

Math Final practice test: Applications

**Geometric:**

Equations of a curve: Find the equation of a curve that satisfies the statements given.

- 1) The subnormal at any point  $(x,y)$  and the line joining the origin to that point form a scalene triangle having the x-axis as a base. Erik Fallon
- 2) Find all curves with equal subtangents and subnormals. Erik Fallon
- 3) Find all curves so that the subtangent is equal to the square root of the abscissa. Erik Fallon
- 4) The length of the perpendicular from the origin to a normal line of the curve is equal to twice the abscissa of the point of contact  $(x,y)$ . Erik Fallon
- 5) The length of the subnormal is proportional to the square of the coordinate. Kristen Schandall

Compute the tangent, and normal lines to the graph.  $y=4x^2-8x+2$

Kristen Schandall

Find the intersection of the tangent line with the y-axis of  $y=2x$

Kristen Schandall

Find the intersection of the normal line with the x-axis of  $y=2x$

Kristen Schandall

Orthogonal trajectories: Find the curves that intersect the given curves at 90 degree angles.

- 1)  $y=2x-5+ke^{3x}$  Erik Fallon
- 2)  $\frac{3}{2}y=2x^2/(k-x)^2$  Erik Fallon
- 3)  $y=2k/(3x^n)$  ( $n>0$ ) Erik Fallon
- 4)  $y^3=2x^4(3-2kx)$  Erik Fallon
- 5)  $8x+4y=k$  Kristen Schandall
- 6)  $\frac{1}{2}y=6kx^n$  ( $n>0$ ) Kristen Schandall

**Populations:**

A certain city had a population of 20,000 in 1940 and a population of 35,000 in 1980. Assume that its population will continue to grow exponentially at a constant rate. What population can its city planners expect in the year 2010? Austin Scampini

For a logistic population  $P(t)$  of fish in a lake, suppose  $K = 1$  &  $M = 4$ , measured in hundreds, after  $(t)$  years. Suppose that we allow harvesting in the pond at a rate of 300 fish per year. Study the model. ( $H = 3$ ) **Austin Scampini**

Suppose that the population  $P(t)$  of a country satisfies the differential equation  $dP/dt = kP(300 - P)$  with  $k$  constant. Its population in 1950 was 200 million and was then growing at the rate of 2 million per year. Predict this country's population for the year 2010. **Austin Scampini**

As the salt  $KNO_3$  dissolves in methanol, the number  $x.t/$  of grams of the salt in a solution after  $t$  seconds satisfies the differential equation  $dx/dt = 0.4x - 0.002x^2$ . **Austin Scampini**

A population  $P(t)$  of small rodents has birth rate  $B = (.001)P$  (births per month per rodent) and constant death rate  $\delta$ . If  $P(0) = 100$  and  $P'(0) = 8$ , how long (in months) will it take this population to double to 200 rodents? (Suggestion: First find the value of  $\delta$ .) **Austin Scampini**

### **Springs:**

(free damped) A 10 kg object is stretched by spring 4m. Assume a damped force of 20 N, determine the equation of motion initially released from 2 m with a velocity of 5 m/s. **Frank Mitchell**

(free damped) A 1 kg object is attached to a 20 N/m spring with 9 N of friction. Find  $x(t)$  if the object is initially at rest and has a velocity of 2 m/s. Express it in phase/angle notation and graph it. **Frank Mitchell**

(free undamped) A spring with a spring constant 15 N/m is attached to a 3 kg mass with negligible friction. Find the frequency, period, phase angle and lag of the system. **Frank Mitchell**

(free undamped) A 4 lb object is supported by a 100 lb/ft spring. Determine the frequency, period, and amplitude of this system. **Frank Mitchell**

(forced) A spring with a spring constant of 8 N/m is attached to a 1 kg mass with 6 N of friction is under a constant force of  $4 \cos(t)$  N.

- Find  $x(t)$  and express it in phase/angle notation
- Find the particular solution that satisfies  $x(0) = 1$  and  $x'(0) = 3$

**Frank Mitchell**

(forced) A 2 kg object is attached to a spring with a spring constant 18 N/m with 12 N of friction and a constant force of  $3 \cos(t)$  N.

- Find  $x(t)$  when the object is initially at rest then moves with a velocity of 1 m/s
- Graph the function and label amplitude, frequency, period, and lag.

**Frank Mitchell**

### **Torricelli:**

A tank of fuel has a hole 25 m below the surface. The hole has a diameter .4 m. If the coefficient of discharge is .98 when the hole is opened. What is the flow rate of the fuel.

Frank Mitchell

A tank of water has a hole 9 m below the surface with a diameter of 2 m. How long does it take the tank to become empty.

Frank Mitchell

A full hemispherical water tank has a radius of 4m has its flat side as its bottom. It has a bottom hole of radius 2cm. If the bottom hole is opened at 2pm when will the tank be empty?

Kristen Schandall

A water tank has the shape of obtained by  $y=x^2$  around the y axis. The water depth is 4ft at 1pm. When the circular plug on the bottom of the tank is removed. At 2 pm the depth of the water is 1 ft.

- a) What time will the tank be empty?
- b) If the top surface of the water has a radius of 2ft, what is the radius of the circular hole on the bottom ?

Kristen Schandall