

Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has six (6) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

| Page | Max. points | Your points |
|--------------|-------------|-------------|
| 2 | 20 | |
| 3 | 15 | |
| 4 | 15 | |
| 5 | 25 | |
| 6 | 25 | |
| Total | 100 | |

Problem 1 (5 pts). Find $f(0)$ and $f(\pi/2)$ for $f(x) = \begin{cases} \sqrt{x+1} & \text{if } x \geq 1, \\ 3 & \text{if } x < 1. \end{cases}$

$$f(0) = \boxed{}$$

$$f(\pi/2) = \boxed{}$$

Problem 2 (10 pts). Find the domain of $f(x) = \sqrt{(x-1)(x-2)}$.

$$\text{domain} = \boxed{}$$

Problem 3 (5 pts). Express the function $f(x) = |x-1|$ in piecewise form without using absolute values.

$$f(x) = \left\{ \boxed{} \right.$$

Problem 4 (5 pts). Let $f(x) = x^2 + 4$ and $g(x) = \sqrt{x}$. Find $(g \circ f)(x)$.

$$(g \circ f)(x) =$$

Problem 5 (10 pts). Recall the “ ε - δ ” definition of limit:

We write $\lim_{x \rightarrow a} f(x) = L$ if for all $\varepsilon > 0$ there exists $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - L| < \varepsilon$.

Use this definition to prove that $\lim_{x \rightarrow 2} (-x - 2) = -4$.

Problem 6 (5 pts). Solve for x :

$$\ln x + \ln(x - 1) = 1$$

$x =$

Problem 7 (10 pts). Compute the derivatives of the following functions.

(a) $f(x) = \pi\sqrt{x}(x^4 - 4x^3 + 6x^2 - 4x^1 + 1 - x^{-1})$

$f'(x) =$

(b) $g(t) = \frac{t^2 - 5}{t^{-1}}$

$g'(t) =$

Problem 8 (15 pts). Compute the following limits:

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 2x - 8}{x^2 - 4} =$

(b) $\lim_{x \rightarrow -\infty} \frac{x^2 - 2x - 8}{x^2 - 4} =$

(b) $\lim_{x \rightarrow -2} \frac{x^2 - 2x - 8}{x^2 - 4} =$

Problem 9 (10 pts). Find the value of the constant k for which the following function is continuous everywhere:

$$f(x) = \begin{cases} 2k^2x^3 & \text{if } x < 2, \\ x + 32k - 18 & \text{if } x \geq 2. \end{cases}$$

$k =$

Problem 10 (15 pts). Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - \frac{9}{2}y = 3$.

Problem 11 (10 pts). How many tangent lines to the curve $y = x/(x+1)$ pass through the point $(0,0)$.

HINT: *You do not have to compute the equations of the lines.*