

**Name:** \_\_\_\_\_

**VIP ID:** \_\_\_\_\_

- Write your name and your VIP ID in the space provided above.
- The test has five (5) pages, including this one.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, or notes may be used on this test.
- An approved calculator may be used on this test.

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Page	Max. points	Your points
2	25	
3	25	
4	25	
5	25	
<b>Total</b>	100	

**Problem 1** (15 pts). The function  $f(x) = x^4 - 5x^3 + 11x$  has a critical point at  $x = 1$ . Identify what kind of critical point it is.

- ☐  $f(x)$  has a local maximum at  $x = 1$ .
- ☐  $f(x)$  has a local minimum at  $x = 1$ .
- ☐  $f(x)$  has an inflection point at  $x = 1$ .
- ☐  $x = 1$  is neither maximum, minimum, nor inflection point of  $f(x)$ .

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**Problem 2** (10 pts). The function  $y = f(x)$  is shown below. How many inflection points does this function have on the interval shown?

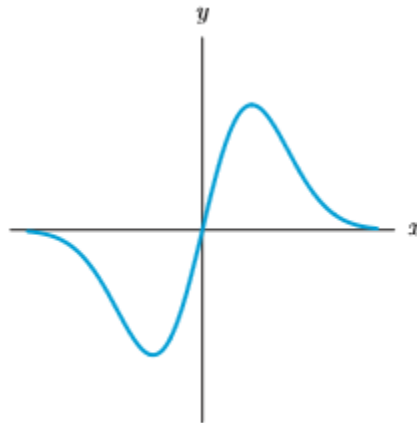


Figure 4.5

- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4

**Problem 3** (10 pts). Concerning the graph of the function below, which of the following statements is true?

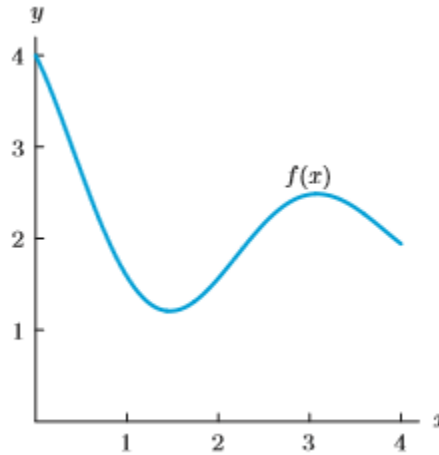


Figure 4.1

- ☐ The derivative is zero at two values of  $x$ , both being local maxima.
- ☐ The derivative is zero at two values of  $x$ , one is a local maximum, while the other is a local minimum.
- ☐ The derivative is zero at two values of  $x$ , one is a local maximum on the interval, while the other is neither a local maximum nor a minimum.
- ☐ The derivative is zero at two values of  $x$ , one is a local minimum on the interval, while the other is neither a local maximum nor a minimum.
- ☐ The derivative is zero only at one value of  $x$ , where it is a local minimum.

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**Problem 4** (15 pts). Find all local maxima, minima and inflection points of the function  $f(x) = 2x^3 + 3x^2 - 180x + 9$ .

**Problem 5** (10 pts). If the graph below is that of  $f'(x)$ , which of the following statements is true concerning the function  $f(x)$ ?

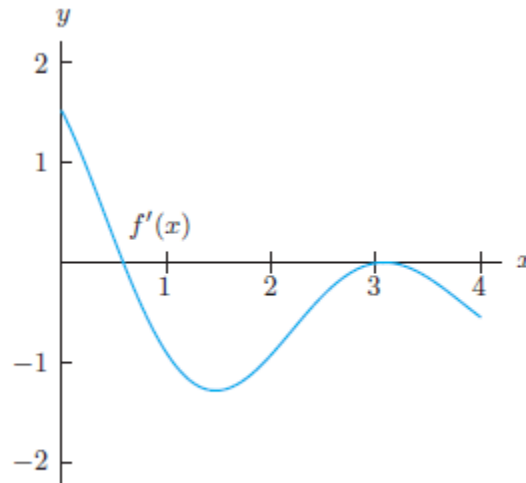


Figure 4.2

- ☐ The derivative is zero at two values of  $x$ , both being local maxima.
- ☐ The derivative is zero at two values of  $x$ , one is a local maximum, while the other is a local minimum.
- ☐ The derivative is zero at two values of  $x$ , one is a local maximum on the interval, while the other is neither a local maximum nor a minimum.
- ☐ The derivative is zero at two values of  $x$ , one is a local minimum on the interval, while the other is neither a local maximum nor a minimum.
- ☐ The derivative is zero only at one value of  $x$ , where it is a local minimum.

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**Problem 6** (15 pts). Find the global maximum and the global minimum of the function  $f(x) = 2x^3 - 9x^2$  over the interval  $-1 \leq x \leq 6$ .

**Problem 7** (10 pts). Find constants  $a$  and  $b$  so that the minimum of the parabola  $f(x) = x^2 + ax + b$  is at the point  $(3, 10)$ .

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**Problem 8** (15 pts). Find the quantity that maximizes profit if the total revenue and total cost (in dollars) are given (resp.) by

$$R(q) = 5q - 0.003q^2,$$

$$C(q) = 300 + 1.1q,$$

where  $q$  is quantity and  $0 \leq q \leq 1000$  units.