Name: _____

VIP ID: _____

- Write your name and VIP ID in the space provided above.

- The test has four (4) pages, including this one.

- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.

- Credit for each problem is given in parentheses at the right of the problem number.

Page	Max. points	Your points
2	50	
3	30	
4	20	
Total	100	

Problem 1 (50 pts—10 pts each part). Consider the 2nd-degree polynomial
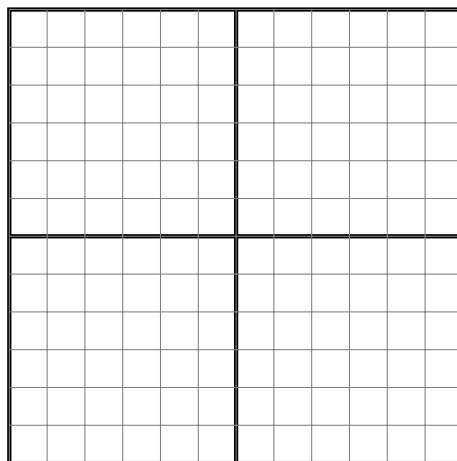
$$p_2(x, y) = 4x^2 + 25y^2 - 20xy.$$

(a) The polynomial p_2 is a quadratic form. Find a symmetric matrix A so that

$$p_2(x, y) = \mathcal{Q}_A(x, y).$$

(b) Classify the symmetric matrix A.

(c) Sketch the level line $p_2(x, y) = 0$.

(d) Is f a coercive function? Why?

(e) Find all critical points of p_2, and classify them.

Problem 2 (30 pts—10 pts each part). Consider the function

$$f(x, y, z) = x^2 + y^2 + z^2 + \frac{1}{x^2 + y^2 + z^2}$$

(a) Is f a convex function? Why?

(b) What is the global minimum value of f? Why?

(c) Find all global minima of f.

Problem 3 (20 pts). Consider the function $f(x, y) = x^3 + e^{3y} - 3xe^y$. Show that f has exactly one critical point, and that this point is a local minimum but not a global minimum.