Name: ____________________

4-digit code: ____________________

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has six (6) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	**Max. points**	**Your points**
2	20	
3	10	
4	30	
5	20	
6	20	
Total	100	

Problem 1 (15 pts)**.** Compute the following limit without using L'Hôpital's rule:

$$\lim_{x\to 0} \frac{\tan(5x^2) + \sin^2(2x)}{x^2} = \boxed{}$$

Hint: $\lim_{x\to 0} \frac{\sin x}{x} = 0.$

Problem 2 (5 pts)**.** Compute the following limit:

$$\lim_{x\to 0} \frac{e^x - 1}{\tan x} = \boxed{}$$

Problem 3 (10 pts)**.** Answer the following questions:

(a) Use the definition of derivative to find $f'(x)$ for $f(x) = 2x^3 + 2$.

$f'(x) = $ $\boxed{}$

(b) Find the point-slope equation of the tangent line to the graph of $f(x)$ at $x = 1$.

Problem 4 (30 pts)**.** Find the derivative of the following functions:

(a) $y = \dfrac{x^3 + x^2 + x - 1}{x^{3/2}}$.

$\dfrac{dy}{dx} =$ $\boxed{}$

(b) $f(x) = \sec^2(x^2) - \tan^2(x^2)$.

$f'(x) =$ $\boxed{}$

(c) $g(t) = 3\cos^{-1}(t^5)$.

Hint: The derivative of $y = \cos^{-1} t$ is $\dfrac{dy}{dt} = \dfrac{-1}{\sqrt{1 - t^2}}$.

$g'(t) =$ $\boxed{}$

Problem 5 (10 pts)**.** An aircraft is climbing at 30^o angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi/h?

The aircraft is gaining altitude at a speed of $\boxed{}$

Problem 6 (10 pts)**.** Show that $y = e^{2x} - e^{-4x}$ satisfies the equation

$$y'' + 2y' - 8y = 0.$$

Problem 7 (10 pts)**.** Find $\dfrac{dy}{dy}$ by implicit differentiation.

$$\sin\left(x^3y^3\right) = y.$$

$$\frac{dy}{dx} = \boxed{}$$

Problem 8 (10 pts)**.** Use logarithmic differentiation to find $\dfrac{dy}{dx}$.

$$y = \frac{\sin x \cos x \tan^3 x}{\sqrt{x}}.$$

$$\frac{dy}{dx} = \boxed{}$$