Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.

- The test has eleven (11) pages, including this one.

- For multi-choice questions, you should circle the answer you select. On the other problems, you should enter your answer in the box(es) provided.

- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.

- Credit for each problem is given in parentheses at the right of the problem number.

- No books, notes or calculators may be used on this test.

- **A:** 243–270 pts. **B+:** 230–242 pts. **B:** 216–229 pts. **C+:** 203–215 pts. **C:** 189–202 pts. **D+:** 175–188 pts. **D:** 160–174 pts. **F:** less than 160 pts.

Page	Max. points	Your points	Page	Max. points	Your points
2	50		7	40	
3	50		8	30	
4	50		9	30	
5	50		10	40	
6	—		11	60	
Total	200		**Total**	200	

Problem 1 (10 pts). Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.

$$A = \boxed{}$$

Problem 2 (10 pts). Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval $[1, 4]$ is revolved about the x-axis.

$$V = \boxed{}$$

Problem 3 (10 pts). Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$ and $x = 0$ is revolved about the y-axis.

$$V = \boxed{}$$

Problem 4 (10 pts). Find the arclength of the curve $y = x^{3/2}$ from $x = 1$ to $x = 2$.

$$L = \boxed{}$$

Problem 5 (10 pts). Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

$A =$

Problem 6 (10 pts). Find the average value of the function $f(x) = 1/x$ over the interval $[1, e]$.

$f_{ave} =$

Problem 7 (15 pts). Find a positive value of k such that the average value of $f(x) = \dfrac{1}{\sqrt{k^2 - x^2}}$ over the interval $[-k, k]$ is π.

You may find the following table useful:

angle θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

$k =$ [   ]

Problem 8 (10 pts). Evaluate the integral $\displaystyle\int x^2 \sqrt{x - 1}\, dx$.

Problem 9 (15 pts). A spring exerts a force of $4N$ when stretched $2\ m$ beyond its natural length.

(a) How much work was performed in stretching the spring to this length?

$$W = \boxed{}$$

(b) How far beyond its natural length can the spring be stretched with $36J$ of work?

$$b = \boxed{}$$

Problem 10 (40 pts). Evaluate each integral:

(a) $\int \csc^2 x \, dx =$

(b) $\int \dfrac{1}{\csc x} \, dx =$

(c) $\int \dfrac{x+1}{x} \, dx =$

(d) $\int \dfrac{x}{x+1} \, dx =$

Problem 11 (10 pts). Use **integration by parts** to evaluate the integral $\int xe^{2x}\,dx$.

$$\int xe^{2x}\,dx = \boxed{}$$

Problem 12 (10 pts). Evaluate the improper integral $\int_1^\infty \dfrac{dx}{x^3}$.

$$\int_1^\infty \frac{dx}{x^3} = \boxed{}$$

Problem 13 (10 pts). Use a **trigonometric substitution** to evaluate the integral $\displaystyle \int \frac{dx}{\sqrt{x^2-9}}$.

$$\int \frac{dx}{\sqrt{x^2-9}} = \boxed{}$$

Problem 14 (10 pts). Evaluate the integral $\int \sin^2 x \cos^2 x \, dx$.

Use trigonometric simplification and one of the following reduction formulas.

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$\int \sin^2 x \cos^2 x \, dx = \boxed{}$$

Problem 15 (20 pts). Use **partial fractions** to evaluate the integral $\displaystyle\int \frac{dx}{x^2 + x - 2}$.

$$\int \frac{dx}{x^2 + x - 2} = \boxed{}$$

Problem 16 (10 pts). Find a formula for the general term of the following sequences:

(a) $\dfrac{1}{2}, \dfrac{3}{4}, \dfrac{5}{6}, \dfrac{7}{8}, \dots$

$$x_n = \boxed{}$$

(b) $1 - \dfrac{1}{2}, \dfrac{1}{3} - \dfrac{1}{2}, \dfrac{1}{3} - \dfrac{1}{4}, \dfrac{1}{5} - \dfrac{1}{4}, \dots$

$$x_n = \boxed{}$$

Problem 17 (10pts). Write out the first five terms of the sequence $\left\{ \dfrac{(-1)^{n+1}}{n^2} \right\}_{n=1}^{\infty}$
Determine whether the sequence converges, and if so find its limit.

First five terms: $\boxed{}$

$$\lim_{n \to \infty} x_n = \boxed{}$$

Problem 18 (5 pts). Use $x_{n+1} - x_n$ to show that the sequence $\{n - 2^n\}_{n=1}^{\infty}$ is strictly increasing or strictly decreasing.

Problem 19 (5 pts). Use x_{n+1}/x_n to show that the sequence $\left\{\dfrac{n^n}{n!}\right\}_{n=1}^{\infty}$ is strictly increasing or strictly decreasing.

Problem 20 (5 pts). Use **differentiation** to show that the sequence $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{\infty}$ is strictly increasing or strictly decreasing.

Problem 21 (20 pts). Determine whether the series converge, and if so find their sum:

(a) $\sum_{k=1}^{\infty}\left(-\frac{3}{4}\right)^{k-1}$

$$\sum_{k=1}^{\infty}\left(-\frac{3}{4}\right)^{k-1} = \boxed{}$$

(b) $\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)}$

$$\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)} = \boxed{}$$

Problem 22 (5 pts)**.** Apply the **divergence test** and state what it tells you about the series.

$$\sum_{k=1}^{\infty} \frac{k^2 + k + 3}{2k^2 + 1}.$$

Problem 23 (10 pts)**.** Use the **integral test** to determine whether the series $\sum_{k=1}^{\infty} \frac{1}{5k + 2}$ converges.

Problem 24 (10 pts)**.** Use the **ratio test** to determine whether the series $\sum_{k=1}^{\infty} \frac{4^k}{k^2}$ converges. If the test is inconclusive, then say so.

Problem 25 (10 pts). Use the **root test** to determine whether the series $\sum\limits_{k=1}^{\infty} \left(\dfrac{3k+2}{2k-1}\right)^{k}$ converges. If the test is inconclusive, then say so.

Problem 26 (10 pts). Classify the series $\sum\limits_{k=1}^{\infty}(-1)^{k}\,\dfrac{4k^{2}+1}{k^{3}+2}$ as absolutely convergent, convergent or divergent.