Lesson 22: Systems of differential equations: Numerical methods

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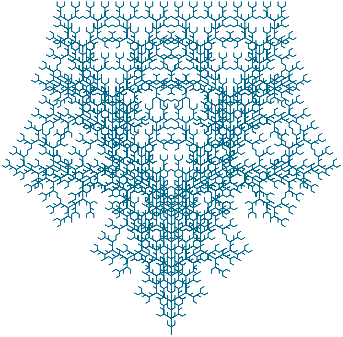

What do we know?

- The concepts of differential equation and initial value problem
- The concept of order of a differential equation.
- The concepts of general solution, particular solution and singular solution.
- Slope fields
- Approximations to solutions via Euler's Method and Improved Euler's Method

- First-Order Differential Equations
  - Separable equations
  - Homogeneous First-Order Equations
  - Linear First-Order Equations
  - Bernoulli Equations
  - General Substitution Methods
  - Exact Equations
- Second-Order Differential Equations
  - Reducible Equations
  - General Linear Equations (Intro)
  - Linear Equations with Constant Coefficients
    - Characteristic Equation
    - Variation of Parameters
    - Undetermined Coefficients

What do we know?

Laplace Transforms

$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x)\, dx$		$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x)\, dx$	
1	$\frac{1}{s}$	$s > 0$	$cf(x) \pm g(x)$	$cF(s) \pm G(s)$	$s > max(a, b)$
x^p	$\frac{\Gamma(p+1)}{s^{p+1}}$	$s > 0$	$x^n f(x)$	$(-1)^n F^{(n)}$	$s > a$
x^n	$\frac{n!}{s^{n+1}}$	$s > 0$	$e^{\alpha x} f(x)$	$F(s-\alpha)$	$s > a + \alpha$
$e^{\alpha x}$	$\frac{1}{s-\alpha}$	$s > \alpha$	$\frac{f(x)}{x}$	$\int_s^\infty F(\sigma)\, d\sigma$	$s > a$
$\sin \beta x$	$\frac{\beta}{s^2+\beta^2}$	$s > 0$	$f \star g$	$F(s)G(s)$	$s > \max(a, b)$
$\cos \beta x$	$\frac{s}{s^2+\beta^2}$	$s > 0$	$f'(x)$	$sF(s) - f(0)$	$s > a$

What do we know?

Systems of Differential Equations

$$\underbrace{\begin{cases} y_1^{(n)} = F_1\left(x, y_1, y_1', \ldots, y_1^{(n-1)}, y_2, y_2', \ldots, y_2^{(n-1)}, \cdots, y_r, y_r', \ldots, y_r^{(n-1)}\right) \\ y_2^{(n)} = F_2\left(x, y_1, y_1', \ldots, y_1^{(n-1)}, y_2, y_2', \ldots, y_2^{(n-1)}, \cdots, y_r, y_r', \ldots, y_r^{(n-1)}\right) \\ \ldots \\ y_r^{(n)} = F_r\left(x, y_1, y_1', \ldots, y_1^{(n-1)}, y_2, y_2', \ldots, y_2^{(n-1)}, \cdots, y_r, y_r', \ldots, y_r^{(n-1)}\right) \end{cases}}_{\text{order } n, r \text{ functions}}$$

- Transformation to First-Order systems
- Solution by elimination

Systems of Differential Equations

Initial Value Problems

Solve the Initial Value Problem. $x = x(t), y = y(t)$

$$\begin{cases} x' = 3x - 5y \\ y' = x - y \end{cases} \qquad \begin{cases} x(0) = 1 \\ y(0) = 3 \end{cases}$$

Systems of Differential Equations

Initial Value Problems

Solve the Initial Value Problem. $x = x(t), y = y(t)$

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We solve the system of differential equations first, by elimination. The *easier* equation is $y' = x - y$, which gives $x = y' + y$. It is then

$$y'' = x' - y'$$

Systems of Differential Equations

Initial Value Problems

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$$y'' = x' - y' = 3x - 5y - y'$$

Systems of Differential Equations

Initial Value Problems

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We solve the system of differential equations first, by elimination. The *easier* equation is $y' = x - y$, which gives $x = y' + y$. It is then

$$y'' = x' - y' = 3x - 5y - y' = 3(y' + y) - 5y - y'$$

Systems of Differential Equations
Initial Value Problems

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$$y'' = x' - y' = 3x - 5y - y' = 3(y' + y) - 5y - y' = 2y' - 2y$$

Systems of Differential Equations

Initial Value Problems

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$$y'' = x' - y' = 3x - 5y - y' = 3(y' + y) - 5y - y' = 2y' - 2y$$

We need to solve the homogeneous linear equation of second order with constant coefficients $y'' - 2y' + 2y = 0$

Systems of Differential Equations

Initial Value Problems

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We need to solve the homogeneous linear equation of second order with constant coefficients $y'' - 2y' + 2y = 0$

$$r^2 - 2r + 2 = 0, \qquad r = \frac{2 \pm \sqrt{4 - 4 \cdot 2}}{2} = 1 \pm i$$

Systems of Differential Equations

Initial Value Problems

Solve the Initial Value Problem. $x = x(t), y = y(t)$

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It is then $y = e^t \big(A \cos t + B \sin t\big)$

Systems of Differential Equations

Initial Value Problems

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Systems of Differential Equations

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$$x = y' + y$$

Systems of Differential Equations

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$$r^2 - 2r + 2 = 0, \qquad r = \frac{2 \pm \sqrt{4 - 4 \cdot 2}}{2} = 1 \pm i$$

It is then $y = e^t\big(A\cos t + B\sin t\big)$, and

$$x = y' + y = e^t\big(A\cos t + B\sin t\big) + e^t\big(B\cos t - A\sin t\big) + e^t\big(A\cos t + B\sin t\big)$$

Systems of Differential Equations

Initial Value Problems

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$$r^2 - 2r + 2 = 0, \qquad r = \frac{2 \pm \sqrt{4 - 4 \cdot 2}}{2} = 1 \pm i$$

It is then $y = e^t\big(A\cos t + B\sin t\big)$, and

$$\begin{aligned} x = y' + y &= e^t\big(A\cos t + B\sin t\big) + e^t\big(B\cos t - A\sin t\big) + e^t\big(A\cos t + B\sin t\big) \\ &= e^t\big((2A + B)\cos t + (2B - A)\sin t\big) \end{aligned}$$

Systems of Differential Equations

Initial Value Problems

Solve the Initial Value Problem

$$\begin{cases} x' = 3x - 5y \\ y' = x - y \end{cases} \qquad \begin{cases} x(0) = 1 \\ y(0) = 3 \end{cases}$$

$$x = e^t\big((2A + B)\cos t + (2B - A)\sin t\big)$$
$$y = e^t\big(A\cos t + B\sin t\big)$$

We need to impose the initial conditions, to find the values of A and B that solve the initial value problem.

$$1 = x(0) = 2A + B$$
$$3 = y(0) = A$$

Systems of Differential Equations

Initial Value Problems

Solve the Initial Value Problem

$$\begin{cases} x' = 3x - 5y \\ y' = x - y \end{cases} \qquad \begin{cases} x(0) = 1 \\ y(0) = 3 \end{cases}$$

$$x = e^t\big((2A + B)\cos t + (2B - A)\sin t\big)$$
$$y = e^t\big(A\cos t + B\sin t\big)$$

We need to impose the initial conditions, to find the values of A and B that solve the initial value problem.

$$1 = x(0) = 2A + B$$
$$3 = y(0) = A$$

A quick computation gives $A = 3, B = -5$, and thus

$$x = e^t\big(\cos t - 13\sin t\big)$$
$$y = e^t\big(3\cos t - 5\sin t\big)$$

Systems of Differential Equations

Numerical Methods

Euler's method for Systems of Differential Equations of First Order

Given an initial value problem consisting on a system of r differential equations of first order, with initial conditions

$$\begin{cases} y_1' = F_1(x, y_1, y_2, \dots, y_r) \\ y_2' = F_2(x, y_1, y_2, \dots, y_r) \\ \dots \\ y_r' = F_r(x, y_1, y_2, \dots, y_r) \end{cases} \qquad \begin{cases} y_1(a_1) = b_1 \\ y_2(a_2) = b_2 \\ \dots \\ y_r(a_r) = b_r \end{cases}$$

a set number of steps n, and a time-step $h > 0$, we compute an approximation to the solution $\{y_1, y_2, \dots, y_r\}$ with the formula

$$y_j(a_j + hk) = y_j\big(a_j + h(k-1)\big) + h \cdot y_j'\big(a_j + h(k-1)\big)\Big|_{\substack{j=1,\dots,r \\ k=1,\dots,n}}$$

Systems of Differential Equations

Numerical Methods

What we actually compute is a matrix of $r \times n$ values

$$
\begin{array}{cccc}
y_{1,1} & y_{2,1} & \cdots & y_{r,1} \\
y_{1,2} & y_{2,2} & \cdots & y_{r,2} \\
y_{1,3} & y_{2,3} & \cdots & y_{r,3} \\
y_{1,4} & y_{2,4} & \cdots & y_{r,4} \\
\cdots & \cdots & \cdots & \cdots \\
y_{1,n} & y_{2,n} & \cdots & y_{r,n}
\end{array}
$$

Systems of Differential Equations

Numerical Methods

What we actually compute is a matrix of $r \times n$ values

$$
\begin{array}{llll}
y_{1,1} = b_1 & y_{2,1} = b_2 & \cdots & y_{r,1} = b_r \\
y_{1,2} & y_{2,2} & \cdots & y_{r,2} \\
y_{1,3} & y_{2,3} & \cdots & y_{r,3} \\
y_{1,4} & y_{2,4} & \cdots & y_{r,4} \\
\cdots & \cdots & \cdots & \cdots \\
y_{1,n} & y_{2,n} & \cdots & y_{r,n}
\end{array}
$$

Systems of Differential Equations
Numerical Methods

What we actually compute is a matrix of $r \times n$ values

$$
\begin{array}{llll}
y_{1,1} & y_{2,1} & \cdots & y_{r,1} \\
y_{1,2} = y_{1,1} + h \cdot y_1'(a_1) & y_{2,2} = y_{2,1} + h \cdot y_2'(a_2) & \cdots & y_{r,2} = y_{r,1} + h \cdot y_r'(a_r) \\
y_{1,3} & y_{2,3} & \cdots & y_{r,3} \\
y_{1,4} & y_{2,4} & \cdots & y_{r,4} \\
\cdots & \cdots & \cdots & \cdots \\
y_{1,n} & y_{2,n} & \cdots & y_{r,n}
\end{array}
$$

Systems of Differential Equations

Numerical Methods

What we actually compute is a matrix of $r \times n$ values

$$
\begin{array}{llll}
y_{1,1} & y_{2,1} & \cdots & y_{r,1} \\
y_{1,2} & y_{2,2} & \cdots & y_{r,2} \\
y_{1,3} = y_{1,2} + h \cdot y_1'(a_1 + h) & y_{2,3} = y_{2,2} + h \cdot y_2'(a_2 + h) & \cdots & y_{r,3} = y_{r,2} + h \cdot y_r'(a_r + h) \\
y_{1,4} & y_{2,4} & \cdots & y_{r,4} \\
\cdots & \cdots & \cdots & \cdots \\
y_{1,n} & y_{2,n} & \cdots & y_{r,n}
\end{array}
$$

Systems of Differential Equations

Numerical Methods

What we actually compute is a matrix of $r \times n$ values

$$
\begin{array}{llll}
y_{1,1} & y_{2,1} & \cdots & y_{r,1} \\
y_{1,2} & y_{2,2} & \cdots & y_{r,2} \\
y_{1,3} & y_{2,3} & \cdots & y_{r,3} \\
y_{1,4} = y_{1,3} + h \cdot y_1'(a_1 + 2h) & y_{2,4} = y_{2,3} + h \cdot y_2'(a_2 + 2h) & \cdots & y_{r,4} = y_{r,3} + h \cdot y_r'(a_r + 2h) \\
\cdots & \cdots & \cdots & \cdots \\
y_{1,n} & y_{2,n} & \cdots & y_{r,n}
\end{array}
$$

Systems of Differential Equations

Numerical Methods

What we actually compute is a matrix of $r \times n$ values

$$
\begin{array}{llll}
y_{1,1} & y_{2,1} & \cdots & y_{r,1} \\
y_{1,2} & y_{2,2} & \cdots & y_{r,2} \\
y_{1,3} & y_{2,3} & \cdots & y_{r,3} \\
y_{1,4} & y_{2,4} & \cdots & y_{r,4} \\
\cdots & \cdots & \cdots & \cdots \\
y_{1,n} = y_{1,n-1} + h \cdot y_1'\big(a_1 + (n-2)h\big) & y_{2,n} & \cdots & y_{r,n}
\end{array}
$$

Systems of Differential Equations

Numerical Methods

What we actually compute is a matrix of $r \times n$ values

$$
\begin{array}{llll}
y_{1,1} & y_{2,1} & \cdots & y_{r,1} \\
y_{1,2} & y_{2,2} & \cdots & y_{r,2} \\
y_{1,3} & y_{2,3} & \cdots & y_{r,3} \\
y_{1,4} & y_{2,4} & \cdots & y_{r,4} \\
\cdots & \cdots & \cdots & \cdots \\
y_{1,n} & y_{2,n} = y_{2,n-1} + h \cdot y_2'\big(a_2 + (n-2)h\big) & \cdots & y_{r,n}
\end{array}
$$

Systems of Differential Equations
Numerical Methods

What we actually compute is a matrix of $r \times n$ values

$$\begin{array}{llll}
y_{1,1} & y_{2,1} & \cdots & y_{r,1} \\
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\cdots & \cdots & \cdots & \cdots \\
y_{1,n} & y_{2,n} & \cdots & y_{r,n} = y_{r,n-1} + h \cdot y'_r\big(a_r + (n-2)h\big)
\end{array}$$

Systems of Differential Equations
Numerical Methods

Use four steps of Euler's method with time-step $h = 0.5$ to solve numerically the following IVP:

$$\begin{cases} y_1' = 3y_1 - 5y_2 \\ y_2' = y_1 - y_2 \end{cases} \qquad \begin{cases} y_1(0) = 1 \\ y_2(0) = 3 \end{cases}$$

Systems of Differential Equations

Numerical Methods

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We have $n = 4$, $r = 2$, $h = 0.5$, $(a_1, a_2) = (0, 0)$, and $(b_1, b_2) = (1, 3)$. We will use a table to compute all values

$$\begin{array}{cc} y_{1,1} & y_{2,1} \\ y_{1,2} & y_{2,2} \\ y_{1,3} & y_{2,3} \\ y_{1,4} & y_{2,4} \end{array}$$

Systems of Differential Equations
Numerical Methods

Use four steps of Euler's method with time-step $h = 0.5$ to solve numerically the following IVP:

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We have $n = 4$, $r = 2$, $h = 0.5$, $(a_1, a_2) = (0, 0)$, and $(b_1, b_2) = (1, 3)$. We will use a table to compute all values

n	x	y_1	y_2	y_1'	y_2'
1	0	1	3		
2					
3					
4					

Systems of Differential Equations

Numerical Methods

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n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	$3 \cdot 1 - 5 \cdot 3$	$1 - 3$
2					
3					
4					

Systems of Differential Equations

Numerical Methods

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n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5				
3					
4					

Systems of Differential Equations
Numerical Methods

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n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	$1 + 0.5 \cdot (-12)$	$3 + 0.5 \cdot (-2)$		
3					
4					

Systems of Differential Equations
Numerical Methods

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n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	-5	2		
3					
4					

Systems of Differential Equations

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n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	-5	2	$3 \cdot (-5) - 5 \cdot (2)$	$-5 - 2$
3					
4					

Systems of Differential Equations

Numerical Methods

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n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	-5	2	-25	-7
3	1				
4					

Systems of Differential Equations

Numerical Methods

Use four steps of Euler's method with time-step $h = 0.5$ to solve numerically the following IVP:

$$\begin{cases} y_1' = 3y_1 - 5y_2 \\ y_2' = y_1 - y_2 \end{cases} \qquad \begin{cases} y_1(0) = 1 \\ y_2(0) = 3 \end{cases}$$

We have $n = 4$, $r = 2$, $h = 0.5$, $(a_1, a_2) = (0, 0)$, and $(b_1, b_2) = (1, 3)$. We will use a table to compute all values

n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	-5	2	-25	-7
3	1	$-5 + 0.5 \cdot (-25)$	$2 + 0.5 \cdot (-7)$		
4					

Systems of Differential Equations
Numerical Methods

Use four steps of Euler's method with time-step $h = 0.5$ to solve numerically the following IVP:

$$\begin{cases} y_1' = 3y_1 - 5y_2 \\ y_2' = y_1 - y_2 \end{cases} \qquad \begin{cases} y_1(0) = 1 \\ y_2(0) = 3 \end{cases}$$

We have $n = 4$, $r = 2$, $h = 0.5$, $(a_1, a_2) = (0, 0)$, and $(b_1, b_2) = (1, 3)$. We will use a table to compute all values

n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	-5	2	-25	-7
3	1	-17.5	-1.5		
4					

Systems of Differential Equations

Numerical Methods

Use four steps of Euler's method with time-step $h = 0.5$ to solve numerically the following IVP:

$$\begin{cases} y_1' = 3y_1 - 5y_2 \\ y_2' = y_1 - y_2 \end{cases} \qquad \begin{cases} y_1(0) = 1 \\ y_2(0) = 3 \end{cases}$$

We have $n = 4$, $r = 2$, $h = 0.5$, $(a_1, a_2) = (0, 0)$, and $(b_1, b_2) = (1, 3)$. We will use a table to compute all values

n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	-5	2	-25	-7
3	1	-17.5	-1.5	$3 \cdot (-17.5) - 5 \cdot (-1.5)$	$-17.5 + 1.5$
4					

Systems of Differential Equations

Numerical Methods

Use four steps of Euler's method with time-step $h = 0.5$ to solve numerically the following IVP:

$$\begin{cases} y_1' = 3y_1 - 5y_2 \\ y_2' = y_1 - y_2 \end{cases} \qquad \begin{cases} y_1(0) = 1 \\ y_2(0) = 3 \end{cases}$$

We have $n = 4$, $r = 2$, $h = 0.5$, $(a_1, a_2) = (0, 0)$, and $(b_1, b_2) = (1, 3)$. We will use a table to compute all values

n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	-5	2	-25	-7
3	1	-17.5	-1.5	-45	-16
4	1.5				

Systems of Differential Equations

Numerical Methods

Use four steps of Euler's method with time-step $h = 0.5$ to solve numerically the following IVP:

$$\begin{cases} y_1' = 3y_1 - 5y_2 \\ y_2' = y_1 - y_2 \end{cases} \qquad \begin{cases} y_1(0) = 1 \\ y_2(0) = 3 \end{cases}$$

We have $n = 4$, $r = 2$, $h = 0.5$, $(a_1, a_2) = (0, 0)$, and $(b_1, b_2) = (1, 3)$. We will use a table to compute all values

n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	-5	2	-25	-7
3	1	-17.5	-1.5	-45	-16
4	1.5	$-17.5 + 0.5 \cdot (-45)$	$-1.5 + 0.5 \cdot (-16)$		

Systems of Differential Equations
Numerical Methods

Use four steps of Euler's method with time-step $h = 0.5$ to solve numerically the following IVP:

$$\begin{cases} y_1' = 3y_1 - 5y_2 \\ y_2' = y_1 - y_2 \end{cases} \qquad \begin{cases} y_1(0) = 1 \\ y_2(0) = 3 \end{cases}$$

We have $n = 4$, $r = 2$, $h = 0.5$, $(a_1, a_2) = (0, 0)$, and $(b_1, b_2) = (1, 3)$. We will use a table to compute all values

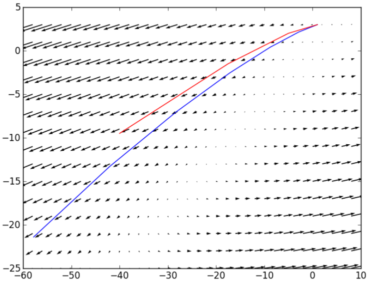
n	x	y_1	y_2	y_1'	y_2'
1	0	1	3	-12	-2
2	0.5	-5	2	-25	-7
3	1	-17.5	-1.5	-45	-16
4	1.5	-40	-9.5		

Systems of Differential Equations

Numerical Methods

Use four steps of Euler's method with time-step $h = 0.5$ to solve numerically the following IVP:

$$\begin{cases} y_1' = 3y_1 - 5y_2 \\ y_2' = y_1 - y_2 \end{cases} \qquad \begin{cases} y_1(0) = 1 \\ y_2(0) = 3 \end{cases}$$



Actual solution $\boldsymbol{y}(x) = \big(y_1(x), y_2(x)\big)$

Numerical Solution with Euler's Method