Equation Sheet

Scalar Component: $\frac{\vec{u}\bullet\vec{v}}{|\vec{v}|}$

$(\frac{\vec{u}\bullet\vec{v}}{|\vec{v}|^2})\vec{v}$: Projection

$\frac{\vec{PQ}\bullet\vec{v}}{|\vec{v}|}$: Distance from point to plane

$\frac{\vec{PQ}\times\vec{v}}{|\vec{v}|}$: Distance from point to line

Standard parametric equations of line thru P= (x_0, y_0, z_0):

x-$x_0 = (t)\overrightarrow{v_1}$

y-$y_0 = (t)\overrightarrow{v_2}$

z-$z_0 = (t)\overrightarrow{v_3}$

equation of a plane: $\overrightarrow{v_1}$(x-x_0)+ $\overrightarrow{v_2}$(y-y_0)+ $\overrightarrow{v_3}$(z-z_0)= 0

Area of parallelogram: $|\vec{u}\ x\ \vec{v}|$

Area of triangle: $\frac{|\vec{u}\ x\ \vec{v}|}{2}$

Volume of a parallelepiped: $\vec{u}\bullet(\vec{v}\ x\ \vec{w})$

Angle between two vectors:

1.) dot = $\vec{u}\ \bullet\ \vec{v} = (|\vec{u}|)(|\vec{v}|)cos\theta$

2.) cross=$|\vec{u}\ x\ \vec{v}| = (|\vec{u}|)(|\vec{v}|)sin\theta$

u x v (cross) = $< (u_3)(v_2) - (u_2)(v_3), (u_3)(v_1) - (u_1)(v_3), (u_1)(v_2) - (u_2)(v_1) >$

Distance= $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

u • v (dot) = $(u_1)(v_1)$+ $(u_2)(v_2)$

$\vec{r}'(t) = \vec{v}(t)$

$\vec{r}''(t) = \vec{v'}(t) = \vec{a}(t)$

$|\vec{r}'(t)| = \vec{s}(t)$

$\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} \qquad \vec{N} = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} \qquad \vec{B} = \frac{\vec{T}(t)\times\vec{N}(t)}{|\vec{T}(t)\times\vec{N}(t)|}$

$\kappa = \frac{|\vec{r}'(t)\times\vec{r}''(t)|}{|\vec{r}'(t)|^3}$

$Length\ of\ curve = \int_a^b |\vec{r}'(t)|dt$

Techniques for finding limits:

1. *Use the limit evaluation rules and hope for the best*
2. *Rewrite the expression in a different way and evaluate*
3. *Try taking the limit from different directions*
4. *Convert the expression to polar coordinates and evaluate*

$$\nabla f(x_o,y_o) =< \frac{\partial f}{\partial x}(x_o,y_o), \frac{\partial f}{\partial y}(x_o,y_o) >$$

$$Hessian\ \ f(x_o,y_o) = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2}(x_o,y_o) & \frac{\partial^2 f}{\partial x \partial y}(x_o,y_o) \\ \frac{\partial^2 f}{\partial y \partial x}(x_o,y_o) & \frac{\partial^2 f}{\partial y^2}(x_o,y_o) \end{vmatrix}$$

$Critical\ point\ on\ f(x,y)$: $\nabla f(x_o,y_o) =< 0,0 >$

$If\ Hess\ f(x_o,y_o) < 0, the\ point\ is\ a\ saddle\ point$

$If\ Hess\ f(x_o,y_o) > 0, and\ \frac{\partial^2 f}{\partial x^2}(x_o,y_o) > 0, then\ point\ is\ a\ local\ min$

$If\ Hess\ f(x_o,y_o) > 0, and\ \frac{\partial^2 f}{\partial x^2}(x_o,y_o) < 0, then\ point\ is\ a\ local\ max$

$For\ absolute$ max $and\ min: (1) set\ \nabla = 0, (2)\ Find\ points\ on\ borders\ and\ their\ values, (3) find\ values\ of\ vertices, (4) choose$ max $and\ min$

$Given\ the\ graph\ of\ z = f(x,y), the\ tangent\ plane\ to\ the\ the\ point\ (x_o,y_o,f(x_o,y_o))\ is\ z - f(x_o,y_o) = \frac{\partial f}{\partial x}(x_o,y_o)(x-x_o) + \frac{\partial f}{\partial y}(x_o,y_o)(y-y_o)$

Cartesian to Cylindrical

$x \to rcos\theta$

$y \to rsin\theta$

$z \to z$

$\int_{\theta_o}^{\theta}\int_{r_o}^{r}\int_{z_o}^{z} r\ dzdrd\theta$

$x^2 + y^2 = r^2$

$tan\theta = \frac{y}{x}$

Cartesian to Spherical

$x \to \rho sin\phi cos\theta$

$y \to \rho sin\phi sin\theta$

$z \to \rho cos\phi$

$\int_{\theta_o}^{\theta}\int_{\phi_o}^{\phi}\int_{\rho_o}^{\rho} \rho^2 sin\phi\ d\rho d\phi d\theta$

$\sqrt{x^2+y^2+z^2} = \rho$

$Type\ 1, D = (x,y) \in \mathbb{R}^2: a \le x \le b, bottom(x) \le y \le top(x)\}$

$Type\ 2, D = (x,y) \in \mathbb{R}^2: c \le y \le d, left(y) \le x \le right(y)\}$

$\int_a^b \int_{bottom(x)}^{top(x)} f(x,y)dydx \qquad \int_c^d \int_{left(x)}^{right(x)} f(x,y)dxdy$

$Polar, D = (r,\theta) \in \mathbb{R}^2: r_1 \le r \le r_2, \alpha \le \theta \le \beta\}$

$\int_\alpha^\beta \int_{r_1}^{r_2} f(rcos\theta, rsin\theta)rdrd\theta$

$Type\ 1, D = (x,y,z) \in \mathbb{R}^3: (x,y) \in D, bottom(xy) \le z \le top(xy)\}$

$Type\ 2, D = (x,y,z) \in \mathbb{R}^3: (x,z) \in D, left(xz) \le y \le right(xz)\}$

$Type\ 3, D = (x,y,z) \in \mathbb{R}^3: (y,z) \in D, left(yz) \le x \le right(yz)\}$

$\int_{z_o}^{z}\int_{y_o}^{y}\int_{x_o}^{x} f(x,y,z)\ dxdydz$

Integrals

$\int e^x sinx\ dx = \frac{1}{2}e^x(sinx - cosx)$

$\int xe^x dx = (x-1)e^x$

$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b|$

$\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$

$\int \cos^2 ax\ dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$

$\int \sin^2 ax\ dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$

Compute projection of $\vec{v}=<2,4>$ over $\vec{u}=<3,1>$

$\frac{\vec{u}}{|\vec{u}|}=\frac{<3,1>}{\sqrt{10}}$ ….

$comp_{\vec{u}}\vec{v}=\frac{\vec{u}\bullet\vec{v}}{|\vec{u}|}\quad=\frac{<2,4>\bullet<3,1>}{\sqrt{10}}=\frac{10}{\sqrt{10}}$

Find parametric equation for line through (6,7,-1) and perpendicular to 4x+5y+10z=3 $\quad\vec{v}=<4,5,10>$

Answer: x=4t+6, y=5t+7, z=10t-1

Intersection of a plane with a line:
x+2y-z=0 and x=5-t, y=t, z=2+t
(5-t)+2(t)-(2+t)=0
3≠ 0…. They do not intersect

Are these 2 vectors parallel, perpendicular or neither? $\vec{u}=<3,1,4>$ and $\vec{v}=1,1,-1>$ $\quad\vec{u}\bullet\vec{v}=(|\vec{u}|)(|\vec{v}|)cos\theta \qquad |\vec{u}|=\sqrt{26},|\vec{v}|=\sqrt{3},\vec{u}\bullet\vec{v}=0$

0=$\cos\theta,\theta=\frac{\pi}{2}$ *... vectors are perpendicular*

$$\lim_{\substack{(x,y)\to(0,0)\\x\neq y}}\frac{x^2-y^2}{x-y}$$

$$\lim_{\substack{(x,y)\to(0,0)\\x\neq y}}\frac{(x-y)(x+y)}{(x-y)}$$

$$\lim_{(x,y)\to(1,1)}(x+y)$$

$$(1+1)$$

$$2$$

$$\lim_{\substack{(x,y)\to(0,0)\\\text{along } y=-x\\x<0}}\frac{-x}{\sqrt{x^2+y^2}}$$

$$=\lim_{x\to0^-}\frac{-x}{\sqrt{x^2+x^2}}$$

$$=\lim_{x\to0^-}\frac{-x}{\sqrt{2x^2}}$$

$$=\lim_{x\to0^-}\frac{-x}{\sqrt{2}|x|}$$

$$=\lim_{x\to0^-}\frac{-x}{\sqrt{2}(-x)}$$

$$=\lim_{x\to0^-}\frac{x}{\sqrt{2}x}$$

$$=\frac{1}{\sqrt{2}}$$

$$\lim_{\substack{(x,y)\to(0,0)\\\text{along } y=-x\\x>0}}\frac{-x}{\sqrt{x^2+y^2}}$$

$$=\lim_{x\to0^+}\frac{-x}{\sqrt{x^2+x^2}}$$

$$=\lim_{x\to0^+}\frac{-x}{\sqrt{2x^2}}$$

$$=\lim_{x\to0^+}\frac{-x}{\sqrt{2}|x|}$$

$$=\lim_{x\to0^+}\frac{-x}{\sqrt{2}x}$$

$$=\frac{-1}{\sqrt{2}}$$

Determine whether the function $f(x,y)=x^2+xy+2+y-1$ *has saddle points.*

$\frac{\partial f}{\partial x}=2x+y+2\quad\frac{\partial f}{\partial y}=x+1\qquad$ x = -1 y = 0 *Find critical points using the gradient

$\frac{\partial f}{\partial x^2}=2\quad\frac{\partial f}{\partial y^2}=0\quad\frac{\partial f}{\partial x\partial y}=1\quad\frac{\partial f}{\partial yx}=1$

Hessian $f_{xx}f_{yy}-f_{xy}f_{yx}=(2)(0)-(1)(1)=-1$

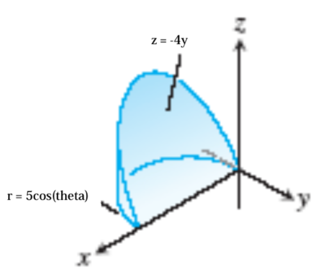
The point (-1,0) is a saddle point because the Hessian is less than 0.

Find the local max, min and saddle points of the function $f(x,y)=-7x^2-2xy-8y^2+64x-38y+2$

1. Find the partial derivative and set them equal to 0 to find critical points.
   a. $f_x(x,y)=-14x-2y+64=0 \qquad f_y(x,y)=-2x-16y-38=0$
   b. *use substitution (x,y) = (5, -3)
2. Find the hessian
   a. $f_{xx}(x,y)=-14 \qquad f_{yy}(x,y)=-16 \qquad f_{xy}(x,y)=-2$ (*this will also equal f_{yx})
   b. Use Hessian Formula: hess = 220
3. Since hess > 0 and $f_{xx}<0$, (5,-3) is a local max.
4. Find the value by plugging (5,-3) into f(x,y). f(x,y) = 219

---

1. Find the volume of the solid bounded by the xy plane, $r=5\cos\theta$, *and* $z=-4y$



$$0\le r\le 5cos\theta$$
$$0\le z\le -4rsin\theta$$
$$-\frac{\pi}{2}\le\theta\le0$$
$$\int_{-\frac{\pi}{2}}^{0}\int_{0}^{5cos\theta}\int_{0}^{-4rsin\theta}r\,dzdrd\theta$$

Evaluate the double integral bounded by
$$x^2+y^2=9,\qquad x^2+y^2=4$$
in the first quadrant
$$\iint_R(4x+2y^2)dA$$
$$\iint_R(4rcos\theta+2(rsin\theta)^2)r\,drd\theta$$
$$\int_0^{\frac{\pi}{2}}\int_2^3(4r^2cos\theta+2r^3\sin^2\theta\,drd\theta$$