

**Name:** \_\_\_\_\_

**VIP ID:** \_\_\_\_\_

- Write your name and VIP ID in the space provided above.
- The test has seven (7) pages, including this one.
- Enter your answer in the box(es) provided.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given at the right of each problem number.

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Page	Max	Points
2	20	
3	10	
4	20	
5	20	
6	20	
7	10	
<b>Total</b>	100	

Skills tested on this page:

Terminology	Notation	Background Algebra
Derivatives	Integration	Management of constants

**Problem 1** (20 pts). Consider the following *first-order separable* differential equation:

$$y' = x(y + 1)$$

(a) [5 pts] Find an *implicit form* of its **general solution**.

(b) [5 pts] Find a *particular solution* that solves the following **initial value problem**

$$y' = x(y + 1), \quad y(0) = e - 1$$

$y =$

(c) [5 pts] Are there any **singular solutions**? Find at least one.

$y =$

(d) [5 pts] Find an explicit *particular solution* that solves this other **initial value problem**

$$y' = x(y + 1), \quad y(0) = -1$$

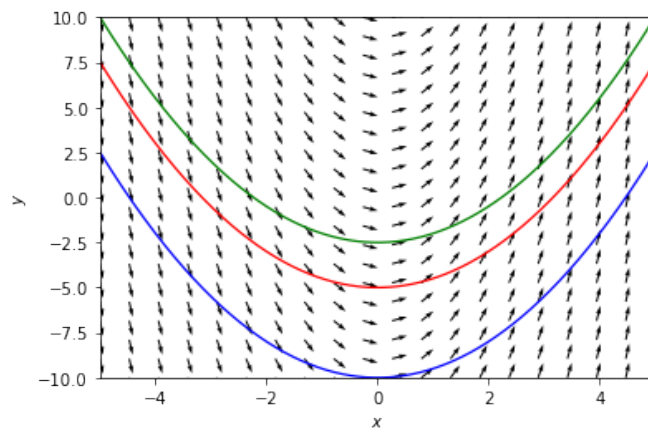
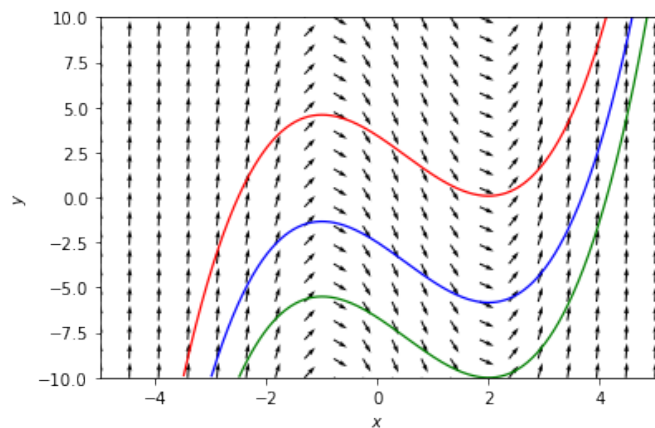
$y =$

## Skills on this page: Slope fields, Euler's Method

**Problem 2** (5 pts—all or nothing). Which of the following is the slope field of the following differential equation?

$$y' = x^2 - x - 2$$

(You do not need to show work)



**Problem 3** (5 pts). Use Euler's method with a step size  $h = 0.5$  to obtain a numerical approximation of the following **initial value problem**

$$y' = x^2 y, \quad y(0) = 1$$

$n$	$x_n$	$y_n$	$f(x_n, y_n)$
0			
1			
2			

**Skills on this page: First-order differential equations**

**Problem 4** (10 pts). Solve the following differential equation

$$(e^x + ye^{xy}) + (e^y + xe^{xy})y' = 0$$

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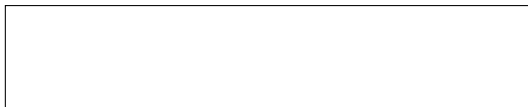
**Problem 5** (10 pts). Solve the following differential equation:

$$xy' = \sqrt{x^2 - y^2} + y$$

**Skills on this page: First-order differential equations**

**Problem 6** (20 pts). Solve the following differential equation:

$$2x^2y - x^3y' = y^3$$



**Skills on this page: Second-order differential equations**

**Problem 7** (10 pts). Solve the following initial value problem:

$$y'' + 4y = 2x - 3, \quad y(0) = 1, \quad y'(0) = 2$$

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**Problem 8** (10 pts). Solve the following differential equation (assume  $y, y' > 0$ ):

$$yy'' + (y')^2 = 0$$

**Skills on this page: Second-order differential equations**

**Problem 9** (10 pts—5 pts each part). Given the differential equation  $x^2y'' - 2xy' + 2y = 0$ ,

- (a) Verify that the functions  $y_1 = x$  and  $y_2 = x^2$  are particular solutions.
- (b) Find a particular solution if initial conditions are given by  $y(1) = 3, y'(1) = 1$ .