Problem 1 (50 pts)**.** Compute the following limits:

(a) $\lim_{x \to 3} \dfrac{x^2 - 2x}{x + 1} =$ $\boxed{}$

(b) $\lim_{x \to \infty} \left(1 + \dfrac{5}{x}\right)^{3x} =$ $\boxed{}$

(c) $\lim_{x \to -\infty} \sqrt{5 - x} =$ $\boxed{}$

(d) $\lim_{x \to \infty} \dfrac{5x^2 + 7}{3x^2 - x} =$ $\boxed{}$

(e) $\lim_{x \to 0} \dfrac{e^x - 1}{\tan x} =$ $\boxed{}$

Problem 2 (50 pts)**.** Find the derivative of the following functions:

(a) $y = \dfrac{x^3 + x^2 + x - 1}{x^{3/2}}$.

$$\frac{dy}{dx} = \boxed{}$$

(b) $f(x) = \cos^2(e^x) + \sin^2(e^x)$.

$$f'(x) = \boxed{}$$

(c) $f(x) = e^{9x+3}$

$$f'(x) = \boxed{}$$

(d) $g(t) = \ln\big(\sin^{-1}(t)\big)$. **Hint:** $\dfrac{d}{dx}\big(\sin^{-1} y\big) = \dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}$.

$$g'(t) = \boxed{}$$

(e) $f(x) = \dfrac{\sec x + x^{-2}}{x^{-2}\sec x}$.

$$f'(x) = \boxed{}$$

Problem 3 (50 pts)**.** Evaluate each integral:

(a) $\displaystyle\int \left(x^2 + \frac{2}{x} - e^{x-1}\right) dx =$ $\boxed{}$

(b) $\displaystyle\int_0^{\pi/4} \left(3\sec^2 x - 2\cos^2 x\right) dx =$ $\boxed{}$

(c) $\displaystyle\int (3 + \sin t)^3 \cos t \, dt =$ $\boxed{}$

(d) $\displaystyle\int \frac{3x^2}{(x^3 - 3)^3} \, dx =$ $\boxed{}$

(e) $\displaystyle\int_0^2 x \sin(x^2) \, dx =$ $\boxed{}$

Problem 4 (50 pts). Let $f(x) = \dfrac{3(x+1)(x-3)}{(x+2)(x-4)}$. Given that

$$f'(x) = \frac{-30(x-1)}{(x+2)^2(x-4)^2}, \text{ and } f''(x) = \frac{90(x^2-2x+4)}{(x+2)^2(x-4)^3},$$

sketch the graph of f (in the next page), and determine the following properties:

(a) The x- and y-intercepts are

(b) The vertical asymptotes are

(c) The horizontal asymptote is

(d) The graph is above the x-axis on the intervals

(e) The graph is increasing on the intervals

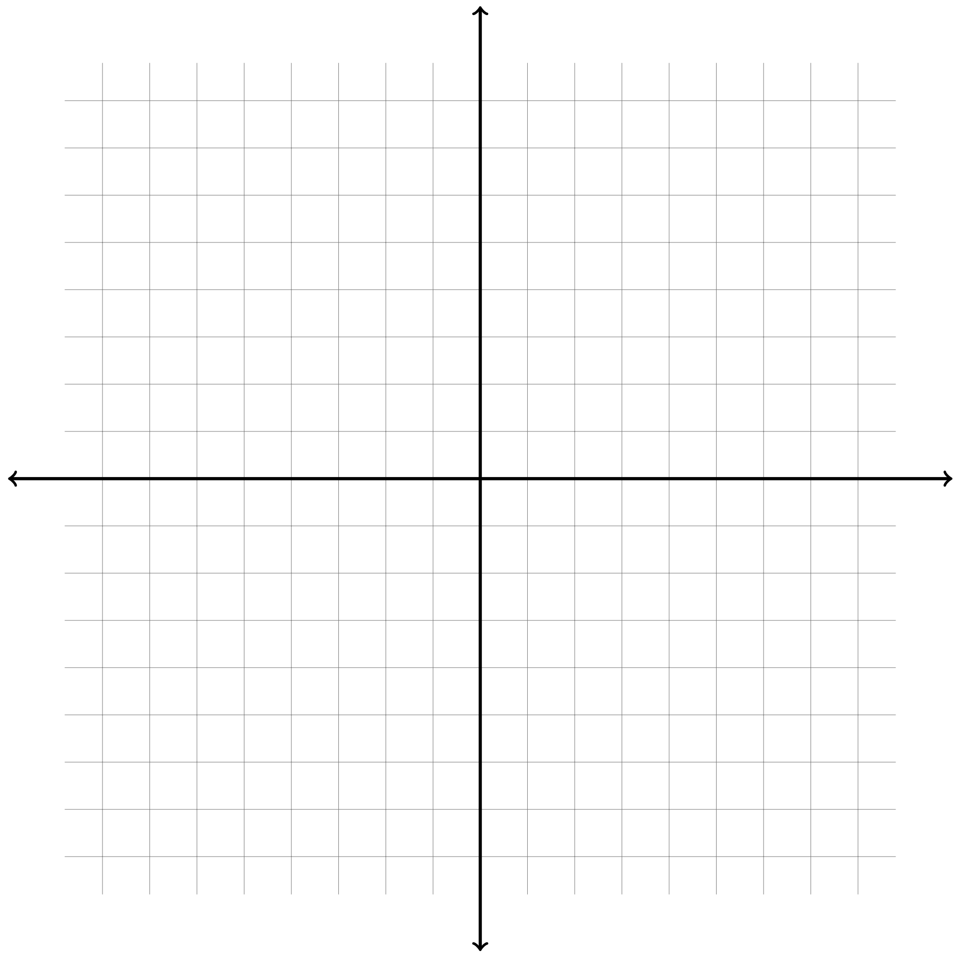
(f) The graph is decreasing on the intervals

(g) The graph is concave-up on the intervals

(h) The graph is concave-down on the intervals

(i) A relative maximum point on the graph is

Use the next page for scratch. Support your claims with sign-charts.



Problem 5 (10 pts)**.** Find the natural domain of the function $f(x) = \sqrt{x^2 - 2x + 5}$.

(a) All reals.

(b) $x < 5$

(c) $x \geq 5$

(d) $x \neq 5$

(e) $x^2 - 2x \neq 5$

Problem 6 (10pts)**.** Express $f(x) = |x^2 - 3x + 5|$ as a composition of two functions; that is, find g and h such that $f = g \circ h$.

$g =$ $\boxed{}$

$h =$ $\boxed{}$

Problem 7 (10pts)**.** Find the amplitude and period of

$$y = 5\cos(2x + \pi).$$

period = $\boxed{}$

Amplitude = $\boxed{}$

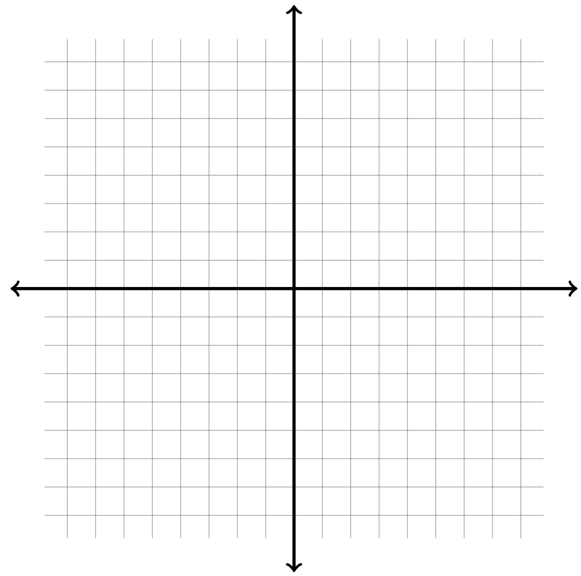
Problem 8 (10 pts)**.** Solve for x:

$$\log x^2 + \log x = 30.$$

$x =$ $\boxed{}$

Problem 9 (10 pts)**.** *Sketch the curve by eliminating the parameter (i.e. try to write $y = f(x)$.) Label the axes accordingly.*

$$x = 3t - 1, \quad y = 6t + 2.$$



Problem 10 (10 pts)**.** Find the value of the constant k for which the following function is continuous everywhere:

$$f(x) = \begin{cases} 7x - 2 & \text{if } x \leq 1, \\ kx^2 & \text{if } x < 1. \end{cases}$$

$k =$ $\boxed{}$

Problem 11 (10 pts)**.** Recall the "ε–δ" definition of limit:

> We say $\lim_{x \to a} f(x) = b$ if for all $\varepsilon > 0$ there exists $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - b| < \varepsilon$.

Use this definition to prove that $\lim_{x \to 4} (2x - 2) = 6$.

Problem 12 (10 pts)**.** Use the **definition of derivative** to find $f'(x)$ for $f(x) = 2x + 2$.

The function $f'(x)$ defined by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

is called the **derivative of** $\boldsymbol{f}$ **with respect to** $\boldsymbol{x}$.

Problem 13 (10pts)**.** Find the equation of the tangent line to the graph of $f(x) = x^2 - 4$ at $x = 1$.

$y = \boxed{}$

Problem 14 (10 pts)**.** Find $\dfrac{dy}{dx}$ by implicit differentiation.

$$5y^2 + \sin y = x^2.$$

$\dfrac{dy}{dx} = \boxed{}$

Problem 15 (10 pts)**.** Let $f(x) = x^2 - x$. Verify that the hypotheses of the Mean-Value Theorem are satisfied on the interval $[-3, 5]$.

Problem 16 (10 pts)**.** Express the sum $\sum_{k=1}^{n}(3+k)^2$ in closed form (you do **NOT** need to simplify.)

Hint: Use the following formulas.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\sum_{k=1}^{n}(3-k)^2 = \boxed{}$$

Problem 17 (10pts)**.** Given that $\ln a = 2$ and $\ln b = 5$, find $\int_1^{ac} \frac{1}{t}\, dt$.

$$\int_1^{ac} \frac{1}{t}\, dt = \boxed{}$$

Problem 18 (10pts)**.** Find the derivative $\frac{d}{dx}\int_x^0 \frac{1}{(t^2+1)^2}\, dt$.

$$\frac{d}{dx}\int_x^0 \frac{1}{(t^2+1)^2}\, dt = \boxed{}$$

Problem 19 (30 pts)**.** Let A be the area of a square whose sides have length x, and assume that x varies with the time t.

(a) Write an equation that relates A and x.

(b) Use the equation in part (a) to find an equation that relates $\dfrac{dA}{dt}$ and $\dfrac{dx}{dt}$.

(c) At a certain instant the sides are 3 ft long and increasing at a rate of 2 ft/min. How fast is the area increasing at that instant?

Problem 20 (15pts)**.** If the sum of two positive numbers is 10, then the largest their product could be is. . .

Largest product is

Problem 21 (15pts)**.** A particle moves with acceleration $a(t) = \sin t$ m/s^2 along an s-axis and has velocity $v_0 = 1$ m/s at time $t = 0$. Find the displacement of the particle.

$s(t) =$