

**Name:** \_\_\_\_\_

**VIP ID:** \_\_\_\_\_

- Write your name and VIP ID in the space provided above.
- The test has four (4) pages, including this one.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.

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Page	Max. points	Your points
2	50	
3	30	
4	20	
<b>Total</b>	100	

**Problem 1** (50 pts—10 pts each part). Consider the 2nd-degree polynomial

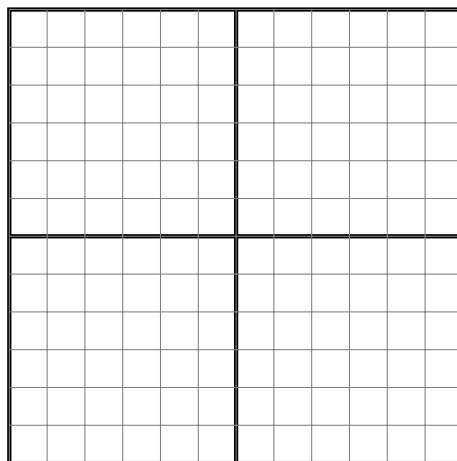
$$p_2(x, y) = 4x^2 + 25y^2 - 20xy.$$

- (a) The polynomial  $p_2$  is a quadratic form. Find a symmetric matrix  $\mathbf{A}$  so that

$$p_2(x, y) = \mathcal{Q}_{\mathbf{A}}(x, y).$$

- (b) Classify the symmetric matrix  $\mathbf{A}$ .

- (c) Sketch the level line  $p_2(x, y) = 0$ .



- (d) Is  $p_2$  a coercive function? Why?

- (e) Find all critical points of  $p_2$ , and classify them.

**Problem 2** (30 pts—10 pts each part). Consider the function

$$f(x, y, z) = x^2 + y^2 + z^2 + \frac{1}{x^2 + y^2 + z^2}$$

- (a) Is  $f$  a convex function? Why?
  
  
  
  
  
  
  
  
  
  
- (b) What is the global minimum value of  $f$ ? Why?
  
  
  
  
  
  
  
  
  
  
- (c) Find all global minima of  $f$ .

**Problem 3** (20 pts). Consider the function  $f(x, y) = x^3 + e^{3y} - 3xe^y$ . Show that  $f$  has exactly one critical point, and that this point is a local minimum but not a global minimum.