Path Method

Problem: Most projects can be divided into several subtasks, each of which has its own estimated duration and requires completion of certain predecessor subtasks. When should each task be scheduled so to not delay the overall completion?

Example

Activity	Description	Immediate Predecessors	Duration (weeks)
A	Build foundation	–	5
B	Build walls & ceiling	A	8
C	Build roof	B	10
D	Do electrical wiring	B	5
E	Put in windows	B	4
F	Put on siding	E	6
G	Paint house	C, F	3

Model

- T: set of tasks with estimated duration d_t, $t \in T$
- for each $t \in T$, let $P_t \subseteq T$ be the set of predecessor tasks
- Variables: x_t start time of t

  y project completion time

$$\min \; y \quad \hat{=} \quad \min \; \max_{t \in T} \{x_t + d_t\}$$

$$\text{s.t.} \quad x_t \geq x_p + d_p \quad \forall t \in T, \forall p \in P_t$$

$$x_t \geq 0 \quad \forall t \in T$$

not an LP!

min-max conversion

$$\min \; y$$

$$\text{s.t.} \quad x_t \geq x_p + d_p \quad \forall t \in T, \forall p \in P_t$$

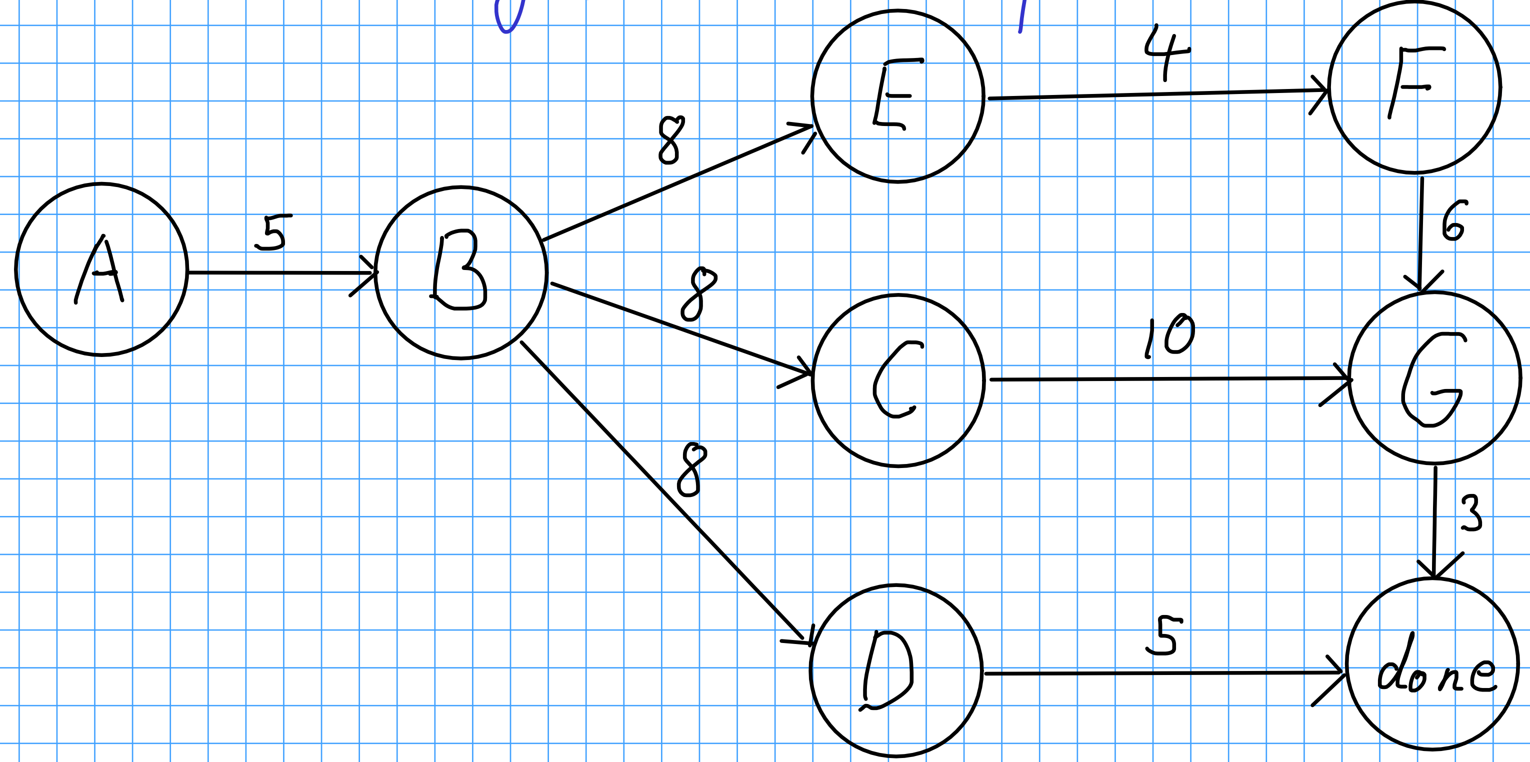
$$x_t \geq 0 \quad \forall t \in T$$

$$y \geq x_t + d_t \quad \forall t \in T$$

difference in feasible region, but same opt. solutions

Network Model and the Critical Path Method

Many task scheduling problems can be represented as networks and solved using the "critical path method"



The <u>critical path</u> of a project is the <u>longest</u> path through the project network (from A to "done"). The activities along a critical path are called <u>critical activities</u>. A delay of such an activity will delay overall project completion.

→ possible final project topic

Transportation and Assignment

Problem: Given a set of m supply points with supplies s_i, a set of n demand points with demand d_j and transportation costs c_{ij} between supply point i and demand point j, find the cheapest (shortest, fastest) route assignment to supply all demands.

Model · Variables x_{ij} denote amount shipped from i to j

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot x_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{m} x_{ij} = d_j && \forall j = 1, \ldots, n \\
& \sum_{j=1}^{n} x_{ij} = s_i && \forall i = 1, \ldots, n \\
& x_{ij} \geq 0 && \forall i = 1, \ldots, m, \ \forall j = 1, \ldots, n
\end{aligned}$$

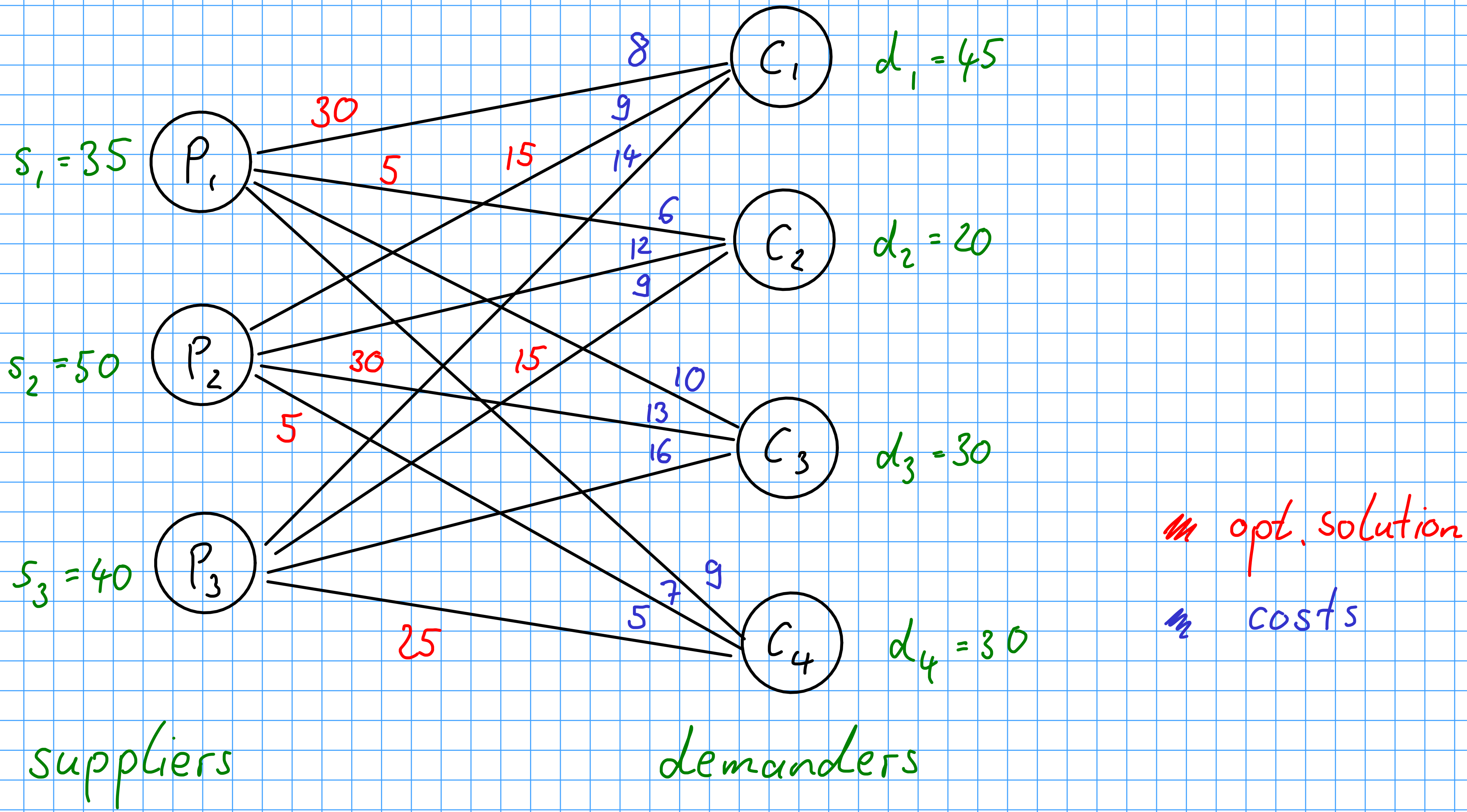
classic transportation problem assumes a "closed system": $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$

assignment problem: $s_i = d_j = 1 \quad \forall i, j$

Example (Power Generation): A local energy provider has 3 electric power plants that supply the needs of 4 cities. The total power supplied by each plant cannot exceed the plant's capacity, and each city must receive sufficient power to meet its demand during peak time. The cost of sending 1 million kwh of electricity between plants and cities is shown below.

From \ To (c_{ij})	City 1	City 2	City 3	City 4	Supply s_i
Plant 1	8	6	10	9	35
Plant 2	9	12	13	7	50
Plant 3	14	9	16	5	40
Demand d_j	45	20	30	30	125

Power Generation Network: A Bipartite Graph



can be solved with a maximum flow algorithm ($\rightarrow$ feasible solution) and a minimum-cost flow algorithm ($\rightarrow$ optimal solution)