

Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has twelve (12) pages, including this one. You have 150 minutes to complete this exam.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit. Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.
- **A:** 243–270 pts. **B+:** 230–242 pts. **B:** 216–229 pts. **C+:** 203–215 pts. **C:** 189–202 pts. **D+:** 175–188 pts. **D:** 160–174 pts. **F:** less than 160 pts.

Page	Max. points	Your points	Page	Max. points	Your points
1	—	—	7	30	
2	20		8	20	
3	20		9	25	
4	30		10	25	
5	30		11	30	
6	20		12	20	
Total	120		Total	150	

Problem 1 (5 pts). Find $f(0)$ and $f(\pi/2)$ for $f(x) = \begin{cases} \sqrt{x+1} & \text{if } x \geq 1, \\ 3 & \text{if } x < 1. \end{cases}$

$$f(0) =$$

$$f(\pi/2) =$$

Problem 2 (10 pts). Find the domain of $f(x) = \sqrt{(x-1)(x-2)}$.

$$\text{domain} =$$

Problem 3 (5 pts). Let $f(x) = x^2 + 4$ and $g(x) = \sqrt{x}$. Find $(g \circ f)(x)$.

$$(g \circ f)(x) =$$

Problem 4 (10 pts). Solve for x :

$$\ln x + \ln(x - 1) = 1$$

$x =$

Problem 5 (10 pts). Compute the derivatives of the following functions.

(a) $f(x) = \pi\sqrt{x}(x^4 - 4x^3 + 6x^2 - 4x^1 + 1 - x^{-1})$

$f'(x) =$

(b) $g(t) = \frac{t^2 - 5}{t^{-1}}$

$g'(t) =$

Problem 6 (15 pts). Compute the following limits:

$$(a) \lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x^2 - 4} = \boxed{}$$

$$(b) \lim_{x \rightarrow -\infty} \frac{x^2 - 2x - 8}{x^2 - 4} = \boxed{}$$

$$(b) \lim_{x \rightarrow -2} \frac{x^2 + 2x - 8}{x^2 - 4} = \boxed{}$$

Problem 7 (15 pts). Find the value of the constant k for which the following function is continuous everywhere:

$$f(x) = \begin{cases} 2k^2x^3 & \text{if } x < 2, \\ x + 32k - 18 & \text{if } x \geq 2. \end{cases}$$

$$k = \boxed{}$$

Problem 8 (15 pts). Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - \frac{9}{2}y = 3$.

Problem 9 (15 pts). Find an equation of the tangent line to the curve $y = \ln(xe^{x^2})$ at the point $(1, 1)$.

Problem 10 (20 pts). Evaluate each limit:

$$\lim_{x \rightarrow 0} (\cos 6x)^{1/x} =$$

$$\lim_{x \rightarrow 0} \frac{\sin(4x^2)}{x^2} =$$

Problem 11 (30 pts). Sketch the graph of the rational function $f(x) = \frac{6x^2}{3 - 3x^2}$.

Indicate clearly: Domain; x - and y -intercepts; vertical and horizontal asymptotes; intervals of increase, decrease and different concavity; location of relative extrema and inflection points.

Problem 12 (10 pts). Find the absolute extrema of $f(x) = \frac{8}{3}x^{4/3} - \frac{4}{3}x^{1/3}$ on the interval $[-1, 1]$.

Absolute maxima at

Absolute minima at

Problem 13 (10 pts). Use logarithmic differentiation to find the derivative of the function

$$y = \frac{\tan^2 x \sin^4 x}{e^{3x}(x^2 + 1)}$$

$$\frac{dy}{dx} =$$

Problem 14 (10 pts). The volume of a cube is increasing at a rate of $300 \text{ cm}^3/\text{min}$. How fast are the edges increasing when the length of an edge is 10 cm?

The edges are increasing at a speed of

Problem 15 (15 pts). A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

Dimensions of most economic fence:

Problem 16 (25 pts). Evaluate each integral:

(a) $\int_1^2 \left(5x + \frac{2}{3x^5} - \sqrt{2}e^x \right) dx$

(b) $\int (3 \sin x - 2 \cos x) dx$

(c) $\int (t^{4/5} - \sin t) dt$

(d) $\int_0^1 5x^4(x^5 + 1)^2 dx$

(e) $\int \left(\frac{1}{x} - 2^x \right) dx$

Problem 17 (30 pts). Express the following expressions of n in closed form and then find the limit.

(a) $\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \cdots + n^2}{n^3}$

(b) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{5k}{n^2}$

Problem 18 (10 pts). Use the Fundamental Theorem of Calculus to find the derivative of the following functions.

(a) $F(x) = \int_1^x \frac{1}{t^4 + 1} dt$

$$F'(x) =$$

(b) $F(x) = \int_{\sin x}^{\pi} \sqrt{e^t + t^8} dt$

$$F'(x) =$$

Problem 19 (10 pts). Find the antiderivative F of $f(x) = 4 - 3(1 + x^2)$ that satisfies $F(1) = 6$.

$$F(x) =$$