Name:	________________
VIP ID:	________________

- Write your name and VIP ID in the space provided above.
- Each of the propositions below is worth 20 points.
- If in your previous exam there are **-20** comments associated to a specific kind of proof technique on the front page, choose a proposition below with the same characteristics, and give a proof. You may choose one proposition for each such instance.

  Proof by Contrapositive: Instead of proving $P \implies Q$, try $\neg Q \implies \neg P$.

  Proof by Contradiction: To prove $P \implies Q$, start with the assumption that P and $\neg Q$ are true.

  If and only if statement: To prove $P \iff Q$, prove both $P \implies Q$ and $Q \implies P$.

  Existence statement: List, construct, or prove by contradiction.

  Proof by Induction: $\forall n \geq n_0, P(n)$. Prove the *basis step* $P(n_0)$, and use the *inductive hypothesis* to prove the *inductive step* $P(n) \implies P(n+1)$.

- Make sure to **box** your proofs, to differentiate them from your exploration and planning. I will only grade for boxed content on each submission.
- Books, and calculators are allowed, but not notes.

Proposition 1. *For every $n \in \mathbb{N}$, it follows that*

$$(1+2+3+\cdots+n)^2 = 1^3+2^3+3^3+\cdots+n^3.$$

Proposition 2. *For every $n \in \mathbb{N}$, it follows that*

$$\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{2n} \geq \frac{1}{2}.$$

Proposition 3. *We say that a number $p \in \mathbb{N}$ is* ***perfect*** *if it equals the sum of its positive divisors less than itself. There is a perfect number less than 10.*

Proposition 4. *For any $n \in \mathbb{N}$, the product of any n consecutive positive integers is divisible by $n!$.*

Proposition 5. *Suppose $a, b \in \mathbb{Z}$. If ab is odd, then a^2+b^2 is even.*

Proposition 6. *There exists a real number $x \in \mathbb{R}$ such that $x^3-4x^2=7$.*

Proposition 7. *If $n \in \mathbb{Z}$, then $4|n^2$ or $4|(n^2-1)$.*

Proposition 8. *Suppose $x \in \mathbb{R}$. If $x^3-x>0$ then $x>-1$.*

Proposition 9. *Suppose $a, b \in \mathbb{Z}$. Then $a \equiv b \pmod{10}$ if and only if $a \equiv b \pmod{2}$ and $a \equiv b \pmod{5}$.*

Proposition 10. *The number $\log_2(1)+\log_2(2)+\log_2(3)$ is irrational.*

Proposition 11. *If $a, b \in \mathbb{R}$ are positive real numbers, then $a+b \geq 2\sqrt{ab}$.*

Proposition 12. *If $p, q \in \mathbb{Q}$ are rational numbers with $p<q$, then there exists another rational number $x \in \mathbb{Q}$ that satisfies $p<x<q$.*