Name: ______________________________

VIP ID: ______________________________

- Write your name and your VIP ID in the space provided above.
- The test has nine (9) pages, including this one, and the formula sheet attached at the end.
- You have 150 minutes to complete this test.
- Each problem is worth 10 points.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- No books, notes or calculators may be used on this test.

Page	**Max**	**Points**
2	20	
3	20	
4	20	
5	10	
6	10	
7	10	
8	10	
Total	100	

Problem 1. Find the distance from the point $(-4, -2, 1)$ to the line L with parametric equations

$$L : \begin{cases} x = 5 - t \\ y = 1 - 2t \\ z = -6 + 3t \end{cases}$$

$d =$ $\boxed{}$

Problem 2 (10 pts)**.** Find the intersection of the plane $x + y + z = -3$ with the line

$$L : \begin{cases} x = 3 + 2t \\ y = -3 + 4t \\ z = -3 + 6t \end{cases}$$

Intersection: $\boxed{}$

Problem 3. Consider the helix obtained as the graph of the following vector function

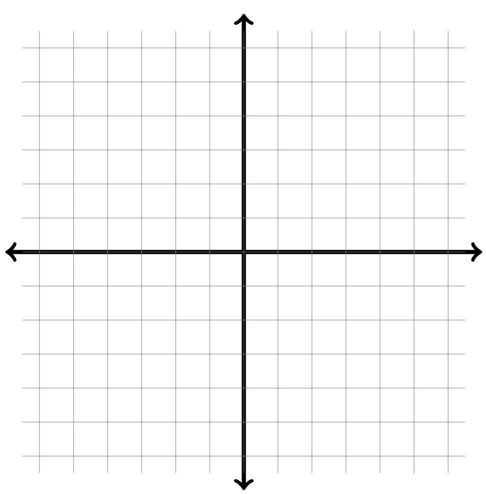
$$\boldsymbol{r}(t) = \cos\left(\tfrac{t}{2}\right)\boldsymbol{i} + \sin\left(\tfrac{t}{2}\right)\boldsymbol{j} + \tfrac{t}{2}\boldsymbol{k}.$$

(a) (5 pts) Prove that this helix lies on the cylinder $x^2 + y^2 = 1$.

(b) (5 pts) Calculate the length of the section of the helix for $0 \le t \le 4\pi$.

$\ell =$

Problem 4 (10 pts)**.** Find and sketch the domain of the function $f(x, y) = \sqrt{y - 6x - 5}$



The domain is

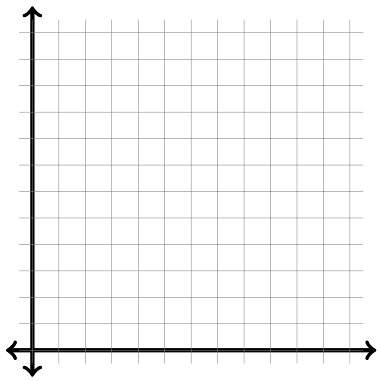
Problem 5 (10 pts)**.** Find all the local maxima, local minima and saddle points of the function

$$f(x, y) = x^3 + y^3 + 3x^2 - 9y^2 - 1.$$

Problem 6 (10 pts)**.** Evaluate $\int_C (xy + y + z)\, ds$ along the curve $\boldsymbol{r}(t) = 2t\boldsymbol{i} + t\boldsymbol{j} + (6 - 2t)\boldsymbol{k}$, for $0 \le t \le 1$.

$$\int_C (xy + y + z)\, ds = \boxed{}$$

Problem 7 (10 pts)**.** Find the absolute maximum and minimum (location and value) of the function $f(x,y) = 2x^2 - 4x + y^2 - 6y + 3$ on the closed triangular region bounded by the lines $x = 0$, $y = 3$, $y = 3x$ in the first quadrant. Sketch the region.

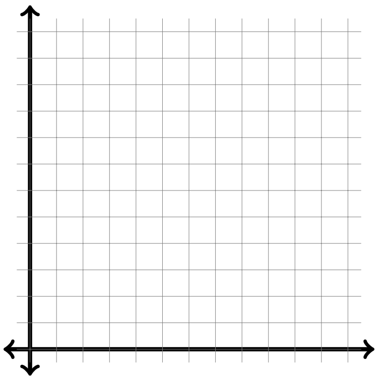


Maximum:

Minimum:

Problem 8 (10 pts)**.** Sketch the domain of integration, convert the polar integral to a Cartesian integral, and evaluate

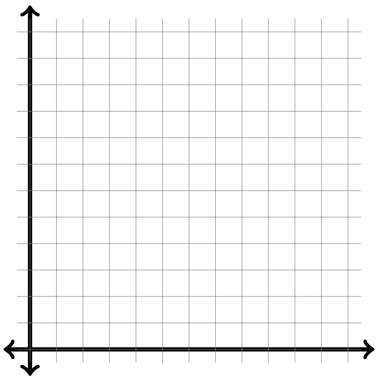
$$\int_0^{\pi/4}\int_0^{3\sec\theta} r^5 \sin^2\theta \, dr \, d\theta$$



$$\int_0^{\pi/4}\int_0^{3\sec\theta} r^5 \sin^2\theta \, dr \, d\theta = \boxed{} = \boxed{}$$

Problem 9 (10 pts)**.** Sketch the domain of integration, reverse the order of integration and evaluate

$$\int_0^{2\sqrt{\ln 10}} \int_{y/2}^{\sqrt{\ln 10}} 8e^{x^2}\,dx\,dy$$



$$\int_0^{2\sqrt{\ln 10}} \int_{y/2}^{\sqrt{\ln 10}} 8e^{x^2}\,dx\,dy = \boxed{\qquad\qquad\qquad} = \boxed{\qquad}$$

Problem 10 (10 pts)**.** We want to find the volume of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 360$ and below by the paraboloid $2x = x^2 + y^2$. Sketch that solid, find an integral expression that computes its volume (double or triple integral, your choice), and evaluate that integral to obtain that volume.

$$V = \iint_D f(x, y)dA = \iiint_R dV = \boxed{}$$