Name: ______________________________

VIP ID: ______________________________

- Write your name and your VIP ID in the space provided above.
- The test has twelve (12) pages, including this one.
- You have 150 minutes to complete it.
- Each problem is worth 5 points.
- There is an extra page containing a extra credit problem worth 20 points.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- No books, notes or calculators may be used on this test.

Problem 1 (5 pts)**.** Find the distance d from the point $(3, 7, -5)$ to the z–axis.

$d =$ $\boxed{}$

Problem 2 (5 pts)**.** Find an exact expression for the angle θ between the vectors $\boldsymbol{v} = \langle 3, -1, 5\rangle$ and $\boldsymbol{w} = \langle -2, 4, 3\rangle$.

$\theta =$ $\boxed{}$

Problem 3 (5 pts)**.** Find the length ℓ of the curve $\boldsymbol{r}(t) = \boldsymbol{i} + t^2\boldsymbol{j} + t^3\boldsymbol{k}$ for $0 \leq t \leq 1$.

$\ell =$ $\boxed{}$

Problem 4 (5 pts)**.** At what points does the helix $\boldsymbol{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

points: $\boxed{}$

Problem 5 (5 pts)**.** Find a unit vector $\boldsymbol{v}$ that is orthogonal to both $\boldsymbol{i}+\boldsymbol{j}$ and $\boldsymbol{i}+\boldsymbol{k}$.

$\boldsymbol{v} =$ $\boxed{}$

Problem 6 (5 pts)**.** Determine whether the points $A = (0, -5, 5)$, $B = (1, -2, 4)$ and $C = (3, 4, 2)$ lie on a straight line.

Problem 7 (5 pts)**.** Find parametric equations for the line of intersections of the planes $x+y+z=1$ and $x+2y+2z=1$.

$\theta =$ $\boxed{}$

$\boxed{}$

Problem 8 (5 pts)**.** Sketch the domain of $f(x,y) = \dfrac{\sqrt{4-x^2}}{y^2+3}$.

Problem 9 (5 pts)**.** Evaluate the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4+y^4}$$

Problem 10 (5 pts)**.** Find the first partial derivatives of the function $f(x, y, z, t) = \frac{xy^2}{t+2z}$

Problem 11 (5 pts)**.** Use the method of Lagrange multipliers to find the dimensions of a rectangle with perimeter 250 cm and maximum area.

width: $\boxed{}$

height: $\boxed{}$

Problem 12 (5 pts)**.** Recall the formula for the volume of a right circular cone of radius r and height h. Suppose that the height decreases from 20 to 19.95 inches, and the radius increases from 4 to 4.05 inches. Compare the change in volume of the cone with an approximation of this change using a total differential.

$dV =$ $\boxed{}$

$\Delta V =$ $\boxed{}$

Problem 13 (5 pts)**.** Find an equation for the tangent plane to the surface $z = xe^{-y}$ at the point $P = (1, 0, 1)$.

tangent plane: $\boxed{}$

Problem 14 (5 pts)**.** Find the absolute extrema of the function $f(x, y) = xy - x - 3y$ on the triangular region R with vertices $(0, 0)$, $(0, 4)$ and $(5, 0)$.

absolute max:

absolute min:

Problem 15 (5 pts)**.** Evaluate $\iint_R y \sin(xy)\, dA$, where $R = [1,2] \times [0,\pi]$.

Problem 16 (5 pts)**.** Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$, and the three coordinate planes.

$V =$

Problem 17 (5 pts)**.** Evaluate the integral $\iint_D \sin(y^2)\, dA$ where D is the triangle with vertices $(0,0)$, $(1,1)$ and $(0,1)$.

Problem 18 (5 pts)**.** Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy–plane, and inside the cylinder $x^2 + y^2 = 2x$.

$V =$

Problem 19 (5 pts)**.** Evaluate $\iiint_E \sqrt{x^2+z^2}\,dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.

Problem 20 (5 pts)**.** Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx$ by passing the description of the region E in terms of cylindrical coordinates (Trust me, it is **way** easier than evaluating the integral above directly)

Problem 21 (Extra credit—20 pts). A transformation is defined by the equations $x = u^2 - v^2, y = 2uv$.

(a) Find the image of the square $S = \{(u, v) : 0 \leq u \leq 1, 0 \leq v \leq 1\}$, and sketch it.

(b) Use the same change of variables to evaluate the integral $\iint_R y\,dA$, where R is the region bounded by the x–axis and the parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x$, $y \geq 0$.

$\iint_R y\,dA =$ $\boxed{}$