Lesson 2: Intercepts, Change and Average Rate of Change

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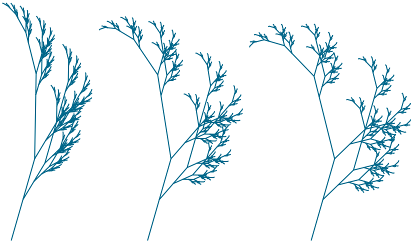

What do we know?

The General Program

- Background Material (Basic Algebra)
- Functions
    - Definition

Warm-up

A somewhat advanced Example

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The solid waste W generated each year in the cities of the US is increasing.
The solid waste generated (in millions of tons) was 238.3 in 2000, and 251.3 in 2006.
Find a formula for this function by using the equation of the line through these two points.

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$$a = f(0) = 238.3 \qquad m = \frac{251.3 - 238.3}{6 - 0} = \frac{13}{6} \approx 2.167$$

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Solution:

$$W = f(t) = 238.3 + \tfrac{13}{6}t$$

Warm-up

Another somewhat advanced Example

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A city's population was 30,700 in the year 2000, and is growing by 850 people a year.

- Give a formula for the city's population, P, as a function of the number of years, t, since 2000.
- What is the population predicted to be in 2020?
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Solution: The population of this city will reach 55000 sometime during 2028.

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- The average rate of change of f between $x = a$ and $x = b$:

  $$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} \text{ (units = dep. variable/ind. variable)}$$

Examples

Example (Winning Height in Men's Olympic Pole Vault)

Year	1960	1964	1968	...	1992	1996	2000
Height (in)	185	201	213	...	228	233	232

- What was the change in height from 1960 to 1968?
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PCB (parts per million)	87	147	204	289	356	452
thickness (mm)	0.44	0.39	0.28	0.23	0.22	0.14

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  Only between 1996 and 1997.