Math 141—Make-up Final Exam Cameron Kahn (Section 012)

Answer each problem completely and show all work in the space provided to get full credit. Carefully read the directions for each problem. No books, notes or calculator allowed during the exam. You have 150 minutes to complete the test.

Problem 1. (5 pts each) Evaluate the following limits. If the limit is infinite, or does not exist, explain why.

(1) $\lim\limits_{x \to -1} \dfrac{x^3 - x^2}{x - 1}$

(2) $\lim\limits_{x \to 1} \dfrac{x^3 - x^2}{x - 1}$

(3) $\lim\limits_{x \to 2^-} \dfrac{1 - x}{x - 2}$

(4) $\lim_{x \to +\infty} \frac{2x^3 + 4x^2 + 1}{x^4 + 2x^2 - 10}$

(5) $\lim_{x \to +\infty} \left(1 + \frac{1}{2x}\right)^{6x}$

(6) $\lim_{x \to +\infty} \frac{\ln(x)}{x}$

Problem 2. (10 pts) Is the function

$$f(x) = \begin{cases} 2x+3 & x \le 4 \\ 7 + 16/x & x > 4 \end{cases}$$

continuous at $x = 4$? (Explain your answer using limits...)

Problem 3. (10 pts) Use the definition of the derivative to find the derivative of the function $f(x) = x^3 - 2x$.

Problem 4. (10 pts) How many tangent lines to the curve $y = x/(x+1)$ pass through the point $(0, 0)$?

Problem 5. (5 pts each) Find $\dfrac{dy}{dx}$

(1) $y = \arctan(x^2)$

(2) $x^3 + y^3 = 3xy$,

(3) $y = \dfrac{x^3 e^x}{\sqrt{x^2+1}}$

Problem 6. (5 pts each) Find derivatives of the following functions.

(1) $f(x) = (\ln(x))^2$

(2) $g(x) = x^2 e^{x^2}$

(3) $j(x) = \dfrac{x^2 + 1}{2x + \sin(x)}$

Problem 7. (10 pts) A 13 foot ladder is leaning against a wall. The top of the ladder is slipping down the wall at a constant rate of 2 ft/sec. How fast is the bottom of the ladder moving away from the wall when the top of the ladder is 5 feet from the ground?

Problem 8. (10 pts) Find the absolute maximum and absolute minimum of $g(x) = x + \dfrac{1}{x}$ on the interval $(0, +\infty)$. If one or both does not exist, explain why.

Problem 9. (15 pts) Find the intervals of increase and decrease, all relative maxima and minima, intervals of concavity, and inflection points for the function $f(x) = 3x^4 - 4x^3$.

Problem 10. (10 pts) A field is shaped like a right triangle, with the hypotenuse lying along a straight stream. The other two sides are made of 1000 feet of fencing. What is the largest area possible for such a field? (Be sure to clearly state what function you are maximizing, and the domain.)

Problem 11. (10 pts) Write the limit-sum definition of the integral $\int_1^7 1/x\,dx$ using right hand endpoints.

Problem 12. (6 pts each) Evaluate the following integrals.

(1) $\int x^{-2/3} - 5e^x \, dx$

(2) $\int_0^{\pi/4} \tan(x) \sec^2(x) \, dx$

(3) $\int \frac{x+1}{2x^2+4x} \, dx$

(4) $\int_{-1}^{2} x(1+x^3) \, dx$

(5) $\int \frac{e^x}{\sqrt{1-e^{2x}}} \, dx$ Hint: $e^{2x} = (e^x)^2$.