Name:	
4-digit code:	

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has fourteen (14) pages, including this one.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit. Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.
- **A:** 243–270 pts. **B+:** 230–242 pts. **B:** 216–229 pts. **C+:** 203–215 pts. **C:** 189–202 pts. **D+:** 175–188 pts. **D:** 160–174 pts. **F**: less than 160 pts.

Page	Max. points	Your points	Page	Max. points	Your points
1	—		8	30	
2	20		9	20	
3	15		10	30	
4	15		11	25	
5	25		12	30	
6	25		13	20	
7	20		14	25	
Total	120		**Total**	180	

Problem 1 (5 pts)**.** Find $f(0)$ and $f(\pi/2)$ for $f(x) = \begin{cases} \sqrt{x+1} & \text{if } x \geq 1, \\ 3 & \text{if } x < 1. \end{cases}$

$f(0) =$ $\boxed{}$

$f(\pi/2) =$ $\boxed{}$

Problem 2 (10 pts)**.** Find the domain of $f(x) = \sqrt{(x-1)(x-2)}$.

domain $=$ $\boxed{}$

Problem 3 (5 pts)**.** Express the function $f(x) = |x-1|$ in piecewise form without using absolute values.

$f(x) = \begin{cases} \boxed{} \end{cases}$

Problem 4 (5 pts)**.** Let $f(x) = x^2 + 4$ and $g(x) = \sqrt{x}$. Find $\big(g \circ f\big)(x)$.

$$\big(g \circ f\big)(x) = \boxed{}$$

Problem 5 (10 pts)**.** Recall the "ε–δ" definition of limit:

> We write $\lim_{x \to a} f(x) = L$ if for all $\varepsilon > 0$ there exists $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - L| < \varepsilon$.

Use this definition to prove that $\lim_{x \to 2} (-x - 2) = -4$.

Problem 6 (5 pts)**.** Solve for x:

$$\ln x + \ln(x-1) = 1$$

$x = \boxed{}$

Problem 7 (10 pts)**.** Compute the derivatives of the following functions.

(a) $f(x) = \pi\sqrt{x}(x^4 - 4x^3 + 6x^2 - 4x^1 + 1 - x^{-1})$

$f'(x) = \boxed{}$

(b) $g(t) = \dfrac{t^2 - 5}{t^{-1}}$

$g'(t) = \boxed{}$

Problem 8 (15 pts)**.** Compute the following limits:

(a) $\displaystyle\lim_{x\to 2} \frac{x^2-2x-8}{x^2-4} = \boxed{}$

(b) $\displaystyle\lim_{x\to -\infty} \frac{x^2-2x-8}{x^2-4} = \boxed{}$

(b) $\displaystyle\lim_{x\to -2} \frac{x^2-2x-8}{x^2-4} = \boxed{}$

Problem 9 (10 pts)**.** Find the value of the constant k for which the following function is continuous everywhere:

$$f(x) = \begin{cases} 2k^2x^3 & \text{if } x < 2, \\ x + 32k - 18 & \text{if } x \geq 2. \end{cases}$$

$k = \boxed{}$

Problem 10 (15 pts)**.** Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - \frac{9}{2}y = 3$.

Problem 11 (10 pts)**.** How many tangent lines to the curve $y = x/(x+1)$ pass through the point $(0, 0)$.
HINT: *You do not have to compute the equations of the lines.*

Problem 12 (10 pts)**.** Evaluate each limit:

$$\lim_{x\to 0} \frac{e^x - 1}{\tan x} = \boxed{}$$

$$\lim_{x\to 0} \frac{\tan(5x^2)}{x^2} = \boxed{}$$

Problem 13 (10 pts)**.** Find an equation of the normal line to the curve $y = \ln(xe^{x^2})$ at the point $(1, 1)$.

Problem 14 (30 pts)**.** Sketch the graph of the rational function $f(x) = \dfrac{2x^2 - 8}{x^2 - 16}$.

Indicate clearly:

- Domain
- x- and y-intercepts.
- Vertical and horizontal asymptotes (any holes?).
- Intervals of increase, decrease and different concavity.
- Location of relative extrema and inflection points.

Problem 15 (10 pts)**.** Find the absolute extrema of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$.

Absolute maxima at $\boxed{}$

Absolute minima at $\boxed{}$

Problem 16 (10 pts)**.** Use logarithmic differentiation to find the derivative of the function

$$y = \frac{\sin^2 x \tan^4 x}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \boxed{}$$

Problem 17 (10 pts). An aircraft is climbing at 30^o angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi/h?

The aircraft is gaining altitude at a speed of

Problem 18 (20 pts). A container with square base, vertical sides, and open top is to be made from 300 ft^2 of material. Find the dimensions of the container with greatest volume.

Dimensions of container:

Problem 19 (25 pts)**.** Evaluate each integral:

(a) $\displaystyle\int_0^2 \left(5x + \frac{2}{3x^5} - \sqrt{2}e^x\right) dx$

(b) $\displaystyle\int \left(3\sin x - 2\sec^2 x\right) dx$

(c) $\displaystyle\int (1 + \sin t)^{90} \cos t \, dt$

(d) $\displaystyle\int_0^1 \frac{5x^4}{(x^5+1)^2}\, dx$

(e) $\displaystyle\int \frac{3x-2}{(x-1)(x+1)^2}\, dx$

Problem 20 (20 pts)**.** Express the following functions of n in closed form and then find the limit.

(a) $\displaystyle\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$

(b) $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \frac{5k}{n^2}$

Problem 21 (10 pts)**.** Use the definition of **definite integral** to express $\int_{-\pi/2}^{\pi/2}(1+\cos x)\,dx$ as a limit.

Problem 22 (10 pts)**.** Use the Fundamental Theorem of Calculus to find the derivative of the following functions.

(a) $g(x) = \displaystyle\int_1^x \frac{1}{t^3+1}\,dt$

$g'(x) =$ $\boxed{}$

(b) $g(y) = \displaystyle\int_x^\pi \sqrt{1+\sec t}\,dt$

$g'(y) =$ $\boxed{}$

Problem 23 (10 pts)**.** Find the antiderivative F of $f(x) = 4 - 3(1+x^2)^{-1}$ that satisfies $F(1) = 0$.

$F(x) =$ $\boxed{}$

Problem 24 (25 pts)**.** Sketch the region enclosed by the curves $y = x^2$, $y = 4x - x^2$, and find the corresponding area.

Area: $\boxed{}$