Name:	______________________________
VIP ID:	______________________________

- Write your name and VIP ID in the space provided above.
- The test has six (6) pages, including this one.
- Each of the pages 2–6 contain a 30-point problem. You may try as many problems as you wish, but I will stop adding once you reach 100 points (*if*, I meant *if*). As it is customary,

score	90–100	85–89	80–84	75–79	70–74	65–69	60–64	0–59
grade	**A**	**B+**	**B**	**C+**	**C**	**D+**	**D**	**F**

- The test is fifty (50) minutes long.
- Enter your answer in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	**Max. points**	**Your points**
2	30	
3	30	
4	30	
5	30	
6	30	
Total	100	

Problem 1 (30 pts—10 pts each part). We want to compute the volume of the solid bounded below by the xy–plane, above by the plane $y + z = 3$, and on the sides by the cylinders $r = \cos\theta$ and $r = 2\cos\theta$.

(a) Sketch the object to the best of your ability.

(b) Express the volume of the object as a triple integral in either cylindrical or spherical coordinates (your choice).

$$V(R) = \iiint_R \mathbf{1}\, dV = \boxed{}$$

(c) Evaluate that integral to obtain the volume of the object.

$$V(R) = \iiint_R \mathbf{1}\, dV = \boxed{}$$

Problem 2 (30 pts—10 pts each part). Let's assume that we are using the spherical coordinates from the textbook: For $\rho \geq 0$, $0 \leq \theta \leq 2\pi$ and $0 \leq \phi \leq \pi$,

$$\begin{cases} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \end{cases}$$

We want to compute the volume of the solid bounded below by the xy–plane, on the sides by the sphere $x^2 + y^2 + z^2 = 4$, and above by the cone $\phi = \pi/3$.

(a) Sketch the object described above to the best of your ability.

(b) Express the volume of the object as a triple integral in either cylindrical or spherical coordinates (your choice).

$$V(R) = \iiint_R \mathbf{1}\, dV = \boxed{}$$

(c) Evaluate that integral to obtain the volume of the object.

$$V(R) = \iiint_R \mathbf{1}\, dV = \boxed{}$$

Problem 3 (30 pts—10 pts each part)**.** Integrate the function $f(x, y, z) = 3xy$ on the solid bounded above by the paraboloid $z = 5 - x^2 - y^2$ and below by the paraboloid $z = 4x^2 + 4y^2$.

(a) Sketch the object to the best of your ability.

(b) Express as a triple integral in either cylindrical or spherical coordinates (your choice).

$\iiint_R f(x, y, z)\, dV = \boxed{}$

(c) Evaluate that integral to obtain the volume of the object.

$\iiint_R f(x, y, z)\, dV = \boxed{}$

Problem 4 (30 pts—10 pts each part)**.** Find the volume of the solid cut from the cylinder $x^2+y^2 \leq 1$ by the sphere $x^2+y^2+z^2=4$.

(a) Sketch the object to the best of your ability.

(b) Express the volume of the object as a triple integral.

$$V(R) = \iiint_R \mathbf{1}\, dV = \boxed{}$$

(c) Evaluate that integral to obtain the volume of the object.

$$V(R) = \iiint_R \mathbf{1}\, dV = \boxed{}$$

Problem 5 (30 pts—10 pts each part)**.** Integrate the function $f(x, y, z) = 4x^2(z-2)$ over the solid cut from the thick-walled cylinder $1 \le x^2 + y^2 \le 2$ in the first octant by the cone $z = \sqrt{x^2 + y^2}$.

(a) Sketch the object to the best of your ability.

(b) Express as a triple integral in either cylindrical or spherical coordinates (your choice).

$$\iiint_R f(x, y, z)\, dV = \boxed{}$$

(c) Evaluate that integral to obtain the volume of the object.

$$\iiint_R f(x, y, z)\, dV = \boxed{}$$