% change $\frac{f(b)-f(a)}{f(a)}$ ①

Average rate of change $\frac{f(b)-f(a)}{b-a}$ ②

③ Exponential Growth

$P = P_0 a^t$ (P_0 = quantity when $t=0$; $a = r+1$)

④ Continuous Growth

$P = P_0 e^{rt}$

r = rate

t = time

Point-slope formula

$y - y_1 = m(x - x_1)$

Intercept-slope form

$y = mx + b$

Group #1

Doubling Time $t = \frac{\ln 2}{k}$

Half-Life $t = \frac{-\ln 2}{k}$ ⑤

cost = fixed cost + variable cost

revenue = price × quantity

profit = revenue − cost

⑥

Properties of Natural Logarithms

$\ln(ab) = \ln a + \ln b$ $\quad \ln(a^r) = r\ln a$ $\quad \ln 1 = 0$

$\ln(a/b) = \ln a - \ln b$ $\quad \ln e^r = r$ $\quad \ln 0 = 1$

Operation of f(x)		Effect on the Graph
f(x)+c	c +	vertical shift up by c units
f(x)-c	c +	vertical shift down
af(x)	a>1	vertical stretch by a factor of a
af(x)	0<a<1	vertical shrink by a factor of a
-f(x)		reflection across x-axis
f(x+c)	c +	horizontal shift left by c units
f(x-c)	c +	horizontal shift right by c units
f(ax)	a>1	horizontal shrink by a factor of a
f(ax)	0<a<1	horizontal stretch by a factor of a
f(-x)		reflection across y-axis

MC(q)=M'(q) MR(q)=R'(q)

Relative Rate of Change=

y=f(x) at x=a is defined as f'(a)/f(a)

if the derivative is...	then the function is...
f'(x)>0	increasing at x
f'(x)<0	decreasing at x
f'(x)=0	stationary at x

f has a critical point at x if f(x)=0,or f'(x) is not defined

if the 2nd derivative is...	then the function is...
f''(x)>0	concave upward
f''(x)<0	concave downward
f''(x)=0	concavity might change (inflection point)

D6) $f(x)=x^n$ $\quad f'(x)= n*x^{n-1}$

D7) $f(x)=e^x$ $\quad f'(x)= e^x$

D8) $f(x)=a^x$ $\quad f'(x)= a^x\ln(a)$

D9) $f(x)=\ln x$ $\quad f'(x)=1/x$

D10) a) $f(x)= g(x)^n$ $\quad f'(x)=ng(x)^{n-1}g'(x)$

b) $f(x)=e^{g(x)}$ $\quad f'(x)=g'(x)\,e^{g(x)}$

c) $f(x)=a^{g(x)}$ $\quad f'(x)= g'(x)\,a^{g(x)}\ln a$

d) $f(x)=\ln g(x)$ $\quad f'(x)= g'(x)/g(x)$

D11) The Product Rule

$h(x) = f(x) * g(x) \qquad h'(x) = f'(x) * g(x) + f(x) * g'(x)$

D12) The Quotient Rule

$h(x) = \frac{f(x)}{g(x)} \qquad h'(x) = \frac{f'(x)*g(x)-f(x)*g'(x)}{g(x)^2}$

F(0)=#, find C.

Antiderivative Rules

1. $F(x)=1$, $f(x)=x$
2. $F(x)+G(x)$, $f(x)+g(x)$
3. $F(x)-G(x)$, $f(x)-g(x)$
4. $c \cdot F(x) = c \cdot f(x)$
5. $F(x) = x^n$, $f(x) = x^{n+1}/n+1$
6. $F(x) = e^x$, $f'(x) = e^x$
7. $F(x) = a^x$, $f(x) = a^x/\ln a$
8. $F(x) = 1/x$, $f(x) = \ln x$

* deriv. of a constant = 0

More...

- $f(x) = g(x)^n$, $f'(x) = n \cdot g(x)^{n-1} \cdot g'$
- $e^{g(x)} \rightarrow g'(x)e^{g(x)}$
- $a^{g(x)} \rightarrow g'(x)a^{g(x)} \cdot \ln a$
- $\ln g(x) \rightarrow g'(x)/g(x)$

$\boxed{PR}$ $h(x) = f(x) \cdot g(x)$

$h'(x) = f'(x) \cdot g(x) + g'(x) \cdot f(x)$

$\boxed{QR}$ $h(x) = f(x)/g(x)$

$h'(x) = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{g(x)^2}$

ln = u, always

$\int u\,dv = u \cdot v - \int v\,du \rightarrow$ int. by parts

Global Max/Min

- closed interval - $[a,b]$ or $a \le x \le b$
  1. $x=a$, $x=b$ candidates
  2. CP's (where $f'=0$)
  3. highest = max lowest = min.
- open interval - use sign charts!

ex. f' [0 − 50 + → ∞] $x=50$, global min.

$f'(x)>0$ - inc. @ x $\qquad f''(x)>0$ - upwards concave

$f'(x)<0$ - decreasing $\qquad f''(x)<0$ - downward

$f'(x)=0$ - stationary $\qquad f''(x)=0$ - concavity change → inflection point

$f'(x)=0$ = critical point

2-5.4, 1, 6.2

Right hand sum = $\sum_{i=1}^{n} f(t_i)\Delta t = f(t_1)\Delta t + f(t_2)\Delta t \ldots + f(t_n)\Delta t$

Left hand sum = $\sum_{i=0}^{n-1} f(t_i)\Delta t = f(t_0)\Delta t + f(t_1)\Delta t \ldots + f(t_{n-1})\Delta t$

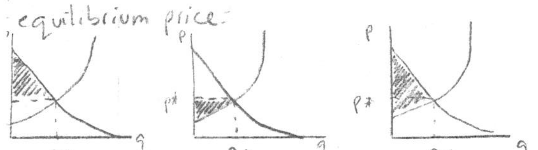
Area under a graph between a & b

$\int_a^b f(x)\,dx$

Area between 2 graphs: $A = \int_a^b f(x) - g(x)\,dx$ when $f(x)$ is on top and $g(x)$ is on bottom

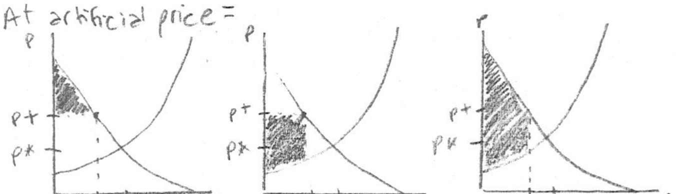
Average of $f(x)$ in the interval $x=a$ & $x=b$: $\frac{1}{b-a} \cdot \int_a^b f(x)\,dx$

equilibrium price:



consumer surplus $\qquad$ producer surplus $\qquad$ total gains from trade

At artificial price =



consumer surplus $\qquad$ producer surplus $\qquad$ total gains from trade

consumer surplus = $\int_0^{q^*} D(q) - p^*\,dq$

q^* p^* = equilibrium

producer surplus = $\int_0^{q} p - S(q)\,dq$

remember: $\int a^x = \frac{a^x}{\ln a}$

The population of Greenville is 85,000. Find the percent change if the population increases by 1,000.

$\frac{86{,}000 - 85{,}000}{85{,}000} = .01176$

(3) A bank advertises an interest rate of 8% per year. If you deposit \$10,000. How much is there after 6 years?

$P = 10{,}000(1.08)^6$

(5) A sample of an element is decaying exponentially at a continuous rate of .12% per year. What is the half-life?

$t = \frac{-\ln 2}{.0012} = -577.6$ years

Age	5	6	7	8	9	10
Weight	41	46	50	57	61	70

Find average rate of change from 5-10yrs.

$\frac{70-41}{10-5} = \frac{29}{5} = 5.8$

(4) \$15,000 is deposited into an account paying 2% interest per year compounded continuously, how long before the balance is \$20,000?

$\frac{\$20{,}000}{15{,}000} = \frac{\$15{,}000 e^{.02t}}{15000}$

$\ln 1.33 = \ln e^{.02t}$

(6) Solve for t:

$5e^{3t} = 8e^{2t}$

$\ln 5e^{3t} = \ln 8e^{2t}$

$3t \ln 5e = 2t \ln 8e$

$\frac{\ln 8+1}{\ln 5+1} = .47$

n order to find the line tangent to a unction you must use the equation -y'=m(x-x') Ex: for x^3 when x=3
) Graph it 2) Look at table and find -value 27 when x=3. 3) Plug in values or x' and y' in equation: y-27=m(x-3)
) solve for slope = f'(x)=27
) Plug it in formula. y-27=27(x-3)
) Simplify, y=27x-54

The function $f(x)= x^4-7x^3+17x$ has a critical point at x=1. Use 2[nd] Derivative Test to identify it as a local max/min.
$f'(x)=4x^3-21x^2+17$ $f''(x)=12x^2-42x$
-if f'(c)=0,and f''(c)>0, then x=c is a local min
-if f'(c)=0,and f''(c)<0, then x=3 is a local max
f''(x)=12-42= -30
-30<0…. Then x=1 is a local max

Use 1[st] derivative to find all critical points and 2[nd] derivative to find all inflection points
$f(x)=2x^3+3x^2-180x+3$
$f'(x)=6x^2+6x-180$

$x = \frac{-6 \pm \sqrt{36-4(6)(180)}}{12} = [-6, 5]$

max, min

$-\infty$ -6 0 5 ∞

$f''(x) = 12x+6$ … $x = -1/2$

$-\infty$ $-1/2$ ∞ ← inflection point

$\int \frac{6x}{3x^2-7}\,dx = \ln(3x^2-7)+C$

$\int \underbrace{x}_{u}\underbrace{e^x dx}_{dv}$ $u=x$ $du=1dx$ $v=\int e^x = e^x$ $dv=e^x$

$= xe^x - e^x + C$

The demand eq. for a product is p=26-.01q. Write the revenue as a function of q & find the # that max. rev

$R(q) = D(q)\cdot q$ (revenue, # made, \$)

$R(q) = q(26-.01q)$ $R' = 26-.02q = 0$

$= 26q - .01q^2$ check for global max $26 = .02q$ $\boxed{1300 = q}$

R' + 0 ↗ 1300 ↘ ∞ → global max

1300 units = max prof.

$D(1300) = 26 - .01(1300) = 13$

1300 units x \$13 = \$16,900 MAX PROFIT

$\int \underbrace{(x-3)}_{u}\underbrace{\sqrt{4x+7}\,dx}_{dv}$ $u = x-3$ $dv = \sqrt{4x+7}$ $du = 1$ $v = \int\sqrt{4x+7}$

$= (x-3)\frac{1}{4}\frac{(4x+7)^{3/2}}{3/2} - \int \frac{\frac{1}{4}(4x+7)^{3/2}}{3/2}\,dy$

$= \frac{1}{6}(x-3)(4x+7)^{3/2} - \frac{1}{24}\frac{(4x+7)^{5/2}}{5/2} + C$

answer must be free of integrals
- if you have ln, make it u

$F(x)=x^3$ $f(x)=\frac{1}{4}x^4$

$15x^{23} \rightarrow 15x^{24}/24$

$\pi x^{231} - e^x \rightarrow \pi\frac{x^{232}}{232} - e^x$

$\sqrt{x} \rightarrow x^{1/2+1}/\,{1/2+1}$

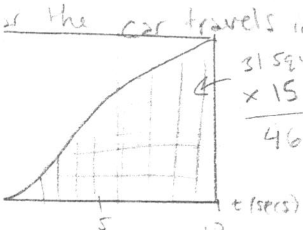
$\int(e^x-3x)dx = e^x - \frac{3}{2}x^2$

$\int(6x-9)(3x^2-9x+7)^{24}dx = \frac{(3x^2-9x+7)^{25}}{25}$

$f(x) = \pi x^2 - 3ex + 13$
$f'(x) = 2\pi x - 3e$
$f''(x) = 2\pi$

$g(x) = \ln(3x^2-4)$
$g'(x) = 6x/3x^2-4$
$g''(x) = \frac{6(3x^2-4)-6x(6x)}{(3x^2-4)^2}$

car accelerates from 0 to 90 mph in 10 seconds with the velocity given. Estimate how far the car travels in 10 seconds



31 squares
x 15 mph(sec)
465 m/hr sec.

$\frac{465\text{mi}\cdot s}{hr}\left(\frac{5280\text{ft}}{1\text{ mi}}\right)\cdot\left(\frac{1\text{ hr}}{3600\text{ sec}}\right) = 682$ ft.

Throughout the 20[th] Century yearly consumption of electricity increased exponentially at a continuous rate of 7% per year. Assume this trend continues and the energy consumed in 1900 was 1.4 mill m.w. hours.

$E(t) = 1.4e^{.07t}$ mill m.w. hrs

Avg yearly consumption for the century

$\frac{1}{100}\int_0^{100} 1.4e^{.07t}\,dt$

= 219 mill mw hrs.

Complete the signed area of $\int_1^5 x^2 \ln x^2$

x	1	2	3	4	5
f(x)	0	5.54	19.78	44.36	80.47

left sum = 69.68
right sum = 150.15
Reimann sum = 109.9