Problem 1. Use the definition of **definite integral** to express the integrals as limits.

(a) $\displaystyle\int_1^2 2x\,dx$

(b) $\displaystyle\int_0^1 \frac{x}{x+1}\,dx$

(c) $\displaystyle\int_1^2 \sqrt{x}\,dx$

(d) $\displaystyle\int_{-\pi/2}^{\pi/2} (1+\cos x)\,dx$

Problem 2. Sketch the region whose signed area is represented by the following definite integral, and evaluate the integral using an appropriate formula from geometry.

(a) $\displaystyle\int_0^3 x\,dx$

(b) $\displaystyle\int_0^2 \left(1-\tfrac{1}{2}x\right)dx$

(c) $\displaystyle\int_0^5 2\,dx$

(d) $\displaystyle\int_{-1}^2 |2x-3|\,dx$

Problem 3. Use the properties of definite integrals and appropriate formulas from geometry to evaluate the following integrals.

(a) $\displaystyle\int_{-1}^3 (4-5x)\,dx$

(b) $\displaystyle\int_{-2}^2 (1-3|x|)\,dx$

(c) $\displaystyle\int_0^1 \left(x+2\sqrt{1-x^2}\right)dx$

(d) $\displaystyle\int_{-3}^0 \left(2+\sqrt{9-x^2}\right)dx$

Problem 4. Find $\displaystyle\int_{-1}^2 \big(f(x)+2g(x)\big)\,dx$ if $\displaystyle\int_{-1}^2 f(x)\,dx = 5$ and $\displaystyle\int_{-1}^2 g(x)\,dx = -3$.

Problem 5. Find $\displaystyle\int_1^4 \big(3f(x)-g(x)\big)\,dx$ if $\displaystyle\int_1^4 f(x)\,dx = 2$ and $\displaystyle\int_1^4 g(x)\,dx = 10$.

Problem 6. Evaluate the following limits by expressing them as a definite integral over the interval $[a,b]$, and applying appropriate formulas from geometry.

(a) $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}(3x_k^*+1)\Delta x$; $a=0, b=1$.

(b) $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{4-(x_k^*)^2}\,\Delta x$; $a=-2, b=2$.