Name: _____

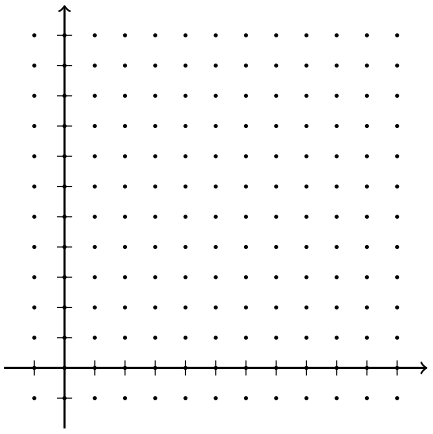
4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has four (4) pages, including this one.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given at the right of each problem number.
- No books, notes or calculators may be used on this test.

Page	**Max**	**Points**
2	30	
3	40	
4	30	
Total	100	

Problem 1 (15 pts)**.** Find the equilibria of the autonomous equation below, and use a slope field to determine stability.

$$\frac{dy}{dx} = y^2 - 5y + 4.$$



Problem 2 (15 pts)**.** Apply Euler method to solve numerically the differential equation $y' = 10y$ with initial condition $y(0) = 1$ in the interval $[0, 0.5]$ with a time-step $h = 0.1$. Fill the table below with the corresponding values.

n	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
x_n	$x_0 = 0$					
y_n	$y_0 = 1$					

Problem 3 (15 pts)**.** The skid marks made by an automobile indicated that its brakes were fully applied for a distance of 10 m before it came to a stop. The car was known to have at that moment a constant deceleration of $20\,\mathrm{m/s^2}$. How fast (in km/h) was the car traveling when the brakes were first applied?

Problem 4 (25 pts)**.** Consider a prolific breed of rabbits whose birth and death rates, β and δ, are each proportional to the rabbit population $P = P(t)$, with $\beta > \delta$.

[15 pts] Show that $P(t)$ is, for some constant k,

$$P(t) = \frac{P_0}{1 - kP_0t}.$$

[10 pts] Suppose that $P_0 = 6$ and that there are nine rabbits after ten months. When does doomsday occur?

Problem 5 (30 pts)**.** Suppose that a car starts from rest, its engine providing an acceleration of 10 ft/s^2, while air resistance provides 0.1 ft/s^2 of deceleration for each foot per second of the car's velocity.

[10 pts] Find the car's maximum possible (limiting) velocity.

[20 pts] Find how long it takes the car to attain 90% of its limiting velocity, and how far it travels while doing so.

Hint: Since you are not allowed to use calculators, the answer of this problem has integer numbers. Write everything as fractions and simplify as much as possible in order to get simple integrals, and you will not have trouble finding those. You may also want to use here the approximation $\ln 0.1 \approx -2.3$ and its corresponding inverse, $e^{-2.3} \approx 0.1$.