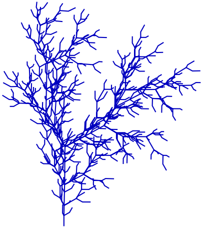
Lesson 5: The Natural Logarithm

Francisco Blanco-Silva

University of South Carolina



WHAT DO WE KNOW?

- Functions
  - $x-$ and $y-$intercepts ($f(x) = 0, f(0)$)
  - Change from $x = a$ to $x = b$

$$\Delta y = f(b) - f(a)$$

  - Average Rate of Change from $x = a$ to $x = b$

$$ARC = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

  - Relative Change from $x = a$ to $x = b$

$$RC = \frac{\Delta y}{f(a)} = \frac{f(b) - f(a)}{f(a)}$$

- Linear Functions:
  $f(x) = b + mx$
- Exponential Functions
  $P = P_0\, a^t = P_0\,(1 + r)^t$

Warm-up

The Natural Logarithm

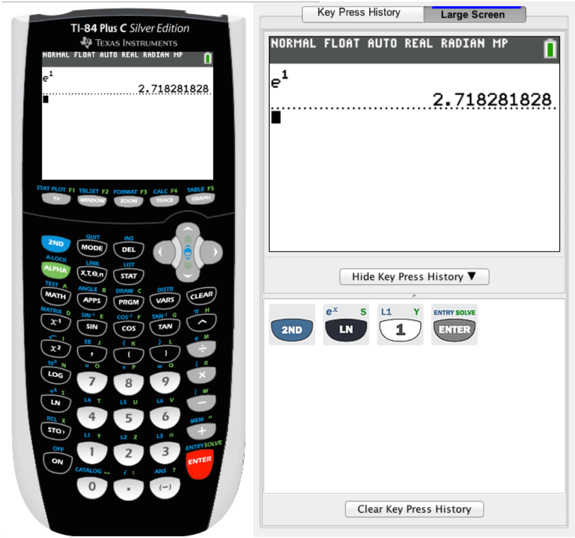
Definition

The natural logarithm of x, written $\ln x$, is the power of e needed to get x:

$$\ln x = c \text{ means } x = e^c, \text{ where } e \approx 2.7182818285$$

Warm-up

The Natural logarithm



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1. $\ln(ab) = \ln a + \ln b$.

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2. $\ln(a/b) = \ln a - \ln b$.

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4. $\ln e^r = r$, and $e^{\ln r} = r$.

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4. $\ln e^r = r$, and $e^{\ln r} = r$.
5. $\ln 1 = 0$, and $\ln e = 1$.

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1. $\ln(ab) = \ln a + \ln b$.
2. $\ln(a/b) = \ln a - \ln b$.
3. $\ln(a^r) = r \ln a$.
4. $\ln e^r = r$, and $e^{\ln r} = r$.
5. $\ln 1 = 0$, and $\ln e = 1$.
6. $\ln x$ is not defined for $x \leq 0$.

Warm-up

The Natural Logarithm

Example

Simplify the following expressions:

- $\ln e^6$
- $\ln 4x + 2\ln x$
- $-\ln x^{-1/3}$
- $\ln 3x - 3\ln x^{-1/3} - \ln 27$

Warm-up

The Natural Logarithm

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Simplify the following expressions:

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WARM-UP
THE NATURAL LOGARITHM

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$$\begin{aligned}\ln 3x - 3\ln x^{-1/3} - \ln 27 &= \ln 3x + \ln x - \ln 27 \\ &= \ln 3x^2 - \ln 27\end{aligned}$$

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$$\begin{aligned}\ln 3x - 3\ln x^{-1/3} - \ln 27 &= \ln 3x + \ln x - \ln 27\\ &= \ln 3x^2 - \ln 27\\ &= \ln \frac{3x^2}{27}\end{aligned}$$

WARM-UP
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- $\ln 3x - 3\ln x^{-1/3} - \ln 27$

$$\begin{aligned}\ln 3x - 3\ln x^{-1/3} - \ln 27 &= \ln 3x + \ln x - \ln 27 \\ &= \ln 3x^2 - \ln 27 \\ &= \ln \frac{3x^2}{27} = \ln \frac{x^2}{9}\end{aligned}$$

Warm-up

The Natural Logarithm

Example

Solve for t:

$$3^t = 10$$

Warm-up

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Apply the natural logarithm to both sides of the equation:

WARM-UP

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Warm-up

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Solve for t:

$$3^t = 10$$

Apply the natural logarithm to both sides of the equation:

$$\begin{aligned} \ln 3^t &= \ln 10 \\ t \cdot \ln 3 &= \ln 10 \end{aligned}$$

Warm-up

The Natural Logarithm

Example

Solve for t:

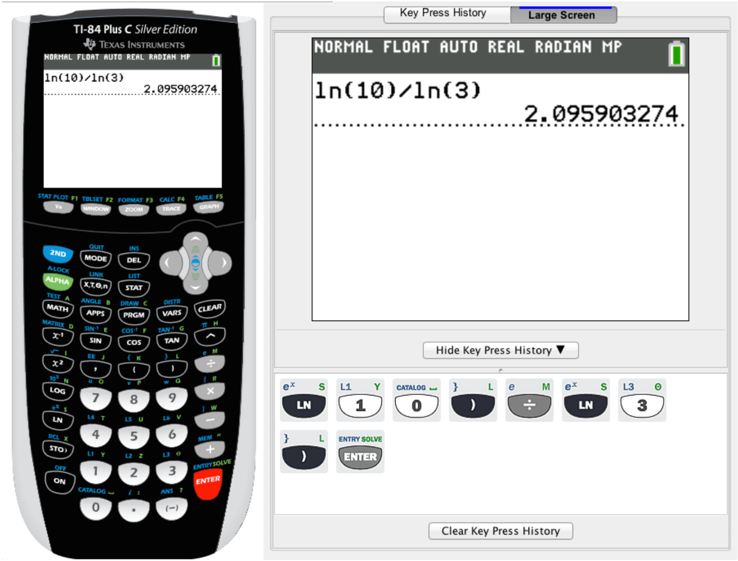
$$3^t = 10$$

Apply the natural logarithm to both sides of the equation:

$$\begin{aligned} \ln 3^t &= \ln 10 \\ t \cdot \ln 3 &= \ln 10 \\ t &= \frac{\ln 10}{\ln 3} \approx 2.0959032737 \end{aligned}$$

Warm-up

The Natural Logarithm



Warm-up

The Natural Logarithm

Example

Solve for t:

$$12 = 5e^{3t}$$

Warm-up

The Natural Logarithm

Example

Solve for t:

$$12 = 5e^{3t}$$

Before applying the natural logarithm, we force one side of the equation to be of the form a^r, without extra coefficients:

$$12/5 = e^{3t}$$

Warm-up

The Natural Logarithm

Example

Solve for t:

$$12 = 5e^{3t}$$

Before applying the natural logarithm, we force one side of the equation to be of the form a^r, without extra coefficients:

$$2.4 = e^{3t}$$

Now we proceed as in the previous example:

Warm-up

The Natural Logarithm

Example

Solve for t:

$$12 = 5e^{3t}$$

Before applying the natural logarithm, we force one side of the equation to be of the form a^r, without extra coefficients:

$$2.4 = e^{3t}$$

Now we proceed as in the previous example:

$$\ln e^{3t} = \ln 2.4$$

Warm-up

The Natural Logarithm

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Now we proceed as in the previous example:

$$\begin{aligned} \ln e^{3t} &= \ln 2.4 \\ 3t &= \ln 2.4 \end{aligned}$$

Warm-up

The Natural Logarithm

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Before applying the natural logarithm, we force one side of the equation to be of the form a^r, without extra coefficients:

$$2.4 = e^{3t}$$

Now we proceed as in the previous example:

$$\begin{aligned} \ln e^{3t} &= \ln 2.4 \\ 3t &= \ln 2.4 \\ t &= \frac{\ln 2.4}{3} \approx 0.2918229125 \end{aligned}$$

Warm-up

The Natural Logarithm

Example

Write the number $a = 1234$ in the form e^k

Warm-up

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$$e^k = 1234$$

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Write the number $a = 1234$ in the form e^k

$$e^k = 1234$$
$$\ln e^k = \ln 1234$$

Warm-up

The Natural Logarithm

Example

Write the number $a = 1234$ in the form e^k

$$e^k = 1234$$
$$\ln e^k = \ln 1234$$
$$k = 7.118016204$$

Warm-up

The Natural Logarithm

Example

Write the number $a = 1234$ in the form e^k

$$e^k = 1234$$
$$\ln e^k = \ln 1234$$
$$k = 7.118016204$$

Therefore,

$$1234 = e^{7.118016204}$$

Exponential Functions

Examples

Example

The population of Nevada in 2000 was 2.02 million, and in 2006, 2.498 million. Assuming that the growth obbeys an exponential law, find a formula as a function of t years after 2000. When will the population reach 4 million?

Exponential Functions

Examples

Example

The population of Nevada in 2000 was 2.02 million, and in 2006, 2.498 million. Assuming that the growth obbeys an exponential law, find a formula as a function of t years after 2000. When will the population reach 4 million?

We need a function of the form $P = P(t) = P_0 a^t$ (in millions), with t in years after 2000. It must be $P_0 = 2.02$.

Exponential Functions

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We need a function of the form $P = P(t) = P_0 a^t$ (in millions), with t in years after 2000. It must be $P_0 = 2.02$. To find the value of a, we use:

$$P(6) = 2.02a^6 = 2.498 \qquad a = \left(\frac{2.498}{2.02}\right)^{1/6} \approx 1.036$$

Exponential Functions

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It is then $P = 2.02(1.036)^t$.

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In order to answer the second question, we must then solve for t in the following equation:

$$2.02(1.036)^t = 4$$

The Natural Logarithm

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The Natural Logarithm

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$$1.036^t = \frac{4}{2.02}$$

The Natural Logarithm

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$$1.036^t = 1.9801980198$$

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$$\ln(1.036^t) = \ln 1.9801980198$$

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$$1.036^t = \frac{4}{2.02}$$

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$$\ln(1.036^t) = \ln 1.9801980198$$

$$\ln(1.036) \cdot t = 0.6831968497$$

The Natural Logarithm

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$$\begin{aligned} \ln(1.036^t) &= \ln 1.9801980198 \\ \ln(1.036) \cdot t &= 0.6831968497 \\ t &= \frac{0.6831968497}{0.035367143845} \end{aligned}$$

The Natural Logarithm

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Solution: In 2019.

Exponential Functions

Applications of the Natural Logarithm

We are able to *rewrite* the expression of any exponential function $P = P_0 a^t$ using the number e as the base, instead. The function will look different, but will keep its value!

Exponential Functions

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$$e^k = a$$
$$\ln e^k = \ln a$$

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All we need to do is re-write a as a power of e ($e^k = a$) and use logarithms to find the value of k:

$$\begin{aligned} e^k &= a \\ \ln e^k &= \ln a \\ k &= \ln a \end{aligned}$$

And so, $P = P_0\, a^t = P_0\, (e^k)^t = P_0\, e^{kt}$

Exponential Functions

Applications of the Natural Logarithm

In Summary

Every exponential function can be written in three different ways:

$$P = P(t) = P_0\, a^t = P_0\,(1+r)^t = P_0\, e^{kt}$$

- P_0 is the initial value
- a is the base
- r is the growth/decay rate
- We refer to k as the continuous growth/decay rate

The values of a, r and k are related, of course:

$$a = 1 + r \qquad k = \ln a \qquad k = \ln(1+r)$$

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$$a = 1 + r \qquad k = \ln a \qquad k = \ln(1+r)$$

$$r = a - 1 \qquad a = e^k \qquad r = e^k - 1$$

Exponential Functions

Applications of the Natural Logarithm

Example

- Write the function $P = 15(1.5)^t$ in the form $P = P_0 e^{kt}$
- Write the function $P = 174e^{0.3t}$ in the form $P = P_0 a^t$

Exponential Functions

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$$k = \ln 1.5 = 0.4054651081$$

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Therefore, the solution of this part is

$$P = 15e^{0.4054651081\,t}$$

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$$a = e^k = e^{0.3} = 1.349858808$$

Exponential Functions

Applications of the Natural Logarithm

Example

- Write the function $P = 15(1.5)^t$ in the form $P = P_0 e^{kt}$
- Write the function $P = 174e^{0.3t}$ in the form $P = P_0 a^t$

- They are giving us a and asking for k. We need to use the relation $k = \ln a$.

$$k = \ln 1.5 = 0.4054651081$$

Therefore, the solution of this part is

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Therefore, the solution of this part is

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Exponential Functions

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- What is the continuous percent growth rate?
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The annual growth rate is then 4.081%