Math 242 Final Exam, Saturday 3 May

Name: **Last 4 digits of SSN:**

Show all **work clearly**, **make sentences**. No work means no credit. The points are: ex1: 10, ex2: 10, ex3: 15, ex4: 10, ex5: 10, ex6: 15, ex7: 10, ex8: 10, ex9: 15, ex10: 10, ex11: 20, ex15: (Total=150 pts).

Exercise 1 (10 points) Solve the initial value problem:

$$y' = 2xy + 3x^2 \exp x^2, \quad y(0) = 5.$$

Exercise 2 (10 points) Find a general solution of the differential equation

$$2xy^2 + x^2y' = y^2.$$

Exercise 3 (15 points) We consider the following differential equation:

$$xy' = 6y + 12x^4y^{2/3},\ x > 0.$$

1. What kind of equation is it?

2. What substitution do we have to do?

3. What differential equation do we obtain after the substitution?

4. Solve this last differential equation and then find the expression of y.

Exercise 4 (10 points) We consider the following differential equation:

$$x^2y - x^3y' = y^3.$$

1. Write this differential equation as a homogeneous one.

2. Then solve this differential equation.

Exercise 5 (10 points) Show that the differential equation

$$e^y + y\cos x + (xe^y + \sin x)\, y' = 0,$$

is exact and then solve it.

Exercise 6 (15 points) We give the differential equation:

$$\frac{dx}{dt} = x^2 - 5x + 4.$$

1) What are the critical points ? Use a phase diagram to determine whether each critical point is stable or unstable.

2) Solve this differential equation with $x(0) = 2$.

Exercise 7 (10 points) We give an initial value problem:

$$y' = 20x, \quad y(0) = 1.$$

Write the algorithm of the Euler's method with a step size h, and apply it to find approximate values of the solution on the interval $[0, 0.5]$ with step size $h = 0.1$.

x	0	0.1	0.2	0.3	0.4	0.5
approx solution						

Exercise 8 (10 points) Solve the differential equation

$$y^{(3)} - 6y'' + 9y' - 54y = 0,$$

using the fact that the function $x \mapsto e^{6x}$ is solution of this differential equation.
Then find the unique solution satisfying the initial conditions:

$$y(0) = 0, \; y'(0) = 3, \; y''(0) = 90.$$

Exercise 9 (15 points) Find a particular solution of the differential equation

$$y'' - 3y' + 2y = e^{-x}(12x + 8).$$

Then solve completely this differential equation with the initial values $y(0) = 3$ and $y'(0) = 6$.

Exercise 10 (10 points) Give the form of a particular solution in each case, but DO NOT determine the values of the coefficients:

1) $y^{(114)} + 789y'' = x^3 + 2$,

2) $y^{(3)} + 3y'' - 4y' - 12y = (x^2 - 7x + 1)e^{-2x}$,
You will use that -3 is a root of the characteristic equation.

3) $y^{(3)} + 3y'' - 4y' - 12y = e^{7x}(x - 78)\sin(13x)$.
The characteristic equation is the same as in the previous question !

Exercise 11 (20 points) 1) Find the inverse Laplace transform of:

$$F_1(s) = \frac{7}{(s-8)^3}, \quad F_2(s) = \frac{3s - 34}{s^2 - 8s + 25}.$$

2) Give the form of the decomposition into partial fractions, but DO NOT determine the coefficients, of:

$$g_1(x) = \frac{x^3 + 10121492}{(x-3)^2(x+4)^3 x}, \quad g_2(x) = \frac{x^2 + 07201969}{(x-1)(x^2-3x+2)}, \quad g_3(x) = \frac{x + 10241929}{x(x^2+x+1)^2}.$$

Question with no point: can you recognize those dates !!

Exercise 12 (15 points) Solve the initial value problem using the Laplace transform:

$$x'' + x = \sin(2t), \qquad x(0) = 1,\ x'(0) = 0.$$