

Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has five (5) pages, including this one and the table of Laplace transforms at the end.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given at the right of each problem number.
- No books, notes or calculators may be used on this test.

Page	Max	Points
2	35	
3	30	
4	35	
Total	100	

Problem 1 (20 pts). Use exclusively techniques based on the Laplace transform to solve the initial value problem $x'' + 3x' + 2x = t$ that satisfies $x(0) = 0, x'(0) = 2$.

Problem 2 (15pts). Find the Laplace transform of $f(x) = \sin 3x \cos 3x$.

Problem 3 (20 pts). Use exclusively the technique of variation of parameters to solve the differential equation $y'' + 3y' + 2y = x$.

Problem 4 (10pts). Find the Laplace transform of $f(x) = \sin(x)/x$.

Problem 5 (15 pts). Use exclusively the method of undetermined coefficients to solve the differential equation $\frac{d^2 f}{d\omega^2}(\omega) + 3\frac{df}{d\omega}(\omega) + 2f(\omega) = \omega$.

Problem 6 (20). Find the inverse Laplace transform of $F(s) = (s^2 + b^2)^{-2}$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. $t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > a $
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > a $
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
11. $t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$