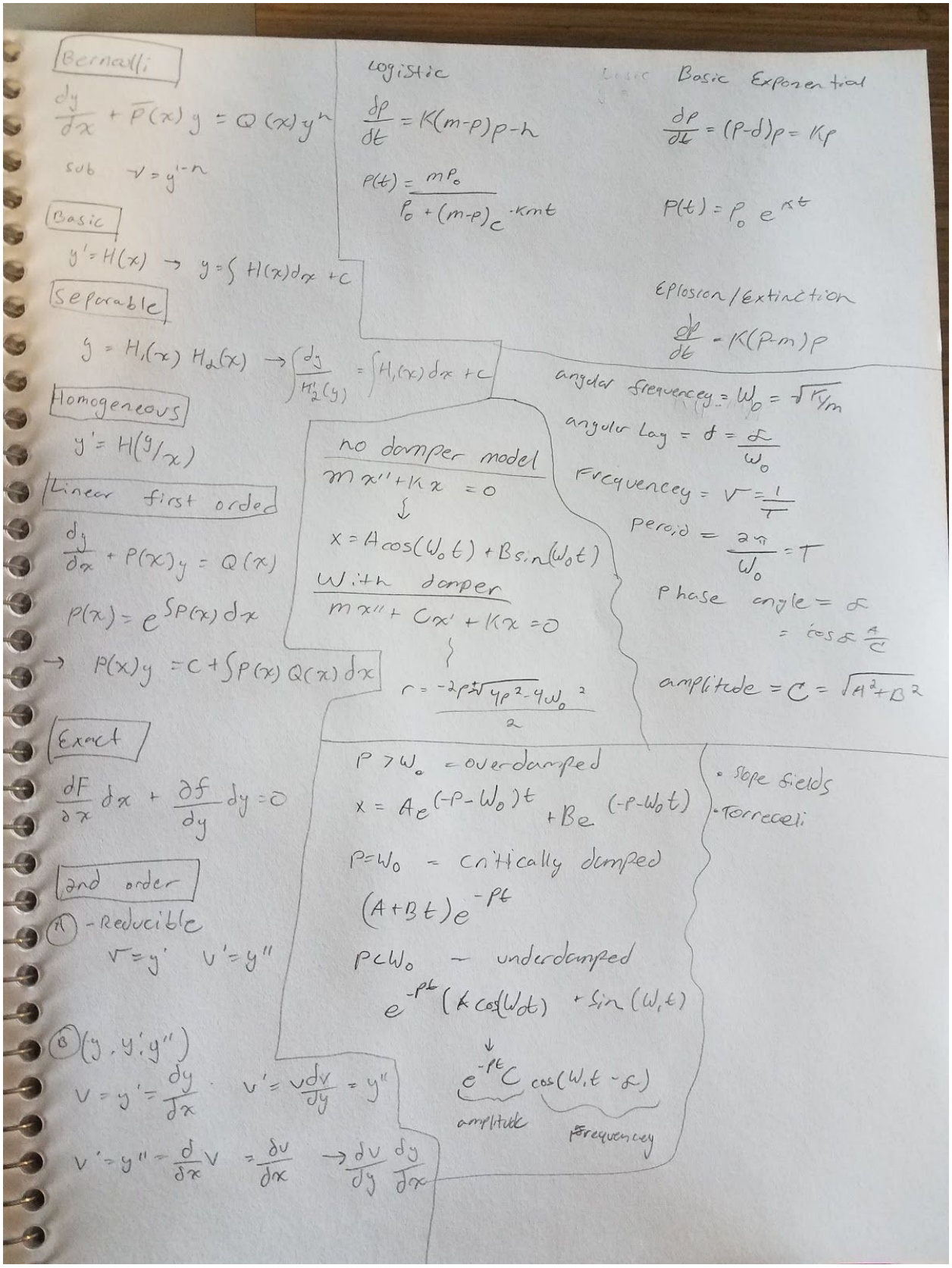
Formula sheet - Robby Carff



Bernalli

$\frac{dy}{dx} + P(x)y = Q(x)y^n$

sub $v = y^{1-n}$

Basic

$y' = H(x) \rightarrow y = \int H(x)dx + C$

Separable

$y = H_1(x)\,H_2(x) \rightarrow \int \frac{dy}{H_2(y)} = \int H_1(x)dx + C$

Homogeneous

$y' = H(y/x)$

Linear first order

$\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = e^{\int P(x)dx}$

$\rightarrow P(x)y = C + \int P(x)Q(x)dx$

Exact

$\frac{dF}{dx}dx + \frac{\partial f}{dy}dy = 0$

2nd order

(A) - Reducible

$v = y' \quad v' = y''$

(B) (y, y', y'')

$v = y' = \frac{dy}{dx} \quad v' = v\frac{dv}{dy} = y''$

$v' = y'' = \frac{d}{dx}v = \frac{\partial v}{dx} \rightarrow \frac{dv}{dy}\frac{dy}{dx}$

Logistic

$\frac{dP}{dt} = K(m-P)P - h$

$P(t) = \frac{mP_0}{P_0 + (m-P)_C \cdot Kmt}$

Basic Exponential

$\frac{dP}{dt} = (P-d)P = KP$

$P(t) = P_0 e^{Kt}$

Eplosion / extinction

$\frac{dP}{dt} = K(P-m)P$

angular frequencey $= W_0 = \sqrt{k/m}$

angular Lag $= \delta = \frac{\alpha}{W_0}$

Frequencey $= \nu = \frac{1}{T}$

period $= \frac{2\pi}{W_0} = T$

Phase angle $= \alpha$

$= \cos\alpha \frac{A}{C}$

amplitude $= C = \sqrt{A^2 + B^2}$

no damper model

$mx'' + kx = 0$

$\downarrow$

$x = A\cos(W_0 t) + B\sin(W_0 t)$

With damper

$mx'' + Cx' + kx = 0$

$r = \frac{-2p \pm \sqrt{4p^2 - 4W_0^2}}{2}$

$P > W_0$ = overdamped

$x = Ae^{(-P-W_0)t} + Be^{(-P-W_0 t)}$

$P = W_0$ = critically damped

$(A+Bt)e^{-Pt}$

$P < W_0$ - underdamped

$e^{-Pt}(k\cos(W_0 t) + \sin(W_1 t)$

$\downarrow$

$e^{-Pt}C\cos(W_1 t - \alpha)$

amplitude — frequencey

- Slope fields
- Torrecelli

Populations

- Basic exponential
- Logistic
- Logistic harvesting
- explosion/extinction

Geometric

- Equations for curves
  The formula sheet given to us is important, it lists
- Orthogonal trajectories

Springs

- Free undamped
- Free damped

Torricelli's

- Find the curve
- Find the time to drain