1.1
Slope = $\frac{y_2-y_1}{x_2-x_1}$
Quadratic Equation = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
P.S. = $y-y_1 = m(x-x_1)$

1.2, 1.3, 1.4
ARC = $\frac{f(b)-f(a)}{b-a}$
RC = $\frac{f(b)-f(a)}{f(a)} \cdot 100(\%)$
Total cost = fixed c + variable c
Rev = $p \cdot q$
Prof = rev − cost

1.5 $P = P_0 a^t = P_0(1+r)^t$

1.6 (log rules)
1) $\ln(ab) = \ln(a)+\ln(b)$
2) $\ln(a/b) = \ln(a)-\ln(b)$
3) $\ln(a^r) = r(\ln)a$
4) $\ln(e^r) = r$ & $e^{\ln(r)} = r$
5) $\ln(1) = 0$ & $\ln(e) = 1$

1.7 HALF LIFE: $t^{1/2} = \frac{(-)\ln 2}{k}$
DOUBLING TIME: $T = \frac{\ln 2}{k}$
EXP. GROWTH & DECAY
$P = P_0 e^{kt}$ ↳continuous
↳not continuous, NO K: $P = P_0(1+r)^t$

COMPOUND INTEREST:
↳annually = $P = P_0(1+r)^t$
↳continuously = $P = P_0 e^{rt}$

1.8 REFLECTIONS, SHIFTS, STRETCHES:
- vert. shift up by c units: $f(x)+c$ $(c>0)$
- vert. shift down c units: $f(x)-c$ $(c>0)$
- vert. stretch by a: $af(x)$ $(a>1)$
- vert. compress. by 1/a: $af(x)$ $(0<a<1)$
- reflec. w/ x-axis: $-f(x)$
- reflec. w/ y-axis: $f(-x)$
- hor. shift ← by c: $f(x+c)$ $(c>0)$
- hor. shift → by c: $f(x-c)$ $(c>0)$
- hor. compress. by 1/a: $f(ax)$ $a>1$
- hor. stretch by a: $f(ax)$ $(0<a<1)$

1.9 POWER FUNCTIONS & PROPORTIONALITY
PROPOR: $y = kx$
inverse PROPOR: $y = k/x$
POWER FUNC: $Q(x) = k \cdot x^p$
POLYNO: $y = p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$

Derivatives

Instantaneous rate of change: at $x=a$, $f'(a)$

Leibnitz notation: $f'(x) = \frac{df}{dx}$ (derivative of f w/respect to x)

Rules:

D1: derivative of a constant function is 0. $f(x)=c$, $f'(x)=0$

D2: derivative of $f(x)=x$ is $f'(x)=1$

D3: $h(x)=f(x)+g(x)$, $h'(x)=f'(x)+g'(x)$

D4: $h(x)=f(x)-g(x)$, $h'(x)=f'(x)-g'(x)$

D5: $h(x)=c\cdot f(x)$, $h'(x)=c\cdot f'(x)$

D6: power rule: $f(x)=x^n$, $f'(x)=nx^{n-1}$

D7: $f(x)=e^x$, $f'(x)=e^x$

D8: only if $a>0$; $f(x)=a^x$, $f'(x)=a^x\ln(a)$

D9: $f(x)=\ln x$, $f'(x)=\frac{1}{x}$

D10: chain rules: $f(x)=g(x)^n$, $f'(x)=ng(x)^{n-1}g'(x)$
$f(x)=e^{g(x)}$, $f'(x)=g'(x)e^{g(x)}$
$f(x)=a^{g(x)}$, $f'(x)=g'(x)a^{g(x)}\ln(a)$
$f(x)=\ln g(x)$, $f'(x)=\frac{g'(x)}{g(x)}$

Relative Rate of Change
$\frac{f'(a)}{f(a)} \cdot 100 = \%$

1st Derivative
$f'(x)>0$ ~ increase @x
$f'(x)<0$ ~ decrease @x
$f'(x)=0$ ~ stationary @x

Elasticity of Demand
$E = \left|\frac{p}{q}\cdot f'(p)\right| = \left|\frac{\Delta q}{q} / \frac{\Delta p}{p}\right|$

Marginal Cost
$C(q)$ = Cost $R(q)$ = Revenue
$MC(q)=C'(q)$ $MR(q)=R'(q)$
Profit = Rev − Cost

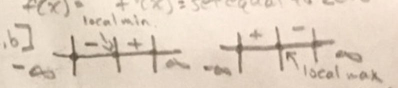
2nd Derivative
$f''(x)>0$ ~ concave up @x
$f''(x)<0$ ~ concave down @x
$f''(x)=0$ ~ concavity shifts @x (inflection pt.)

Logistic Growth
$P = f(t) = \frac{L}{1+Ce^{-kt}}$

Global Extrema/Optimization
1) Gather candidates $x=a, x=b$ $[a,b]$
2) Evaluate all candidates $f(a)=?$ $f(b)=?$ $f(c)=?$
3) Choose pt. w/ largest value → global max
Pt. w/ smallest value → global min.

Maximizing Revenue
$R=p\cdot q$ $P=R-C$
1) Set profit equal to zero
2) Find the first derivative
3) Set equation equal to zero and solve
4) Calculate to find max profit

1st Derivative Test (Critical Pts)
$f(x)=$ $f'(x)=$ set equal to zero



2nd Derivative Test (Inflection Pts)
$f(x)=?$ $f'(x)=?$
$f''(x)=?$ >0 then local min
$f''(x)=?$ <0 then local max

$\int x\,dx = 1$
$\int x^n dx = \frac{x^{n+1}}{n+1}+C$
$\int e^x dx = e^x + C$
$\int e^{kx}dx = \frac{1}{k}e^{kx}+C$
$\int a^x dx = \frac{a^x}{\ln a}+C$ $(a>0)$
$\int \frac{1}{x}dx = \ln|x|+C$
$\int x^{-1}dx = \ln|x|+C$

$\int f(x)^n \cdot f'(x)\,dx = \frac{1}{n+1}\cdot F(x)^{n+1}+C$ $(n\neq -1)$
$\int a^{f(x)}\cdot f'(x)\,dx = \frac{a^{f(x)}}{\ln a}+C$ $(a>0)$
$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)|+C$
$\int f(x)\pm g(x)\,dx = \int f(x)dx \pm \int g(x)dx$
$\int c\cdot f(x)\,dx = c\int f(x)dx$
$\int 0\,dx = C$
$\int w^n \cdot w'dw = \frac{w^{n+1}}{n+1}+C$

Calculator: MATH 9: fnint( )
fnint (f(x), x, lower bound, upper bound)

When finding c, set equation F(x) equal to y, and plug in x value

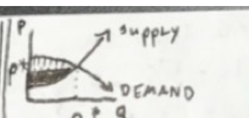
F(x) is referred to as the indefinite integral of f(x)
$F(x) = \int f(x)dx$, with respect to x

RIEMANN SUM

LEFT		RIGHT	
x_2-x_1	$\times$ y_1 = #	x_2-x_1	$\times$ y_2 = #
x_3-x_2	$\times$ y_2 = +#	x_3-x_2	$\times$ y_3 = +#
	a		b

x	x_1	x_2	x_3
y	y_1	y_2	y_3

average = $\frac{a+b}{2}$



[ ] - CONSUMER SURPLUS
[■] - PRODUCER SURPLUS

CONSUMER SURPLUS:
AREA = $\int_0^{q^*}(\text{demand equation} - p^*)\,dq$
PRODUCER SURPLUS:
AREA = $\int_0^{q^*}(p^* - \text{supply equation})\,dq$
(IF EQUATION IS NOT IN $p=f(q)$ form change the equation so that it is!!)*

TOTAL CHANGE OF A RATE OF CHANGE FUNCTION IS THE SAME AS AREA: $\left(\int_a^b f(t)dt\right)$

TOTAL AREA = $G(b)-G(a) = \int_a^b |f(x)|dx$
SIGNED AREA = $F(b)-F(a) = \int_a^b f(x)dx$
$(g(x)=|f(x)|)$

e.g. AREA BETWEEN $t=0$ and $t=9$ UNDER $P=95(0.6)^t$
$A = \int_0^9 95(0.6)^t dt = 184.10$

AVERAGE: $\frac{1}{b-a}\int_a^b f(x)dx$

AREA BETWEEN TWO CURVES $\int_a^b |f(x)-g(x)|dx$

Ex.

1.1
1) For the function in this problem, find f(7):
$f(x)=3x-9x^2$
$\rightarrow f(7)=\underline{-420}$

1.2
1) The value of a car in thousands of \$, v, is a function of the age of the car in years, a. →

1.2 (cont'd)
A car's value is $v=f(a)=13.78-.8a$. After how many years is the car's value \$0?
$\rightarrow f(0)=13.78$
$f(a)=0$
$13.78-.8a=0$
$13.78=.8a$
$13.78/.8=a$
$17.22=a$, • After $\underline{17}$ $\underline{\text{years}}$, car value is \$0

1.3 1) Find the ARC of $f(x)=6x^2+4$ between $x=1$ & $x=3$.
$\rightarrow ARC=\frac{f(3)-f(1)}{3-1}=\frac{6(3)^2+4-(6(1)^2+4)}{3-1}=\frac{58-10}{2}=\frac{48}{2}=\underline{24}=ARC$

1.4 1) An online t-shirt seller pays \$500 to start up a website & \$7/t-shirt, then sells the shirts for \$7 each. What are fixed/variable costs? $\underline{F.C.=\$500}$ for web, V.C.=\$7/t-shirt site

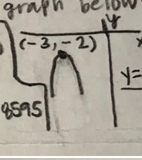
1.5
Which function has the largest % growth rate?
a) $P=1100(1.01)^t$
b) $P=100(1.07)^t$
c) $P=900(0.40)^t$
d) $P=500(1.12)^t$
→ d, 12%

1.6 Solve for x:
$4+3^{x+1}=8$
$\rightarrow$ 1) $4+3^{x+1}=8$ (isolate x)
2) $3^{x+1}=4$ (take ln)
3) $(x+1)\ln 3=\ln 4$ (solve for x)
4) $x=\frac{\ln 4}{\ln 3}-1\approx .2618595$

1.7 The ozone quality, Q, is decaying exponentially at a continuous rate of 0.25% / year. What is the 1/2 life of the ozone?
$\rightarrow \frac{-\ln(2)}{-0.0025}\approx \underline{277.258 \text{ years}}$

1.8 Write an equation for a graph obtained by vert. stretching the graph of $y=x^3$ by a factor of 3, followed by vert. shift up by 2.
$\rightarrow y=x^3$
1) vert. stretch $\rightarrow y=3x^3$
2) shift up $\rightarrow \underline{y=3x^3+2}$

1.9 1) Find a possible formula for the graph below:



$\underline{y=(x+3)^2-2}$

2) Y is directly proportional to x, and when $x=3$, $y=15$. What's the constant of proportionality?
$\rightarrow y=kx=15=k(3)$
• Solve for k
$k=5$, so
$\underline{y=5(x)}$

D11: product rule
$h(x)=f(x)\cdot g(x)$
$h'(x)=f'(x)\cdot g(x)+f(x)\cdot g'(x)$
$(u\cdot v)'=u'v+uv'$

D12: quotient rule
$h(x)=\frac{f(x)}{g(x)}$ $h'(x)=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g(x)^2}$
$\left(\frac{u}{v}\right)=\frac{u'v-uv'}{v^2}$

Examples;
D1: $f(x)=5$, $f'(x)=0$
D2: $f(x)=x$, $f'(x)=1 \rightarrow f(x)=x+\pi$, $f'(x)=1$

* D3: $f(x)=x^2+4$
$f'(x)=2x$
* D4: $f(x)=x^3-x^2$
$f'(x)=3x^2-2x$
* D5: $f(x)=3x^3$
$f'(x)=9x^2$
* D6: $f(x)=x^5$
$f'(x)=5x^4$
* D7: $f(x)=6e^x+5x^2$
$f'(x)=10x+6e^x$

* D8: $f(x)=8^x+8\cdot 5^x$
$f'(x)=8^x\ln(8)+8\cdot 5^x\ln(5)$
* D9: $R(q)=q^7-7\ln(q)$
$R'(q)=7q^6-\frac{7}{q}$
* D10: $f(x)=(3-5)^6$
$f'(x)=6(3x-5)^6\cdot(3-0)$
$=18(3x-5)^6$
* D11: $f(x)=(3x^3-\ln x)(x^5-x^{1/2})$
$u=3x^3-\ln x$, $u'=9x^2-\frac{1}{x}$
$v=x^5-x^{1/2}$, $v'=5x^4-\frac{1}{2x}^{-1/2}$
$f'(x)=u'v+uv'$

* D12: $\frac{3x^2+\ln x}{1-3x^{-4/3}+2^x}$
$u=3x^2+\ln x$
$v=1-3x^{-4/3}+2^x$
$u'=6x+\frac{1}{x}$
$v'=-4x^{1/3}+2^x\ln(2)$
$f'(x)=\frac{u'v-uv'}{v^2}$

Marginal Cost
If $C(50)=4300$ and $MC(50)=24$ estimate $C(52)$
$C(52)=C(50)+2\cdot MC(50)=4300+(2\cdot 24)=4348$

Use 1st Derivate to find all critical pts and use 2nd Der. for inflection pts. $f(x)=2x^3+3x^2-180x+3$
$f'(x)=6x^2+6x-180$ (set equal to zero to get critical pts. (-6,5)
$6(-7)^2-6(-7)-180=72>0$ Use 1st Der. equ

+ − − +
−6 0 5

Change in signs is inflection pt. Use 2nd Der equ.

− +
−1 −.5 0

Relative Rate of Change
$f(8)=253$ $f'(8)=17$
$\frac{f'(8)}{f(8)}=\frac{17}{253}\cdot 100\approx 6.72\%$

(20,300) (25,270) $300=-6(20)+b$
$\frac{270-300}{25-20}=-6$ $b=420$
$p=-6p+420$
$=-6p^2+420p$
Max Rev $=-12p+420$
@ 35 units $p=35$

1st + 2nd Derivative
Find all critical pts.
Find all inflection pts.
$f(x)=2x^3+3x^2-180x+3$
$f'(x)=6x^2+6x-180+3$
$\rightarrow \frac{-6\pm\sqrt{36-4\cdot 6(-180)}}{12}=\{-6,5\}$
$6(-7)^2+6(-7)-180=72>0$
$6(0)^2+6(0)-180=-180<0$
$6(6)^2+6(6)-180=72>0$

$f(x)=x^4-7x^3+17x$ has critical pt. @ $x=1$
Use 2nd Der. Test to identify it as local max or min.
$f''(x)=12x^2-42x$ (plug in critical pt.) $=x=-30$, min

$p=29-.01q$
$=29q-.01q^2$
$f'(x)=29-.02q$
$29=.02q$
$q=1450=$ Max Rev

Plug Max Rev into original
$=29q-.01q^2$
$=29(1450)-.01(1450)^2$
Total Rev. = 21025

Elasticity of Demand
$q=400-10p$ $p=10$ 10 units
$\left|\frac{p}{q}\cdot\frac{dq}{dp}\right|=\left|\frac{10}{300}(-10 \text{ units})\right|=10/3$

Logistic Growth
If it is t years after 1935, the proportion of land in use for a town
$f(t)=\frac{100}{1+3e^{-.0275t}}$ How much was used in 1935?
$\frac{100}{1+3e^{-.0275\cdot 0}}=25\%$

$f(x)=13x^{17}-3\sqrt[7]{x}+\frac{4}{\sqrt[6]{x}}$
$f(x)=13x^{17}-3x^{1/7}+4x^{-1/6}$
$F(x)=13\left(\frac{1}{18}\right)x^{18}-3\left(\frac{7}{8}\right)x^{8/7}+4\left(\frac{6}{5}\right)x^{5/6}$

$\int\frac{3+x}{x^2}dx$
$=\int\left(\frac{3}{x^2}+\frac{x}{x^2}\right)dx$
$=3\int x^{-2}dx+\int x^{-1}dx$
$F(x)=\frac{-3}{x}+\ln|x|$

$\int\left(\frac{2}{x}-\frac{5}{x^5}\right)dx$ that satisfies $F(1)=29$
$=2\int\frac{1}{x}dx-5\int x^{-5}dx$
$F(x)=2\ln|x|+\frac{5}{4x^4}+C$
$29=2\ln|1|+\frac{5}{4(1)^4}+C$
$C=27.75$

$\int 3(x^2-x^4)^{21}(x-2x^3)dx$
$du=2x-4x^3$
$=2(x-2x^3)$
$\frac{1}{2}du=x-2x^3dx$
$=3\int u^{21}\frac{1}{2}du$
$=\frac{3}{2}\int u^{21}du$
$=\frac{3}{2}\left(\frac{1}{22}\right)u^{22}$
$F(x)=\frac{3}{44}(x^2-x^4)^{22}$

$\int 23e^{23x}dx$ $w=23x$
$=\int e^w dw$ $\frac{dw}{dx}=23$
$F(x)=e^{23x}$ $dw=23dx$

$\int 26x\cdot 2^{13x^2-4}$ $w=13x^2-4$
$=\int 2^w dw$ $dw=26x\,dx$
$F(x)=\frac{2^{13x^2-4}}{\ln 2}$

$\int\frac{3x}{\sqrt{25x^2+4}}dx=\int 3x\cdot(25x^2+4)^{-1/2}dx$
$u=25x^2+4$
$du=50x$
$\frac{3}{50}\int u^{-1/2}du$
$=\frac{3}{50}\cdot\frac{u^{1/2}}{1/2}=2\cdot\frac{3}{50}\cdot(25x^2+4)^{1/2}+C$

---

e.g. TOTAL CHANGE
$C(q)=15000+320q$
WHAT IS CHANGE IN COST OF PRODUCING 50 UNITS VS. 100 UNITS?
CHANGE = $C(100)-C(50)$
$=320\cdot 100-320\cdot 50$
$=\$16,000$

$MC(q)=C'(q)$

e.g. AVERAGE
$f(x)=11+10x-x^2$
between $x=0$ & $x=3$
$\frac{1}{3}\int_0^3(11+10x-x^2)dx$
AVERAGE = 23

CALC: MATH → 9: fnInt(
$\rightarrow \int_{\square}^{\square}(\square)d\square$

FUNDAMENTAL THEOREM
$\int_a^b f(x)dx=F(b)-F(a)$
e.g. $\int_2^5 x^2dx=\left[\frac{1}{3}x^3+C\right]_2^5$
$=\left[\frac{1}{3}(5)^3+C\right]-\left[\frac{1}{3}(2)^3+C\right]$
$=\frac{125}{3}-\frac{8}{3}$
$=39$

IF $F'(t)$ IS CONTINUOUS
$a\le t\le b$
$\int_a^b F'(t)dt=F(b)-F(a)$

e.g. AREA (TOTAL CHANGE)
VELOCITY IS $f(t)=100e^t$ m/s
WHAT IS DISTANCE BETWEEN $t=2$ and $t=15$ (seconds)?
$=\int_2^{15}100e^t dt$
$=100\int_2^{15}e^t dt$
$=326,900,998.3$ m