Equivalency Between CIs and Hypotheses Tests

CIs can be used to test hypotheses.

A CI for a parameter θ with level of confidence $100(1-\alpha)\%$ can be used to test the hypotheses

$$H_0 : \theta = \theta_0$$
$$H_a : \theta \neq \theta_0$$

with significance level α by invoking the following decision rule:

Reject H_0 if the ci doesn't contain θ_0

Fail to reject H_0 if it does contain θ_0

The CI approach will always reach the same conclusion as the associated hypothesis test.

0.1 Example

For a test of

$$H_0 : \mu = 50$$
$$H_a : \mu \neq 50$$

The p-value for this is 0.16. Thus, H_0 would not be rejected at either the 5% or the 10% significance levels.

Question: Would a 95% CI for μ contain the value 50?

Answer Yes. Since we failed reject the null under an $\alpha = 0.05$, that means that 50 is contained in the confidence interval.

1 Identifying Causality: Experiments vs Observational Studies

Many studies are carried out to examine whether two variables (called explanatory and response variables) are related to each other.

Such studies can be either of two types

Observational study: The investigator merely observes whether the two variables vary together. The big thing here is that no attempt is made to induce changes in the response variable.

Experiment: Treatments are imposed on individuals. A deliberate attempt is made to induce change in the response variable.

An observational study on its own CANNOT establish cause and effect. This is because observational studies suffer from the possible presence of variables whose effects on the response are confounded the effect (if any) of the explanatory variable.

1.1 Exercise

An observational study shows that people who eat foods reach in antioxidants like fruits and vegetables have lower rates of colon cancer than those who dont eat those foods.

Question: Can we conclude that eating such food reduces the risk of colon cancer?

Answer: No because this is an observational study.

Confounding variables: Health consciousness (e.g. exercise, access to medical care, amount of drinking/smoking, income, etc.)

Example

In a clinical trial (**experiment** on human subjects) to investigate whether dietary antioxidants (vitamins A, C, and E) would lower colon cancer rates, **864** subjects were **randomized** to four treatment groups given different amounts of antioxidants:

Group 1: Daily beta carotene (vitamin A)
Group 2: Daily vitamins C and E
Group 3: All three vitamins daily
Group 4: No vitamin supplements.

After four years, researchers were surprised to find no significant difference in colon cancer among these groups.