Name: _____

VIP ID: _____

- Write your name and VIP ID in the space provided above.
- The test has six (6) pages including this one.
- Enter your answers in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books or notes may be used on this test. An approved calculator is allowed (and recommended).

Page	**Max. points**	**Your points**
2	10	
3	30	
4	30	
5	20	
6	10	
Total	100	

Problem 1 (10 pts)**.** The expression below gives parametric equations and parameter interval for the motion of a particle in the xy–plane.

$$\begin{cases} x(t) = t^2 \\ y(t) = t + 1 \end{cases} \qquad (-1 \leq t \leq 1)$$

(a) Identify the particle's path by finding a Cartesian equation for it (one in terms of x and y, without the t).

(b) Graph the Cartesian equation.

(c) Indicate the portion of the graph traced by the particle, and the direction of motion.

Problem 2 (10 pts)**.** Find the tangent to the curve $x(t) = \sec t$, $y(t) = \tan t$, $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ at the point $(\sqrt{2}, 1)$ (where $t = \pi/4$).

Problem 3 (10 pts)**.** Consider the region enclosed by the astroid $x(t) = \cos^3 t$, $y(t) = \sin^3 t$, $0 \leq t \leq 2\pi$. Express its area as an integral on the parameter variable t, and compute its value.

Problem 4 (10 pts)**.** Consider the astroid $x(t) = 2\cos^3 t$, $y(t) = 2\sin^3 t$, $0 \leq t \leq 2\pi$. Express its length as an integral on the parameter variable t, and compute its value.

Problem 5 (10 pts)**.** Find Cartesian coordinates of the point given in polar coordinates by $P(\sqrt{2}, \pi/4)$.

Problem 6 (10 pts)**.** Find a polar equation for the circle $x^2 + (y-3)^2 = 9$.

Problem 7 (10 pts)**.** Sketch the graph and find a Cartesian equation for the curve given in polar coordinates by $r\cos\theta = -4$.

Problem 8 (10 pts)**.** Consider the region in the xy–plane enclosed by the cardioid with polar equation $r = 2(1 + \cos\theta)$ (assume $0 \leq \theta \leq 2\pi$). Express its area with an integral in terms of the parameter θ, and compute its value.

Problem 9 (10 pts)**.** Consider the cardioid $r = 1 - \cos\theta$ (assume again $0 \leq \theta \leq 2\pi$). Express its length as an integral of the parameter θ, and compute its value.

Problem 10 (10 pts)**.** Find the area and length of one of the leaves of the eight-leaved rose $r = \cos 4\theta$. Sketch the graph, and make sure to label all intersections with the x and y–axis.