Rules

There are 10 problems assigned in this session, each of them worth 20 points.

(a) You must submit **only five of those problems.**

(b) But beware: if two or more students choose exactly the same five problems, then all those students will receive a 50-point penalty. You must therefore coordinate with every single other student in the class, to avoid penalties.

For extra credit (10 points): how many possible selections are there?

(c) You are also required not to submit any three consecutive problems. For instance, it is acceptable to submit problems 1, 2, 4, 5 and 9. It is not acceptable to submit problems 2, 3, 4, 6 and 9 (since problems 2, 3, 4 are consecutive).

For extra credit (10 points): How many possible selections are there with this extra constraint?

The Island of Knights and Knaves[1]

In the Island of Knights and Knaves, "knights" always tell the truth, and "knaves" always lie. Every inhabitant of the island is either a knight or a knave. You are a visitor.

Sample Problem. Two of the inhabitants, A and B, standing together in a garden. You ask A, "Are you a knight or a knave?" A answered, but rather indistinctly and you cannot make out what he said. You then ask B, "What did A say?" B replies "A said that he is a knave." What is B? Can you determine what A is?

Solution: A can never say "I am a knave." (Why?) This implies that B must be a knave. We cannot determine whether A is a knight or a knave with the given information. □

Three people A, B, C in a garden

Problem 1. Three people, A, B, C, each of whom is either a knight or a knave. A and B make the following statements:

A: All of us are knaves.

B: Exactly one of us is a knight.

What are A, B, C?

Problem 2. Suppose instead, A and B say the following:

A: All of us a knaves.

B: Exactly one of us is a knave.

Can it be determined what B is? Can it be determined what C is?

Problem 3. Two people are said to be of the *same type* if they are both knights or both knaves. A and B make the following statements:

A: B is a knave.

B: A and C are of the same type.

What is C?

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[1] All problems in this assignment come from Robert Smullian's *What is the Name of this Book?*, Chapter 3.

Two people A,B in a garden

Problem 4. A makes the following statement: "At least one of us is a knave." What are A and B?

Remark. *Recall the definition of* exclusive OR *(`XOR`), and the difference with* OR*:*

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Problem 5. Suppose A says, "Either I am a knave or B is a knight." What are A and B?

Problem 6. Suppose A says, "Either I am a knave or else two plus two equals five." What would you conclude?

Knights, Knaves and Normals

Three types of people in a new island: knights, who always tell the truth; knaves, who always lie; and normal people, who sometimes lie and sometimes tell the truth. On this island, knaves are said to be of the *lowest rank*, normals of *middle rank*, and knights of *highest rank*.

Problem 7. We are given three people, A, B, C, one of whom is a knight, one a knave, and one normal (but not necessarily in that order). They make the following statements:

A: I am normal.

B: That is true.

C: I am not normal.

What are A, B, and C?

Problem 8. Given two people A, B, each of whom is a knight, a knave or a normal, they make the following statements:

A: I am of lower rank than B.

B: That's not true!

Can the ranks of either A or B be determined? Can it be determined of these statements whether it is true or false?

Problem 9. Given three people A, B, C, one of whom is a knight, one a knave, and one normal. A, B make the following statements:

A: B is of higher rank than C.

B: C is of higher rank than A.

Then C is asked: "Who has a higher rank, A or B?" What does C answer?

The Island of Bahava

In the island of Bahava women are also called knights, knaves or normal. A knight can only marry a knave, and a knave can only marry a knight. (Hence a normal can only marry a normal.)

Problem 10. Consider a married couple, Mr. and Mrs. A. They make the following statements:

Mr. A: My wife is not normal.

Mrs. A: My husband is not normal.

What are Mr. and Mrs. A?