Name: _____

VIP ID: _____

- Write your name and VIP ID in the space provided above.

- The test has seven (7) pages, including this one.

- Six of those pages contain one or several problems, for a total worth of 25 points.

- Mark in the table below the four pages that you want me to grade. I will not grade any page that is not marked in the table below. If you mark more than four pages, I will only grade the first four.

- Credit for each problem is given at the right of each problem number.

- For each problem, enter your answer in the box(es) provided.

- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.

- No books or notes are allowed. You may use a graphing calculator with no Computer Algebra System.

Page	Max	Grade it?	Points
2	25		
3	25		
4	25		
5	25		
6	25		
7	25		
Total	100		

Problem 1 (25 pts). Consider the following differential equation:

$$y' = x^2(3 - y)$$

(a) **[5pts]** What kind of equation is it?

This equation is

(b) **[5 pts]** Find an *implicit form* of its **general solution**.

(c) **[5 pts]** Find a *particular solution* that solves the following **initial value problem**.

$$y' = x^2(3 - y), \quad y(0) = 3 - e$$

$$y =$$

(d) **[5 pts]** Are there any **singular solutions**? Find at least one.

$$y =$$

(e) **[5 pts]** Find an explicit *particular solution* that solves this other **initial value problem**.
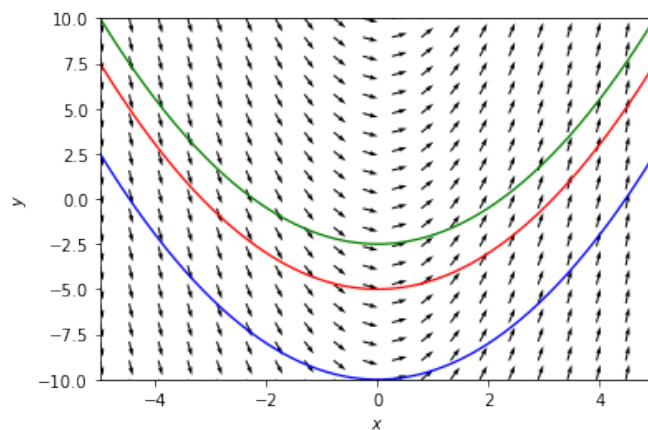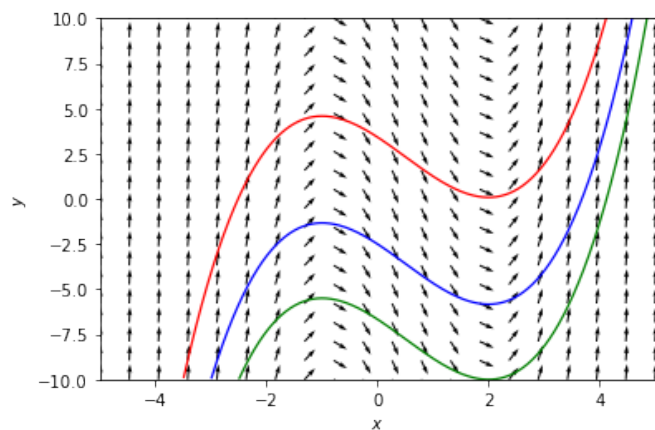
$$y' = x^2(3 - y), \quad y(0) = 3$$

$$y =$$

Problem 2 (10 pts—all or nothing). Which of the following is the slope field of the following differential equation?

$$y' = x^2 - x - 2$$

(You do not need to show work)

Problem 3 (15 pts). Use Euler's method with a step size $h = 0.5$ to obtain a numerical approximation of the following **initial value problem**:

$$y' = xy^2, \quad y(0) = 1$$

n	x_n	y_n	$f(x_n, y_n)$
0			
1			
2			
3			

Problem 4 (10 pts). Solve the following differential equation. (Assume $x > 0$)

$$\left(x^3 + \frac{y}{x}\right) + \left(y^2 + \ln x\right)y' = 0$$

Problem 5 (15 pts). Solve the following differential equation:

$$\frac{dy}{dx} = \frac{y}{x} + e^{y/x}$$

Problem 6 (25 pts)**.** Solve the following differential equation. (Assume $x > 0$)

$$xy' + y = xy^2 \ln x$$

Problem 7 (10 pts). Find a general solution to the following differential equation.

$$y'' - 4y' + 4y = 3e^{2x}$$

Problem 8 (15 pts). Solve the following differential equation. (Assume $y, y' > 0$)

$$yy'' + (y')^2 = 0$$

Problem 9 (25 pts). Given the differential equation $x^2 y'' - 2xy' + 2y = 0$,

(a) [**10 pts**] Verify that the functions $y_1 = x$ and $y_2 = x^2$ are particular solutions.

(b) [**5 pts**] Compute the Wronskian of y_1 and y_2. Indicate where we are guaranteed a solution of the differential equation in the form $y = Ay_1 + By_2$.

(c) [**10 pts**] Find a particular solution if initial conditions are given by $y(1) = 3, y'(1) = 1$.