Math 242 Final Exam, Monday 10 December

Name: **Last 4 digits of SSN:**

Show all **work clearly**, **make sentences**. No work means no credit. The points are: Ex1: 10, Ex2: 10, Ex3: 15, Ex4: 10, Ex5: 15 and Problem: 40.

Exercise 1 Solve the initial value problem:

$$y' = 2xy + 3x^2 e^{x^2}, \quad y(0) = 5.$$

Exercise 2 Find a general solution of the differential equation

$$y' = 1 + x^2 + y^2 + x^2 y^2.$$

Exercise 3 Show that the differential equation

$$\left(x^3 + \frac{y}{x}\right) dx + (y^2 + \ln x)dy = 0,$$

is exact and then solve it.

Exercise 4 We give an initial value problem and its exact solution $y(x)$:

$$y' = 4x^2 + 2y, \quad y(0) = 2, \quad y(x) = -1 - 2x - 2x^2 + 3e^{2x}.$$

Apply Euler's method to approximate the solution first on the interval $[0, 1]$ with step size $h = 0.25$, and on the interval $[0, 0.5]$ with the step size $h = 0.1$. Write the formula you use for the computation. Then compare the four-decimal-place values of the approximate solution with the values of the exact solution using the following arrays. What do you think of this two different cases ?

step size $h = 0.25$

x	0	0.25	0.5	0.75	1
approx solution					
exact solution					

step size $h = 0.1$

x	0	0.1	0.2	0.3	0.4	0.5
approx solution						
exact solution						

Exercise 5 Solve the differential equation

$$y^{(3)} + y'' - 16y' + 20y = 0,$$

using the fact that the function $x \mapsto e^{-5x}$ is solution of this differential equation.
Then find the unique solution satisfying the initial conditions:

$$y(0) = 0,\ y'(0) = 9,\ y''(0) = -13.$$

Problem

We consider the initial value problem

$$y'' - 3y' + 2y = 20x\cos(2x), \tag{1}$$

with the initial values $y(0) = 0$, $y'(0) = 1$.
We want to solve it by two different ways.

I : Laplace transform

1) Use the theorem of differentiation of transforms to find the Laplace transform of $x\cos(2x)$.

2) Let $K(s) = 20\dfrac{s+2}{(s-1)(s^2+4)^2}$. Show that

$$K(s) = \frac{12}{5}\frac{1}{s-1} + \frac{-12s+8}{(s^2+4)^2} + \frac{1}{5}\frac{-12s-12}{s^2+4}.$$

3) Then solve the initial value problem using Laplace transform.

II : Classical way

1) Find the complementary solution of (1).

2) Use the sheet about the particular solution to find one for (1).

3) Then solve the initial value problem.

III : Conclusion What method do you prefer ?