Name:	
4-digit code:	

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has six (6) pages, including this one.
- Enter your answers in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	Max. points	Your points
2	20	
3	20	
4	20	
5	20	
6	20	
Total	100	

Spring 2015

Page 2/6

**Problem 1** (5 pts—all or nothing). Find the domain of  $f(x) = \sqrt{(1-x)(2-x)}$ .

domain =

**Problem 2** (5 pts—all or nothing). Let  $f(x) = x^2 + 4$ ,  $g(x) = \sqrt{x}$ . Find  $g \circ f$ 

 $(g \circ f)(x) =$ 

**Problem 3** (5pts). Evaluate the following limit:

 $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{5k}{n^2}$ 

**Problem 4** (5 pts). Assume y is a function of x given implicitly by  $\sin(x+y) = x+y$ . Find y'.

**Problem 5** (10 pts). Use logarithmic differentiation to find the derivative of the function

$$y = \frac{\tan^2 x \sin^4 x}{e^{3x}(x^2 + 1)}$$

$$y' =$$

**Problem 6** (10 pts). Find an equation of the tangent line to the curve  $y = \ln(xe^x)$  at the point (1,1).

**Problem 7** (20 pts). Sketch the graph of the rational function  $f(x) = \frac{x^2}{1 - x^2}$ .

Indicate clearly: Domain; x- and y-intercepts; vertical and horizontal asymptotes; intervals of increase, decrease and different concavity. Indicate also the location of relative extrema, if any.

Page 5/6

**Problem 9** (10 pts). A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

Dimensions of most economic fence:

**Problem 10** (1/2/3/4 pts). Compute the following limits:

(a) 
$$\lim_{x \to -\infty} \frac{x^2 - 2x - 8}{x^2 - 4} =$$

(b) 
$$\lim_{x\to 2} \frac{x^2 + 2x - 8}{x^2 - 4} =$$

(c) 
$$\lim_{x \to -2} \frac{x^2 - 2x - 8}{x^2 - 4} =$$

(d) 
$$\lim_{x \to 0} x^{1/x} =$$

**Problem 11** (3/3/4 pts). Evaluate each integral:

(a) 
$$\int \left(\frac{1}{x} - 2^x\right) dx =$$

(b) 
$$\int (3\sin x - 2\cos x) dx =$$

(c) 
$$\int_{1}^{2} \left(5x + \frac{2}{3x^5} - \sqrt{2}e^x\right) dx =$$