Math 242 Test 2, Friday 2 November

Name: **Last 4 digits of SSN:**

Show all work clearly, **MAKE SENTENCES**. No work means no credit. The points
are:
ex1: 25, ex2: 25, ex3: 20, ex4: 15 and the course questions are over 15 points.

Course Questions

1. Write the the algorithm of the Euler's method and precise what is the order of the
   cumulative error.

2. Write the definition of the linearly dependence and independence of n functions.
   What is the characterization with the Wronskian ?

3. Write a general solution of a nonhomogeneous linear differential equation (use the
   sum of two functions).

Exercise 1 We give the differential equation:

$$\frac{dx}{dt} = 6x - 2x^2.$$

1. How do we call this differential equation ?

2. What are the critical points ? Use a phase diagram to determine wether each critical point is stable or unstable.

3. Solve this differential equation with $x_0 = 1$.

Exercise 2 Solve the differential equation:

$$y^{(3)} + y'' - y' + 15y = 0,$$

using the fact that the function $x \mapsto e^{-3x}$ is solution of this differential equation. Then find the unique solution satisfying the initial conditions:

$$y(0) = 0, \; y'(0) = 1, \; y''(0) = 3.$$

Exercise 3 Solve the initial value problem:

$$y^{(3)} - 4y'' + 4y' = 0, \quad y(0) = 0, \ y'(0) = 1, \ y''(0) = 2.$$

Exercise 4 Find a linear homogeneous constant-coefficient equation with the general solution:

$$y(x) = (A + Bx + Cx^2)e^x + B\cos(3x) + C\sin(3x).$$

The solution requires characteristic roots $r = 1$ (multiplicity 3) and $r = \pm 3i$, giving the characteristic polynomial

$$(r-1)^3(r^2+9) = r^5 - 3r^4 + 12r^3 - 28r^2 + 27r - 9 = 0.$$

Therefore a corresponding equation is

$$y^{(5)} - 3y^{(4)} + 12y''' - 28y'' + 27y' - 9y = 0.$$