Mathematical Imaging

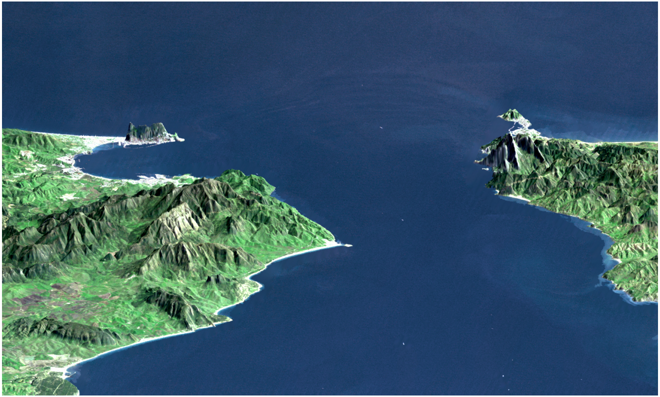
Francisco Blanco-Silva

October 21, 2013

An Example

Image generated from a Landsat satellite image draped over an elevation model produced by the Shuttle Radar Topography Mission (SRTM).

Natural colors of the scene are enhanced by image processing. The scene includes some infrared reflectance (as green) to highlight the vegetation pattern as well as shading of the elevation model to further highlight the topographic features.

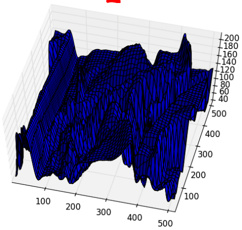


Mathematical Imaging

Image → Mathematical Objects

Image

$$\begin{pmatrix} & \vdots & \vdots & \vdots & \\ \cdots & 230 & 200 & 198 & \cdots \\ \cdots & 200 & 100 & 98 & \cdots \\ \cdots & 200 & 110 & 68 & \cdots \\ \cdots & 130 & 190 & 48 & \cdots \\ & \vdots & \vdots & \vdots & \end{pmatrix}$$



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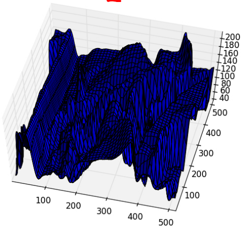

Outline

Optical Imaging

- Chemical Imaging
- Computational Imaging (Digital Photography/Video)
- Dark-field Imaging (Scattered-light registration)

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L. J. M. Daguerre.
First photography, taken in 1839

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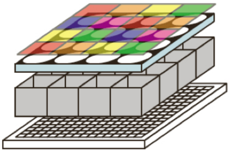

Fig. 3. TOMBO system with different interference filter on each unit.

Optical Imaging

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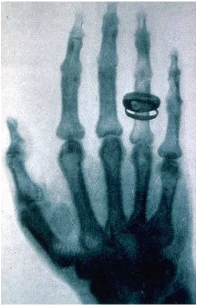
Imaging from Wave Propagation

- Transmission Imaging
- Radar Imaging/Interferometry
- Volume Imaging



The hand of Mrs. Wilhelm Röntgen: The first X-ray image, 1895.

In Otto Glasser, "*Wilhelm Conrad Röntgen and the early history of the Röntgen rays.*" London, 1933. National Library of Medicine.

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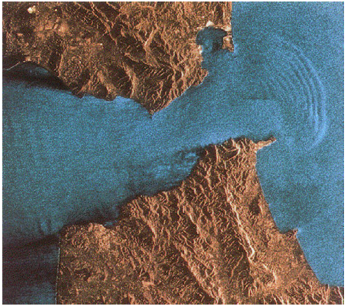

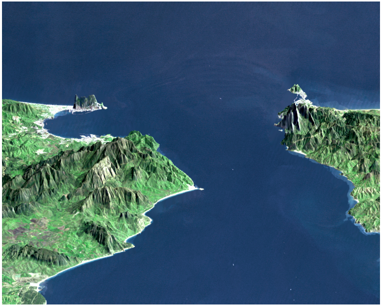

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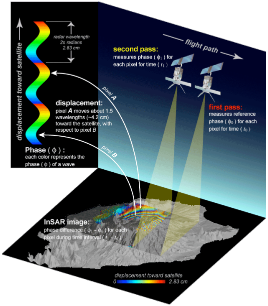

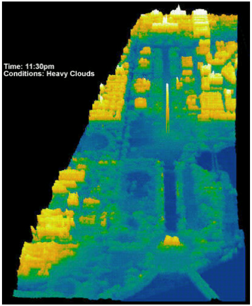

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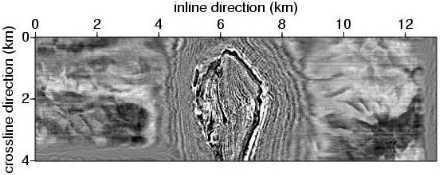


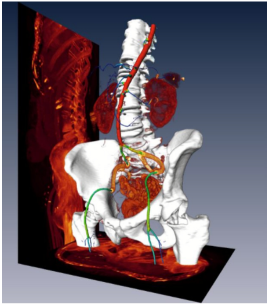


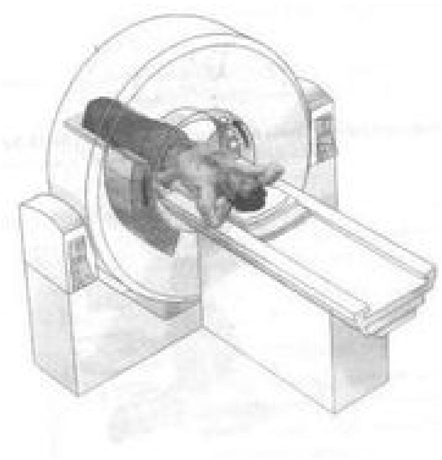
Image from Wave Propagation

Volume Imaging

- Atom probe tomography
- Confocal laser scannig microscopy
- Cryo-electron tomography
- Electrical capacitance tomography
- Electrical impedance tomography
- Functional magnetic resonance imaging
- Magnetic induction tomography
- Magnetic resonance imaging
- Optical projection tomography
- Photoacoustic tomography
- Positron emission tomography
- Quantum tomography
- Single photon emission computed tomography
- Seismic tomography
- Ultrasound transmission tomography
- X-ray tomography
- Zeeman-Doppler

Acquisition

Example: X-ray Tomography



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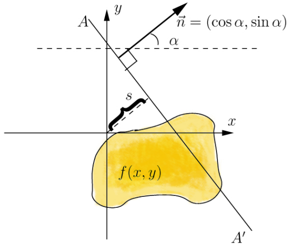

Basic Idea

X-ray with intensity A on a line $\big(x(t), y(t)\big)$ with equation $t(\sin\alpha, -\cos\alpha) + s(\cos\alpha, \sin\alpha)$ shot through a flat object Ω with density $f(x,y)$.

Recorded intensity A' satisfies:

$$\log(A/A') = \underbrace{\int_{-\infty}^{\infty} f\big(x(t), y(t)\big)\, dt}_{\mathcal{R}[f]}\,.$$

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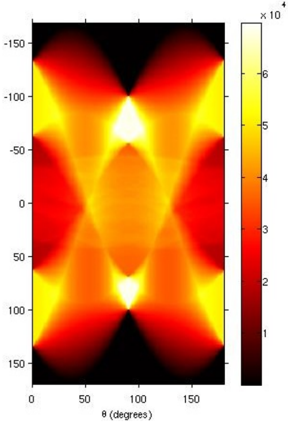

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Example: X-ray Tomography

Filtered back-projection

$$f = \frac{1}{4\pi}\mathcal{R}^* H\big[\mathcal{R}f\big],$$

where H is a ramp-filter function, and $\mathcal{R}^*[f]$ is the formal adjoint of the Radon transform of a function f on the plane: $\mathcal{R}^*[g](\boldsymbol{x}) = \int_{-\pi}^{\pi} g\big(\alpha, \boldsymbol{n}(\alpha)\cdot\boldsymbol{x}\big)\,d\alpha$.

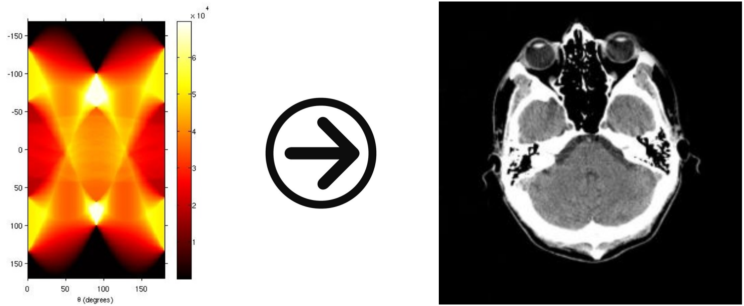

Image Compression

$1,024 \times 1,024 = 1,048,576$ pixels

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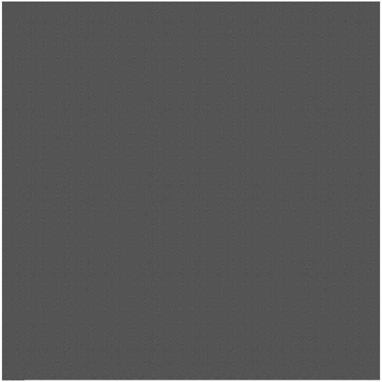

1 wavelet coefficient

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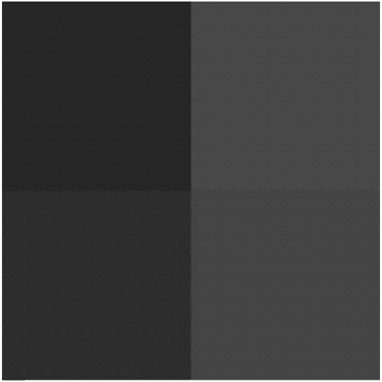

$1 + 4 = 5$ wavelet coefficients

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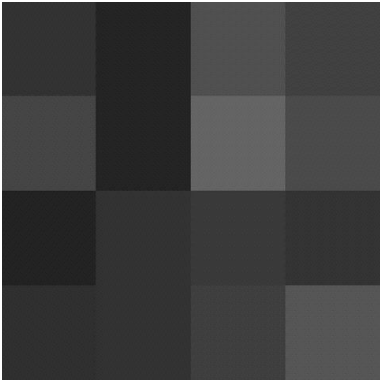

$1 + 4 + 16 = 21$ wavelet coefficients

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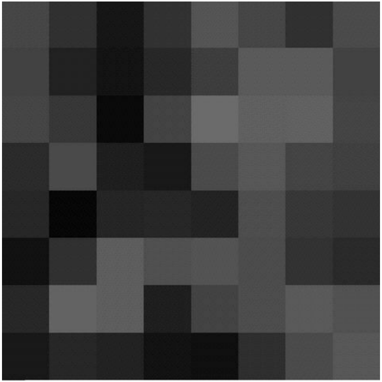

$1 + 4 + 16 + 64 = 85$ wavelet coefficients

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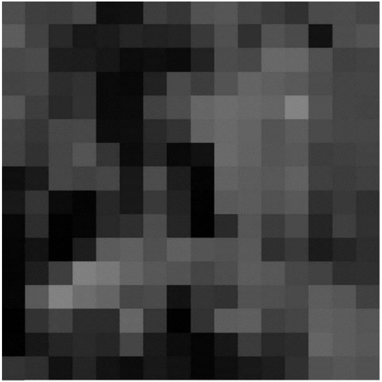

$1 + 4 + 16 + 64 + 256 = 341$ wavelet coefficients

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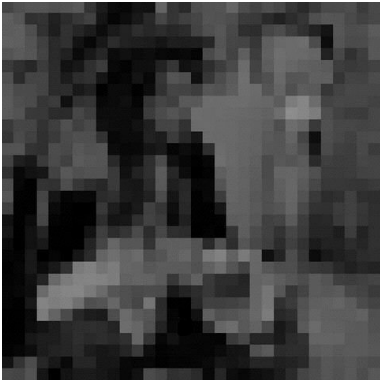

$1 + 4 + 16 + 64 + 256 + 1024 = 1365$ wavelet coefficients

Image Compression

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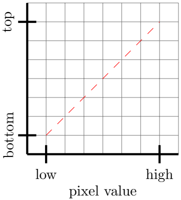
$1 + 4 + 16 + 64 + 256 + 1024 + 4096 = 5461$ wavelet coefficients

Edition, Enhancing and Restoration

- Intensity Adjustment
- Denoising
- Inpainting
- Deblurring



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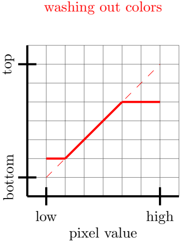

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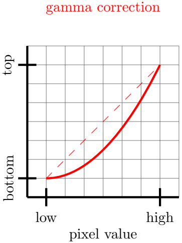

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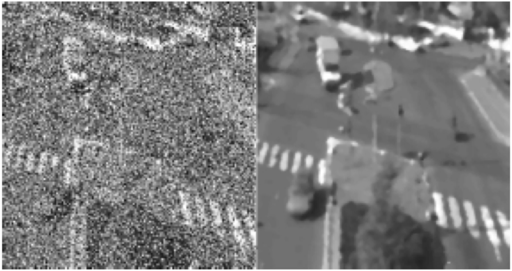

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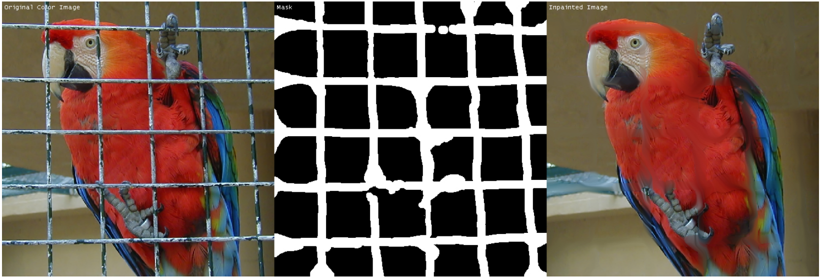

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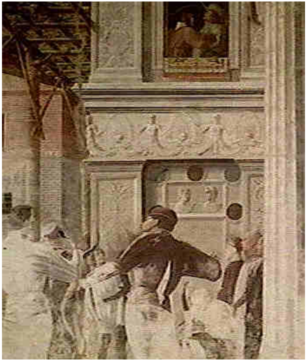

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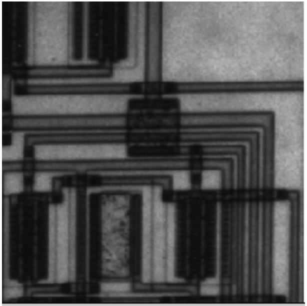
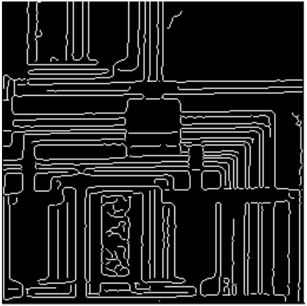
Image Analysis

Low-level Image Analysis

Low level analysis aims at extracting reliable, local or global geometric information from a mathematical image:

- Edge/Corner detection. Level lines, curvature
- Smoothness
- Quadtree decomposition
- Perimeter Determination/Image Area
- Topology/Euler number





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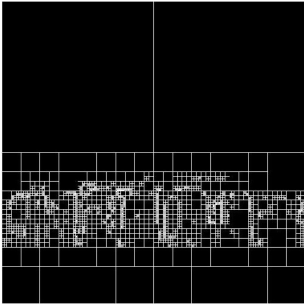

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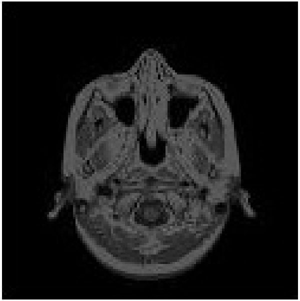

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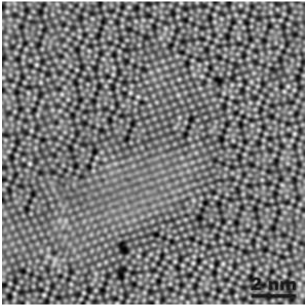


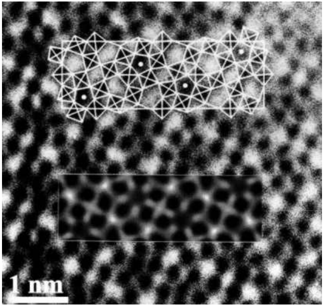
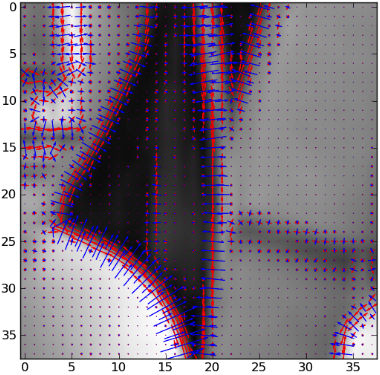


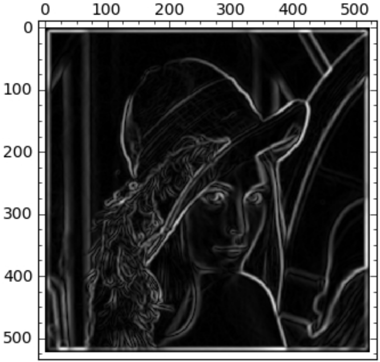
Image Analysis

Example: Edge detection ala Hildreth-Marr

Definition (Hildreth-Marr's edge points)

Smooth function $f\colon [0,1]^2 \to \mathbb{R}$. Edges are points (x,y) with $|\nabla f(x,y)|$ maximal





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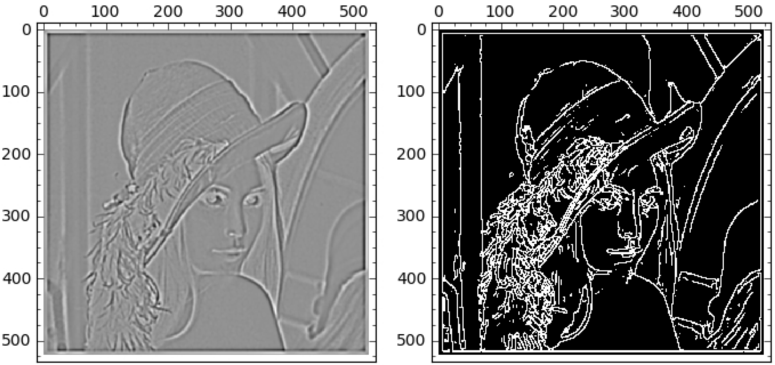

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Visual Learning and Recognition

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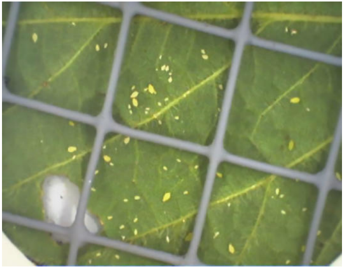
- Pattern Recognition
- Motion Recognition



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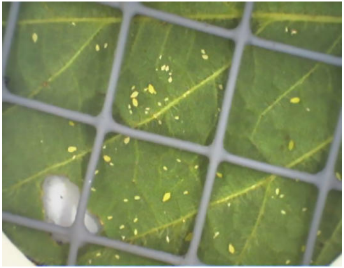

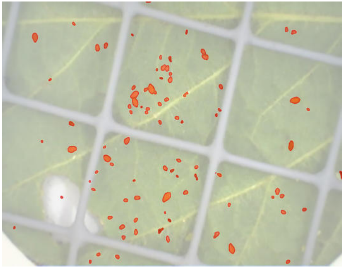

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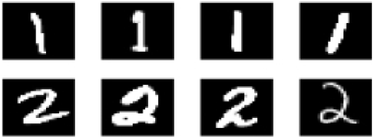

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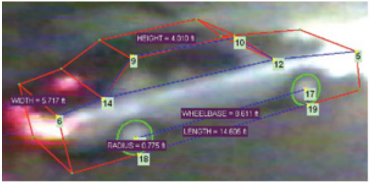

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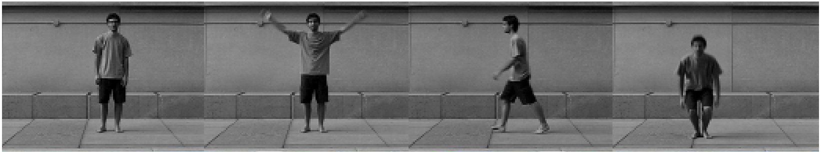

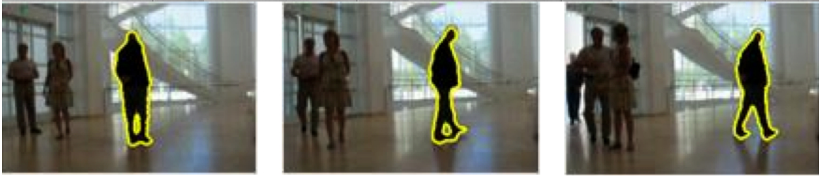

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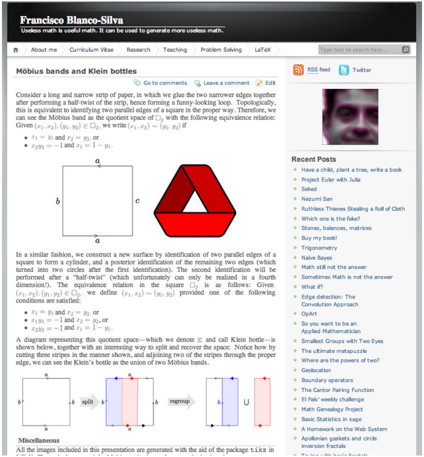
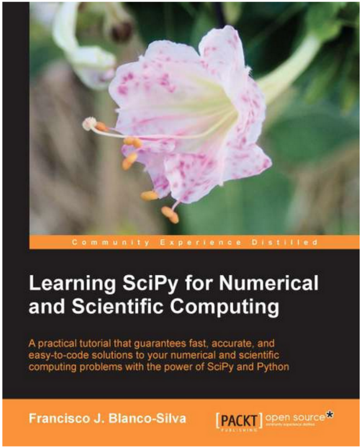
For more information, examples, ideas, . . .





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