

Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has fourteen (14) pages, including this one, and your help sheet.
- For multi-choice questions, you should circle the answer you select. On the other problems, you should enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.
- **A:** 243–270 pts. **B+:** 230–242 pts. **B:** 216–229 pts. **C+:** 203–215 pts. **C:** 189–202 pts. **D+:** 175–188 pts. **D:** 160–174 pts. **F:** less than 160 pts.

Page	Max	Points	Page	Max	Points	Page	Max	Points
2	30		6	30		10	30	
3	25		7	25		11	25	
4	25		8	25		12	25	
5	20		9	20		13	20	
Total	100		Total	100		Total	100	

Problem 1 (15 pts). Find the distance d from the point $(3, 7, -5)$ to the z -axis.

 $d =$

Problem 2 (15 pts). Find an exact expression for the angle θ between the vectors $\mathbf{v} = \langle 3, -1, 5 \rangle$ and $\mathbf{w} = \langle -2, 4, 3 \rangle$.

 $\theta =$

Problem 3 (15 pts). Find the length ℓ of the curve $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ for $0 \leq t \leq 1$.

$\ell =$

Problem 4 (10 pts). At what points does the helix $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

points:

Problem 5 (15 pts). Find a unit vector \mathbf{v} that is orthogonal to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$.

$\mathbf{v} =$

Problem 6 (10 pts). Determine whether the points $A = (0, -5, 5)$, $B = (1, -2, 4)$ and $C = (3, 4, 2)$ lie on a straight line.

Problem 7 (20 pts). Find parametric equations for the line of intersections of the planes $x+y+z = 1$ and $x + 2y + 2z = 1$. Find the angle θ between the two planes.

 $\theta =$

Problem 8 (15 pts). Sketch the domain of $f(x, y) = \frac{\sqrt{4 - x^2}}{y^2 + 3}$.

Problem 9 (15 pts). Evaluate the limit, if it exists

$$\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln (x^2 + y^2)$$



Problem 10 (15 pts). The volume of a right circular cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$. Show that if the height remains constant while the radius changes, then the volume satisfies

$$\frac{\partial V}{\partial r} = \frac{2V}{r}.$$

Problem 11 (10 pts). Use the method of Lagrange multipliers to find the dimensions of a rectangle with perimeter p and maximum area.

width:

height:

Problem 12 (15 pts). Recall the formula for the volume of a right circular cone of radius r and height h . Suppose that the height decreases from 20 to 19.95 inches, and the radius increases from 4 to 4.05 inches. Compare the change in volume of the cone with an approximation of this change using a total differential.

$$dV =$$

$$\Delta V =$$

Problem 13 (10 pts). Find an equation for the tangent plane to the surface $z = xe^{-y}$ at the point $P = (1, 0, 1)$.

tangent plane:

Problem 14 (20 pts). Find the absolute extrema of the function $f(x, y) = xy - x - 3y$ on the triangular region R with vertices $(0, 0)$, $(0, 4)$ and $(5, 0)$.

absolute max:

absolute min:

Problem 15 (15 pts). Evaluate $\iint_R y \sin(xy) dA$, where $R = [1, 2] \times [0, \pi]$.

Problem 16 (15 pts). Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$, and the three coordinate planes.

$V =$

Problem 17 (15 pts). Evaluate the integral $\iint_D \sin(y^2) dA$ where D is the triangle with vertices $(0, 0)$, $(1, 1)$ and $(0, 1)$.

Problem 18 (10 pts). Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy -plane, and inside the cylinder $x^2 + y^2 = 2x$.

$V =$

Problem 19 (10 pts). Evaluate $\iiint_E \sqrt{x^2 + z^2} dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.

Problem 20 (15 pts). Evaluate $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2 + y^2) dz dy dx$ by passing the description of the region E in terms of cylindrical coordinates (Trust me, it is **way** easier than evaluating the integral above directly)

Problem 21 (20 pts). A transformation is defined by the equations $x = u^2 - v^2, y = 2uv$.

- (a) Find the image of the square $S = \{(u, v) : 0 \leq u \leq 1, 0 \leq v \leq 1\}$.
- (b) Use the same change of variables to evaluate the integral $\iint_R y \, dA$, where R is the region bounded by the x -axis and the parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x, y \geq 0$.

Image of S :

$\iint_R y \, dA =$