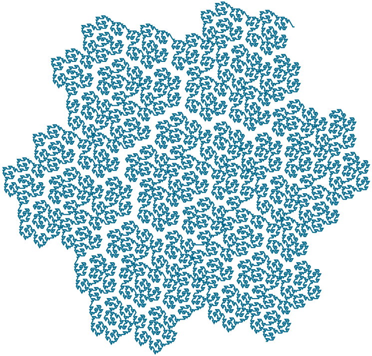
Lesson 11: Rules of Differentiation—Product and Quotient Rules

Francisco Blanco-Silva

University of South Carolina



What do we know?
The General Program

- Functions
  - $x-$ and $y-$intercepts $(f(x) = 0, f(0))$
  - Change from $x = a$ to $x = b$
    $$\Delta y = f(b) - f(a)$$
  - Average Rate of Change from $x = a$ to $x = b$
    $$ARC = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$
  - Relative Change from $x = a$ to $x = b$
    $$RC = \frac{\Delta y}{f(a)} = \frac{f(b) - f(a)}{f(a)}$$
  - Instantaneous Rate of Change at $x = a$
    $$f'(a)$$
- Linear Functions:
  $f(x) = b + mx$
- Exponential Functions
  $P_0a^t = P_0(1 + r)^t = P_0e^{kt}$
- Power Functions
  $$kx^p$$
- Polynomials
  $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$

What do we know?

Rules of Differentiation

D1 $f(x) = c, \quad f'(x) = 0$

D2 $f(x) = x, \quad f'(x) = 1$

D3 $h(x) = f(x) + g(x), \quad h'(x) = f'(x) + g'(x)$

D4 $h(x) = f(x) - g(x), \quad h'(x) = f'(x) - g'(x)$

D5 $h(x) = c \cdot f(x), \quad h'(x) = c \cdot f'(x)$

D6 $f(x) = x^n, \quad f'(x) = nx^{n-1}$

D7 $f(x) = e^x, \quad f'(x) = e^x$

D8 $f(x) = a^x, \quad f'(x) = a^x \ln a$

D9 $f(x) = \ln x, \quad f'(x) = \frac{1}{x}$

D10
- $f(x) = g(x)^n, \quad f'(x) = ng(x)^{n-1}g'(x)$
- $f(x) = e^{g(x)}, \quad f'(x) = g'(x)e^{g(x)}$
- $f(x) = a^{g(x)}, \quad f'(x) = g'(x) \ln a \, a^{g(x)}$
- $f(x) = \ln g(x), \quad f'(x) = \frac{g'(x)}{g(x)}$

The Product and Quotient Rules

Two more rules

D11 The Product Rule

If $h(x) = f(x) \cdot g(x)$, then $h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

D12 The Quotient Rule

If $h(x) = \dfrac{f(x)}{g(x)}$, then $h'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$.

The Product and Quotient Rules

Two more rules

D11 The Product Rule
If $h(x) = f(x) \cdot g(x)$, then $h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

D12 The Quotient Rule
If $h(x) = \dfrac{f(x)}{g(x)}$, then $h'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$.

You may have seen these two rules with the u, v notation:

D11 $(u \cdot v)' = u'v + uv'$.

D12 $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$.

The Product and Quotient Rules

Two more rules

D11 The Product Rule

If $h(x) = f(x) \cdot g(x)$, then $h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

D12 The Quotient Rule

If $h(x) = \dfrac{f(x)}{g(x)}$, then $h'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$.

You may have seen these two rules with the u, v notation:

D11 $(u \cdot v)' = u'v + uv'$.

D12 $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$.

No matter what technique you use, make sure you get the order of the parts and the sign right.

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = (x^2 - 4)(4x^6 - 7x + 2)$

- $f(x) = \left(x + \sqrt[3]{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(x^2 - 4)}_{u}\underbrace{(4x^6 - 7x + 2)}_{v}$

- $f(x) = \left(x + \sqrt[3]{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(x^2-4)}_{u}\underbrace{(4x^6-7x+2)}_{v}$

$$u = x^2 - 4$$
$$v = 4x^6 - 7x + 2$$

- $f(x) = \left(x + \sqrt[3]{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(x^2 - 4)}_{u}\underbrace{(4x^6 - 7x + 2)}_{v}$

$$u = x^2 - 4 \qquad u' = 2x$$
$$v = 4x^6 - 7x + 2 \qquad v' = 24x^5 - 7$$

- $f(x) = \left(x + \sqrt[3]{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(x^2-4)}_{u}\underbrace{(4x^6-7x+2)}_{v}$

$$u = x^2 - 4 \qquad u' = 2x$$

$$v = 4x^6 - 7x + 2 \qquad v' = 24x^5 - 7$$

$$f'(x) = u'v + uv'$$

- $f(x) = \left(x + \sqrt[3]{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(x^2 - 4)}_{u}\underbrace{(4x^6 - 7x + 2)}_{v}$

$$u = x^2 - 4 \qquad u' = 2x$$
$$v = 4x^6 - 7x + 2 \qquad v' = 24x^5 - 7$$

$$f'(x) = u'v + uv' = 2x(4x^6 - 7x + 2) + (x^2 - 4)(24x^5 - 7)$$

- $f(x) = \left(x + \sqrt[3]{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(x^2 - 4)}_{u}\underbrace{(4x^6 - 7x + 2)}_{v}$

$$u = x^2 - 4 \qquad u' = 2x$$
$$v = 4x^6 - 7x + 2 \qquad v' = 24x^5 - 7$$

$$f'(x) = u'v + uv' = 2x(4x^6 - 7x + 2) + (x^2 - 4)(24x^5 - 7)$$

- $f(x) = \underbrace{(x + x^{1/3})}_{u}\underbrace{(5x^2 + e^x)}_{v}$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(x^2 - 4)}_{u}\underbrace{(4x^6 - 7x + 2)}_{v}$

$$u = x^2 - 4 \qquad u' = 2x$$
$$v = 4x^6 - 7x + 2 \qquad v' = 24x^5 - 7$$

$$f'(x) = u'v + uv' = 2x(4x^6 - 7x + 2) + (x^2 - 4)(24x^5 - 7)$$

- $f(x) = \underbrace{(x + x^{1/3})}_{u}\underbrace{(5x^2 + e^x)}_{v}$

$$u = x + x^{1/3}$$
$$v = 5x^2 + e^x$$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(x^2 - 4)}_{u}\underbrace{(4x^6 - 7x + 2)}_{v}$

$$u = x^2 - 4 \qquad u' = 2x$$
$$v = 4x^6 - 7x + 2 \qquad v' = 24x^5 - 7$$

$$f'(x) = u'v + uv' = 2x(4x^6 - 7x + 2) + (x^2 - 4)(24x^5 - 7)$$

- $f(x) = \underbrace{(x + x^{1/3})}_{u}\underbrace{(5x^2 + e^x)}_{v}$

$$u = x + x^{1/3} \qquad u' = 1 + \tfrac{1}{3}x^{-2/3}$$
$$v = 5x^2 + e^x \qquad v' = 10x + e^x$$

$$f'(x) = u'v + uv'$$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(x^2 - 4)}_{u}\underbrace{(4x^6 - 7x + 2)}_{v}$

$$u = x^2 - 4 \qquad u' = 2x$$
$$v = 4x^6 - 7x + 2 \qquad v' = 24x^5 - 7$$

$$f'(x) = u'v + uv' = 2x(4x^6 - 7x + 2) + (x^2 - 4)(24x^5 - 7)$$

- $f(x) = \underbrace{(x + x^{1/3})}_{u}\underbrace{(5x^2 + e^x)}_{v}$

$$u = x + x^{1/3} \qquad u' = 1 + \tfrac{1}{3}x^{-2/3}$$
$$v = 5x^2 + e^x \qquad v' = 10x + e^x$$

$$f'(x) = u'v + uv' = (1 + \tfrac{1}{3}x^{-2/3})(5x^2 + e^x) + (x + x^{1/3})(10x + e^x)$$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = (3x^3 - \ln x)\left(x^5 - \sqrt{x}\right)$
- $f(x) = \left(x^3 + \sqrt{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(3x^3 - \ln x)}_{u} \underbrace{\left(x^5 - x^{1/2}\right)}_{v}$
- $f(x) = \left(x^3 + \sqrt{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(3x^3 - \ln x)}_{u} \underbrace{\left(x^5 - x^{1/2}\right)}_{v}$

$$u = 3x^3 - \ln x$$
$$v = x^5 - x^{1/2}$$

- $f(x) = \left(x^3 + \sqrt{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(3x^3 - \ln x)}_{u}\underbrace{(x^5 - x^{1/2})}_{v}$

$$u = 3x^3 - \ln x \qquad u' = 9x^2 - \frac{1}{x}$$

$$v = x^5 - x^{1/2} \qquad v' = 5x^4 - \frac{1}{2}x^{-1/2}$$

- $f(x) = (x^3 + \sqrt{x})(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(3x^3 - \ln x)}_{u}\underbrace{\left(x^5 - x^{1/2}\right)}_{v}$

$$u = 3x^3 - \ln x \qquad u' = 9x^2 - \frac{1}{x}$$

$$v = x^5 - x^{1/2} \qquad v' = 5x^4 - \frac{1}{2}x^{-1/2}$$

$$f'(x) = u'v + uv'$$

- $f(x) = \left(x^3 + \sqrt{x}\right)(5x^2 + e^x)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(3x^3 - \ln x)}_{u}\underbrace{(x^5 - x^{1/2})}_{v}$

$$u = 3x^3 - \ln x \qquad u' = 9x^2 - \frac{1}{x}$$

$$v = x^5 - x^{1/2} \qquad v' = 5x^4 - \frac{1}{2}x^{-1/2}$$

$$f'(x) = u'v + uv' = \left(9x^2 - \tfrac{1}{x}\right)\left(x^5 - x^{1/2}\right) + \left(3x^3 - \ln x\right)\left(5x^4 - \tfrac{1}{2}x^{-1/2}\right)$$

- $f(x) = \left(x^3 + \sqrt{x}\right)\left(5x^2 + e^x\right)$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(3x^3 - \ln x)}_{u}\underbrace{(x^5 - x^{1/2})}_{v}$

$$u = 3x^3 - \ln x \qquad u' = 9x^2 - \frac{1}{x}$$
$$v = x^5 - x^{1/2} \qquad v' = 5x^4 - \frac{1}{2}x^{-1/2}$$

$$f'(x) = u'v + uv' = \left(9x^2 - \tfrac{1}{x}\right)\left(x^5 - x^{1/2}\right) + \left(3x^3 - \ln x\right)\left(5x^4 - \tfrac{1}{2}x^{-1/2}\right)$$

- $f(x) = \underbrace{(x^3 + x^{1/2})}_{u}\underbrace{(5x^2 + e^x)}_{v}$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(3x^3 - \ln x)}_{u}\underbrace{(x^5 - x^{1/2})}_{v}$

$$u = 3x^3 - \ln x \qquad u' = 9x^2 - \frac{1}{x}$$
$$v = x^5 - x^{1/2} \qquad v' = 5x^4 - \frac{1}{2}x^{-1/2}$$

$$f'(x) = u'v + uv' = \left(9x^2 - \tfrac{1}{x}\right)\left(x^5 - x^{1/2}\right) + \left(3x^3 - \ln x\right)\left(5x^4 - \tfrac{1}{2}x^{-1/2}\right)$$

- $f(x) = \underbrace{(x^3 + x^{1/2})}_{u}\underbrace{(5x^2 + e^x)}_{v}$

$$u = x^3 + x^{1/2}$$
$$v = 5x^2 + e^x$$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(3x^3 - \ln x)}_{u}\underbrace{(x^5 - x^{1/2})}_{v}$

$$u = 3x^3 - \ln x \qquad u' = 9x^2 - \frac{1}{x}$$
$$v = x^5 - x^{1/2} \qquad v' = 5x^4 - \frac{1}{2}x^{-1/2}$$

$$f'(x) = u'v + uv' = \left(9x^2 - \frac{1}{x}\right)\left(x^5 - x^{1/2}\right) + \left(3x^3 - \ln x\right)\left(5x^4 - \frac{1}{2}x^{-1/2}\right)$$

- $f(x) = \underbrace{(x^3 + x^{1/2})}_{u}\underbrace{(5x^2 + e^x)}_{v}$

$$u = x^3 + x^{1/2} \qquad u' = 3x^2 + \frac{1}{2}x^{-1/2}$$
$$v = 5x^2 + e^x \qquad v' = 10x + e^x$$

$$f'(x) = u'v + uv'$$

The Product and Quotient Rule

Examples

Find the derivative of the following functions

- $f(x) = \underbrace{(3x^3 - \ln x)}_{u}\underbrace{(x^5 - x^{1/2})}_{v}$

$$u = 3x^3 - \ln x \qquad u' = 9x^2 - \frac{1}{x}$$
$$v = x^5 - x^{1/2} \qquad v' = 5x^4 - \frac{1}{2}x^{-1/2}$$

$$f'(x) = u'v + uv' = \left(9x^2 - \frac{1}{x}\right)\left(x^5 - x^{1/2}\right) + \left(3x^3 - \ln x\right)\left(5x^4 - \frac{1}{2}x^{-1/2}\right)$$

- $f(x) = \underbrace{(x^3 + x^{1/2})}_{u}\underbrace{(5x^2 + e^x)}_{v}$

$$u = x^3 + x^{1/2} \qquad u' = 3x^2 + \frac{1}{2}x^{-1/2}$$
$$v = 5x^2 + e^x \qquad v' = 10x + e^x$$

$$f'(x) = u'v + uv' = (3x^2 + \frac{1}{2}x^{-1/2})(5x^2 + e^x) + (x^3 + x^{1/2})(10x + e^x)$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{3x^2 + \ln x}{1 - 3x^{4/3} + 2^x}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{3x^2 + \ln x}{1 - 3x^{4/3} + 2^x} \qquad \begin{array}{l} \longleftarrow u \\ \longleftarrow v \end{array}$$

$$u = 3x^2 + \ln x$$

$$v = 1 - 3x^{4/3} + 2^x$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{3x^2 + \ln x}{1 - 3x^{4/3} + 2^x} \qquad \begin{matrix} \longleftarrow u \\ \longleftarrow v \end{matrix}$$

$$u = 3x^2 + \ln x \qquad u' = 6x + \tfrac{1}{x}$$

$$v = 1 - 3x^{4/3} + 2^x \qquad v' = -4x^{1/3} + 2^x \ln 2$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{3x^2 + \ln x}{1 - 3x^{4/3} + 2^x} \qquad \begin{matrix} \longleftarrow u \\ \longleftarrow v \end{matrix}$$

$$u = 3x^2 + \ln x \qquad u' = 6x + \frac{1}{x}$$

$$v = 1 - 3x^{4/3} + 2^x \qquad v' = -4x^{1/3} + 2^x \ln 2$$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{3x^2 + \ln x}{1 - 3x^{4/3} + 2^x} \quad \begin{array}{l} \longleftarrow u \\ \longleftarrow v \end{array}$$

$$u = 3x^2 + \ln x \qquad u' = 6x + \tfrac{1}{x}$$

$$v = 1 - 3x^{4/3} + 2^x \qquad v' = -4x^{1/3} + 2^x \ln 2$$

$$\begin{aligned} f'(x) &= \frac{u'v - uv'}{v^2} \\ &= \frac{\left(6x + \frac{1}{x}\right)\left(1 - 3x^{4/3} + 2^x\right) - \left(3x^2 + \ln x\right)\left(-4x^{1/3} + 2^x \ln 2\right)}{\left(1 - 3x^{4/3} + 2^x\right)^2} \end{aligned}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\pi + 2\pi x - \ln \pi + \pi^x}{e - e^x + 3x^2}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\pi + 2\pi x - \ln\pi + \pi^x}{e - e^x + 3x^2} \qquad \begin{array}{l} \longleftarrow u \\ \longleftarrow v \end{array}$$

$$u = \pi + 2\pi x - \ln\pi + \pi^x$$
$$v = e - e^x + 3x^2$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\pi + 2\pi x - \ln\pi + \pi^x}{e - e^x + 3x^2} \quad \begin{matrix} \longleftarrow u \\ \longleftarrow v \end{matrix}$$

$$u = \pi + 2\pi x - \ln\pi + \pi^x \qquad u' = 2\pi + \pi^x \ln\pi$$

$$v = e - e^x + 3x^2 \qquad v' = -e^x + 6x$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\pi + 2\pi x - \ln \pi + \pi^x}{e - e^x + 3x^2} \quad \begin{array}{l} \longleftarrow u \\ \longleftarrow v \end{array}$$

$$u = \pi + 2\pi x - \ln \pi + \pi^x \qquad u' = 2\pi + \pi^x \ln \pi$$

$$v = e - e^x + 3x^2 \qquad v' = -e^x + 6x$$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\pi + 2\pi x - \ln \pi + \pi^x}{e - e^x + 3x^2} \quad \begin{matrix} \longleftarrow u \\ \longleftarrow v \end{matrix}$$

$$u = \pi + 2\pi x - \ln \pi + \pi^x \qquad u' = 2\pi + \pi^x \ln \pi$$

$$v = e - e^x + 3x^2 \qquad v' = -e^x + 6x$$

$$\begin{aligned} f'(x) &= \frac{u'v - uv'}{v^2} \\ &= \frac{(2\pi + \pi^x \ln \pi)(e - e^x + 3x^2) - (\pi + 2\pi x - \ln \pi + \pi^x)(-e^x + 6x)}{(e - e^x + 3x^2)^2} \end{aligned}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \left(e^{x^3} - \ln(4x^6 - 5)\right)\left(3 + \frac{1}{\sqrt{3x-9}}\right)$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \underbrace{\left(e^{x^3} - \ln(4x^6 - 5)\right)}_{u} \underbrace{\left(3 + \frac{1}{\sqrt{3x-9}}\right)}_{v}$$

$$u = e^{x^3} - \ln(4x^6 - 5)$$

$$v = 3 + (3x - 9)^{-1/2}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \underbrace{\left(e^{x^3} - \ln(4x^6 - 5)\right)}_{u}\underbrace{\left(3 + \frac{1}{\sqrt{3x-9}}\right)}_{v}$$

$$u = e^{x^3} - \ln(4x^6 - 5) \qquad u' = 3x^2e^{x^3} - \frac{24x^5}{4x^6 - 5}$$

$$v = 3 + (3x-9)^{-1/2}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \underbrace{\left(e^{x^3} - \ln(4x^6 - 5)\right)}_{u} \underbrace{\left(3 + \frac{1}{\sqrt{3x-9}}\right)}_{v}$$

$$u = e^{x^3} - \ln(4x^6 - 5) \qquad u' = 3x^2 e^{x^3} - \frac{24x^5}{4x^6 - 5}$$

$$v = 3 + (3x-9)^{-1/2} \qquad v' = -\tfrac{3}{2}(3x-9)^{-3/2}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \underbrace{\left(e^{x^3} - \ln(4x^6 - 5)\right)}_{u} \underbrace{\left(3 + \frac{1}{\sqrt{3x-9}}\right)}_{v}$$

$$u = e^{x^3} - \ln(4x^6 - 5) \qquad u' = 3x^2 e^{x^3} - \frac{24x^5}{4x^6 - 5}$$

$$v = 3 + (3x-9)^{-1/2} \qquad v' = -\tfrac{3}{2}(3x-9)^{-3/2}$$

$$f'(x) = u'v + uv'$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \underbrace{\left(e^{x^3} - \ln(4x^6 - 5)\right)}_{u} \underbrace{\left(3 + \frac{1}{\sqrt{3x-9}}\right)}_{v}$$

$$u = e^{x^3} - \ln(4x^6 - 5) \qquad u' = 3x^2 e^{x^3} - \frac{24x^5}{4x^6 - 5}$$

$$v = 3 + (3x-9)^{-1/2} \qquad v' = -\tfrac{3}{2}(3x-9)^{-3/2}$$

$$\begin{aligned} f'(x) &= u'v + uv' \\ &= \left(3x^2 e^{x^3} - \frac{24x^5}{4x^6 - 5}\right)\left(3 + (3x-9)^{-1/2}\right) \\ &\quad + \left(e^{x^3} - \ln(4x^6 - 5)\right)\left(-\tfrac{3}{2}(3x-9)^{-3/2}\right) \end{aligned}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\left(5x^7 - 4x + 3\sqrt{x}\right)^2}{\ln\left(3 - 4x^{2/3}\right)}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\left(5x^7 - 4x + 3\sqrt{x}\right)^2}{\ln\left(3 - 4x^{2/3}\right)} \quad \begin{array}{l} \longleftarrow u \\ \longleftarrow v \end{array}$$

$$u = \left(5x^7 - 4x + 3x^{1/2}\right)^2$$

$$v = \ln\left(3 - 4x^{2/3}\right)$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\left(5x^7 - 4x + 3\sqrt{x}\right)^2}{\ln\left(3 - 4x^{2/3}\right)} \quad \begin{array}{l} \longleftarrow u \\ \longleftarrow v \end{array}$$

$$u = \left(5x^7 - 4x + 3x^{1/2}\right)^2 \qquad u' = 2\left(5x^7 - 4x + 3x^{1/2}\right)\left(35x^6 - 4 + \tfrac{3}{2}x^{-1/2}\right)$$

$$v = \ln\left(3 - 4x^{2/3}\right)$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\left(5x^7 - 4x + 3\sqrt{x}\right)^2}{\ln\left(3 - 4x^{2/3}\right)} \quad \begin{array}{l} \longleftarrow u \\ \longleftarrow v \end{array}$$

$$u = \left(5x^7 - 4x + 3x^{1/2}\right)^2 \qquad u' = 2\left(5x^7 - 4x + 3x^{1/2}\right)\left(35x^6 - 4 + \tfrac{3}{2}x^{-1/2}\right)$$

$$v = \ln\left(3 - 4x^{2/3}\right) \qquad v' = \frac{-\frac{8}{3}x^{-1/3}}{3 - 4x^{2/3}}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\left(5x^7 - 4x + 3\sqrt{x}\right)^2}{\ln\left(3 - 4x^{2/3}\right)} \quad \begin{array}{l} \longleftarrow u \\ \longleftarrow v \end{array}$$

$$u = \left(5x^7 - 4x + 3x^{1/2}\right)^2 \qquad u' = 2\left(5x^7 - 4x + 3x^{1/2}\right)\left(35x^6 - 4 + \tfrac{3}{2}x^{-1/2}\right)$$

$$v = \ln\left(3 - 4x^{2/3}\right) \qquad v' = \frac{-\frac{8}{3}x^{-1/3}}{3 - 4x^{2/3}}$$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

The Product and Quotient Rule

Examples

Find the derivative of the following function

$$f(x) = \frac{\left(5x^7 - 4x + 3\sqrt{x}\right)^2}{\ln\left(3 - 4x^{2/3}\right)} \quad \begin{array}{l} \longleftarrow u \\ \longleftarrow v \end{array}$$

$$u = \left(5x^7 - 4x + 3x^{1/2}\right)^2 \qquad u' = 2\left(5x^7 - 4x + 3x^{1/2}\right)\left(35x^6 - 4 + \tfrac{3}{2}x^{-1/2}\right)$$

$$v = \ln\left(3 - 4x^{2/3}\right) \qquad v' = \frac{-\frac{8}{3}x^{-1/3}}{3 - 4x^{2/3}}$$

$$f'(x) = \frac{u'v - uv'}{v^2} = \text{ain't nobody got time for that!}$$