Name:	______________________
VIP ID:	______________________

- Write your name and VIP ID in the space provided above.
- Each of the propositions below is worth 20 points. Present a proof of **exactly** five of them. Mark clearly in the next page which propositions you have chosen.
- You much choose at least one of each of the following categories:

  Proof by Contrapositive: Instead of proving $P \implies Q$, try $\neg Q \implies \neg P$.

  Proof by Contradiction: To prove $P \implies Q$, start with the assumption that P and $\neg Q$ are true.

  If and only if statement: To prove $P \iff Q$, prove both $P \implies Q$ and $Q \implies P$.

  Existence statement: List, construct, or prove by contradiction.

  Proof by Induction: $\forall n \geq n_0, P(n)$. Prove the *basis step* $P(n_0)$, and use the *inductive hypothesis* to prove the *inductive step* $P(n) \implies P(n+1)$.

- Make sure to **box** your proofs, to differentiate them from your exploration and planning. I will only grade for boxed content on each submission.
- Books, notes and calculators are allowed.

Proposition #	**Max**	**Points**
	20	
	20	
	20	
	20	
	20	
Total	100	

Proposition 1. *For every $n \in \mathbb{N}$, it follows that*

$$3^1 + 3^2 + 3^3 + 3^4 + \cdots + 3^n = \frac{3^{n+1} - 3}{2}.$$

Proposition 2. *Concerning the Fibonacci sequence $\{F_n\}_{n \in \mathbb{N}}$, it follows that*

$$F_2 + F_4 + F_6 + F_8 + \cdots + F_{2n} = F_{2n+1} - 1.$$

Proposition 3. *For every $n \in \mathbb{N}$, it follows that*

$$(1 + 2 + 3 + \cdots + n)^2 = 1^3 + 2^3 + 3^3 + \cdots + n^3.$$

Proposition 4. *For every $n \in \mathbb{N}$, it follows that*

$$\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2}.$$

Proposition 5. *For any integer $n \geq 0$, it follows that $3|(5^{2n} + 2)$.*

Proposition 6. *For every $n \in \mathbb{N}$, it follows that*

$$\sum_{k=1}^{n} (4k - 3) = n(2n - 1)$$

Proposition 7. *Given an integer $a \in \mathbb{Z}$, then $a^3 + a^2 + a$ is even if and only if a is even.*

Proposition 8. *There exist three positive real numbers $x \in \mathbb{R}$ for which $x^2 < \sqrt{x}$.*

Proposition 9. *There exists a real number $x \in \mathbb{R}$ such that $x^3 - 4x^2 = 7$.*

Proposition 10. *If $n \in \mathbb{Z}$, then $4|n^2$ or $4|(n^2 - 1)$.*

Proposition 11. *Suppose $x, y \in \mathbb{R}$. Then $x^3 + x^2 y = y^2 + xy$ if and only if $y = x^2$ or $y + x = 0$.*

Proposition 12. *There exist no integers $a, b \in \mathbb{Z}$ for which $21a + 30b = 1$.*

Proposition 13. *The number $\sqrt[3]{2}$ is irrational.*

Proposition 14. *The number $\log_3 4$ is irrational.*

Proposition 15. *If $a, b \in \mathbb{R}$ are positive real numbers, then $a + b \geq 2\sqrt{ab}$.*

Proposition 16. *For each positive real number $x \in \mathbb{R}$, if x is irrational, then $\sqrt{x}$ is irrational.*

Proposition 17. *For each integer $n \in \mathbb{Z}$, n is even if and only if $4|n^2$.*

Proposition 18. *If $p, q \in \mathbb{Q}$ are rational numbers with $p < q$, then there exists another rational number $x \in \mathbb{Q}$ that satisfies $p < x < q$.*