Math 242 Test 2, Tuesday 31 March

Name: **Last 4 digits of SSN:**

Show all work **clearly**, **make sentences**. No work means no credit. The points are:
ex1: 20, ex2: 15, ex3: 15, ex4: 15, ex5: 15.

Exercise 1 We give the differential equation:

$$\frac{dx}{dt} = x^2 - 5x + 4.$$

1. What are the critical points ? Use a phase diagram to determine whether each critical point is stable or unstable.

2. Solve this differential equation with $x_0 = 2$.

Exercise 2 We give an initial value problem:

$$y' = 10(x + y), \quad y(0) = 1.$$

Write the algorithm of the Euler's method with a step size h, and apply it to find approximate values of the solution on the interval $[0, 0.5]$ with step size $h = 0.1$. At last, give the order of this method.

x	0	0.1	0.2	0.3	0.4	0.5
approx solution						

Exercise 3 Find a linear homogeneous constant-coefficient differential equation with, in each cases, the given general solution:

a) $y(x) = Ae^{-x} + Be^{2x} + Ce^{x}$,

b) $y(x) = A + Bx + Cx^2 + De^{17x}$,

c) $y(x) = Ae^{3x} + B\cos(2x) + C\sin(2x) + x(D\cos(2x) + E\sin(2x))$.

Exercise 4 Solve the differential equation:

$$y^{(3)} - 5y'' - 16y' + 80y = 0,$$

using the fact that the function $x \mapsto e^{5x}$ is solution of this differential equation.
Then find the unique solution satisfying the initial conditions:

$$y(0) = 9/4,\ y'(0) = -3,\ y''(0) = 18.$$

Exercise 5 We consider the initial value problem:

$$y'' + y' - 2y = (20x + 4)e^{3x}, \quad y(0) = 4,\ y'(0) = 1.$$

Using that a particular solution is given by $y_p(x) = (2x - 1)e^{3x}$, solve this problem.