MA241 Group 1 Study Guide

12.1 Three-Dimensional Coordinate System

1. Objects in 3D
    a. Sketch and describe(inequalities and equalities)
        i. Point
        ii. Line
        iii. Plane
        iv. Circles
        v. Spheres
        vi. Cylinders
2. Distance
    a. Points to point/planes/axis
    b. Center and Radius from equation for a sphere
        i. Completing the square

12.2 Vectors

3. Vectors
    a. Sketching vectors
    b. Vector Form
    c. Component form
    d. Finding direction and magnitude
    e. Vector Operations
        i. Addition
        ii. Subtraction
        iii. Scalar Multiplication
        iv. Unit Vector
    f. Midpoint of Line Segment

12.3 The Dot Product

4. Dot Product
    a. Dot Product Formula
        i. Find angle between vectors
            1. Angle between non-zero vectors
            2. Triangles
        ii. Orthogonal Vectors
        iii. Vector Projection
        iv. Scalar Components
        v. Work Equation(Not sure If we covered this but I saw it in the book) didnt cover this

12.4 The Cross Product

5. Cross Product
    i. Find Perpendicular vectors
    ii. Find Parallel vectors
    iii. Area of Triangles
    iv. Area of Parallelograms
    v. Vector Projections
    vi. Vector Components
    vii. Determinant Formula
    viii. Triple Scalar or Box Product
    ix. Coplanar tests

12.5 Lines and Planes in Space

6. Lines and Line Segments in Space
    a. Vector Equation for a line
    b. Parametric Equation for a line
    c. Distance from a point to a line in space
    d. Plane Equation in space
    e. Lines of Intersection
        i. Spheres to Planes
        ii. Intersection between Lines
        iii. Points to Planes
    f. Angles Between Planes

12.6 Cylinders and Quadric Surfaces

From looking in the book I don't remember this being on the 1st test. Not sure if we should include it. Would like some input from the group. we had a day of notes on this but never actually did anything with it so we would probably be fine without it

Group Assignments

Chris	12.3
Cameron	12.5 Equations of lines and planes: Distance
Evan	12.2
Luke	12.1
Maxwell	12.4
Alyssa	12.5 Equations of lines and planes:

	Intersections
Susan	12.5 Equations of lines and planes: Angles

Questions to be submitted

12.1

Give a geometric description of a set of points in space whose coordinates satisfy the given pairs of equations

1) x=2, y=3
2) $x^2 + z^2 = 4$, y=0
3) $x^2 + y^2 + (z+3)^2 = 25,\ y = -4$
4) $z = y^2,\ x = 1$

Describe set of points in space whose coordinates satisfy the given inequalities or combination of equations and inequalities

1) **a**. $x^2 + y^2 \le 1,\ z = 0$ **b**. $x^2 + y^2 \le 1,\ z = 3$ **c**. $x^2 + y^2 \le 1,\ no\ restriction\ on\ z$
2) **a.** $z = 1 - y,\ no\ restriction\ on\ x$ **b.** $z = y^3,\ x = 2$

Describe a set of points with a single equations or a set of equations

1) A plane through the point (3,-1,2) perpendicular to the x-axis
2) A circle of radius 2 centered at the point (0,2,0) lying in the yz-plane
3) A circle on which the through the point (1,1,3) perpendicular to the z-axis meets a sphere of radius 5 centered at the origin.

Write inequalities to describe the sets

1) The solid cube in the first octant bounded by the coordinate planes and the planes x=2, y=2, z=2

FInd the distance between $p_1\ and\ p_2$

1) $p_1(-1, 1, 5), \quad p_2(2, 5, 0)$

Find the center and radius of a sphere

1) $x^2 + y^2 + z^2 - 6y + 8z = 0$

12.2

Sketch the following u=<2,4,3> v=<1,2,1>

1) U+v
2) u-v
3) u*v

Find the component form, direction and magnitude of the vector 8u

1) u=<2,3>

Find the component form, direction and magnitude of the vector 5u-8v

1) u=<5,6> and v=<8,8>

Find the component form, direction and magnitude of the vector 11/12u+6/12v

1) u=<-1,6> v=<7,-5>

Let u=<2,3,8> and v=<0,8,9> express u+v, u-v, and u*v

Express the midpoint of the following

1) u=<1,3,1> v=<-5,2,6>

12.3

Find $u \bullet v$, $|u|$, $|v|$,

Cosine of the angle between v and u,

Scalar component of u in the direction of v, and

The vector $proj_v u$ for:

1) v = (3/5)i + (4/5)k u = 5i + 12j

2) $v = < \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} >$ $u = < \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} >$

3) v = -i + j u = $\sqrt{2}i + \sqrt{3}j + 2k$

Find the measures of the angles of the triangle whose vertices are A=(-1,0) B=(2,1) C=(1, -2)

Find the measures of the angles between the diagonals of the rectangle whose vertices are A = (1,0), B = (0,3), C = (3,4), D = (4,1)

Use the dot product to determine if points P(0,1,0), Q(2,2,1), and R(5,-1,0) fall on the same line.

Determine if the following vectors are orthogonal

1) u = <2, 3, 1> v = <3, 1, -9>

2) u = <3, -1> v =<7, 5>

12.4

1) (Easy) Find the Cross Product of v_1 = <2,1,4> and v_2 = <3,1,1>
2) (Slightly Harder) Find the Cross Product of r_1 = 5*i + 6*j + 7*k and r_2 = 1*i + 2*j + 3*k
3) (Hardest) Find the volume of the Parallelepiped defined by the points (0,0,0) (4,0,0) (3,1,0) (2,1,4)

12.5 (Distances)

Find the distance between the following:

1) The point (3,1,4) and the plane x+y+z=9.
2) The line x=-7t, y=3t, z=-t and the point (2,3,-6).
3) The plane x+4y+2z=15 and the plane x+4y+2z=25.
4) The point (0,4,0) and the plane 7x+3y+z=21.
5) The sphere $(x+3)^2+(y-5)^2+z^2=16$ and the plane 8x+y+3z=12.

12.5 (Angles)

1) Find the angle between the planes:

a. x+y+(z-2) = 0 and 4x-2y+6(z-2) = 0
b. 5x+7y = -15 and 9x+10y+9z = -12

c. x+7-z = 7 and 4x+y+z = 9
d. 2x+y+6z = 7 and 3x-y-z = 8

12.5 (Intersections)

a. Consider the plane 2x+y-4z=4. Find all points of intersection of the plane with the line x=t, y=2+3t, z=t
b. Find the intersection of the line x=3t y=1+2t z=2-t and the plane 2x+3y-z=4
c. Find the plane determined by the intersecting lines: x=-1+4t y=2+t z=1-3t and x=1-4s y=1+2s z=2-2s
d. Find the point at which the line intersects the plane: x=-7-8t y=-4+9t z=7-3t ; 4x-8y+2z=8

Formulas to be submitted

12.1

Point on x-axis- (x,0,0)
Point on y-axis- (0,y,0)
Point on z-axis- (0,0,z)
xy-plane- z=0
yz-plane- x=0
zx-plane- y=0
$|p_1p_2|$ = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
Sphere of radius a- $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=a^2$

12.2

Magnitude equals

12.3

$u \bullet v = u_1v_1 + u_2v_2 + u_3v_3$
$\theta = cos^{-1}(\frac{u \bullet v}{|u||v|})$
Vectors U and V are orthogonal if $u \bullet v = 0$
$proj_v u = (\frac{u \bullet v}{|v^2|})v$
Scalar Component of U in the direction of V = $u * \frac{v}{|v|}$

12.5

Plane: $V_1(X-X_0) + V_2(Y-Y_0) + V_3(Z-Z_0)=0$
Line: $z=z_0 + tv_3$
$y=y_0 + tv_2$
$x=x_0 + tv_1$

12.5 (Distance)

From point P to point Q: $|\{PQ\}|$
From point P to line {r(t)} = Q +t{u}: $|\{PQ\} \times \{u\}| / |\{u\}|$
From point P to plane {n}{x}=d containing point Q: $|\{PQ\} * \{n\}| / |\{n\}|$
From plane {n}{x}=d to plane {n}{x}=f: |d-f|/|{n}|

From sphere $(x-a)^2+(y-b)^2+(z-c)^2=r^2$ where point P is (a,b,c) to plane {n}{x}=d containing point Q:
$(|\{PQ\} * \{n\}| / |\{n\}|) - r$

12.5 (Angles)
$< v > * < w > = |< v >| * |< w >| * cos\Theta$

Example Problems to go on Cheat Sheet

Comments

Chris - Starting to organize subjects by chapter sections