

Answers ①

$$\textcircled{1} |a| = \sqrt{2^2 + 0^2 + 1^2} = \sqrt{5}$$

$$|b| = \sqrt{4^2 + (-2)^2 + 0^2} = \sqrt{20}$$

$$a \cdot b = (2)(4) + (0)(-2) + (1)(0) = 8$$

$$\cos \theta = \frac{a \cdot b}{|a||b|} = \frac{8}{\sqrt{5} \cdot \sqrt{20}} = \frac{4}{5}$$

$$\theta = \cos^{-1}(4/5) \approx \boxed{37^\circ}$$

$$\textcircled{2} r'(t) = -2\sin(2t), 4, 2\cos(2t)$$

$$|r'(t)| =$$

$$\sqrt{(-2\sin(2t))^2 + 4^2 + (2\cos(2t))^2}$$

$$|r'(0)| = \sqrt{4 + 4^2} = \sqrt{20}$$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \left\langle \frac{-2\sin(2t)}{\sqrt{20}}, \frac{4}{\sqrt{20}}, \frac{2\cos(2t)}{\sqrt{20}} \right\rangle$$

simplified \downarrow

$$T(t) = \left\langle \frac{\sqrt{20}(-4\cos(2t)) - 0}{20}, 0, \frac{\sqrt{20}(-4\sin(2t))}{20} \right\rangle$$
$$T(t) = \left\langle \frac{-\sqrt{20}\cos(2t)}{5}, 0, \frac{-\sqrt{20}\sin(2t)}{5} \right\rangle$$

$$|T'(t)| = \sqrt{\left(\frac{-\sqrt{20}\cos(2t)}{5}\right)^2 + 0^2 + \left(\frac{-\sqrt{20}\sin(2t)}{5}\right)^2}$$

$$|T'(t)| = \sqrt{20/25}$$

$$\frac{|T'(t)|}{|r'(t)|} = \boxed{\frac{\sqrt{\frac{20}{25}}}{\sqrt{20}}}$$