HW 6: Exercise 2-6 from AMPL book.

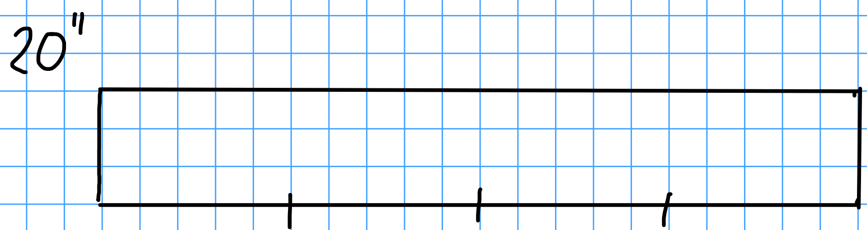
Reading: AMPL Chapter 4

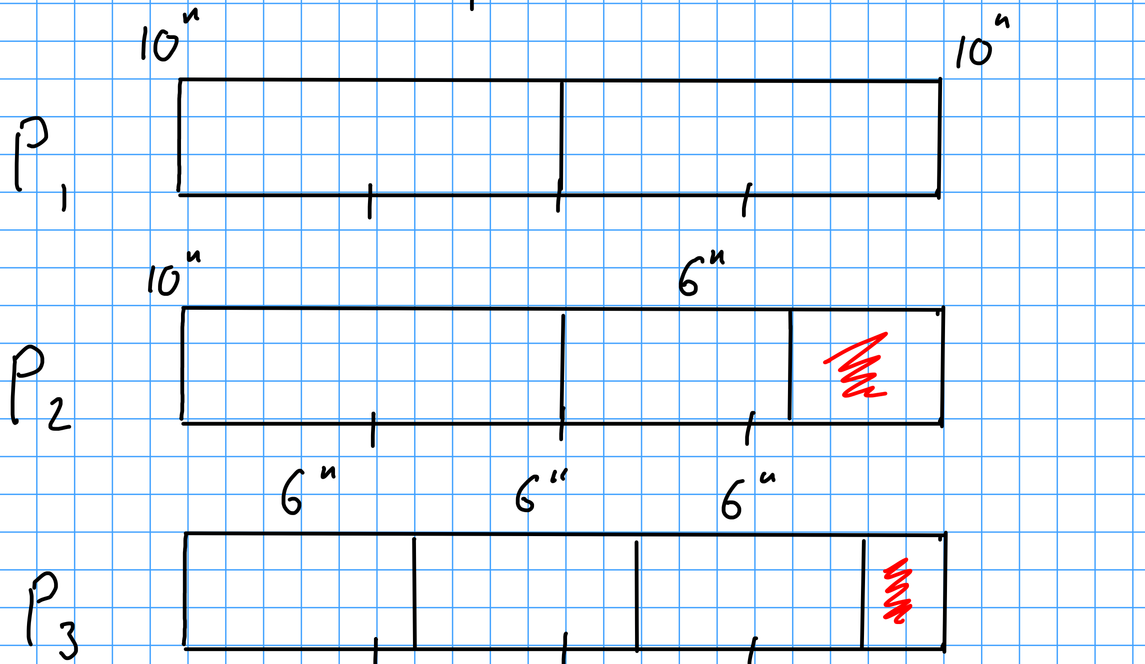
Cutting-Stock Problems

for scheduling

Famous problem with an important modeling idea

Problem: Cut large paper rolls of a given size (or shape/width) in order to meet demands of smaller rolls with as little left over ("trim loss") as possible.





Model

- P : set of cutting patterns (indexed by j)
- W : set of required shapes (indexed by i)
- a_{ij} : number of shapes i obtained from pattern j
- b_i : demand of shape i
- x_j : number of times pattern j is applied.

size of P can grow exponentially for larger data/parameters

↓

choice of good patterns is hard

$$\min \sum_{j \in P} x_j$$

$$\text{s.t.} \quad \sum_{j \in P} a_{ij} x_j \geq b_i \quad \forall i \in W$$

$$x_j \geq 0 \qquad (x_j \in \mathbb{Z}^+) \leftarrow \text{not an LP}$$

minimizing total number of rolls $\hat{=}$ minimizing total material used

→ implicitly min. total trim loss (if we consider overproduced shapes also as trim loss)

Work Scheduling

Problem: A chain of fast-food restaurants operates 7 days a week and requires the following minimum number of kitchen employees from Monday through Sunday: 45, 45, 40, 50, 65, 35, 35.
Each employee is scheduled to work 4 days a week and one weekend day. Management wants to know the minimum number of employees needed and their respective schedules.

Model

- S : set of schedules $(|S| = 10)$, $a_{ij} = 1$ if schedule j includes day i and $a_{ij} = 0$ else
- b_i : number of employees needed on day i
- x_j : number of employees hired to work schedule j

$$\min \sum_{j \in S} x_j \quad \text{s.t.} \quad \sum_{j \in S} a_{ij} x_j \geq b_i \quad \forall \text{ days } i, \quad x_j \geq 0 \;\; (x_j \in \mathbb{Z}^+) \quad \forall j \in S$$

Problem: A restaurant operates 7 days a week and requires the following number of employees from Monday to Sunday: 45, 45, 40, 50, 65, 35, 35. Regular employees are paid \$120/day and work 5 days a week. In addition, some employees have volunteered to work one day extra each week for which they are paid \$250/day overtime.

Model
- S, T: set of 5-day and 6-day schedules
- x_j, y_k: number of 5-day and 6-day employees

$$\min \sum_{j \in S} (5 \cdot 120)\, x_j + \sum_{k \in T} (5 \cdot 120 + 250)\, y_k$$

$$\text{s.t.} \quad \sum_{j \in S} a_{ij} \cdot x_j + \sum_{k \in T} a'_{ik}\, y_k \geq b_i \qquad \forall \text{ days } i$$

$$x_j \geq 0 \quad \forall j \in S \qquad (x_j \in \mathbb{Z}^+)$$

$$y_k \geq 0 \quad \forall k \in T \qquad (y_k \in \mathbb{Z}^+)$$

Problem: A restaurant operates 7 days a week and requires the following minimum number of working hours from Monday to Sunday: 360, 360, 320, 400, 520, 280, 280. Full-time employees work 8hrs/day at \$15/hr, part-time employees 4hrs/day at \$10/hr. Each employee works 5 days a week. Union req. limit part-time labor to 25%.

Model
- S : set of work schedules
- x_j, y_j : number of full-time and part-time employees on schedule j

$$\min \sum_{j\in S} 8\cdot 15\, x_j + \sum_{j\in S} 4\cdot 10\, y_j$$

measured in hours

$$\text{s.t.} \sum_{j\in S} a_{ij}(8x_j + 4y_j) \geq b_i \quad \forall \text{ days } i$$

$$\sum_{j\in S} x_j \geq 3\cdot \sum_{j\in S} y_j$$

$$x_j, y_j \geq 0 \quad \forall j\in S$$

Task Scheduling and the Critical Path Method

Problem: Most projects can be divided into several subtasks, each of which has its own estimated duration and requires completion of certain predecessor subtasks. When should each task be scheduled so to not delay the overall completion?

Example

Activity	Description	Immediate Predecessor	Duration (weeks)
A	Build foundation	–	5
B	Build walls & ceiling	A	8
C	Build roof	B	10
D	Do electrical wiring	B	5
E	Put in windows	B	4
F	Put on siding	E	6
G	Paint house	C, F	3