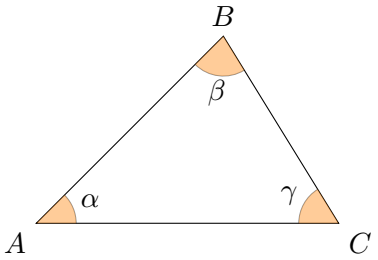
Set-up

Throughout this session, consider the values of m, d given by your birthday (in the form $m/d/Y$). For instance, if you were born today, then $m = 10$ and $d = 8$.

Consider a triangle $\triangle ABC$ that has angles with degree values $\alpha = 4\max(m,d) - 2\min(m,d) + 6$, and $\beta, \gamma \in \mathbb{N}$.



Exploration Stage

Problem 1 (10 pts—all or nothing)**.** Describe the set $\mathcal{B}$ that contains all the possible values of β (in degrees) for the triangle $\triangle ABC$ that you have just constructed. Use set-builder notation, rather than listing its elements.

Problem 2 (10 pts—all or nothing)**.** Once an angle $\beta \in \mathcal{B}$ has been chosen, there is only one possible value for the remaining angle γ. Find a formula for γ in terms of m, d and β.

Planning and Delivery

Problem 3 (20 pts)**.** Prove that in your triangle $\triangle ABC$ with angles (in degrees) with values $\beta, \gamma \in \mathbb{N}$, and $\alpha = 4\max(m,d) - 2\min(m,d) + 6$, if $\underbrace{\beta \text{ is even}}_{P}$, then $\underbrace{\gamma \text{ is also even}}_{Q}$.

Start by completing the following step-by-step table, before you put your proof into words.

Statement	**Reason (Fact)**
$P =$ "β is even."	hypothesis
$\exists b \in \mathbb{N}, \beta = 2b$	Definition of even (natural) number
$\vdots$	$\vdots$
$\gamma = 2c$ for $c = \cdots$	$\cdots$
$Q =$ "γ is even."	Definition of even number

Problem 4 (20 pts)**.** For the same triangle, prove that if β is odd, then γ is also odd.

Problem 5 (40 pts)**.** Write a proof of the following proposition.

> **Proposition.** *In any right triangle with integer-valued angles (in degrees), the non-right angles have the same parity.*