TTAP: $z - z_0 = \frac{\partial F}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial F}{\partial y}(x_0, y_0)(y - y_0)$

LA: $L(x,y) = f(x_0, y_0) + \frac{\partial F}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial F}{\partial y}(x_0, y_0)(y - y_0)$

Imp. Diff $\frac{\partial z}{\partial x} = \frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$ $\frac{\partial z}{\partial y} = \frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$

Diff Fun: $dz = \frac{\partial f}{\partial x}(x_0, y_0)\,dx + \frac{\partial f}{\partial y}(x_0, y_0)\,dy$; $\Delta z = f(x_0, y_0) - f(x, y)$

Grad: $\nabla f = \frac{\partial F}{\partial x}(x_0, y_0) + \frac{\partial F}{\partial y}(x_0, y_0)$

Directi Der. $D_u g = \vec{u} \cdot \nabla f$ [u = unit vector]

Chain Rule $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{dv}{dt} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$ e.g. $z = x^2 + y^2 + xy$; $x = \sin t$ $y = e^t$ $\frac{\partial z}{\partial t} = (2x+y)\cos t + (2y+x)e^t = \sin 2t + 2e^{2t} + e^t(\cos t + \sin t)$

Para. equat: Find $\vec{PQ} \rightarrow \langle a, b, c \rangle$ $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$

Domain: e.g. $\sqrt{1 + x - y^2}$ Domain $= x \geq -1 + y^2$ Range $= [0, \infty)$: $\ln \neq 0$, $\sqrt{\ } = 0$

Contour 

Level Curves $f(x,y) = y^2 + 1$ $k = 1, 5, 10$ Set k to function $y^2 + 1 = 1$, $y^2 + 1 = 10$ then sketch

Limits: $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}$ Plug in diff value for x or y to come from diff dirs if they all equal same = continues

Partial Derv. $f(x,y) = x^2 + y^2$ $\frac{\partial f}{\partial x} = 2x$ $\frac{\partial f}{\partial y} = 2y$; for second Partial $F_{xx} = 2$ $F_{xy} = 0$ $F_{yx} = 0$ $F_{yy} = 2$

TTAP e.g $z = 3x^2 - y^2 + 3y$, $(-3, 5, 17)$; $f(x) = 6x$ $f_y = -2y + 3$
$f(x)(-3,5) = -18$, $F_y(-3,5) = -7$ using equation
$z - 17 = f(x)(-3,5)(x - (-3)) + f_y(-3,5)(y - 5)$
$z - 17 = -18(x - -3) + -7(y - 5)$ or $z = -18x - 7y - 2$

Diff Fun e.g $m = p^3 q^7$ $dm = \frac{\partial m}{\partial p}dp + \frac{\partial m}{\partial q}dq = 3p^2q^7\,dp + 7p^3q^6\,dq$

e.g Find dz & Δz; $z = 5x^2 + y^2$ changes from $(1,1)$ to $(0.95, 1.1)$
$dx = \Delta x = -0.05$, $dy = \Delta y = 0.1$
$z_x = 10x$ $z_y = 2y$
$dz = z_x(1,1)\,dx + z_y(1,1)\,dy = 10(-0.05) + (2)(0.1) = -0.3$
$\Delta z = f(0.95, 1.1) - f(1,1) = 5(0.95)^2 + (1.1)^2 - 5 - 1 \approx -0.278$

Lagrange
Find max rate change & Dir $f(p,q) = 5qe^{-p} + 6pe^{-q}$, $(0,0)$
$\nabla f(p,q) = \langle -5qe^{-p} + 6e^{-q}, 5e^{-p} - 6pe^{-q} \rangle$
$\nabla F(0,0) = (6,5)$ or $=$ to $\langle \frac{6}{\sqrt{61}}, \frac{5}{\sqrt{61}} \rangle$ = direction
Max rate of change $= \sqrt{61}$

Abs. Max & Min

LA e.g. $f(x,y) = \sqrt{y + (\cos(x))^2}$
$f_x(x,y) = \frac{1}{2}(y + \cos^2 x)^{-1/2}(2\cos x)(-\sin x) = \frac{-\cos x \sin x}{\sqrt{y + \cos^2 x}}$
$f_y(x,y) = \frac{1}{2}(y + \cos^2(x))^{-1/2}(1) = \frac{1}{2\sqrt{y + \cos^2 x}}$

Derivatives $e^x = e^x$ | $b^x = b^x \ln(b)$ | $\ln(x) = \frac{1}{x}$
$\sin x = \cos x$ $\cos x = -\sin x$ $\tan x = \sec^2 x$
$\arcsin(x) = \frac{1}{\sqrt{1 - x^2}}$ $\csc x = -\csc x \cot x$
$\arccos(x) = \frac{-1}{\sqrt{1 - x^2}}$ $\arctan = \frac{1}{1 + x^2}$

Dif. Der. e.g
$F(x,y) = 2x\sin(xy) \rightarrow$
$f_x(x,y) = 2x\cos(xy)\cdot y + 2\sin(xy)$
$= 2xy\cos(xy) + 2\sin(xy)$
$f_y(x,y) = 2x\cos(xy)\cdot x$
$= 2x^2\cos(xy)$

Imp. Diff eg.; $F(x,y) = \sin(x - y) - xe^y = 0$
$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{\cos(x-y) - e^y}{\cos(x-y)(-1) - xe^y} = \frac{\cos(x-y) - e^y}{\cos(x-y) + xe^y}$

local M & Min
step #1 find critical points of derivative, Then take Second partial derivative and do cross product
$\frac{\partial^2 f}{\partial x^2}(x_0, y_0)$ $\frac{\partial^2 f}{\partial x \partial y}(x_0, y_0)$
$\frac{\partial^2 f}{\partial x \partial y}(x_0, y_0)$ $\frac{\partial^2 f}{\partial y^2}(x_0, y_0)$

Step #1 Find critical points
step #2 Parametrize border of D
step #3 Evaluate $f(x,y)$ for all canadates (x,y) largest is abs. Max smallest is abs. min

if cross product $= > 0$ then saddle point, if cross product is < 0 and $\frac{\partial^2 f}{\partial x^2} > 0$ = min
" " " " " < 0 = max
Then plug in critical points

$D_u f(x,y) = f_x(x,y)\cos\theta + f_y(x,y)\sin\theta$ $\theta = \pi/6$ - given
$D_u f(2,0) = f_x(2,0)\cos\frac{\pi}{6} + f_y(2,0)\sin\frac{\pi}{6} = 0 + 8(\frac{1}{2}) = 4$