Name: ______________________________

VIP ID: ______________________________

- Write your name and your VIP ID in the space provided above.
- The test has six (6) pages, including this one and the table of Laplace transforms at the end.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given at the right of each problem number.

Page	**Max**	**Points**
2	40	
3	20	
4	20	
5	20	
Total	100	

Problem 1 (40 pts—10 pts each)**.** Find the Laplace transform of the following functions:

(a) $f(x) = 3x^2 - 4x + 7$

$F(s) =$ $\boxed{\qquad\qquad (s > \quad)}$

(b) $f(x) = 2e^{3x} - 8e^{-7x} + \cos(\pi x)$

$F(s) =$ $\boxed{\qquad\qquad (s > \quad)}$

(c) $f(x) = 3x^2 \sin(5x)$

$F(s) =$ $\boxed{\qquad\qquad (s > \quad)}$

(d) $f(x) = 6xe^{-x} \sin x$

$F(s) =$ $\boxed{\qquad\qquad (s > \quad)}$

Problem 2 (20 pts—10 pts each)**.** Find the inverse Laplace transform of the following functions in the given domains.

(a) $F(s) = \dfrac{2s-3}{s^2-2s-15}$, $(s > 5)$

$f(x) = \boxed{}$

(b) $F(s) = \dfrac{s-3}{(s-3)^2+16}$, $(s > 3)$

$f(x) = \boxed{}$

Problem 3 (20 pts)**.** Use the definition of the Laplace transform to find that

$$\mathcal{L}\{4\sin 3x\} = \frac{12}{s^2+9} \text{ for } s > 0.$$

Problem 4 (20 pts)**.** Use techniques based on the Laplace transform to solve the initial value problem $y'' + 3y' + 2y = x$ that satisfies $y(0) = 0, y'(0) = 2$.

$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x)\, dx$				
1	$\frac{1}{s}$	$s > 0$	$cf(x) \pm g(x)$	$cF(s) \pm G(s)$	$s > max(a, b)$
x^n	$\frac{n!}{s^{n+1}}$	$s > 0$	$e^{\alpha x} f(x)$	$F(s - \alpha)$	$s > a + \alpha$
$e^{\alpha x}$	$\frac{1}{s - \alpha}$	$s > \alpha$	$x^n f(x)$	$(-1)^n F^{(n)}(s)$	$s > a$
$\sin \beta x$	$\frac{\beta}{s^2 + \beta^2}$	$s > 0$	$f'(x)$	$sF(s) - f(0)$	
$\cos \beta x$	$\frac{s}{s^2 + \beta^2}$	$s > 0$	$f''(x)$	$s^2 F(s) - sf(0) - f'(0)$	