Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has twelve (12) pages, including this one. You have 120 minutes to complete this exam.
- Each page is worth 20 points. You must choose 5 pages and work on those problems. If you work problems from more than 5 pages, I will grade the last 5 pages with any of your written content.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- No books, notes or calculators may be used on this test.

Page	**Max. points**	**Your points**
1	—	—
2	20	
3	20	
4	20	
5	20	
6	20	
Total	100	

Page	**Max. points**	**Your points**
7	20	
8	20	
9	20	
10	20	
11	20	
12	20	
Total	120	

Problem 1 (5 pts)**.** Find $f(0)$ and $f(\pi/2)$ for $f(x) = \begin{cases} \sqrt{x+1} & \text{if } x \geq 1, \\ 3 & \text{if } x < 1. \end{cases}$

$f(0) =$ _____

$f(\pi/2) =$ _____

Problem 2 (10 pts)**.** Find the domain of $f(x) = \sqrt{(x-1)(x-2)}$.

domain = _____

Problem 3 (5 pts)**.** Let $f(x) = x^2 + 4$ and $g(x) = \sqrt{x}$. Find $\big(g \circ f\big)(x)$.

$\big(g \circ f\big)(x) =$ _____

Problem 4 (10 pts)**.** Solve for x:

$$\ln x + \ln(x-1) = 1$$

$x =$ [ ]

Problem 5 (10 pts)**.** Compute the derivatives of the following functions.

(a) $f(x) = \pi\sqrt{x}(x^4 - 4x^3 + 6x^2 - 4x^1 + 1 - x^{-1})$

$f'(x) =$ [ ]

(b) $g(t) = \dfrac{t^2 - 5}{t^{-1}}$

$g'(t) =$ [ ]

Problem 6 (10 pts)**.** Compute the following limits:

(a) $\lim_{x\to 2} \dfrac{x^2+2x-8}{x^2-4} =$ $\boxed{}$

(b) $\lim_{x\to -\infty} \dfrac{x^2-2x-8}{x^2-4} =$ $\boxed{}$

(b) $\lim_{x\to -2} \dfrac{x^2+2x-8}{x^2-4} =$ $\boxed{}$

Problem 7 (10 pts)**.** Find the value of the constant k for which the following function is continuous everywhere:

$$f(x) = \begin{cases} 2k^2x^3 & \text{if } x < 2, \\ x + 32k - 18 & \text{if } x \geq 2. \end{cases}$$

$k =$ $\boxed{}$

Problem 8 (10 pts)**.** Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - \frac{9}{2}y = 3$.

Problem 9 (10 pts)**.** Find an equation of the tangent line to the curve $y = \ln(xe^{x^2})$ at the point $(1, 1)$.

Problem 10 (20 pts)**.** Evaluate each limit:

$\lim_{x\to 0} \cot 2x \sin 6x =$ $\boxed{}$

$\lim_{x\to 0} \dfrac{\sin(4x^2)}{x^2} =$ $\boxed{}$

Problem 11 (20 pts)**.** Sketch the graph of the rational function $f(x) = \dfrac{6x^2}{3 - 3x^2}$.

Indicate clearly: Domain; x- and y-intercepts; vertical and horizontal asymptotes; intervals of increase, decrease and different concavity; location of relative extrema and inflection points.

Problem 12 (10 pts)**.** Find the absolute extrema of $f(x) = \frac{8}{3}x^{4/3} - \frac{4}{3}x^{1/3}$ on the interval $[-1, 1]$.

Absolute maxima at

Absolute minima at

Problem 13 (10 pts)**.** Use logarithmic differentiation to find the derivative of the function

$$y = \frac{\tan^2 x \sin^4 x}{e^{3x}(x^2+1)}$$

$$\frac{dy}{dx} =$$

Problem 14 (10 pts)**.** The volume of a cube is increasing at a rate of 300 cm^3/min. How fast are the edges increasing when the length of an edge is 10 cm?

The edges are increasing at a speed of

Problem 15 (10 pts)**.** A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

Dimensions of most economic fence:

Problem 16 (20 pts)**.** Evaluate each integral:

(a) $\displaystyle\int_0^2 \left(5x + \frac{2}{3x^5} - \sqrt{2}e^x\right) dx$

(b) $\displaystyle\int \left(3\sin x - 2\cos x\right) dx$

(c) $\displaystyle\int \left(t^{4/5} - \sin t\right) dt$

(d) $\displaystyle\int \left(\tfrac{1}{x} - 2^x\right) dx$

Problem 17 (20 pts)**.** Express the following expressions of n in closed form and then find the limit.

(a) $\lim_{n\to\infty} \dfrac{1^2+2^2+3^2+\cdots+n^2}{n^3}$

(b) $\lim_{n\to\infty} \sum_{k=1}^{n} \dfrac{5k}{n^2}$

Problem 18 (10 pts)**.** Use the Fundamental Theorem of Calculus to find the derivative of the following functions.

(a) $F(x) = \displaystyle\int_1^x \frac{1}{t^4+1}\,dt$

$F'(x) =$ ☐

(b) $F(x) = \displaystyle\int_{\sin x}^{\pi} \sqrt{e^t + t^8}\,dt$

$F'(x) =$ ☐

Problem 19 (10 pts)**.** Find the antiderivative F of $f(x) = 4 - 3(1+x^2)$ that satisfies $F(1) = 6$.

$F(x) =$ ☐