(q^*, p^*) is the equilibrium point (when supply = demand)
supply: quantity of items manufacturer will make
demand: quantity of items buyers will purchase

linear function: y=mx+b

exponential function: $P=P_0(1+r)^t=P_0a^t=P_0e^{kt}$

slope=$\frac{\Delta y}{\Delta x}$

average rate of change=(f(b)-f(a))/(b-a)

relative rate of change=(f(b)-f(a))/a

half-life=$\frac{\ln 0.5}{k}$

doubling time=$\frac{\ln 2}{k}$

a=1+r $r = a - 1$

k=ln a $a = e^k$

$k = \ln(1+r)$ $r = e^k - 1$

$Cost_{total}=fixed\ cost + variable\ cost$

$Revenue = price \times quantity$

$Profit = revenue - cost_{total}$

continuous growth rate=k

annual growth rate=r

[Derivative]

$f(x) = c \quad f'(x) = 0$

$f(x) = x \quad f'(x) = 1$

$h(x) = f(x) + g(x) \quad h'(x) = f'(x) + g'(x)$

$h(x) = f(x) - g(x) \quad h'(x) = f'(x) - g'(x)$

$h(x) = c \cdot f(x) \quad h'(x) = c \cdot f'(x)$

$f(x) = x^n \quad f'(x) = nx^{n-1}$

$f(x) = e^x \quad f'(x) = e^x$

$f(x) = a^x \quad f'(x) = a^x \ln a$

[Chain rules]

$f(x) = g(x)^n \quad f'(x) = ng(x)^{n-1} g'(x)$

$f(x) = e^{g(x)} \quad f'(x) = g'(x) e^{g(x)}$

$f(x) = a^{g(x)} \quad f'(x) = g'(x) a^{g(x)} \ln a$

$f(x) = \ln g(x) \quad f'(x) = \frac{g'(x)}{g(x)}$

[Leibnitz notation] $\frac{dP}{dh}$

[product]

$h(x) = f(x) \cdot g(x)$

$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ or $u'v + uv'$

[Quotient]

$h(x) = \frac{f(x)}{g(x)} \quad h'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$ or $\frac{u'v - uv'}{v^2}$

Sec 1
team 2

relative rate of change : $y = f(x)$ at $x = a$ $\frac{f'(a)}{f(a)}$ %

$MP = MR - MC$
$MC(q) = C'(q)$
$MR(q) = R'(q)$

$f' > 0$ f is increasing

$f' < 0$ f is decreasing

$f'' > 0$ f is concave upward ∪

$f'' < 0$ f is concave downward ∩

$f' = 0$ stationary @ x (critical point)

$f'' = 0$ concavity changes at x (inflection point)

2nd Derivative Test

$f''(x) > 0$ f has a local minimum at x

$f''(x) < 0$ f has a local maximum at x

$f(x) \leq$ all values of f   global min.

$f(x) \geq$ all values of f   global max.

Profit = Total Revenue - Total Cost
Marginal Cost = derivative of cost
Marginal Revenue = derivative of revenue

Antiderivatives

$f(x) = 1 \quad F(x) = x$

$F(x) + G(x)$ antiderivative is sum of antiderivatives

$F(x) - G(x)$ antiderivative is difference of antiderivatives

$c \cdot f(x) \quad c \cdot F(x)$

$f(x) = x^n \quad F(x) = \frac{1}{n+1} x^{n+1} \quad (n \neq -1)$

$f(x) = e^x \quad F(x) = e^x$

$f(x) = a^x \ (a > 0) \quad F(x) = \frac{a^x}{\ln a}$

$f(x) = \ln x \quad F(x) = \ln|x|$

Substitution Rule:

$\int f(x)^n \cdot f'(x) dx = \frac{1}{n+1} f(x)^{n+1}$

$\int a^{f(x)} \cdot f'(x) dx = \frac{a^{f(x)}}{\ln a}$

$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$

$u \cdot v - \int v \cdot du$

$TA = A_1 + A_2$

Net Area $= F(b) - F(a) = \int_a^b f(x)dx$

Avg: $\frac{1}{b-a} \int_a^b f(t)dt$
interval a to b

Graphs:

Area under graph of f b/n a + b $= \int_a^b f(x)dx$

Area b/n 2 curves $= \int_a^b (f(x) - g(x))dx$

Area of curves that intersect but are bound by vertical lines: $\int_a^c f(x) - g(x)dx + \int_c^b g(x) - f(x)dx$

- Avg. rate of change b/n $x=a$ + $x=b$: $\frac{f(b) - f(a)}{b-a}$
- Relative change b/n $x=a$ + $x=b$: $100 \cdot \frac{f(b) - f(a)}{f(a)}$
- Relative rate of change at $x=a$: $100 \cdot \frac{f'(a)}{f(a)}$ %

consumer surplus: $\int_0^{q_0} (f(q) - p_0)\,dq$ in dollars

$p_0 = f(q_0)$

producer surplus: $\int_0^{q_0} (p_0 - g(q))\,dq$ in dollars

at equilibrium: $\int_0^{q^*} (p^* - g(q))\,dq$ in dollars

Riemann Sums

x	5	10	15	20	25	30	35	4[illegible]
y	107	108	107	90	92	100	110	[illegible]

left	right
107.5	108.5
108.5	107.5
107.5	90.5
90.5	92.5
92.5	100.5
100.5	110.5
110.5	112.5
} 3570	} 3595

$\frac{3570 + 3595}{2} = 3582.5$ Riemann sum

Ex.1: You have your choice of receiving \$5000 now or receiving five equal payments of \$1000 each, paid once per year starting now. You can assume a 6% interest rate. Which is the best financial option?

Option 1

$5000e^{.06\times4}=6356$ dollars

Option 2

first year: $1000e^{.06}=1061.84$

second year: $2061.84e^{.06}=2189.3$

third year: $3189.3e^{.06}=3386.551$

fourth year: $4386.55e^{.06}=4657.8$

fifth year: 5657.8 dollars

Answer: Option 1

Ex. 2: Convert the function $P = 750e^{0.04t}$ to the form $P = P_0a^t$.

$e^{kt} = a^{kt}$ **Answer:** $\boldsymbol{P = 750(1.0408)^t}$

$e^{0.04} = a$

$a = 1.0408$

Ex.3: The solution to $200 = 30e^{0.15}t$ is:

$\frac{200}{30} = e^{0.15}t$

$\ln\frac{200}{30} = \ln e^{.15}\ t$

Answer: $t = \frac{\ln\frac{200}{30}}{0.15} \approx 12.65$

Ex.4: The amount, A(mg), of a drug In the body is 25 when it first enters the system decreases by 12% each hour. A possible formula for A as a function of t, in hours after the drug enters the system, is:

$P = P_0a^t$.

$P_0 = 25$ $a = 1 - .12$

Answer: $\boldsymbol{A = 25(0.88)^t}$

The Statement $f'(a)=b$ means that if the independent variable x goes up from a to a+1, then the dependent variable goes up or down by |b| units

At a price of P dollars, a quantity q of an item is sold. $q=f(p)$

Find all x-values for which the tangent line to the graph of the function $y=f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2$ is parallel to the line $12x-2y=41$

1. find the slope of the given line $y=6x-41/2$
2. find the x-values for which the slope of the tangent line of f equals 6.

Compute the derivative of f beforehand. $f'(x)=\frac{1}{3}\cdot 3x^2+\frac{1}{2}\cdot 2x = x^2+x$

Solve for x in the equation $f'(x)=6$ $x^2+x=6$ $x^2+x-6=0$ $x=(-3,2)$

example

or what values of a and does $f(x)=a(x-b\ln x)$ have a local extremum at points (5,8)?

$f(5)=8$ $a(5-b\ln 5)=8$

$f'(5)=0$ $f'(x)=a(1-\frac{b}{x})$

$a(1-\frac{b}{5})=0$

$a=0$ or $b=5$

$(5-b\ln 5)=8$ $a(5-b\ln 5)=8$

$a=0$ $a(5-5\ln 5)=8$

$\varnothing$ $a=\frac{8}{5-5\ln 5}$

$\int(3x^2-4)e^x\,dx = uv-\int v\,du$ {integration by parts}

$\boxed{u=3x^2-4}$ $\boxed{dv=e^x}$ $(3x^2-4)e^x-\int e^x\,6x\,dx$

$\frac{du}{dx}=6x$ $\boxed{v=e^x}$

$\boxed{du=6x\,dx}$ $(3x^2-4)e^x-(uv-\int v\,du)$

$(3x^2-4)e^x-(6xe^x-6\int e^x dx)$

final: $(3x^2-4)e^x-6xe^x+6e^x$

$\int x^{300}\ln x\,dx$ $uv-\int v\,du$

$u=\ln x$ $dv=x^{300}$ $\ln x\frac{1}{301}x^{301}-\int\frac{1}{301}x^{301}\frac{1}{x}dx$

$\frac{du}{dx}=\frac{1}{x}$ $v=\frac{1}{301}x^{301}$ $\ln x\frac{1}{301}x^{301}-\frac{1}{301}\int x^{301}\frac{1}{x}dx$

$du=\frac{1}{x}dx$ $\ln x\frac{1}{301}x^{301}-\frac{1}{301}\cdot\frac{1}{301}x^{301}$

example antiderivative

$\int\frac{x+2}{x^2+4x+5}dx$

$u=x^2+4x+5$

$\frac{du}{dx}=2x+4$ $du=2x+4dx$

$\frac{1}{2}du=x+2dx$

$\int\frac{du}{u}=\ln|u|=\ln|x^2+4x+5|$

examples

$\int_3^{17}(x^3+9x^2-7)dx=\frac{x^4}{4}+3x^3-7x\Big|_3^{17}$

$\int(x^3+9x^2-7)dx = 1/4\,x^4+9/3\,x^3-7x=\frac{x^4}{4}+3x^3-7x$

$\frac{17^4}{4}+3\cdot17^3-7\cdot17)-(\frac{3^4}{4}+3\cdot3^3-7\cdot3$, units

rate $=r(t)=\frac{t}{t^2+1}$ thousands of antibodies per minute; assuming no antibodies @ t = 0, find total after 4 minutes $\int_0^4\frac{t}{t^2+1}dt \to$ plug into calculator ≈ 1.416 thousands

[illegible]

$x=0$ $f(x)=9-x$

Compute area b/n curves $y=9-x$, $y=3+x/2$, $x=0$, + $x=2$.

$\int_0^2 (9-x)-(3+x/2))dx$

What is the average number of bacteria between t = 0 and t = 5 $f(t)=25e^{.3t}$ in millions of bacteria. Avg. of a function between t=a and t=b is given by $\frac{1}{5-2}\int_2^5 25e^{.3t}dt$ in millions of bacteria

Compute consumers' surplus for the demand curve $p=100-q^2$ when 5 units are sold $\int_0^5(100-3q^2-25)dq$ *when supply given for consumer

$p_0=f(q_0)=f(5)=100-3\cdot5^2=25 \to \int_0^5(100-3q^2-25)dq$ dollars

Find the producer surplus for the supply curve $p=3+q^2$ when 50 units are sold $\int_0^{50}(p_0-3+q^2)$ $p_0=3+q_0=3+50^2=2503$

$\int_0^{50}(2503-(3+q^2))dq$ dollars