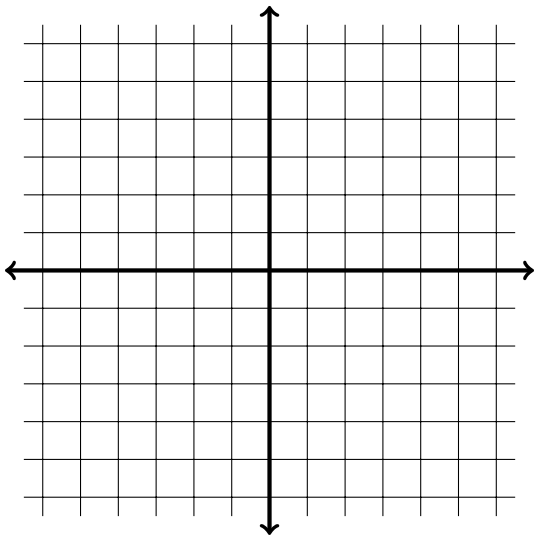
Name: _____

VIP ID: _____

- Write your name and your VIP ID in the space provided above.
- The test has five (5) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	**Max. points**	**Your points**
2	20	
3	30	
4	20	
5	30	
Total	100	

Problem 1 (10 pts)**.** Find (and sketch) the domain of $f(x,y) = \dfrac{\sqrt{4-x^2}}{y^2+3}$



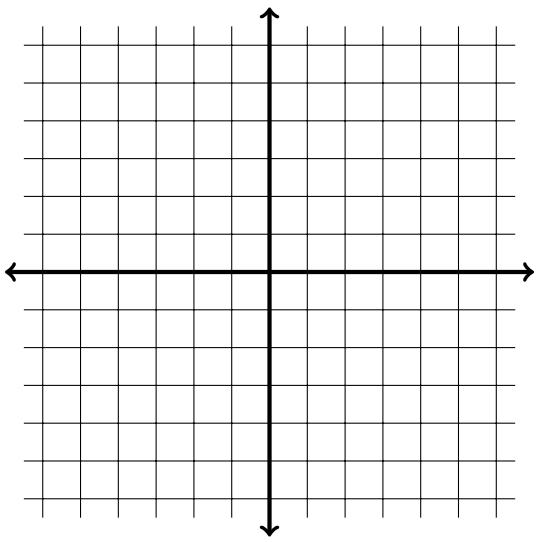
Show work here:

domain:

Problem 2 (10 pts—5 points each part)**.** For the function $f(x,y) = \sqrt{y - x^2 + x}$.

(a) Sketch the level lines $f(x,y) = k$ for $k = -1, 0, 1, 2$
(whenever the equations make sense)

(b) Use the previous information to compute the range of the function f.



Show work here:

range:

Problem 3 (10 pts)**.** Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the function $f(x,y) = \dfrac{xy^2}{y+2x}$.

$\dfrac{\partial f}{\partial x} =$ $\qquad$ $\dfrac{\partial f}{\partial x} =$

Problem 4 (10 pts)**.** Compute the directional derivative $D_{\boldsymbol{v}} f(0,\pi)$ for the function $f(x,y) = e^x \cos(xy^2 - 2y)$ and the direction of the vector $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j}$.

$D_{\boldsymbol{v}} f(0,\pi) =$

Problem 5 (10 pts—5 pts each)**.** Compute the following limits. If necessary, re-write the fraction first.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \cos\frac{x^2+y^3}{x+y+1} =$

(b) $\displaystyle\lim_{(x,y).\to(2,0)} \frac{\sqrt{2x-y}-2}{2x-y-4} =$

Problem 6 (10 pts—5 pts each)**.** Limits that do not exist:

(a) By considering different paths of approach, show that $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y}{x-y}$ does not exist.

(b) By changing the limit to polar coordinates, show that $\lim_{(x,y)\to(0,0)} \dfrac{x-y}{x+y}$ does not exist.

Problem 7 (10 pts)**.** At what points does the vector function $\boldsymbol{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

points:

Problem 8 (10 pts)**.** Find the length ℓ of the curve $\boldsymbol{r}(t) = \boldsymbol{i} + t^2\boldsymbol{j} + t^3\boldsymbol{k}$ for $0 \le t \le 1$.

$\ell =$ ☐

Problem 9 (20 pts)**.** Compute the curvature $\kappa(t)$ of the vector function $\boldsymbol{r}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$.

$\kappa(t) =$ ☐