Math 242 Test 3, Thursday 16 April

Name: **Last 4 digits of SSN:**

Show all work **clearly**, **make sentences**. No work means no credit. The points are:
ex1: 18, ex2: 17, ex3: 15, ex4: 16, ex5: 17,ex6: 17.

Exercise 1
Give the form of a particular solution in each case, but DO NOT determine the values of the coefficients:

i) $y^{(13)} + 2y' = (7x^5 + x^2 + 1)$,

ii) $y^{(3)} - y'' - y' + y = (x^3 - 3x + 2)e^x + 2x + 1$,

iii) $y^{(3)} - y'' - y' + y = e^x(x^2 - 8)\cos(13x)$.

Exercise 2
Find a particular solution of the differential equation

$$y'' - 3y' + 2y = e^{-x}(12x + 8).$$

Then solve completely this differential equation with the initial values $y(0) = 3$ and $y'(0) = 6$.

Exercise 3 (Method of variation of parameters in the case $n = 2$)

1) We consider the second-order linear differential equation

$$y'' + P(x)y' + Q(x)y = f(x),$$

where P, Q and f are continuous. A complementary solution is given by:

$$y_c(x) = c_1 y_1(x) + c_2 y_2(x),$$

where c_1 and c_2 are constants.
What will be the form of a particular solution ? To find this solution, what system of equations, with unknown c_1' and c_2', do we have to solve ?

2) Apply this method to find a particular solution of the differential equation

$$y'' + 9y = 2\sec 3x.$$

Remark: you may use that $\int \tan 3x dx = -1/3 \ln|\cos 3x| + C$.

Exercise 4 (Laplace Transform of $\sin kt$ and $\cos kt$)

1) *a*) Consider the function $f(t) = \sin kt$. Write the differential equation satisfied by f (you will need to differentiate two times). What are the initial conditions satisfied by this function f?

b) Deduce the Laplace transform of $\sin kt$ (don't use the table of Laplace transform !!).

2) Use the Laplace transform of a derivate to deduce the Laplace transform of $\cos kt$.

Exercise 5

Find the inverse Laplace transform of the following functions:

$$F_1(s) = \frac{5}{s^{5/2}} + \frac{4}{s-5}, \quad F_2(s) = \frac{7s-3}{s^2+4}, \quad F_3(s) = \frac{1}{s(s-2)}.$$

Remark: you will use that $\Gamma(1/2) = \sqrt{\pi}$.

Exercise 6
Solve the initial value problem using the Laplace transform:

$$x'' - x - 6x = 0, \quad x(0) = 2, x'(0) = -1.$$

(To find the inverse Laplace transform, you will use the partial fraction as seen with the logistic equation).