Name: ____________________

VIP ID: ____________________

- Write your name and VIP ID in the space provided above.
- The test has six (6) pages, including this one.
- Credit for each problem is given at the right of each problem number.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- No notes are allowed. You may use your book and a graphing calculator (without Computer Algebra System) if needed.

Page	**Max**	**Points**
2	20	
3	25	
4	15	
5	20	
6	20	
Total	100	

Problem 1 (5 pts)**.** Sketch the following set of points in the plane:

$$\big\{(x,y)\in\mathbb{R}^2 : (y-x^2)(y^2-x)(3-x)=0\big\}.$$

Problem 2 (5 pts)**.** Write the expression involving sets A, B and C given by the following Venn diagram:
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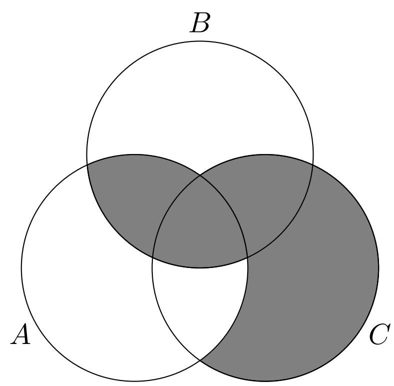

Problem 3 (10 pts–5 pts each part)**.** Write the following sets either in set-builder notation, or by listing its elements. Draw both sets on the plane.

(a) $\displaystyle\bigcup_{\alpha\in[0,2]} [\alpha,2]\times[0,5\alpha^2] =$

(b) $\displaystyle\bigcap_{\alpha\in[0,2]} [\alpha,2]\times[0,5\alpha^2] =$

Problem 4 (5 pts)**.** Decide whether or not the following two statements are logically equivalent:

$$(\neg P) \wedge (P \implies Q) \text{ and } \neg(Q \implies P)$$

Problem 5 (5 pts)**.** Give a statement that is logically equivalent to $\neg(P \implies Q)$ that does not use the symbol $\neg$.

Problem 6 (5 pts)**.** Suppose that P, Q and R are statements and $(P \vee Q) \wedge (Q \implies R)$ is true. If R is false, give all possible combination of truth values of P and Q that work, or state that these cannot be determined.

Problem 7 (10 pts–5 pts each part)**.** Consider the following statement S:

> Given a real number $\varepsilon > 0$, there is a real number $\delta > 0$ so that for all $x \in \mathbb{R}$, if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

(a) Express S in symbolic form.

(b) Express $\neg S$ in symbolic form without using the symbol $\neg$.

Problem 8 (5 pts)**.** Prove the following result:

Theorem. *If* $x \in \mathbb{R}$ *and* $0 < x < 3/2$*, then* $8x(3 - 2x) \leq 9$*.*

Problem 9 (10 pts–5 pts each part)**.** Prove the following result:

Theorem. *If the equation* $ax^2 + bx + c = 0$ *has two different real-valued solutions, and* $b \neq 0$*, then*

(a) The reciprocal of the sum of the two solutions is equal to $-a/b$*.*

(b) The product of the two solutions is equal to c/a*.*

Problem 10 (20 pts–10 pts each part)**.** Prove the following propositions

Proposition 1. *For every $n \in \mathbb{N}$, it follows that*

$$\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2}.$$

Proposition 2. *Suppose $x, y \in \mathbb{R}$. Then $x^3 + x^2y = y^2 + xy$ if and only if $y = x^2$ or $y + x = 0$.*

Problem 11 (10 pts–5 pts each part)**.** Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by $f(m, n) = 7m + 2n$. Prove or disprove the following statements:

(a) f is injective.

(b) f is surjective.

Problem 12 (10 pts–5 pts each part)**.** Consider the set $A = \{a, b, c, d, e, f, g\}$, and let R be an equivalence relation on A.

(a) Is it possible for all equivalence classes to have the same cardinality? Why or why not?

(b) Suppose that aRb, bRf, eRc, dRc and all other relations follow from these. List all the equivalence classes and give the elements of each.