Name: _____

VIP ID: _____

- Write your name and your VIP ID in the space provided above.
- The test has six (6) pages, including this one.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given at the right of each problem number.
- There is an extra-credit problem on the last page, together with a table of Laplace Transforms. Points obtained in the extra-credit problem will be added to the score of exam #1.

Page	**Max**	**Points**
2	40	
3	20	
4	20	
5	20	
Total	100	

Problem 1 (40 pts—10 pts each)**.** Find the Laplace transform of the following functions:

(a) $f(x) = 2x^3 - 7x + 4$

$F(s) =$ $\boxed{\qquad\qquad (s > \quad)}$

(b) $f(x) = 3e^{2x} - 7e^{-8x} + \cos\left(\frac{\pi}{2}x\right)$

$F(s) =$ $\boxed{\qquad\qquad (s > \quad)}$

(c) $f(x) = 5x^2 \sin(3x)$

$F(s) =$ $\boxed{\qquad\qquad (s > \quad)}$

(d) $f(x) = 3xe^{-2x} \sin x$

$F(s) =$ $\boxed{\qquad\qquad (s > \quad)}$

Problem 2 (20 pts—10 pts each)**.** Find the inverse Laplace transform of the following functions in the given domains.

(a) $F(s) = \dfrac{3s-2}{s^2+2s-15}$, $(s > 3)$

$f(x) =$ ____

(b) $F(s) = \dfrac{4(s-2)}{(s-2)^2+16}$, $(s > 2)$

$f(x) =$ ____

Problem 3 (20 pts)**.** Use the definition of the Laplace transform to find that

$$\mathcal{L}\{5\cos 2x\} = \frac{5s}{s^2+4} \text{ for } s > 0.$$

Problem 4 (20 pts)**.** Use techniques based on the Laplace transform to solve the initial value problem $y'' + y' - 2y = e^x$ that satisfies $y(0) = 0, y'(0) = \frac{4}{3}$. Compute the numerical value of all relevant constants.

$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x)\,dx$			
1	$\frac{1}{s}$ $s>0$	$cf(x)\pm g(x)$	$cF(s) \pm G(s)$	$s > max(a,b)$
x^n	$\frac{n!}{s^{n+1}}$ $s>0$	$e^{\alpha x} f(x)$	$F(s-\alpha)$	$s > a+\alpha$
$e^{\alpha x}$	$\frac{1}{s-\alpha}$ $s>\alpha$	$x^n f(x)$	$(-1)^n F^{(n)}(s)$	$s > a$
$\sin \beta x$	$\frac{\beta}{s^2+\beta^2}$ $s>0$	$f'(x)$	$sF(s) - f(0)$	
$\cos \beta x$	$\frac{s}{s^2+\beta^2}$ $s>0$	$f''(x)$	$s^2F(s) - sf(0) - f'(0)$	

Problem 5 (20 pts–extra credit for exam #1)**.** Solve the previous problem using either the method of *variation of parameters*, or *undetermined coefficients*. Do compute the value of all relevant constants.