

**Name:** \_\_\_\_\_

**4-digit code:** \_\_\_\_\_

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has five (5) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

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Page	Max. points	Your points
2	30	
3	25	
4	25	
5	20	
<b>Total</b>	100	

**Problem 1** (15 pts). Sketch the domain of  $f(x, y) = \frac{\sqrt{4 - x^2}}{y^2 + 3}$ .

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**Problem 2** (15 pts). Evaluate the limit, if it exists

$$\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln (x^2 + y^2)$$



**Problem 3** (15 pts). The volume of a right circular cone of radius  $r$  and height  $h$  is  $V = \frac{1}{3}\pi r^2 h$ . Show that if the height remains constant while the radius changes, then the volume satisfies

$$\frac{\partial V}{\partial r} = \frac{2V}{r}.$$

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**Problem 4** (10 pts). Use the method of Lagrange multipliers to find the dimensions of a rectangle with perimeter  $p$  and maximum area.

width:

height:

**Problem 5** (15 pts). Recall the formula for the volume of a right circular cone of radius  $r$  and height  $h$ . Suppose that the height decreases from 20 to 19.95 inches, and the radius increases from 4 to 4.05 inches. Compare the change in volume of the cone with an approximation of this change using a total differential.

$$dV =$$

$$\Delta V =$$

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**Problem 6** (10 pts). Find an equation for the tangent plane to the surface  $z = xe^{-y}$  at the point  $P = (1, 0, 1)$ .

tangent plane:

**Problem 7** (20 pts). Find the absolute extrema of the function  $f(x, y) = xy - x - 3y$  on the triangular region  $R$  with vertices  $(0, 0)$ ,  $(0, 4)$  and  $(5, 0)$ .

absolute max:

absolute min: