

MATH 5310: Probability

Fall 2024

Homework 3

Due before 11:59pm on Tuesday 10/8

**Part1:**

- (1) A main learning outcome for Chapter 2 being able to find the probability distributions of transformations of random variables. For example, assume we know  $F_X(x)$  or  $f_X(x)$  for a random variable  $X$ , and we want to find the CDF and/or PDF/PMF for a random variable  $Y$  that is a transformation of  $X$  through the map  $Y = g(X)$ .
  - (a) Briefly describe the main techniques (methods and approaches) we have learned so far for finding  $F_Y(y)$  and  $f_Y(y)$ .
  - (b) Briefly describe your strategy for deciding which technique(s) you would attempt to use to find  $F_Y(y)$  and  $f_Y(y)$ . Think of questions such as: What properties of  $X$  or  $Y$  lead you to choose one technique over another? What features of  $g(X)$  help you figure out where to start? Does my approach change depending on whether you are trying to find the CDF or the PDF/PMF? Your answer only needs to be a few sentences, but I want you to be forming a general strategy for finding probability distributions for transformations of random variables.
- (2) Another common transformation method is the *Probability Integral Transformation*. For context, I introduced this method indirectly as part of our in-class discussion of how to simulate random variables by sampling from a uniform distribution and then inverting the CDF. In this problem, you will directly work with this method.
  - (a) Read Theorem 2.1.10 and briefly state, in your own words, the main point of the theorem.
  - (b) Use this theorem to solve problem 2.9 in Casella and Berger

**Part 2:** Additional Casella and Berger Problems:

2.2 (b) and (c) only

2.3

2.6. (a) and (c) only

2.18

2.24. (a) and (b) only