

Problem 1. State DeMorgan's Laws for set complements $(A \cup B)^c$ and $(A \cap B)^c$.

Problem 2. Let $A = \{a, b\}$, $B = \{b, 1, 2\}$. Give the elements of $(A \times B) \setminus (A \times \{b\})$ by listing them within braces.

Problem 3. For each $n \in \mathbb{N}$, let I_n be the closed interval $[-1 + \frac{1}{n}, 1 - \frac{1}{n}]$. Describe the set $\bigcup_{n \in \mathbb{N}} I_n$ in either interval or set-builder notation.

Problem 4. For each $n \in \mathbb{N}$, let J_n be the closed interval $[1 + \frac{1}{n}, 2 - \frac{1}{n}]$. Describe the set $\bigcup_{n=2}^{\infty} J_n$ in either interval or set-builder notation.

Problem 5. Let $X = \{a, b, c, d\}$ and $S = \{Y \in \mathcal{P}(X) : b \notin Y, |Y| \leq 2\}$. Give the elements of S .

Problem 6. Let P and Q be statements. Are the following statements equivalent? Justify your answers.

(a) $P \wedge (Q \vee \neg Q)$ and $(\neg P) \implies (Q \wedge \neg Q)$.

(b) $(\neg P) \wedge (P \implies Q)$ and $\neg(Q \vee P)$.

Problem 7. Consider the following statement S :

“All foreign cars are well made.”

Which of the following statements (there may be more than one) correctly negate S ?

(a) “All foreign cars are badly made.”

(b) “All domestic (non-foreign) cars are well made.”

(c) “There are domestic (non-foreign) cars that are well made.”

(d) “Some foreign cars are badly made.”

(e) “If a car is not foreign, then it is not well made.”

Problem 8. Consider the following statement P :

$$\forall X \subset \mathbb{N}, \exists n \in \mathbb{Z}, |X| = n$$

(a) Rewrite $\neg P$ as an affirmative statement (i.e. the symbol \neg should not appear anywhere)

(b) What is $\neg P$ saying in plain English? Is it true or false?

Problem 9. Consider the following statement R :

“An integer n is divisible by 15 only if it is divisible by 5.”

(a) Rewrite R in the form $P \implies Q$.

(b) Use the word *necessary* or *sufficient* as appropriate:

“For an integer n to be divisible by 5 it is _____ that n be divisible by 15.”

(c) Use the word *necessary* or *sufficient* as appropriate:

“For an integer n to be divisible by 15 it is _____ that n be divisible by 5.”

(d) State the converse of R .

(e) State the contrapositive of R .

Problem 10. Let $A = [-1, 0) \cup (0, 1]$, and consider $U = \mathbb{R}$ as the universal set. Describe the set A^c .