Part 1

1.

What is the most rigorous mathematical definition for probability that you know or have learned?

Any of the definitions for probability (that I remember) are some variation of Kolmogorov's Axioms. Here's one from **Mathematical Statistics: An Introduction to Likelihood based Inference** by Richard J Rossi (2018).

Definition (Kolmogorov's Probability Function)**.** *Let Ω be the sample space associated with a chance experiment, and let β be a $\sigma-$algebra of events of Ω. A set function P on β satisfying the following three properties is called a probability function or a probability measure.*
1. $P(\Omega) = 1$
2. $P(A) \geq 0$ *for every event* $A \in \beta$
3. If $\{A_i : i \in N\} \subset \beta$ *is a collection of disjoint events, then:*

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

The triple (Ω, β, P) *is called a probability space.*

2.

How would you describe the purpose of probability to a friend that has a limited math and stats background.

Big picture, probability is all about learning to understand random events. Randomness defines nearly every aspect of our lives, and probability gives us the tools we need to wrangle some understanding and insight into the chaos that surrounds us. With probability, events that would otherwise be incomprehensible can now be broken down and analyzed. I think a lot of people attribute probability to just dice rolls and gambling, but it's so much deeper than that. Probability can help you model the behavior of real world random events like a wildfire or hurricane; potentially saving peoples lives. The world is random, and probability is crucial to learn from the world on its own terms.

Part 2

Exercise 1.4

For events A and B, find formulas for the probabilities of the following events in terms of the quantities $P(A)$, $P(B)$ and $P(A \cap B)$.

A

Either A or B or both

This is just the union of the two events.

$$P(A \cup B)$$

B

Either A or B but not both

We can use the same formula here but we need to be sure to remove the intersection of the two.

$$P(A \cup B) - P(A \cap B)$$

1.11

Show that the intersection of two sigma algebras is a sigma algebra.

For a set to be a sigma algebra, represented by β, it must meet the following requirements:
1. $\emptyset \in \beta$
2. $A \in \beta \implies A^c \in \beta$
3. $\{A_1, A_2, \ldots\} \in \beta \implies \cup_{i=1}^{\infty} A_i \in \beta$

Let X, Y both be sigma algebras of S.
1.
Because X is a sigma algebra, it follows that $\emptyset \in X$.
Because Y is a sigma algebra, it follows that $\emptyset \in Y$.
Therefore, $\emptyset \in X \cap Y$
2.
Let $A \in X \cap Y$
$A \in X$, so $A^c \in X$
$A \in Y$, so $A^c \in Y$
Therefore, $A^c \in X \cap Y$
3.
Let $\{A_1, A_2, \cdots\} \in X \cap Y$
$\{A_1, A_2, \cdots\} \in X$, so $\cup_{i=1}^{\infty} A_i \in X$
$\{A_1, A_2, \cdots\} \in Y$, so $\cup_{i=1}^{\infty} A_i \in Y$
Therefore, $\{A_1, A_2, \cdots\} \in X \cap Y$
Conclusion:
$X \cap Y$ is a sigma algebra as it fulfills all of the necessary requirements.