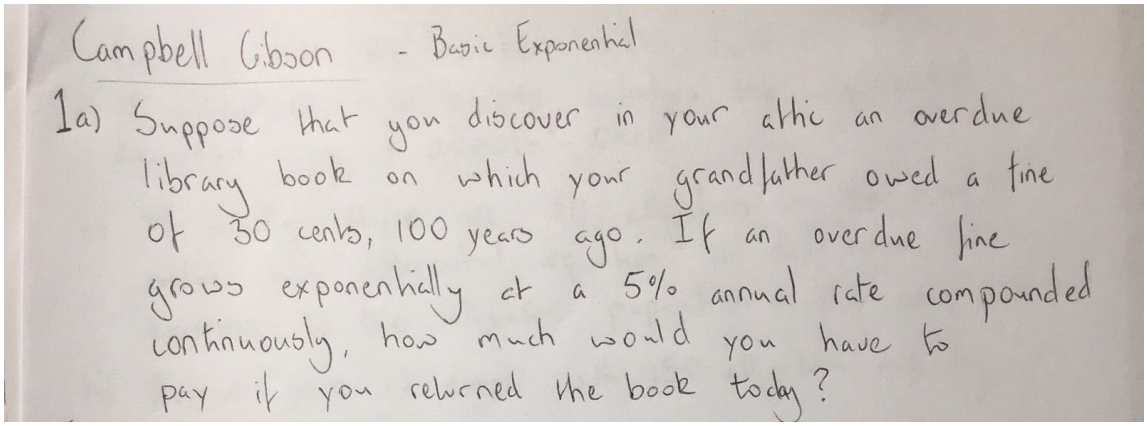
APPLICATION EXAMPLES

Populations

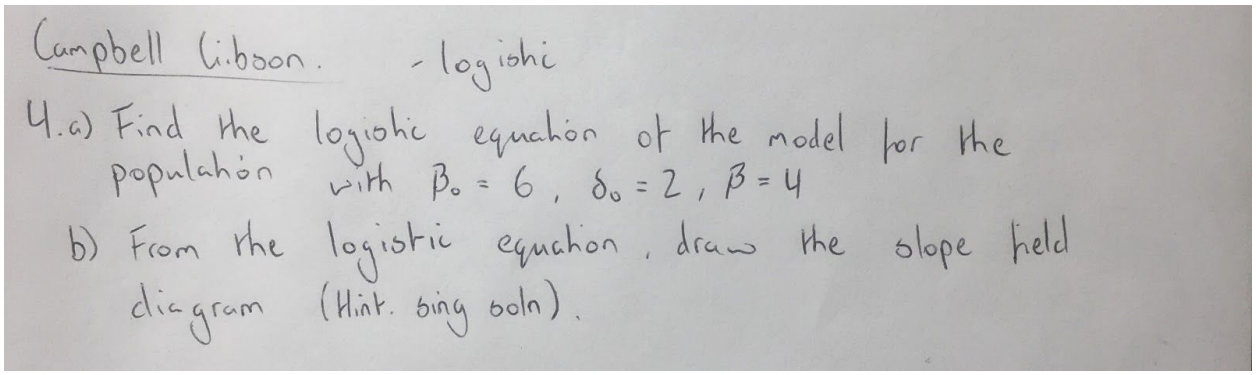
- Basic exponential



Campbell Gibson - Basic Exponential

1a) Suppose that you discover in your attic an overdue library book on which your grandfather owed a fine of 30 cents, 100 years ago. If an overdue fine grows exponentially at a 5% annual rate compounded continuously, how much would you have to pay if you returned the book today?

- Logistic

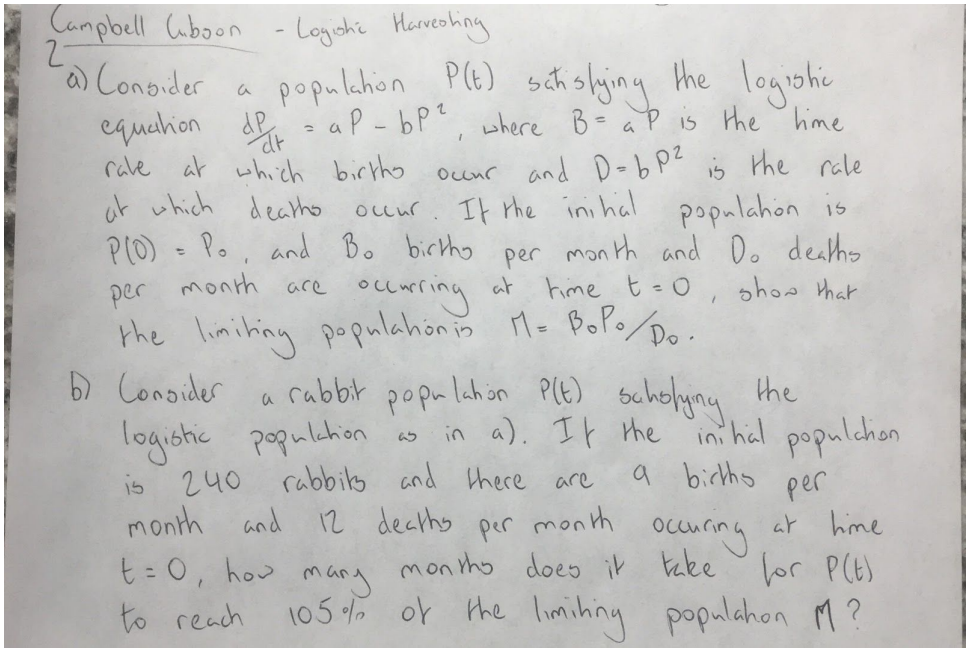


Campbell Gibson. - logistic

4.a) Find the logistic equation of the model for the population with $\beta_0 = 6$, $\delta_0 = 2$, $\beta = 4$

b) From the logistic equation, draw the slope field diagram (Hint. sing soln).

- Logistic harvesting



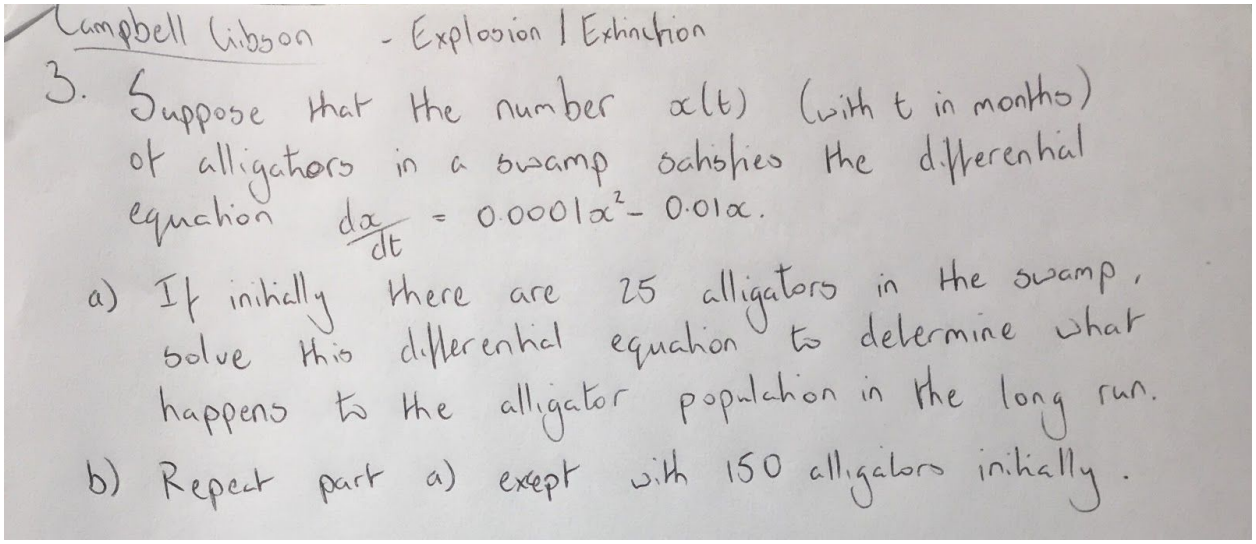
Campbell Gibson - Logistic Harvesting

2

a) Consider a population $P(t)$ satisfying the logistic equation $\frac{dP}{dt} = aP - bP^2$, where $B = aP$ is the time rate at which births occur and $D = bP^2$ is the rate at which deaths occur. If the initial population is $P(0) = P_0$, and B_0 births per month and D_0 deaths per month are occurring at time $t = 0$, show that the limiting population is $M = B_0 P_0 / D_0$.

b) Consider a rabbit population $P(t)$ satisfying the logistic population as in a). If the initial population is 240 rabbits and there are 9 births per month and 12 deaths per month occuring at time $t = 0$, how many months does it take for $P(t)$ to reach 105% of the limiting population M?

- explosion/extinction


Campbell Gibson - Explosion / Extinction

3. Suppose that the number $x(t)$ (with t in months) of alligators in a swamp satisfies the differential equation $\frac{dx}{dt} = 0.0001x^2 - 0.01x$.

a) If initially there are 25 alligators in the swamp, solve this differential equation to determine what happens to the alligator population in the long run.

b) Repeat part a) except with 150 alligators initially.

Geometric

- Equations for curves examples - Robby Carff

  Notes:

  Find the equation for a curve for which the normal at any point (x,y) and the line joining the origin to that point form an isosceles triangle having the x-axis as base.

  Exam:

  Find all curves for which the subtangent at any point (x, y) is equal to five times the square of the abscissa

- Orthogonal trajectories Examples - Robby Carff

  Notes:

  Find the orthogonal trajectories of the following curve.

  $Y = k / x^n$

  Exam:

  Find the orthogonal trajectory of $3x + 5y = k$

Springs

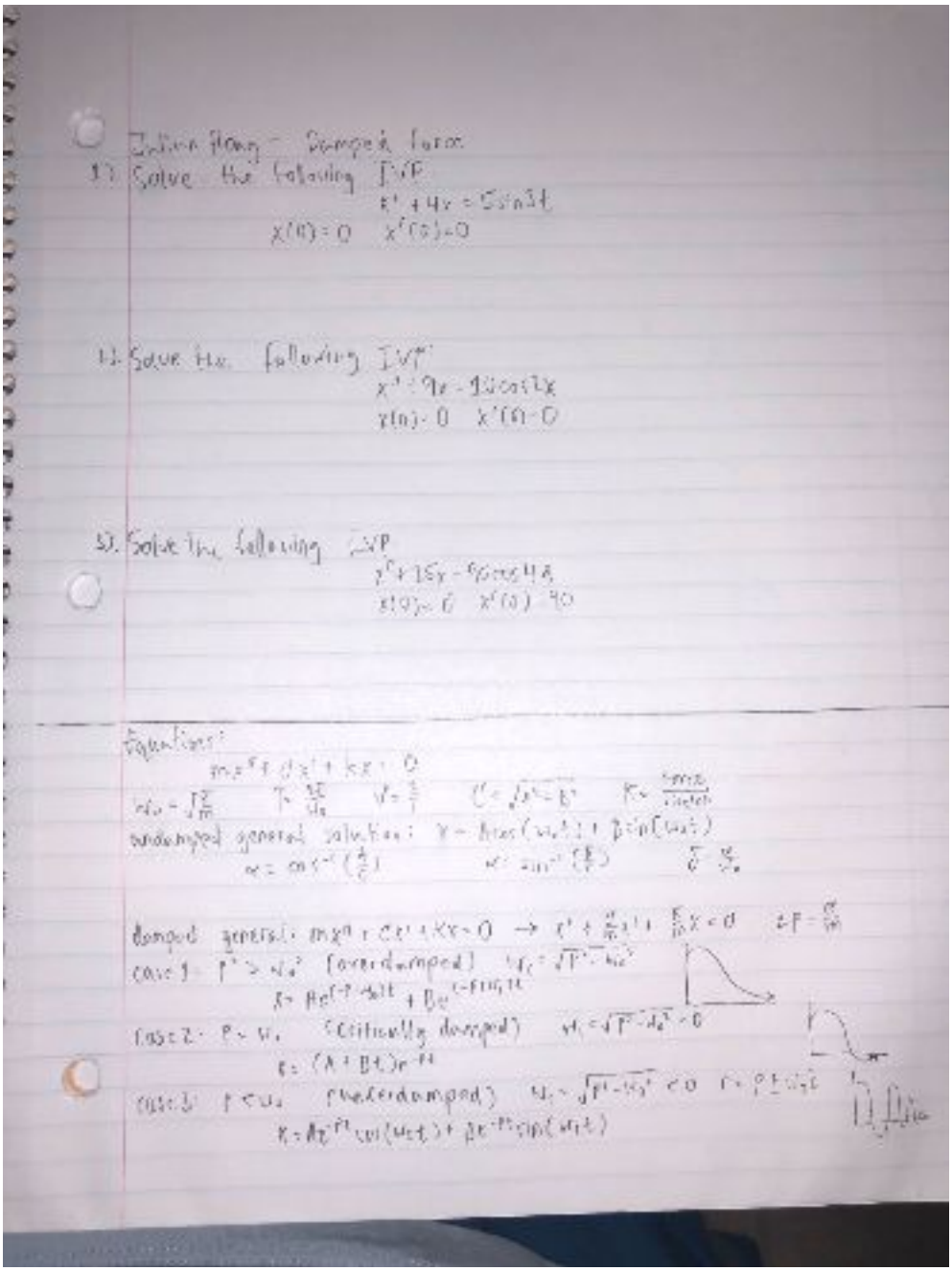
- Free undamped -Dylan Reidy

  From Textbook: “A mass of 3kg is attached to the end of a spring that is stretched 20 cm by a force of 15N. It is set in motion with initial position X_0 = 0 and initial velocity v_0 = -10 m/s. Find the amplitude, period, and frequency of the resulting motion.”

- Free damped - Dylan Reidy

  Modified problem from notes: w_0 = 49, p = 2 Given the equation x’’+10x’+49x = 0, determine whether the system is overdamped, critically damped, or underdamped, and find x(t).

- Forced & equations - Julian Hong



Torricelli's - Robby Carff

A cylindrical tank with length 5ft and radius 3ft is situated with its axis horizontal. If a circular hole with a diameter of 2 in is opened on the bottom, how long will it take for all the water to drain?

A hemispherical bowl has top radius 4 ft and at time t = 0 is full of water. At that moment a circular hole with diameter 1 in. is opened on the bottom of the tank. How long will it take for all the water to drain from the tank?

Population- Daniel Pappas

Bankock's population in 1985 was 5,279,000 and 6,360,000 in 2000. What was the rate of growth between these two periods? Estimate the population in 2016.

The fish population P(t) in a small pond satisfies dp/dt = 0.0317P - 0.00035P^2. The initial population P(0) = 47. If time t is measured in months, how long will it take the population to quadruple?