Part 1

Part 2

1.41

As in Example 1.3.6, consider telegraph signals dot and dash sent in the proportion 3:4, where erratic transmissions cause a dot to become a dash with a probability 1/4, and a dash to become a dot with probability 1/3.

A

If a dash is received, what is the probability that a dash has been sent?
Solution:
To start, let's examine our goal. We're looking for:

$$P(\text{dash sent}|\text{dash received})$$

We need to consider a few things, because there are actually a couple ways for us to receieve a dash because the signals can change. We're going to need to compile some building blocks for this problem.

Notation Note: For the sake of simplicity, I will be referring to dots as o and dashes as a. Sent and receieved will be referred to with s and r respectively. For example, o_s indicates a dot that was sent.

Let's first examine the proportion 3:4 for sent signals. Interpreting this means that for every 3 dots, we get 4 dashes. As a simple example, if we send 7 signals, 3/7 will be dots and 4/7 will be dashes. Those are actually the probabilities for the sent signals even!

$$\begin{aligned}
P(o_s) &= \frac{3}{3+4} = \frac{3}{7} \\
P(a_s) &= \frac{4}{3+4} = \frac{4}{7} \\
P(a \to o) &= \frac{1}{3} \\
P(o \to a) &= \frac{1}{4}
\end{aligned}$$

We still need one more tool to tackle this problem, we need $P(a_r)$. This one requires a bit more effort to sort out. We need the probability that a dash was sent AND NOT changed. We also need the probability a dot was sent AND changed. We then add those together to get our probability.

$$
\begin{aligned}
P(a_r) &= P(a_s \text{ and not } a \to o) + P(o_s \text{ and } o \to a) \\
&= \left(\frac{4}{7} \cdot \frac{2}{3}\right) + \left(\frac{3}{7} \cdot \frac{1}{4}\right) \\
&\approx 0.488
\end{aligned}
$$

For a sanity check, I will also do the same computation for a dot received. These two should add up to 1 as they are the only possible outcomes.

$$
\begin{aligned}
P(o_r) &= P(o_s \text{ and not } o \to a) + P(a_s \text{ and } a \to o) \\
&= \left(\frac{3}{7} \cdot \frac{3}{4}\right) + \left(\frac{4}{7} \cdot \frac{1}{3}\right) \\
&\approx 0.511
\end{aligned}
$$

$0.488 + 0.511 \approx 1$ so we're good! Now we just plug our tools into Baye's Rule.

$$
\begin{aligned}
P(a_s|a_r) &= \frac{P(a_s \cap a_r)}{a_r} \\
&= \frac{(4/7) \cdot 0.488}{0.488} \\
&\approx 0.57
\end{aligned}
$$

B

Assuming independence between signals, if the message dot-dot was receieved, what is the probability distribution of the four possible messages that could have been sent?

Solution

There are 4 ways to get dot-dot.

1. Getting two natural dots.
2. Getting one natural dot and one changed dash
3. Getting one changed dash and one natural dot
4. Getting two changed dashes.

To tackle this problem we need a random variable to make use of these events.

Let's let $X = \#$ of natural o's.

We can handle this using the classic table for breaking down all the steps.

x	$P(X = x)$	$P(X = x)$	$P(X = x \mid \text{two dots received})$
0	$((4/7) \cdot (1/3))^2$	0.036	$0.036/0.261 = .138$
1	$2 \cdot (((3/7) \cdot (3/4)) \cdot ((4/7) \cdot (1/3)))$	0.122	$0.122/0.261 = 0.467$
2	$((3/7) \cdot (3/4))^2$	0.103	$0.103/0.261 = 0.395$
		0.261	1

$$f_X(x) = \begin{cases} 0.138 & x = 0 \\ 0.467 & x = 1 \\ 0.395 & x = 2 \end{cases}$$

$$F_X(x) = \begin{cases} 0.138 & x = 0 \\ 0.605 & x = 1 \\ 0.1 & x = 2 \end{cases}$$