

Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has five (5) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

| Page | Max. points | Your points |
|--------------|-------------|-------------|
| 2 | 20 | |
| 3 | 30 | |
| 4 | 20 | |
| 5 | 30 | |
| Total | 100 | |

Problem 1 (10 pts). Evaluate each limit:

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\tan x} = \boxed{}$$

$$\lim_{x \rightarrow 0} \frac{\tan(5x^2)}{x^2} = \boxed{}$$

Problem 2 (10 pts). Find an equation of the normal line to the curve $y = \ln(xe^{x^2})$ at the point $(1, 1)$.

Problem 3 (30 pts). Sketch the graph of the rational function $f(x) = \frac{2x^2 - 8}{x^2 - 16}$.

Indicate clearly:

- Domain
- x - and y -intercepts.
- Vertical and horizontal asymptotes (any holes?).
- Intervals of increase, decrease and different concavity.
- Location of relative extrema and inflection points.

Problem 4 (10 pts). Find the absolute extrema of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$.

Absolute maxima at

Absolute minima at

Problem 5 (10 pts). Use logarithmic differentiation to find the derivative of the function

$$y = \frac{\sin^2 x \tan^4 x}{(x^2 + 1)^2}$$

$$\frac{dy}{dx} =$$

Problem 6 (10 pts). An aircraft is climbing at 30° angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi/h?

The aircraft is gaining altitude at a speed of

Problem 7 (20 pts). A container with square base, vertical sides, and open top is to be made from 300 ft^2 of material. Find the dimensions of the container with greatest volume.

Dimensions of container: