**Math 242 Test 3, Wednesday 28 November**

**Name:** **Last 4 digits of SSN:**

Show all work clearly, **MAKE SENTENCES**. No work means no credit. The points are:
ex1: 25, ex2: 15, ex3: 25, ex4: 20 and the course questions are over 15 points.

**Course Questions**

1. Method of variation of parameters in the case $n = 2$:
   We consider the second-order linear differential equation

   $$y'' + P(x)y' + Q(x)y = f(x),$$

   where $P$, $Q$ and $f$ are continuous. A general solution is given by:

   $$y_c(x) = c_1 y_1(x) + c_2 y_2(x),$$

   where $c_1$ and $c_2$ are constants.
   Of what form can we search a particular solution ? To find this, we need to impose a condition, what is this condition ?

2. Give the definition of a function of exponential order as $t \to \infty$. Then state the theorem which gives the existence of the Laplace transform.

3. State the theorem about the Laplace transform of derivates.

**Exercise 1** Find a particular solution of the following differential equation:

$$y'' + y' - 2y = 9x^2 e^x.$$

**Exercise 2** Find the inverse Laplace transform of the functions:

$$F(s) = \frac{3s+5}{s^2+4} \quad \text{and} \quad G(s) = \frac{3}{s^2(s^2+9)}.$$

**Exercise 3** Solve the initial value problem using the Laplace transform:

$$y'' + y = \cos(3t), \quad y(0) = 1, y'(0) = 0.$$

**Exercise 4** 1. Use the partial fraction to find the inverse Laplace transform of

$$F(s) = \frac{5 - 2s}{s^2 + 7s + 10}.$$

2. Write the partial fractions of the rational function (we do not ask the value of the coefficients):

$$G(s) = \frac{5s^5 + 4s^4 + 3s^3 + 2s^2 + 1}{(s-1)(s^3 + 2s^2 + 3s)^2}.$$