Lesson 12: Marginal Analysis. Relative Rate of Change

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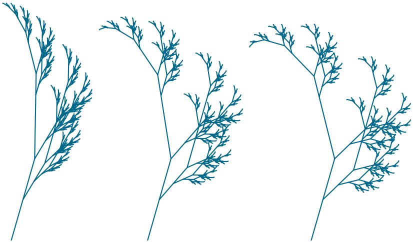

What do we know?

The General Program

- Functions
  - $x-$ and $y-$intercepts $(f(x)=0, f(0))$
  - Change from $x=a$ to $x=b$

$$\Delta y = f(b) - f(a)$$

  - Average Rate of Change from $x=a$ to $x=b$

$$ARC = \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$

  - Relative Change from $x=a$ to $x=b$

$$RC = \frac{\Delta y}{f(a)} = \frac{f(b)-f(a)}{f(a)}$$

  - Instantaneous Rate of Change at $x=a$

$$f'(a)$$

- Linear Functions:
  $f(x) = b + mx$
- Exponential Functions
  $P_0a^t = P_0(1+r)^t = P_0e^{kt}$
- Power Functions
  kx^p
- Polynomials
  $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$

What do we know?

Rules of Differentiation

D1 $f(x) = c$
$f'(x) = 0$

D2 $f(x) = x$
$f'(x) = 1$

D3 $h(x) = f(x) + g(x)$
$h'(x) = f'(x) + g'(x)$

D4 $h(x) = f(x) - g(x)$
$h'(x) = f'(x) - g'(x)$

D5 $h(x) = c \cdot f(x)$
$h'(x) = c \cdot f'(x)$

D6 $f(x) = x^n$
$f'(x) = nx^{n-1}$

D7 $f(x) = e^x$
$f'(x) = e^x$

D8 $f(x) = a^x$
$f'(x) = a^x \ln a$

D9 $f(x) = \ln x$
$f'(x) = \frac{1}{x}$

D10
- $f(x) = g(x)^n$
  $f'(x) = ng(x)^{n-1}g'(x)$
- $f(x) = e^{g(x)}$
  $f'(x) = g'(x)e^{g(x)}$
- $f(x) = a^{g(x)}$
  $f'(x) = g'(x) \ln a \, a^{g(x)}$
- $f(x) = \ln g(x)$
  $f'(x) = \frac{g'(x)}{g(x)}$

D11 $h(x) = f(x) \cdot g(x)$
$h'(x) = f'(x)g(x) + f(x)g'(x)$

D12 $h(x) = f(x)/g(x)$

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Marginal Analysis

Definition

- **Marginal Analysis** is one of the key features that defines the *classical economics* of Adam Smith, and the more mathematical approach of *Neoclassical Economists* pioneered by Alfred Marshall.

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- What's the principle?
  Economic decisions are made at the margin. For example, a consumer might decide to buy one more apple if the price was reduced by 5¢, or a business might decide to buy one more van if the cost was reduced by $1,000.

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- How do we use it?
  The extra cost of an extra unit is known as the marginal cost; the extra revenue of an extra unit is known as the marginal revenue. Marginal Analysis suggests that companies will maximize profit if they produce where marginal cost is equal to marginal revenue, since then they are increasing profit on all previous units produced.

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- In our case, if we know the cost $C(q)$ and revenue $R(q)$, then the marginal cost and revenue (denoted MC and MR respectively) are computed as follows:

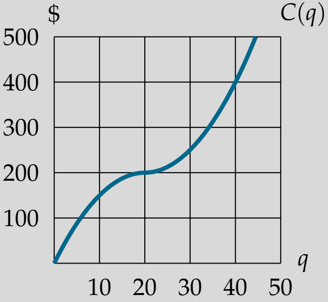
$$MC(q) = C'(q) \qquad\qquad MR(q) = R'(q)$$

Marginal Analysis

Example

Example

In the figure below, is marginal cost greater at $q = 5$ or at $q = 30$?



Marginal Analysis

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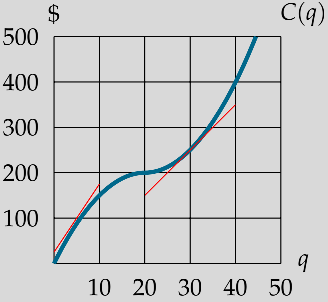

Marginal Analysis

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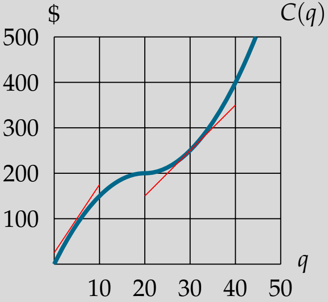

Note that the marginal cost (being the derivative) could be measured by assessing the value of the slopes of tangent lines. From the picture, it is clear that the largest of the two slopes happens at $q = 5$; that is where the marginal cost is greater.

Marginal Analysis

Examples

Example

The cost of producing q items is given by $C(q) = 0.08q^3 + 75q + 1000$.

- Find the marginal cost function.
- Find $C(50)$ and $MC(50)$. Give units and explain what it means about cost of production.

Marginal Analysis

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- $MC(q) = C'(q) = 0.08 \cdot 3q^2 + 75$

Marginal Analysis

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- $MC(q) = C'(q) = 0.08 \cdot 3q^2 + 75$
- $C(50) = 0.08 \cdot 50^3 + 75 \cdot 50 + 1000 = \$14750$,
  $MC(50) = C'(50) = 0.08 \cdot 3 \cdot 50^2 + 75 = \$675/\text{unit}$.

Marginal Analysis

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- $C(50) = 0.08 \cdot 50^3 + 75 \cdot 50 + 1000 = \$14750$,
  $MC(50) = C'(50) = 0.08 \cdot 3 \cdot 50^2 + 75 = \$675/\text{unit}$.
  This means that it costs \$14750 to produce 50 units. And at this production level, if we want to increase the production by one unit, we will have to pay some extra \$675 per unit.

Marginal Analysis

Examples

Example

Assume that $C(q)$ and $R(q)$ represent the cost and revenue (resp.) in dollars, of producing q items.

- If $C(50) = 4300$ and $MC(50) = 24$, estimate $C(52)$.
- If $MC(50) = 24$ and $MR(50) = 35$, approximately how much profit is earned by the 51st item?
- If $C'(100) = 38$ and $R'(100) = 35$, should the company produce the 101st item? Why?

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- $C(52) \approx C(50) + 2 \cdot MC(50) = 4300 + 2 \cdot 24 = \$4348$

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- $MR(50) - MC(50) = 35 - 24 = \$11/\text{item}$

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- $C(52) \approx C(50) + 2 \cdot MC(50) = 4300 + 2 \cdot 24 = \$4348$
- $MR(50) - MC(50) = 35 - 24 = \$11/\text{item}$
- $MR(100) - MC(100) = R'(100) - C'(100) = 35 - 38 = -\$3/\text{item}$. We would be losing \$3 per item. Imma say no.

Marginal Analysis

Examples

Example

An industrial production process costs $C(q)$ million dollars to produce q million units; these units then sell for $R(q)$ million dollars.
If $C(2.1) = 5.2$, $R(2.1) = 6.6$, $MC(2.1) = 0.6$, and $MR(2.1) = 0.7$, calculate the following:

- The profit by producing 2.1 million units.
- The approximate change in revenue if production increases from 2.1 to 2.15 million units (in thousand dollars)
- The approximate change in revenue if production decreases from 2.1 to 2.07 million units (in thousand dollars)
- The approximate change in profit if production increases from 2.1 to 2.15 million units (in thousand dollars)

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- The approximate change in profit if production increases from 2.1 to 2.15 million units (in thousand dollars)

- $\Pi(2.1) = R(2.1) - C(2.1) = 6.6 - 5.2 = 1.4$ million dollars.

Marginal Analysis

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Marginal Analysis

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$$R(2.15) - R(2.1) \approx (2.15 - 2.1) \cdot MR(2.1)$$

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- We need to approximate $R(2.15) - R(2.1)$.

$$R(2.15) - R(2.1) \approx (2.15 - 2.1) \cdot MR(2.1) = 0.05 \cdot 0.7 = 0.035$$

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$$R(2.15) - R(2.1) \approx (2.15 - 2.1) \cdot MR(2.1) = 0.05 \cdot 0.7 = 0.035$$

Therefore, $R(2.15) - R(2.1) \approx \$35,000$.

Marginal Analysis

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- We need to approximate $R(2.07) - R(2.1)$.

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- We need to approximate $R(2.07) - R(2.1)$.

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- We need to approximate $R(2.07) - R(2.1)$.

$$R(2.07) - R(2.1) \approx (2.07 - 2.1) \cdot MR(2.1) = -0.03 \cdot 0.7 = -0.021$$

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- We need to approximate $R(2.07) - R(2.1)$.

$$R(2.07) - R(2.1) \approx (2.07 - 2.1) \cdot MR(2.1) = -0.03 \cdot 0.7 = -0.021$$

Therefore, $R(2.07) - R(2.1) \approx \$21,000$.

Marginal Analysis

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We need to approximate $\Pi(2.15) - \Pi(2.1)$.

$$\Pi(2.15) - \Pi(2.1) = \overbrace{\big(R(2.15) - C(2.15)\big)}^{\Pi(2.15)} - \overbrace{\big(R(2.1) - C(2.1)\big)}^{\Pi(2.1)}$$

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$$C(2.15) - C(2.1) \approx (2.15 - 2.1) \cdot MC(2.1)$$

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$$\begin{aligned}\Pi(2.15) - \Pi(2.1) &= \big(R(2.15) - C(2.15)\big) - \big(R(2.1) - C(2.1)\big)\\ &= \big(R(2.15) - R(2.1)\big) - \big(C(2.15) - C(2.1)\big) = 0.035 - 0.03 = 0.005\end{aligned}$$

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The approximate change in profit is \$5,000

Relative Rate of Change

Definition

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The relative rate of change of $y = f(x)$ at $x = a$ is defined to be

$$\frac{f'(a)}{f(a)}$$

Relative Rate of Change

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The relative rate of change of $y = f(x)$ at $x = a$ is defined to be

$$\frac{f'(a)}{f(a)}$$

Similar to the *relative change*, it has no units. We usually represent it as a percentage (after multiplying times 100, of course)

Relative Rate of Change

Examples

Example

Annual world soybean production, $W = f(t)$, in million tons, is a function of t years since the start of 2000.

- Interpret the statements $f(8) = 253$ and $f'(8) = 17$ in terms of soybean production.
- Calculate the relative rate of change of W at $t = 8$; interpret it in terms of soybean production.

Relative Rate of Change

Examples

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Annual world soybean production, $W = f(t)$, in million tons, is a function of t years since the start of 2000.

- Interpret the statements $f(8) = 253$ and $f'(8) = 17$ in terms of soybean production.
- Calculate the relative rate of change of W at $t = 8$; interpret it in terms of soybean production.

- 253 million tons of soybeans were produced in the year 2008. That year, the annual soybean production was increasing at a rate of 17 million tons per year.

Relative Rate of Change

Examples

Example

Annual world soybean production, $W = f(t)$, in million tons, is a function of t years since the start of 2000.

- Interpret the statements $f(8) = 253$ and $f'(8) = 17$ in terms of soybean production.
- Calculate the relative rate of change of W at $t = 8$; interpret it in terms of soybean production.

- 253 million tons of soybeans were produced in the year 2008. That year, the annual soybean production was increasing at a rate of 17 million tons per year.
- We have

$$\frac{f'(8)}{f(8)} = \frac{17}{253} = 0.067$$

In 2008, annual soybean production was increasing at a rate of 6.7% per year.

Relative Rate of Change

Examples

Compute the relative rate of change of the following functions:

$$f(x) = 5x + 4$$

$$f(x) = 4x^2 + \sqrt{x}$$

$$f(x) = 6e^{5x}$$

$$f(x) = \ln(3x - 5)$$

Relative Rate of Change

Examples

Compute the relative rate of change of the following functions:

$f(x) = 5x + 4 \qquad f'(x) = 5$

$f(x) = 4x^2 + \sqrt{x}$

$f(x) = 6e^{5x}$

$f(x) = \ln(3x - 5)$

Relative Rate of Change

Examples

Compute the relative rate of change of the following functions:

$f(x) = 5x + 4$ $\qquad f'(x) = 5$ $\qquad \dfrac{f'(x)}{f(x)} = \dfrac{5}{5x+4}$

$f(x) = 4x^2 + \sqrt{x}$

$f(x) = 6e^{5x}$

$f(x) = \ln(3x - 5)$

Relative Rate of Change

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$f(x) = 4x^2 + \sqrt{x}$ $\quad f'(x) = 8x + \frac{1}{2}x^{-1/2}$

$f(x) = 6e^{5x}$

$f(x) = \ln(3x - 5)$

Relative Rate of Change

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Compute the relative rate of change of the following functions:

$f(x) = 5x + 4$ $\quad f'(x) = 5$ $\quad \dfrac{f'(x)}{f(x)} = \dfrac{5}{5x+4}$

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$f(x) = 6e^{5x}$

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Relative Rate of Change

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Compute the relative rate of change of the following functions:

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$f(x) = 6e^{5x}$ $\quad f'(x) = 30e^{5x}$

$f(x) = \ln(3x - 5)$

Relative Rate of Change

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$f(x) = 6e^{5x}$ $\quad f'(x) = 30e^{5x}$ $\quad \dfrac{f'(x)}{f(x)} = \dfrac{30e^{5x}}{6e^{5x}} = 5$

$f(x) = \ln(3x - 5)$

Relative Rate of Change

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$f(x) = 6e^{5x}$ $\quad f'(x) = 30e^{5x}$ $\quad \dfrac{f'(x)}{f(x)} = \dfrac{30e^{5x}}{6e^{5x}} = 5$

$f(x) = \ln(3x-5)$ $\quad f'(x) = \dfrac{3}{3x-5}$

Relative Rate of Change

Examples

Compute the relative rate of change of the following functions:

$f(x) = 5x + 4$	$f'(x) = 5$	$\dfrac{f'(x)}{f(x)} = \dfrac{5}{5x+4}$
$f(x) = 4x^2 + \sqrt{x}$	$f'(x) = 8x + \frac{1}{2}x^{-1/2}$	$\dfrac{f'(x)}{f(x)} = \dfrac{8x + \frac{1}{2}x^{-1/2}}{4x^2 + \sqrt{x}}$
$f(x) = 6e^{5x}$	$f'(x) = 30e^{5x}$	$\dfrac{f'(x)}{f(x)} = \dfrac{30e^{5x}}{6e^{5x}} = 5$
$f(x) = \ln(3x - 5)$	$f'(x) = \dfrac{3}{3x-5}$	$\dfrac{f'(x)}{f(x)} = \dfrac{3}{(3x-5)\ln(3x-5)}$