Name: ____________________

4-digit code: ____________________

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has five (5) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	**Max. points**	**Your points**
2	30	
3	20	
4	30	
5	20	
Total	100	

Problem 1 (5 pts)**.** Find a formula for the general term of the sequence:

$$\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$$

$x_n =$ $\boxed{}$

Problem 2 (10 pts)**.** Write out the first five terms of the sequence $\left\{ \frac{(-1)^{n+1}}{n^2} \right\}_{n=1}^{\infty}$

Determine whether the sequence converges, and if so find its limit.

First five terms: $\boxed{}$

$\lim_{n\to\infty} x_n =$ $\boxed{}$

Problem 3 (5 pts)**.** Apply the **zero-limit test** (also known as the divergence test) and state what it tells you about the series.

$$\sum_{n=1}^{\infty} \frac{n^2+n+3}{2n^2+1}.$$

Problem 4 (10pts)**.** The Taylor series expansion for $f(x) = \sin x$ around $a = 0$ is given by

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}, \qquad (-\infty, \infty)$$

Use this, and basic properties of derivatives/integrals of power series, to find a power series expansion of $f(x) = x \cos x$.

$x \cos x =$ $\boxed{}$

Problem 5 (10 pts)**.** The following two series converge. Find their sum:

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^{n-1}$

$$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^{n-1} = \boxed{}$$

(b) The **telescopic** series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(n+2)(n+3)}$

$$\sum_{n=1}^{\infty}\frac{1}{(n+2)(n+3)} = \boxed{}$$

Problem 6 (10 pts)**.** Find the center and radius of convergence of the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{4^n}{n^2}(x-2)^n$$

Problem 7 (10 pts)**.** Use the **integral test** to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n+2}$ converges.

Problem 8 (10 pts)**.** Use any of the **comparison tests** to determine the convergence of the series

$$\sum_{n=0}^{\infty} \frac{3^n}{5^{n+1}+4}$$

Problem 9 (10 pts)**.** Use the **root test** to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n+2}{2n-1}\right)^n$ converges. If the test is inconclusive, then say so.

Problem 10 (10 pts)**.** Classify the series $\sum_{n=1}^{\infty}(-1)^n \frac{4n^2+1}{n!}$ as absolutely convergent, conditionally convergent, or divergent.

Problem 11 (10 pts)**.** Write a power series representation of the rational function

$$f(x) = \frac{3}{2-5x}.$$