Math 242 Test 3, Wednesday 28 November

Name: **Last 4 digits of SSN:**

Show all work clearly, **MAKE SENTENCES**. No work means no credit. The points are:
ex1: 25, ex2: 15, ex3: 25, ex4: 20 and the course questions are over 15 points.

Course Questions

1. Method of variation of parameters in the case $n = 2$:
   We consider the second-order linear differential equation

   $$y'' + P(x)y' + Q(x)y = f(x),$$

   where P, Q and f are continuous. A general solution is given by:

   $$y_c(x) = c_1 y_1(x) + c_2 y_2(x),$$

   where c_1 and c_2 are constants.
   Of what form can we search a particular solution ? To find this, we need to impose a condition, what is this condition ?

2. Give the definition of the Laplace transform.

3. State the theorem about the Laplace transform of integrals.

Exercise 1 Find a particular solution of the following differential equation:

$$y'' - 4y = 13x\cos(3x).$$

You will use the fact that the family $(x^k\cos(3x), x^k\sin(3x))_k$ is linearly independent. This means, for example, that if $\alpha_1 x\cos(3x) + \alpha_0\cos(3x) + \beta_1 x\sin(3x) + \beta_0\sin(3x) = 0$ then $\alpha_1 = \alpha_0 = \beta_1 = \beta_0 = 0$.

Exercise 2 Find the inverse Laplace transform of the functions:

$$F(s) = \frac{3s-7}{s^2+9}, \quad \text{and} \quad H(s) = \frac{4}{s^2(s^2+16)}.$$

Exercise 3 Solve the initial value problem using the Laplace transform:

$$y'' + y = \sin(2t), \quad y(0) = 1, y'(0) = 0.$$

Exercise 4 1. Use the partial fraction to find the inverse Laplace transform of

$$F(s) = \frac{3s+2}{s^2-s-12}.$$

2. Write the partial fractions of the rational function (we do not ask the value of the coefficients):

$$G(s) = \frac{9s^4-5s^2+s}{(s-7)(s+3)^2(s^2+5s+10)^2}.$$