Name: ____________________

VIP ID: ____________________

- Write your name and VIP ID in the space provided above.
- The test has six (6) pages, including this one.
- Credit for each problem is given at the right of each problem number.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- No books, notes or calculators are allowed.

Page	**Max**	**Points**
2	20	
3	20	
4	30	
5	20	
6	10	
Total	100	

Problem 1 (5 pts)**.** Write the following set in set-builder notation:

$$\{\dots, -\tfrac{3}{2}, -\tfrac{3}{4}, 0, \tfrac{3}{4}, \tfrac{3}{2}, \tfrac{9}{4}, 3, \tfrac{15}{4}, \tfrac{9}{2}, \dots\}$$

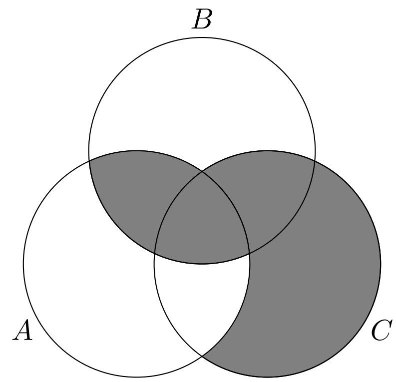
Problem 2 (15 pts–5 pts each)**.** Write out the following set by listing its elements between braces:

(a) $\{n \in \mathbb{Z} : 2 < n < 5\} \times \{n \in \mathbb{Z} : |n| = 5\}$.

(b) $\big\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 2\big\}$

(c) $\big\{X \subseteq \mathscr{P}\big(\{1, 2, 3\}\big) : |X| \leq 1\big\}$

Problem 3 (10 pts)**.** Write the expression involving sets A, B and C given by the following Venn diagram:



Problem 4 (10 pts–5 pts each)**.** Write the following sets either in set-builder notation, or by listing its elements. Draw both sets in the plane.

(a) $\displaystyle\bigcup_{\alpha\in[0,1]} [\alpha, 1] \times [0, 2\alpha^3]$

(b) $\displaystyle\bigcap_{\alpha\in[0,1]} [\alpha, 1] \times [0, 2\alpha^3]$

Problem 5 (10 pts–5 pts each)**.** Suppose $A = \{4, 3, 6, 7, 1, 9\}$ and $B = \{5, 6, 8, 4\}$ have universal set $U = \{0, 1, 2, \ldots, 10\}$.

(a) Find $A \cap B$.

(b) Find $(A^{\complement} \cap B)^{\complement}$.

Problem 6 (10 pts)**.** Translate the following sentence in symbolic logic, negate it, and translate back to English.

If x is prime, then $\sqrt{x}$ is not a rational number.

Problem 7 (10 pts)**.** The *Pierce arrow* $\downarrow$ (also known as the `NOR` operator or `NOR` gate) is a logical operator defined as follows: If P and Q are statements, then $P \downarrow Q$ is true precisely when both P and Q are false. The computer used in the spacecraft that first carried humans to the moon, the Apollo Guidance Computer, was constructed entirely using `NOR` gates with three inputs. Answer the following questions:

(a) Construct a truth table for $P \downarrow Q$.

(b) Show that $P \downarrow P$ is logically equivalent to $\neg P$.

(c) Show that $(P \downarrow Q) \downarrow (P \downarrow Q)$ is logically equivalent to $P \vee Q$.

Problem 8 (10 pts–2 pts each). Let U be the set of all students in your class. Let $C(x)$ be the statement "x has a cat." Let $D(x)$ be the statement "x has a dog." Let $F(x)$ be the statement "x has a ferret." Express each of the statements below in terms of U, $C(x)$, $D(x)$, $F(x)$, quantifiers and logical operations.

(a) A student in your class has a cat, a dog and a ferret.

(b) All students in your class have a cat, a dog or a ferret.

(c) Some student in your class has a cat and a ferret, but not a dog.

(d) No student in your class has a cat, a dog and a ferret.

(e) For each of the three animals (cat, dog, ferret) there is a student in your class who has one of these animals as a pet.

Problem 9 (10 pts). Write each of the following statements in the form "If _____, then _____" in English.

(a) I will remember to send you the address only if you send me an e-mail.

(b) To be a citizen of this country, it is sufficient that you were born in the United States.

(c) If you keep your textbook, it will be useful reference in your future courses.

(d) The Red Wings will win the Stanley Cup if their goalie plays well.

(e) That you get the job implies that you had the best credentials.

(f) The beach erodes when there is a storm.

(g) It is necessary to have a valid passport to log on to the server.

Problem 10 (10 pts–2 pts each)**.** Consider the statement $Q(x)$ about integer numbers $x \in \mathbb{Z}$: "$x+1 > 2x$." Find the truth value of the following:

(a) $Q(-1) \wedge Q(0) \wedge Q(1)$

(b) $\exists x, Q(x)$

(c) $\forall x, Q(x)$

(d) $\exists x, \neg Q(x)$

(e) $\forall x, \neg Q(x)$