Name: ____________________

4-digit code: ____________________

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has twelve (12) pages, including this one. You have 150 minutes to complete this exam.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit. Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.
- **A:** 243–270 pts. **B+:** 230–242 pts. **B:** 216–229 pts. **C+:** 203–215 pts. **C:** 189–202 pts. **D+:** 175–188 pts. **D:** 160–174 pts. **F**: less than 160 pts.

Page	**Max. points**	**Your points**	**Page**	**Max. points**	**Your points**
1	—	—	7	30	
2	20		8	20	
3	20		9	25	
4	30		10	25	
5	30		11	30	
6	20		12	20	
Total	120		**Total**	150	

Problem 1 (5 pts). Find $f(0)$ and $f(\pi/2)$ for $f(x) = \begin{cases} \sqrt{x+1} & \text{if } x \geq 1, \\ 3 & \text{if } x < 1. \end{cases}$

$f(0) =$ $\boxed{}$

$f(\pi/2) =$ $\boxed{}$

Problem 2 (10 pts). Find the domain of $f(x) = \sqrt{(x-1)(x-2)}$.

domain $=$ $\boxed{}$

Problem 3 (5 pts). Let $f(x) = x^2 + 4$ and $g(x) = \sqrt{x}$. Find $\big(g \circ f\big)(x)$.

$\big(g \circ f\big)(x) =$ $\boxed{}$

Problem 4 (10 pts)**.** Solve for x:

$$\ln x + \ln(x-1) = 1$$

$x = \boxed{}$

Problem 5 (10 pts)**.** Compute the derivatives of the following functions.

(a) $f(x) = \pi\sqrt{x}(x^4 - 4x^3 + 6x^2 - 4x^1 + 1 - x^{-1})$

$f'(x) = \boxed{}$

(b) $g(t) = \dfrac{t^2 - 5}{t^{-1}}$

$g'(t) = \boxed{}$

Problem 6 (15 pts)**.** Compute the following limits:

(a) $\lim_{x \to 2} \dfrac{x^2 + 2x - 8}{x^2 - 4} = \boxed{}$

(b) $\lim_{x \to -\infty} \dfrac{x^2 - 2x - 8}{x^2 - 4} = \boxed{}$

(b) $\lim_{x \to -2} \dfrac{x^2 + 2x - 8}{x^2 - 4} = \boxed{}$

Problem 7 (15 pts)**.** Find the value of the constant k for which the following function is continuous everywhere:

$$f(x) = \begin{cases} 2k^2x^3 & \text{if } x < 2, \\ x + 32k - 18 & \text{if } x \geq 2. \end{cases}$$

$k = \boxed{}$

Problem 8 (15 pts)**.** Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - \frac{9}{2}y = 3$.

Problem 9 (15 pts)**.** How many tangent lines to the curve $y = x/(x+1)$ pass through the point $(0, 0)$.
HINT: *You do not have to compute the equations of the lines.*

Problem 10 (10 pts)**.** Evaluate each limit:

$$\lim_{x\to 0} \cot 2x \sin 6x = \boxed{}$$

$$\lim_{x\to 0} \frac{\sin(4x^2)}{x^2} = \boxed{}$$

Problem 11 (10 pts)**.** Find an equation of the tangent line to the curve $y = \ln(xe^{x^2})$ at the point $(1, 1)$.

Problem 12 (30 pts)**.** Sketch the graph of the rational function $f(x) = \dfrac{6x^2}{3-3x^2}$.

Indicate clearly:

- Domain
- x- and y-intercepts.
- Vertical and horizontal asymptotes (any holes?).
- Intervals of increase, decrease and different concavity.
- Location of relative extrema and inflection points.

Problem 13 (10 pts)**.** Find the absolute extrema of $f(x) = \frac{8}{3}x^{4/3} - \frac{4}{3}x^{1/3}$ on the interval $[-1, 1]$.

Absolute maxima at $\boxed{}$

Absolute minima at $\boxed{}$

Problem 14 (10 pts)**.** Use logarithmic differentiation to find the derivative of the function

$$y = \frac{\tan^2 x \sin^4 x}{e^{3x}(x^2+1)}$$

$$\frac{dy}{dx} = \boxed{}$$

Problem 15 (10 pts)**.** The volume of a cube is increasing at a rate of 300 cm^3/min. How fast are the edges increasing when the length of an edge is 10 cm?

The edges are increasing at a speed of

Problem 16 (15 pts)**.** Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs.

Area of largest rectangle:

Problem 17 (25 pts)**.** Evaluate each integral:

(a) $\displaystyle\int_0^2 \left(5x + \frac{2}{3x^5} - \sqrt{2}e^x\right) dx$

(b) $\displaystyle\int \left(3\sin x - 2\cos x\right) dx$

(c) $\displaystyle\int (1+\sin t)^{90} \cos t \, dt$

(d) $\displaystyle\int_0^1 \frac{5x^4}{(x^5+1)^2}\, dx$

(e) $\displaystyle\int (3x-2)^{200}\, dx$

Problem 18 (30 pts)**.** Express the following functions of n in closed form and then find the limit.

(a) $\displaystyle\lim_{n\to\infty} \frac{1^2 + 2^2 + 3^2 + \cdots + n^2}{n^3}$

(b) $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \frac{5k}{n^2}$

Problem 19 (10 pts)**.** Use the Fundamental Theorem of Calculus to find the derivative of the following functions.

(a) $g(x) = \displaystyle\int_1^x \frac{1}{t^4+1}\,dt$

$g'(x) = \boxed{}$

(b) $g(x) = \displaystyle\int_{\sin x}^{\pi} \sqrt{e^t + t^8}\,dt$

$g'(x) = \boxed{}$

Problem 20 (10 pts)**.** Find the antiderivative F of $f(x) = 4 - 3(1+x^2)^{-1}$ that satisfies $F(1) = 0$.

$F(x) = \boxed{}$