

Name: \_\_\_\_\_

4-digit code: \_\_\_\_\_

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has six (6) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

---

Page	Max. points	Your points
2	20	
3	15	
4	20	
5	25	
6	20	
<b>Total</b>	100	

**Problem 1** (10 pts). Find a formula for the general term of the following sequences:

(a)  $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \dots$

$x_n =$

(b)  $1 - \frac{1}{2}, \frac{1}{3} - \frac{1}{2}, \frac{1}{3} - \frac{1}{4}, \frac{1}{5} - \frac{1}{4}, \dots$

$x_n =$

---

**Problem 2** (10pts). Write out the first five terms of the sequence  $\left\{ \frac{(-1)^{n+1}}{n^2} \right\}_{n=1}^{\infty}$ . Determine whether the sequence converges, and if so find its limit.

First five terms:

$\lim_{n \rightarrow \infty} x_n =$

**Problem 3** (5 pts). Use  $x_{n+1} - x_n$  to show that the sequence  $\{n - 2^n\}_{n=1}^{\infty}$  is strictly increasing or strictly decreasing.

---

**Problem 4** (5 pts). Use  $x_{n+1}/x_n$  to show that the sequence  $\left\{\frac{n^n}{n!}\right\}_{n=1}^{\infty}$  is strictly increasing or strictly decreasing.

---

**Problem 5** (5 pts). Use **differentiation** to show that the sequence  $\left\{\frac{n}{2n+1}\right\}_{n=1}^{\infty}$  is strictly increasing or strictly decreasing.

**Problem 6** (20 pts). Determine whether the series converge, and if so find their sum:

(a)  $\sum_{k=1}^{\infty} \left(-\frac{3}{4}\right)^{k-1}$

$$\sum_{k=1}^{\infty} \left(-\frac{3}{4}\right)^{k-1} =$$

(b)  $\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)}$

$$\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)} =$$

**Problem 7** (5 pts). Apply the **divergence test** and state what it tells you about the series.

$$\sum_{k=1}^{\infty} \frac{k^2 + k + 3}{2k^2 + 1}.$$

---

**Problem 8** (10 pts). Use the **integral test** to determine whether the series  $\sum_{k=1}^{\infty} \frac{1}{5k+2}$  converges.

---

**Problem 9** (10 pts). Use the **ratio test** to determine whether the series  $\sum_{k=1}^{\infty} \frac{4^k}{k^2}$  converges. If the test is inconclusive, then say so.

**Problem 10** (10 pts). Use the **root test** to determine whether the series  $\sum_{k=1}^{\infty} \left( \frac{3k+2}{2k-1} \right)^k$  converges. If the test is inconclusive, then say so.

---

**Problem 11** (10 pts). Classify the series  $\sum_{k=1}^{\infty} (-1)^k \frac{4k^2+1}{k^3+2}$  as absolutely convergent, convergent or divergent.