Name: _____

VIP ID: _____

- Write your name and your VIP ID in the space provided above.
- The test has six (6) pages, including this one.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given at the right of each problem number.

Page	**Max**	**Points**
2	30	
3	30	
4	20	
5	20	
Total	100	

Problem 1 (30 pts—10 pts each part)**.** A body with mass 0.5 kg is attached to the end of a spring that is stretched 2 m by a force of 100 N. It is set in motion one meter to the right, and moving to the left at that time with an initial velocity of 5 m/s.

(a) Find the position function of the body.

$x(t) =$ [ ]

(b) Indicate the amplitude, frequency, period of oscillation and time lag of this motion.

Amplitude: [ ] Frequency: [ ] Period: [ ] Time lag: [ ]

(c) Sketch the solution curve. Make sure to label all relevant information (amplitude, time lag and period).

Problem 2 (30 pts—10 pts each part)**.** The mass and spring of the previous problem are now attached also to a dashpot that provides 1 N of resistance for each meter per second of velocity. The mass is set in motion with the same initial position and initial velocity as before.

(a) Find the position function of the body.

$x(t) =$ [ ]

(b) Indicate the amplitude, the new frequency, pseudoperiod of motion and new time lag of this motion.

Amplitude: [ ] Frequency: [ ] Pseudoperiod: [ ] Time lag: [ ]

(c) Sketch the solution curve. Make sure to label all relevant information (amplitude, time lag and pseudoperiod).

Problem 3 (20 pts—10 pts each part)**.** Consider an undamped forced motion with equation $x'' + 9x = 80\cos 5t$, which is set in motion with $x(0) = x'(0) = 0$.

(a) Find the position function of the body.

$x(t) =$ ☐

(b) Sketch the solution curve. Indicate clearly how far the mass moves to the right before starting back toward the origin (show all necessary work to find this value)

Problem 4 (10 pts)**.** A motorboat starts from rest. Its motor provides a constant acceleration of 4 ft/s^2, but water resistance causes a deceleration of $0.0025v^2$. What is the *limiting velocity* of the boat?

$v =$ _____

Problem 5 (10 pts)**.** An arrow is shot straight upward from the ground with an initial velocity of 160 ft/s. It experiences both the deceleration of gravity and deceleration $0.00125v^2$ due to air resistance. How high in the air does it go?

$h =$ _____

$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x)\,dx$			
1	$\frac{1}{s}$ $s>0$	$cf(x) \pm g(x)$	$cF(s) \pm G(s)$	$s > max(a,b)$
x^n	$\frac{n!}{s^{n+1}}$ $s>0$	$e^{\alpha x} f(x)$	$F(s-\alpha)$	$s > a+\alpha$
$e^{\alpha x}$	$\frac{1}{s-\alpha}$ $s>\alpha$	$x^n f(x)$	$(-1)^n F^{(n)}(s)$	$s > a$
$\sin \beta x$	$\frac{\beta}{s^2+\beta^2}$ $s>0$	$f'(x)$	$sF(s) - f(0)$	
$\cos \beta x$	$\frac{s}{s^2+\beta^2}$ $s>0$	$f''(x)$	$s^2F(s) - sf(0) - f'(0)$	