Math 242 Test 3, Friday 21 April

Name: **Last 4 digits of SSN:**

Show all work **clearly**, **make sentences**. No work means no credit. The points are:
ex1: 20, ex2: 20, ex3: 20, ex4: 10, ex5: 30.

Exercise 1 (Particular solutions)

1) Method of variation of parameters in the case $n = 2$:
   We consider the second-order linear differential equation

$$y'' + P(x)y' + Q(x)y = f(x),$$

   where P, Q and f are continuous. A general solution is given by:

$$y_c(x) = c_1 y_1(x) + c_2 y_2(x),$$

   where c_1 and c_2 are constants.
   What will be the form of a particular solution ? To find this solution, what system of equations, with unknown c_1' and c_2', do we have to solve ?

2) Find the form of a particular solution in each cases, but DO NOT determine the coefficients.

   (a) $y^{(13)} - 9y'' = x^4 - x + 41,$

   (b) $y^{(3)} + y'' + 9y = (x^2 - 1)e^x + 3x,$

(c) $y'' - 2y' + 17y = 5e^x(5x - 8)\sin(4x)$.

Exercise 2

Find the inverse Laplace transform of the following functions:

$$a)\ F(s) = \frac{2s+4}{s^2+36}, \qquad b)\ G(s) = \frac{5s+7}{s^2-8s+25}.$$

Exercise 3
Use the theorem of differentiation of Laplace transform to find the Laplace transform of

$$f(t) = te^{-3t}\sin(2t),$$

and to find the inverse Laplace transform of

$$F(s) = \ln\left(1 + \frac{1}{s^2}\right).$$

Exercise 4
Solve the initial value problem using Laplace transform:

$$x'' + x = 0, \quad x(0) = 0 \text{ and } x'(0) = \pi.$$

Exercise 5
Solve the initial value problem using the Laplace transform:

$$x^{(3)} - x'' + 4x' - 4x = 15e^t, \qquad x(0) = 0,\ x'(0) = 6 \text{ and } x''(0) = 16.$$

You will use that 1 is a root of the characteristic equation.