Set Theory

Definition (Sample Space)**.** *The set S, of all possible outcomes of a particular experiment is called the* ***sample space*** *for the experiment.*

Notation: Sample Space $= S = \Omega$
Example: Single coin flip. $S = \{H, T\}$

Definition (Event)**.** *An* ***event*** *is any collection of possible outcomes of an experiment. That is, any possible subset of S (including S itself).*

Example: An experiment where a coin is flipped two times consequtively.
$S = \{HH, HT, TH, TT\}$
Let $A =$ First flip is heads $= \{HH, HT\}$

Elementary Set Operations

Definition (Union)**.** *The* ***union*** *of A and B are the set of elements that belong to either A OR B.*

Notation: $A \cup B$
$A \cup B = \{x : x \in Aor x \in B\}$

Definition (Intersection)**.** *The* ***intersection*** *of A and B are the set of elements that belong to both A AND B.*

Notation: $A \cap B$
$A \cap B = \{x : x \in Aand x \in B\}$

Definition (Complement)**.** *The* ***complement*** *of A is the set of all elements not in A.*

Notation: A^c
$A^c = \{x : x \notin A\}$

Theorem. *1.1.4: Useful Set Properties*
For any events A, B, C, defined on a sample space S.
Commutativity:

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associativity:

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Distributive Laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

DeMorgan's Laws:

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Definition (Disjoint, Pairwise Disjoint)**.** *Two events A and B are disjoint or mutually exclusive if $A \cap B = \emptyset$. The events $A_1, A_2, \cdots$ are pairwise disjoint if $A_i \cap A_j = \emptyset \; \forall \; i \neq j$*

Definition (Partition)**.** *If $A_1, A_2, \cdots$ are pairwise disjoint and $\cup_{i=1}^{\infty} A_i = S$ then the collection $A_1, A_2, \cdots$ forms a partition of S.*

Probability Theory

Definition (Sigma Algebra)**.** *A collection of subsets of S is called a textbfsigma algebra (or Borel field), denoted by β, if it satisfies the following three properties:*

1. *$\theta \in \beta$*
2. *If $A \in$ beta then $A^c \in \beta$ (β is closed under complementation).*
3. *If $A_1, A_2, \cdots \in \beta$ then $\cup_{i=1}^{\infty} A_i \in \beta$ (β is closed under countable unions).*