

Math 242 Test 2, Monday 20 October

Name:

Last 4 digits of SSN:

Show all work **clearly, make sentences**. No work means no credit. The points are:

ex1: 25, ex2: 20, ex3: 20, ex4: 15, ex5: 20.

Exercise 1 We give the differential equation:

$$\frac{dx}{dt} = 2x^2 - 10x + 12.$$

1. What are the critical points ? Use a phase diagram to determine whether each critical point is stable or unstable.

2. Solve this differential equation with $x_0 = 1$.

Exercise 2 We give an initial value problem and its exact solution $y(x)$:

$$y' = 2xy^2, \quad y(0) = 1, \quad y(x) = \frac{1}{1 - x^2}.$$

Apply Euler's method to approximate the solution on the interval $[0, 0.4]$ with step size $h = 0.1$. Write the formula you use for the computation. Then compare the four-decimal-place values of the approximate solution with the values of the exact solution using the following array. Does this step size look good ?

x	0	0.1	0.2	0.3	0.4
approx solution					
exact solution					

Exercise 3 Solve the differential equation:

$$y^{(3)} - 9y'' + 24y' - 20y = 0,$$

using the fact that the function $x \mapsto e^{5x}$ is solution of this differential equation. Then find the unique solution satisfying the initial conditions:

$$y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -1.$$

Exercise 4 Find a linear homogeneous constant-coefficient equation with the general solution:

$$y(x) = Ae^{-3x} + B \cos(2x) + C \sin(2x) + x(D \cos(2x) + E \sin(2x)).$$

Exercise 5 Solve the initial value problem, where y_p is a particular solution of the differential equation:

$$y'' + 2y' - 8y = 14e^{3x}, \quad y(0) = 3, \quad y'(0) = 2 \quad y_p(x) = 2e^{3x}.$$