

Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has six (6) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	Max. points	Your points
2	20	
3	15	
4	15	
5	25	
6	25	
Total	100	

Problem 1 (5 pts). Find $f(3)$ and $f(\pi)$ for $f(x) = \begin{cases} \sqrt{x+1} & \text{if } x \geq 1, \\ 3 & \text{if } x < 1. \end{cases}$

$f(3) =$

$f(\pi) =$

Problem 2 (5 pts). Find the domain of $f(x) = 2 + \sqrt{x-1}$.

$\text{domain} =$

Problem 3 (10 pts). Express the function $f(x) = |x| + 3x + 1$ in piecewise form without using absolute values.

$f(x) = \left\{ \right.$

Problem 4 (5 pts). Let $f(x) = x^2 + 3$ and $g(x) = \sqrt{x}$. Find $(g \circ f)(x)$.

$$(g \circ f)(x) =$$

Problem 5 (10 pts). Recall the “ ε - δ ” definition of limit:

We write $\lim_{x \rightarrow a} f(x) = L$ if for all $\varepsilon > 0$ there exists $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - L| < \varepsilon$.

Use this definition to prove that $\lim_{x \rightarrow 4} (5x - 2) = 18$.

Problem 6 (5 pts). Solve for x :

$$\log(3x) - 3\log(x^{-1/3}) = \log 27.$$

$x =$

Problem 7 (10 pts). Compute the derivatives of the following functions.

(a) $f(x) = \pi x^{-1}(x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1)$

$f'(x) =$

(b) $g(t) = \frac{t^2 - 5}{\sqrt{t}}$

$g'(t) =$

Problem 8 (15 pts). Compute the following limits:

$$(a) \lim_{x \rightarrow 1} \frac{x^2 - 2x - 8}{x^2 - 4} = \boxed{}$$

$$(b) \lim_{x \rightarrow \infty} \frac{x^2 - 2x - 8}{x^2 - 4} = \boxed{}$$

$$(b) \lim_{x \rightarrow -2} \frac{x^2 - 2x - 8}{x^2 - 4} = \boxed{}$$

Problem 9 (10 pts). Find the value of the constant k for which the following function is continuous everywhere:

$$f(x) = \begin{cases} 2k^2x^3 & \text{if } x < 2, \\ x + 32k - 18 & \text{if } x \geq 2. \end{cases}$$

$k =$

Problem 10 (15 pts). Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - 8y = 3$.

Problem 11 (10 pts). How many tangent lines to the curve $y = x/(x+2)$ pass through the point $(0, 0)$.

HINT: *You do not have to compute the equations of the lines.*