Equation Sheet:

Linear First Order Equation: $\frac{dy}{dx} + P(x)y = Q(x)$

Bernoulli Equation: $\frac{dy}{dx} + P(x)y = Q(x)y^n$

Linear Constant Coefficients: $ay'' + by' + cy = f(x)$

3 Cases

Case 1: $r_1 \neq r_2$

$y_1 = e^{r_1 x}, y_2 = e^{r_2 x}$

Case 2: $r_1 = r_2$

$y_1 = e^{rx}, y_2 = xe^{rx}$

Case 3: $r = \alpha \pm \beta i$

$y_1 = e^{\alpha x} cos(\beta x), y_2 = e^{\alpha x} sin(\beta x)$

Non-Homogeneous

Method 1: Variation of Parameters

ay''+by'+cy= f(x)

A(x)= $-\frac{1}{a} \int \frac{y_2(x)f(x)-}{w(y_1, y_2)}$ dx, B(x)= $\frac{1}{a} \int \frac{Y_1(x)f(x)}{w(y1,y2)} dx$

Wronskian: w(y1,y2)= $\begin{vmatrix} y1 & y_2 \\ y1' & y2' \end{vmatrix}$ (matrix)

Method 2: Constant coefficients for solving $ay'' + by' + cy = f(x)$ what integrals general solution: y=Ay1(x)+by2(x)+partial solution

Constant Coefficient Substitution

Polynomial: $Pn(x)$

Particular Solution: $x^s(Qn(x))$

Polynomial: xe^{ax} or $e^{ax}Pn(x)$

Particular Solution: $x^s(Qn(x))e^{ax}$

Polynomial: $e^{ax}Pn(x)sin(\beta x)$ or $e^{ax}Pn(x)cos(\beta x)$

Particular Solution: $x^s e^{ax} Qn(x) sin(\beta x) + x^s e^{ax} Rn(x) cos(\beta x)$

Exact Equations: $\frac{\delta M}{\delta x} = \frac{\delta N}{\delta y}$

2nd Order Differential Equations

Reducible

Case 1: y is missing

$v = y'$

$v' = y''$

Case 2: x is missing

$y' = v = \frac{dv}{dy}$

$y'' = v\frac{dv}{}$ dy