| Name:         |  |
|---------------|--|
| 4-digit code: |  |

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has six (6) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

| Page  | Max. points | Your points |
|-------|-------------|-------------|
| 2     | 20          |             |
| 3     | 15          |             |
| 4     | 20          |             |
| 5     | 25          |             |
| 6     | 20          |             |
| Total | 100         |             |

**Problem 1** (10 pts). Find a formula for the general term of the following sequences:

(a)  $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \dots$ 

 $x_n =$ 

(b)  $1 - \frac{1}{2}, \frac{1}{3} - \frac{1}{2}, \frac{1}{3} - \frac{1}{4}, \frac{1}{5} - \frac{1}{4}, \dots$ 

 $x_n =$ 

**Problem 2** (10pts). Write out the first five terms of the sequence  $\left\{\frac{\ln n}{n}\right\}_{n=1}^{\infty}$  Determine whether the sequence converges, and if so find its limit.

First five terms:

 $\lim_{n \to \infty} x_n = \boxed{}$ 

**Problem 3** (5 pts). Use  $x_{n+1} - x_n$  to show that the sequence  $\{n - n^2\}_{n=1}^{\infty}$  is strictly increasing or strictly decreasing.

**Problem 4** (5 pts). Use  $x_{n+1}/x_n$  to show that the sequence  $\{ne^{-n}\}_{n=1}^{\infty}$  is strictly increasing or strictly decreasing.

**Problem 5** (5 pts). Use **differentiation** to show that the sequence  $\left\{3 - \frac{1}{n}\right\}_{n=1}^{\infty}$  is strictly increasing or strictly decreasing.

**Problem 6** (20 pts). Determine whether the series converge, and if so find their sum:

(a) 
$$\sum_{k=1}^{\infty} \left(-\frac{3}{2}\right)^{k+1}$$

$$\sum_{k=1}^{\infty} \left( -\frac{3}{2} \right)^{k+1} =$$

(b) 
$$\sum_{k=1}^{\infty} \left( \frac{1}{2^k} - \frac{1}{2^{k+1}} \right)$$

$$\sum_{k=1}^{\infty} \left( \frac{1}{2^k} - \frac{1}{2^{k+1}} \right) = \boxed{}$$

**Problem 7** (5 pts). Apply the **divergence test** and state what it tells you about the series.

$$\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)^k.$$

**Problem 8** (10 pts). Use the **integral test** to determine whether the series  $\sum_{k=1}^{\infty} \frac{1}{1+9k^2}$  converges.

**Problem 9** (10 pts). Use the **ratio test** to determine whether the series  $\sum_{k=1}^{\infty} \frac{3^k}{k!}$  converges. If the test is inconclusive, then say so.

**Problem 10** (10 pts). Use the **root test** to determine whether the series  $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$  converges. If the test is inconclusive, then say so.

**Problem 11** (10 pts). Classify the series  $\sum_{k=1}^{\infty} \frac{k \cos k\pi}{k^2 + 1}$  as absolutely convergent, convergent or divergent.