Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has sixteen (16) pages, including this one, and your help sheet.
- For multi-choice questions, you should circle the answer you select. On the other problems, you should enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.
- **A:** 243–270 pts. **B+:** 230–242 pts. **B:** 216–229 pts. **C+:** 203–215 pts. **C:** 189–202 pts. **D+:** 175–188 pts. **D:** 160–174 pts. **F**: less than 160 pts.

Page	Max	Points
2	25	
3	25	
4	25	
5	25	
Total	100	

Page	Max	Points
6	40	
7	20	
8	10	
9	10	
10	20	
Total	100	

Page	Max	Points
11	20	
12	20	
13	25	
14	20	
15	15	
Total	100	

Problem 1 (10 pts)**.** Find the area of the region that is enclosed between the curves $x = 2y^2$ and $x = 4 + y^2$.

$A =$ ☐

Problem 2 (15 pts)**.** Find the volume of the solid that is obtained by rotating the region bounded by the curves $x = 2\sqrt{y}$, $x = 0$ and $y = 9$ about the y-axis.

$V =$ ☐

Problem 3 (15 pts)**.** Find the volume of the solid generated when the region enclosed by the curves $x = 1 + y^2$, $y = 1$, $y = 2$ and $x = 0$ is revolved about the x-axis.

$V =$ ☐

Problem 4 (10 pts)**.** Use the Fundamental Theorem of Calculus to find the derivative of

$$f(x) = \int_{1-3x}^{1} \frac{u^3}{1+u^2}\, du.$$

$f'(x) =$ ☐

Problem 5 (15 pts)**.** Find a positive value of k such that the average value of $f(x) = 2 + 6x - 3x^2$ over the interval $[0, k]$ is equal to 3.

$k =$ _____

Problem 6 (10 pts)**.** Evaluate the integral $\int \sec^3 x \tan x \, dx$.

Problem 7 (15 pts)**.** Evaluate the integral $\displaystyle\int_0^{1/\sqrt{3}} \frac{x^2-1}{x^4-1}\,dx$.

$A =$ ☐

Problem 8 (10 pts)**.** Find the average value of the function $f(x) = 1/x$ over the interval $[1, e]$.

$f_{ave} =$ ☐

Problem 9 (40 pts)**.** Evaluate each integral:

(a) $\int \csc^2 x \, dx =$

(b) $\int \dfrac{1}{\csc x} \, dx =$

(c) $\int \dfrac{x+1}{x} \, dx =$

(d) $\int \dfrac{x}{x+1} \, dx =$

Problem 10 (10 pts)**.** Use **integration by parts** to evaluate the integral $\int xe^{2x}\,dx$.

$$\int xe^{2x}\,dx = \boxed{}$$

Problem 11 (10 pts)**.** Evaluate the improper integral $\int_1^\infty \frac{dx}{x^3}$.

$$\int_1^\infty \frac{dx}{x^3} = \boxed{}$$

Problem 12 (10 pts)**.** Use a **trigonometric substitution** to evaluate the integral $\int \frac{dx}{\sqrt{x^2-9}}$.

$$\int \frac{dx}{\sqrt{x^2-9}} = \boxed{}$$

Problem 13 (10 pts)**.** Evaluate the integral $\int \sin^2 x \cos^2 x \, dx$.

Use trigonometric simplification and one of the following reduction formulas.

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$\int \sin^2 x \cos^2 x \, dx = \boxed{}$$

Problem 14 (20 pts)**.** Use **partial fractions** to evaluate the integral $\displaystyle\int \frac{dx}{x^2+x-2}$.

$$\int \frac{dx}{x^2+x-2} = \boxed{\phantom{\text{answer}}}$$

Problem 15 (10 pts)**.** Find a formula for the general term of the following sequences:

(a) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$

$x_n =$ [ ]

(b) $1 - \frac{1}{2}, \frac{1}{3} - \frac{1}{2}, \frac{1}{3} - \frac{1}{4}, \frac{1}{5} - \frac{1}{4}, \ldots$

$x_n =$ [ ]

Problem 16 (10pts)**.** Write out the first five terms of the sequence $\left\{ \frac{(-1)^{n+1}}{n^2} \right\}_{n=1}^{\infty}$

Determine whether the sequence converges, and if so find its limit.

First five terms: [ ]

$\lim_{n\to\infty} x_n =$ [ ]

Problem 17 (20 pts)**.** Determine whether the series converge, and if so find their sum:

(a) $\displaystyle\sum_{k=1}^{\infty}\Big(-\frac{3}{4}\Big)^{k-1}$

$$\sum_{k=1}^{\infty}\Big(-\frac{3}{4}\Big)^{k-1} = \boxed{}$$

(b) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)}$

$$\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)} = \boxed{}$$

Problem 18 (5 pts)**.** Apply the **divergence test** and state what it tells you about the series.

$$\sum_{k=1}^{\infty} \frac{k^2+k+3}{2k^2+1}.$$

Problem 19 (10 pts)**.** Use the **integral test** to determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{5k+2}$ converges.

Problem 20 (10 pts)**.** Use the **ratio test** to determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{4^k}{k^2}$ converges. If the test is inconclusive, then say so.

Problem 21 (10 pts)**.** Use the **root test** to determine whether the series $\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$ converges. If the test is inconclusive, then say so.

Problem 22 (10 pts)**.** Classify the series $\sum_{k=1}^{\infty} (-1)^k \frac{4k^2+1}{k^3+2}$ as absolutely convergent, convergent or divergent.

Problem 23 (15 pts)**.** Find the MacLaurin series for $f(x) = \sin \pi x$. Find the associated radius of convergence.