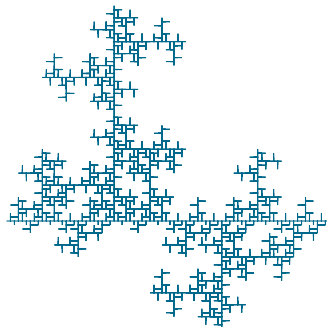


# Lesson 1: Introduction to Functions

Francisco Blanco-Silva

University of South Carolina



## WHAT DO WE NEED TO KNOW?

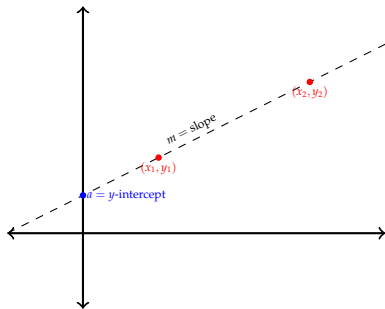
- ▶ A solid background in High-School Algebra:
  - ▶ Simplifying Expressions
  - ▶ Solving Linear Equations and Inequalities
  - ▶ Lines (and their graphs)
  - ▶ Solving Quadratic Equations
  - ▶ Quadratics (and their graphs)
  - ▶ Logarithmic and Exponential Equations
  - ▶ Functions
    - ▶ Evaluation
    - ▶ Combination — Transformation
- ▶ It helps if you have been exposed to:
  - ▶ Polynomials
  - ▶ Exponents — Powers
  - ▶ Problem Solving

# WARM-UP

LINES: SLOPES, INTERCEPTS, ...

A line can be defined by:

- ▶ Two points  $(x_1, y_1)$  and  $(x_2, y_2)$ .
- ▶ A point  $(x_1, y_1)$  and slope  $m$ .
- ▶ Slope  $m$  and  $y$ -intercept  $a$ .

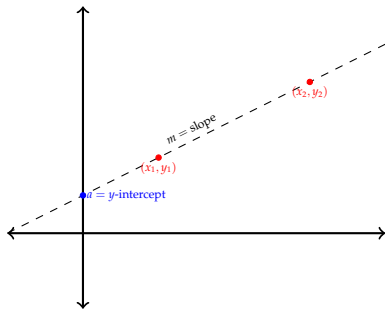


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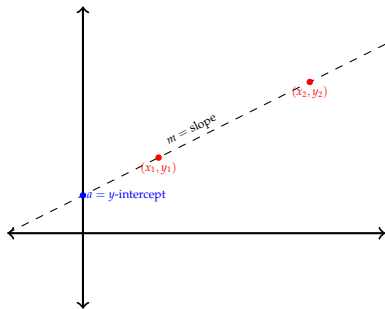
$$(\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2})$$

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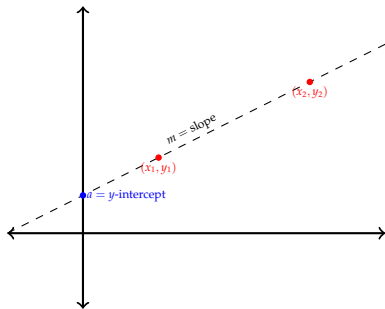
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$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - y_1 = m(x - x_1)$$

$$y = a + mx$$

$$(\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2})$$

# FUNCTIONS

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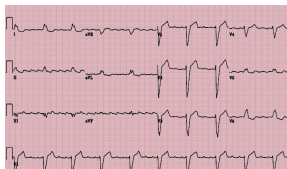
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$$A = \pi r^2$$

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The area of a circle is  $\pi$  times the square of its radius.

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We write  $y = f(t)$  to express that  $y$  (the dependent variable) is a function of  $t$  (the independent variable).

### Example

The value of a car in thousands of dollars,  $V$ , is a function of the age of the car,  $a$ , in years.

$$V = f(a)$$

- What is the independent variable? And the dependent variable?

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After five years, the car is worth \$9,000.

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In about 17 years, this car will be worthless!