We are going to define two operations that can be performed on vector fields. Each operation resembles differentiation, but one produces a vector field, whereas the other produces a scalar field.

**Definition.** If $\boldsymbol{F} = \langle P, Q, R\rangle$ is a vector field on $\mathbb{R}^3$ and the partial derivatives of $P$, $Q$ and $R$ all exists, then the *curl* of $\boldsymbol{F}$ is the vector field on $\mathbb{R}^3$ defined by

$$\operatorname{curl} \boldsymbol{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\boldsymbol{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\boldsymbol{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\boldsymbol{k}.$$

The *divergence* of $\boldsymbol{F}$ is the function of three variables defined by

$$\operatorname{div} \boldsymbol{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.$$

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**Problem 1** (5 pts)**.** Assume that $f$ is a function of three variables that satisfies

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}, \quad \frac{\partial^2 f}{\partial x \partial z} = \frac{\partial^2 f}{\partial z \partial x}, \quad \frac{\partial^2 f}{\partial z \partial y} = \frac{\partial^2 f}{\partial y \partial z}.$$

Prove that in this case, $\operatorname{curl}(\nabla f) = \mathbf{0}$.

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**Problem 2** (5 pts)**.** Note that the previous problem implies that conservative vector fields will have zero curl. Determine, using this trick, whether $\boldsymbol{F} = \langle y^2z^3, 2xyz^3, 3xy^2z^2\rangle$ is a conservative vector field or not.

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**Problem 3** (15 pts)**.** For the vector field $\boldsymbol{F} = \cos(xz)\boldsymbol{j} - \sin(xy)\boldsymbol{k}$, compute:

(a) $\operatorname{curl} \boldsymbol{F}$
(b) $\operatorname{div} \boldsymbol{F}$
(c) $\operatorname{div} \operatorname{curl} \boldsymbol{F}$

*Use the back of this page for this problem*