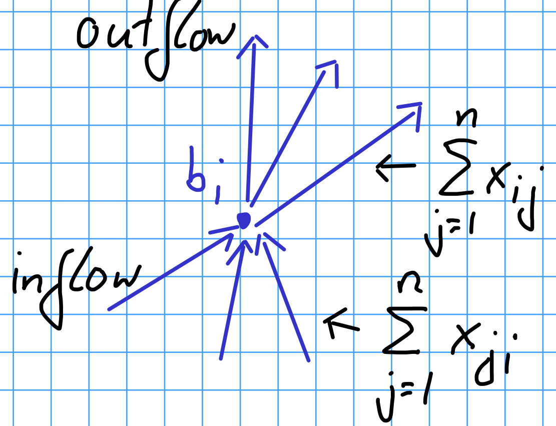
Transshipment and Circulation

Problem: Given a set of n locations with supplies b_i (or demands if b_i is negative) and costs (lengths/times) c_{ij} with lower and upper limits (l_{ij}, u_{ij}) for transportation between i and j, find the cheapest (shortest/fastest) transshipment to meet demands.

Model Variables x_{ij}: amount shipped from i to j

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}$$



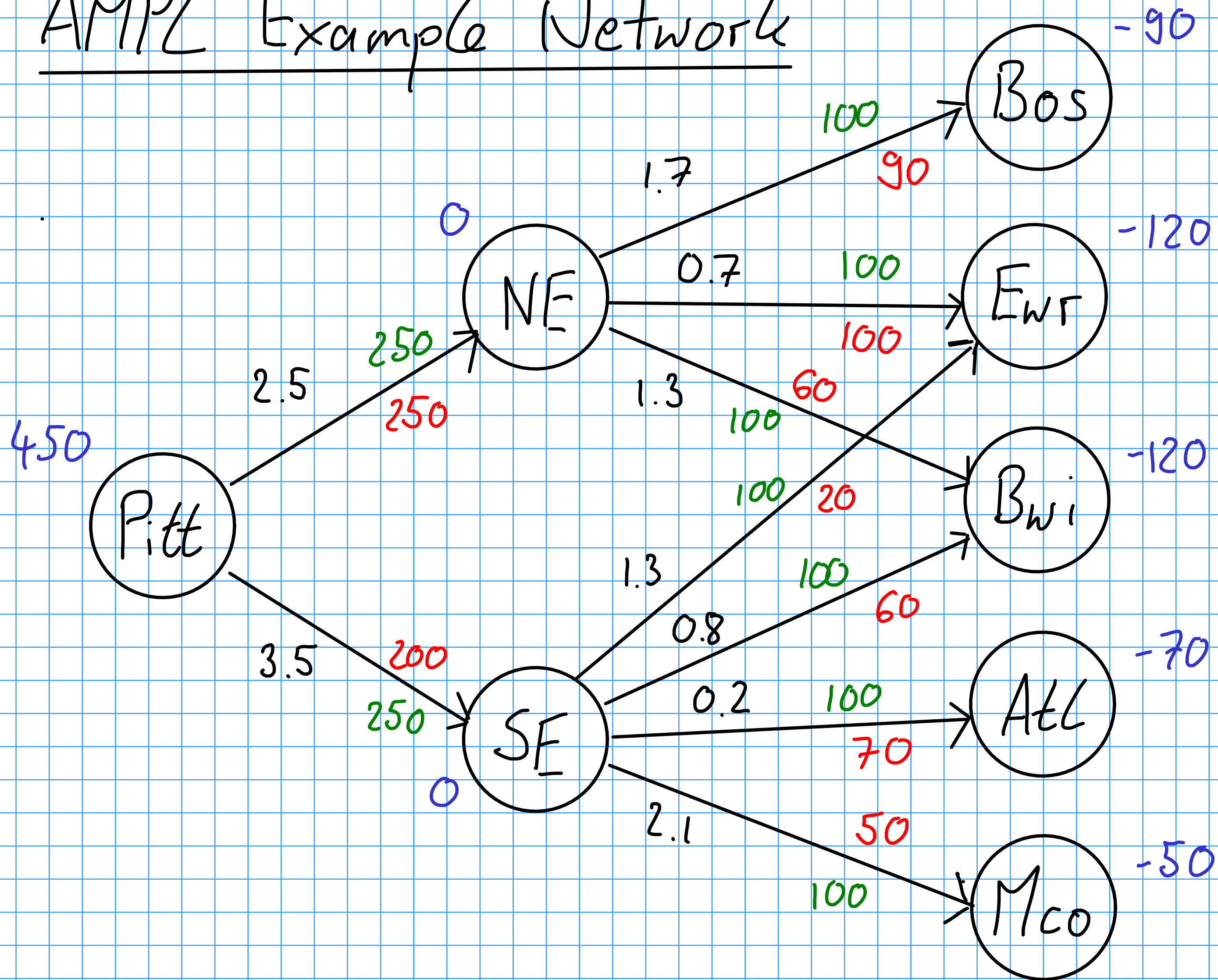
$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} x_{ji} = b_i \quad \forall\, i = 1, \dots, n$$

$$l_{ij} \leq x_{ij} \leq u_{ij} \quad \forall\, i,j = 1, \dots, n$$

The problem is called <u>min-cost (network) flow problem</u> with flow balance constraints.

For $b_i = 0$ everywhere, the problem is called a circulation problem.
For $(l_{ij}, u_{ij}) = (0, \infty)$, the problem is called uncapacitated.

AMPL Example Network



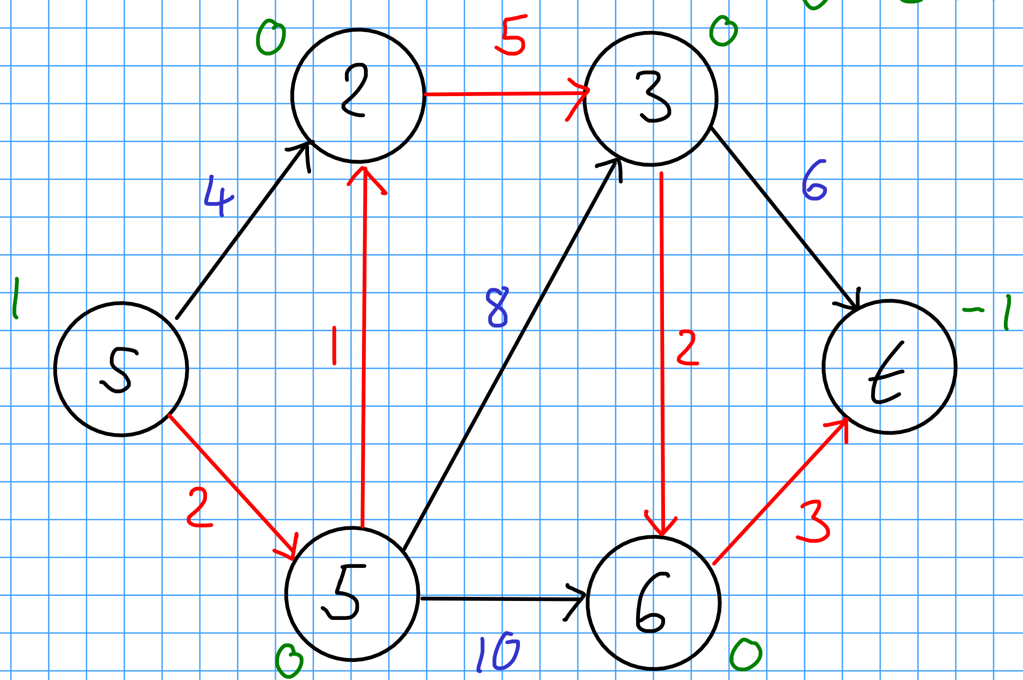
net supply b_i
cost
capacity (upper)
optimal solution

→ final project topic

Special cases of min-cost flow problems

- Transportation: transshipment in a bipartite graph
- Assignment: transportation with unit supplies/demands
- Circulation: transshipment with zero supplies/demands
- Shortest Path: Find the cheapest/shortest path from s to t

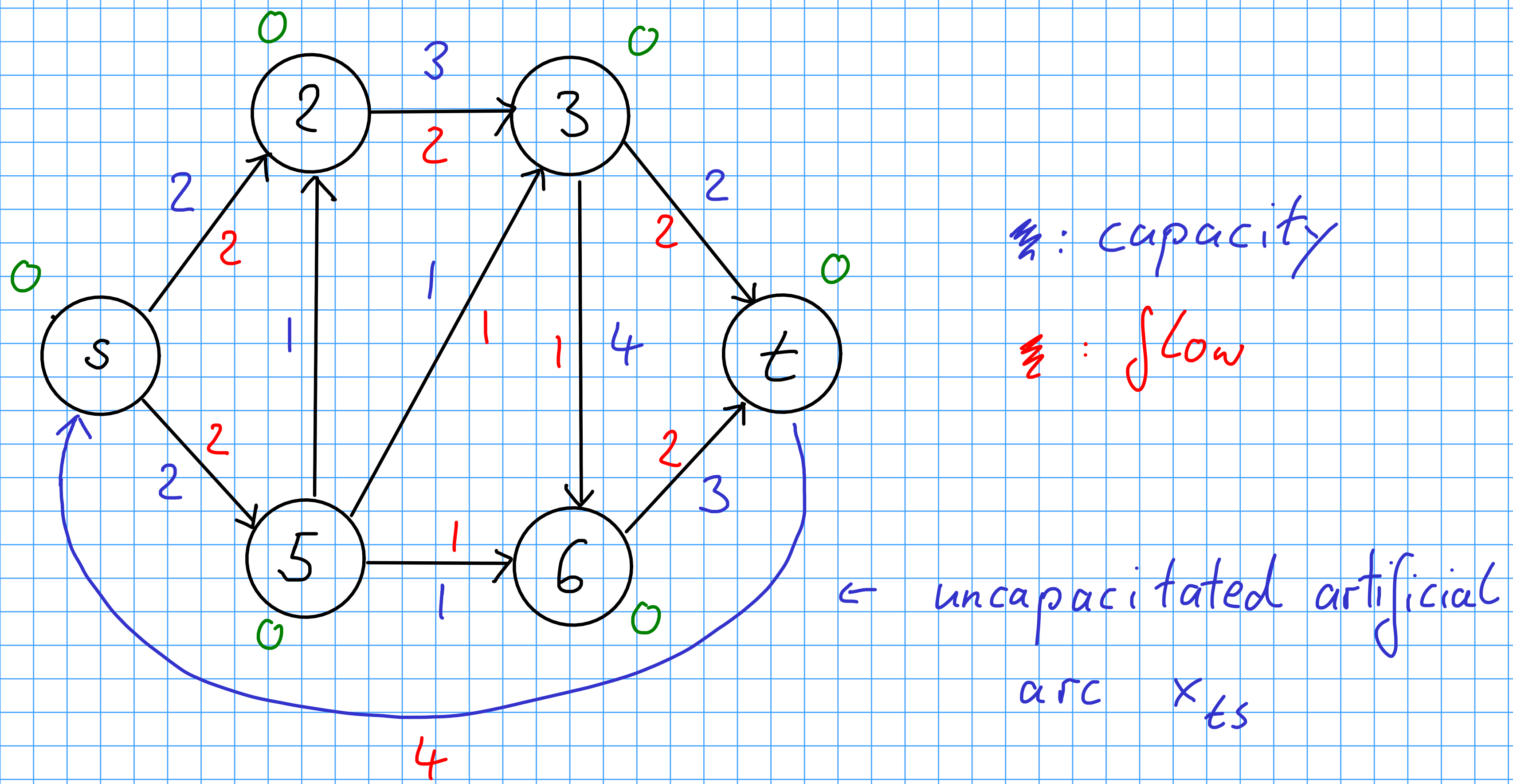
$$\min \sum_i \sum_j c_{ij} x_{ij} \quad \text{s.t.} \quad \sum_j x_{ij} - \sum_j x_{ij} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = t \\ 0 & \text{else} \end{cases} \quad \forall i \in N, \quad x_{ij} \geq 0 \ \forall i,j \in E$$



: cost/distance

• Maximum Flow: Find a largest shipment from s to t

$$\max\ x_{ts} \quad \text{s.t.} \quad \sum_j x_{ij} - \sum_j x_{ji} = 0 \ \forall i \in N, \quad 0 \le x_{ij} \le u_{ij} \ \forall i,j \in E$$



becomes special case of circulation problem

Integer Linear Programs

Integer Programs

Reading: AMPL Chapter 20

Many decision variables must be limited to integers (people, products) or binary values (yes/no, on/off, 0/1).

(Binary) Integer Program (IP)

$$\max \{ c^T x : Ax \geq b,\ x \text{ integer/binary} \}$$

Mixed Integer Programs (MIP)

$$\max \{ c^T x + d^T y : Ax + By \geq b,\ y \text{ integer/binary},\ x \text{ continuous} \}$$