Name: ______________________________

4-digit code: ______________________________

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has thirteen (13) pages, including this one. You have 150 minutes to complete it.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.
- **A:** 243–270 pts. **B+:** 230–242 pts. **B:** 216–229 pts. **C+:** 203–215 pts. **C:** 189–202 pts. **D+:** 175–188 pts. **D:** 160–174 pts. **F**: less than 160 pts.

Page	**Max**	**Points**	**Page**	**Max**	**Points**	**Page**	**Max**	**Points**
2	30		6	30		10	30	
3	25		7	25		11	25	
4	25		8	25		12	25	
5	20		9	20		13	20	
Total	100		**Total**	100		**Total**	100	

Problem 1 (15 pts)**.** Find the distance d from the point $(3, 7, -5)$ to the z–axis.

$d =$ $\boxed{}$

Problem 2 (15 pts)**.** Find an exact expression for the angle θ between the vectors $\boldsymbol{v} = \langle 3, -1, 5\rangle$ and $\boldsymbol{w} = \langle -2, 4, 3\rangle$.

$\theta =$ $\boxed{}$

Problem 3 (15 pts)**.** Find the length ℓ of the curve $\boldsymbol{r}(t) = \boldsymbol{i} + t^2\boldsymbol{j} + t^3\boldsymbol{k}$ for $0 \le t \le 1$.

$\ell =$ $\boxed{}$

Problem 4 (10 pts)**.** At what points does the helix $\boldsymbol{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

points: $\boxed{}$

Problem 5 (15 pts)**.** Find a unit vector $\boldsymbol{v}$ that is orthogonal to both $\boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{i} + \boldsymbol{k}$.

$\boldsymbol{v} = \boxed{}$

Problem 6 (10 pts)**.** Determine whether the points $A = (0, -5, 5)$, $B = (1, -2, 4)$ and $C = (3, 4, 2)$ lie on a straight line.

Problem 7 (20 pts)**.** Find parametric equations for the line of intersections of the planes $x+y+z=1$ and $x+2y+2z=1$. Find the angle θ between the two planes.

$\theta =$

Problem 8 (15 pts)**.** Sketch the domain of $f(x,y) = \dfrac{\sqrt{4-x^2}}{y^2+3}$.

Problem 9 (15 pts)**.** Evaluate the limit, if it exists

$$\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln\left(x^2+y^2\right)$$

Problem 10 (15 pts)**.** The volume of a right circular cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$. Show that if the height remains constant while the radius changes, then the volume satisfies

$$\frac{\partial V}{\partial r} = \frac{2V}{r}.$$

Problem 11 (10 pts)**.** Use the method of Lagrange multipliers to find the dimensions of a rectangle with perimeter p and maximum area.

width: [ ]

height: [ ]

Problem 12 (15 pts)**.** Recall the formula for the volume of a right circular cone of radius r and height h. Suppose that the height decreases from 20 to 19.95 inches, and the radius increases from 4 to 4.05 inches. Compare the change in volume of the cone with an approximation of this change using a total differential.

$dV =$ $\boxed{}$

$\Delta V =$ $\boxed{}$

Problem 13 (10 pts)**.** Find an equation for the tangent plane to the surface $z = xe^{-y}$ at the point $P = (1, 0, 1)$.

tangent plane: $\boxed{}$

Problem 14 (20 pts)**.** Find the absolute extrema of the function $f(x, y) = xy - x - 3y$ on the triangular region R with vertices $(0, 0)$, $(0, 4)$ and $(5, 0)$.

absolute max: $\boxed{}$

absolute min: $\boxed{}$

Problem 15 (15 pts)**.** Evaluate $\iint_R y\sin(xy)\,dA$, where $R = [1,2]\times[0,\pi]$.

Problem 16 (15 pts)**.** Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$, and the three coordinate planes.

$V =$

Problem 17 (15 pts)**.** Evaluate the integral $\iint_D \sin(y^2)\, dA$ where D is the triangle with vertices $(0,0)$, $(1,1)$ and $(0,1)$.

Problem 18 (10 pts)**.** Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy–plane, and inside the cylinder $x^2 + y^2 = 2x$.

$V =$

Problem 19 (10 pts)**.** Evaluate $\iiint_E \sqrt{x^2+z^2}\,dV$, where E is the region bounded by the paraboloid $y = x^2+z^2$ and the plane $y=4$.

Problem 20 (15 pts)**.** Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx$ by passing the description of the region E in terms of cylindrical coordinates (Trust me, it is **way** easier than evaluating the integral above directly)

Problem 21 (20 pts)**.** A transformation is defined by the equations $x = u^2 - v^2, y = 2uv$.

(a) Find the image of the square $S = \{(u, v) : 0 \le u \le 1, 0 \le v \le 1\}$.

(b) Use the same change of variables to evaluate the integral $\iint_R y\,dA$, where R is the region bounded by the x–axis and the parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x$, $y \ge 0$.

Image of S:

$\iint_R y\,dA =$