Name: ______________________________

4-digit code: ______________________________

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has eleven (17) pages, including this one, and your help sheet.
- For multi-choice questions, you should circle the answer you select. On the other problems, you should enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.
- **A:** 243–270 pts. **B+:** 230–242 pts. **B:** 216–229 pts. **C+:** 203–215 pts. **C:** 189–202 pts. **D+:** 175–188 pts. **D:** 160–174 pts. **F**: less than 160 pts.

Page	**Max**	**Points**	**Page**	**Max**	**Points**	**Page**	**Max**	**Points**
2	20		7	40		7	20	
3	20		8	20		8	15	
4	20		9	10		9	20	
5	25		10	10		10	25	
6	15		11	20		11	20	
Total	100		**Total**	100		**Total**	100	

Problem 1 (10 pts)**.** Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.

$A =$ $\boxed{}$

Problem 2 (10 pts)**.** Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval $[1, 4]$ is revolved about the x-axis.

$V =$ $\boxed{}$

Problem 3 (10 pts)**.** Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$ and $x = 0$ is revolved about the y-axis.

$V =$ $\boxed{}$

Problem 4 (10 pts)**.** Find the arclength of the curve $y = x^{3/2}$ from $x = 1$ to $x = 2$.

$L =$ $\boxed{}$

Problem 5 (10 pts)**.** Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

$A =$ $\boxed{}$

Problem 6 (10 pts)**.** Find the average value of the function $f(x) = 1/x$ over the interval $[1, e]$.

$f_{ave} =$ $\boxed{}$

Problem 7 (15 pts)**.** Find a positive value of k such that the average value of $f(x) = \dfrac{1}{\sqrt{k^2 - x^2}}$ over the interval $[-k, k]$ is π.

You may find the following table useful:

angle θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

$k =$ $\boxed{}$

Problem 8 (10 pts)**.** Evaluate the integral $\displaystyle\int x^2\sqrt{x-1}\,dx$.

Problem 9 (15 pts)**.** A spring exerts a force of $4N$ when stretched 2 m beyond its natural length.

(a) How much work was performed in stretching the spring to this length?

$W =$ $\boxed{}$

(b) How far beyond its natural length can the spring be stretched with $36J$ of work?

$b =$ $\boxed{}$

Problem 10 (40 pts)**.** Evaluate each integral:

(a) $\displaystyle\int \csc^2 x\, dx =$ $\boxed{}$

(b) $\displaystyle\int \frac{1}{\csc x}\, dx =$ $\boxed{}$

(c) $\displaystyle\int \frac{x+1}{x}\, dx =$ $\boxed{}$

(d) $\displaystyle\int \frac{x}{x+1}\, dx =$ $\boxed{}$

Problem 11 (10 pts)**.** Use **integration by parts** to evaluate the integral $\int xe^{2x}\,dx$.

$$\int xe^{2x}\,dx = \boxed{}$$

Problem 12 (10 pts)**.** Evaluate the improper integral $\displaystyle\int_1^\infty \frac{dx}{x^3}$.

$$\int_1^\infty \frac{dx}{x^3} = \boxed{}$$

Problem 13 (10 pts)**.** Use a **trigonometric substitution** to evaluate the integral $\int \frac{dx}{\sqrt{x^2-9}}$.

$$\int \frac{dx}{\sqrt{x^2-9}} = \boxed{}$$

Problem 14 (10 pts)**.** Evaluate the integral $\int \sin^2 x \cos^2 x\, dx$.
Use trigonometric simplification and one of the following reduction formulas.

$$\int \sin^n x\, dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\, dx$$
$$\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\, dx$$

$$\int \sin^2 x \cos^2 x\, dx = \boxed{}$$

Problem 15 (20 pts)**.** Use **partial fractions** to evaluate the integral $\displaystyle\int \frac{dx}{x^2+x-2}$.

$$\int \frac{dx}{x^2+x-2} = \boxed{}$$

Problem 16 (10 pts)**.** Find a formula for the general term of the following sequences:

(a) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$

$x_n = \boxed{}$

(b) $1 - \frac{1}{2}, \frac{1}{3} - \frac{1}{2}, \frac{1}{3} - \frac{1}{4}, \frac{1}{5} - \frac{1}{4}, \ldots$

$x_n = \boxed{}$

Problem 17 (10pts)**.** Write out the first five terms of the sequence $\left\{ \frac{(-1)^{n+1}}{n^2} \right\}_{n=1}^{\infty}$ Determine whether the sequence converges, and if so find its limit.

First five terms: $\boxed{}$

$\lim_{n\to\infty} x_n = \boxed{}$

Problem 18 (5 pts)**.** Use $x_{n+1} - x_n$ to show that the sequence $\left\{n - 2^n\right\}_{n=1}^{\infty}$ is strictly increasing or strictly decreasing.

Problem 19 (5 pts)**.** Use x_{n+1}/x_n to show that the sequence $\left\{\dfrac{n^n}{n!}\right\}_{n=1}^{\infty}$ is strictly increasing or strictly decreasing.

Problem 20 (5 pts)**.** Use **differentiation** to show that the sequence $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{\infty}$ is strictly increasing or strictly decreasing.

Problem 21 (20 pts)**.** Determine whether the series converge, and if so find their sum:

(a) $\displaystyle\sum_{k=1}^{\infty}\Big(-\frac{3}{4}\Big)^{k-1}$

$$\sum_{k=1}^{\infty}\Big(-\frac{3}{4}\Big)^{k-1} = \boxed{}$$

(b) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)}$

$$\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)} = \boxed{}$$

Problem 22 (5 pts)**.** Apply the **divergence test** and state what it tells you about the series.

$$\sum_{k=1}^{\infty} \frac{k^2+k+3}{2k^2+1}.$$

Problem 23 (10 pts)**.** Use the **integral test** to determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{5k+2}$ converges.

Problem 24 (10 pts)**.** Use the **ratio test** to determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{4^k}{k^2}$ converges. If the test is inconclusive, then say so.

Problem 25 (10 pts)**.** Use the **root test** to determine whether the series $\sum_{k=1}^{\infty}\left(\frac{3k+2}{2k-1}\right)^k$ converges. If the test is inconclusive, then say so.

Problem 26 (10 pts)**.** Classify the series $\sum_{k=1}^{\infty}(-1)^k\,\frac{4k^2+1}{k^3+2}$ as absolutely convergent, convergent or divergent.