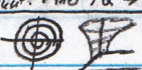
Distance formula d (p1,p2)=$\sqrt{(x1-x2)^2+(y1-y2)^2+(z1-z2)^2}$ p1=(x1,y1,z1) p2=(x2,y2,z2) **Circle** $(x-a)^2+(y-b)^2=r^2$ (a,b)=center r=radius **Sphere** $(x-a)^2+(y-b)^2+(z-c)^2=r^2$ (a,b,c)=center r=radius **P=(x1,y1,z1) Q=(x2,y2,z2)** $\vec{v}=\overrightarrow{PQ}=<x2-x1, y2-y1, z2-z1>$ $|\vec{v}|=d(P,Q)$ $\boldsymbol{\vec{w}\,(unit\ vector)}=\frac{1}{|\vec{v}|}*\vec{v}$ **Dot Product** $\vec{v}=<x1,y1>$ $\vec{w}=<x2,y2>$ $\vec{v}\cdot\vec{w}=x1*x2+y1*y2$ **Angle between 2 vectors** $\vec{v}\cdot\vec{w}=|\vec{v}|\cdot|\vec{w}|\cos\theta$ /// parallel if 0 or 180 degrees, perpendicular if 90, Pi/2 or –Pi/2 **Component** of $\vec{w}$ on $\vec{v}$ = $\frac{\vec{v}\cdot\vec{w}}{|\vec{v}|}$ **Scalar projection** of $\vec{w}$ on $\vec{v}$ = component of $\vec{w}$ on $\vec{v}$ * $\frac{\vec{v}}{|\vec{v}|}$ **Cross product** $\vec{v}=(x1,y1,z1)$ $\vec{w}=(x2,y2,z2)$ $\vec{v}\times\vec{w}$ = (y1*z2-y2*z1) $\vec{\imath}$ – (x1*z2-x2*z1) $\vec{\jmath}$ + (x1*y2-x2*y1) $\vec{k}$ **Triple product** of $\vec{u},\vec{v},\vec{w}=\vec{u}\cdot(\vec{v}\times\vec{w})$ **Parametric equations** x-x1=t(x2-x1) y-y1=t(y2-y1) z-z1=t(z2-z1) $1\le t\le 0$ **Length of a portion of a graph** $L(a,b)=\int_a^b\left|\overrightarrow{r^1}(t)\right|dt=\int_a^b\sqrt{(x^1(t))\text{^}2+(y^1(t))\text{^}2+(z^1(t))\text{^}2}$ dt **Tangent vector** $\vec{T}(t)=\frac{\vec{r}(t)}{|\vec{r}^1(t)|}$ **Normal vector** $\vec{N}(t)=\frac{\overrightarrow{r^{11}}(t)}{\left|\overrightarrow{r^{11}}(t)\right|}$ $\boldsymbol{\vec{B}(t)=\vec{T}(t)*\vec{N}(t)}$ **Curvature** $K(t)=\left|\vec{r}^1(t)*\overrightarrow{r^{11}}(t)\right|/|\vec{r}^1(t)|\text{^}3$ $K=(x)=\frac{|f^{11}(x)|}{(1+f^1(x)^2)\text{^}(\frac{3}{2})}$ **Unit Circle** (0°, 0 radians, sin=0,cos=1,tan=0) (30°, $\pi/6$, sin=1/2, cos=$\frac{\sqrt{3}}{2}$,tan=$\frac{\sqrt{3}}{3}$) (45°,$\pi/4$,sin=$\frac{\sqrt{2}}{2}$=, cos=$\frac{\sqrt{2}}{2}$, tan=1) (60°, $\pi/3$, cos=$\frac{\sqrt{3}}{2}$,sin=1/2, tan=$\sqrt{3}$) (90°, $\pi/2$, sin=1, cos=0, tan=--) **Ellipsoid** $\frac{(x-x0)^2}{a^2}+\frac{(y-y0)^2}{b^2}+\frac{(z-z0)^2}{c^2}=1$ **Elliptic paraboloid** $\frac{(z-z0)^2}{c}=\frac{(x-x0)^2}{a^2}+\frac{(y-y0)^2}{b^2}$ **Cone** $\frac{(z-z0)^2}{c^2}=\frac{(x-x0)^2}{a^2}+\frac{(y-y0)^2}{b^2}$ **Hyperboloid (one sheet)** $\frac{(x-x0)^2}{a^2}+\frac{(y-y0)^2}{b^2}-\frac{(z-z0)^2}{c^2}=1$ **Hyperboloid (two sheets)** $-\frac{(x-x0)^2}{a^2}-\frac{(y-y0)^2}{b^2}+\frac{(z-z0)^2}{c^2}=1$ **Hyperbolic paraboloid** $\frac{(z-z0)^2}{c}=\frac{(x-x0)^2}{a^2}-\frac{(y-y0)^2}{b^2}$ **Double angle formulas** sin(2x)=2sin(x)cos(x) : cos(2x)=cos^2(x)-sin^2(x)=2cos^2(x) -1=1-2sin^2(x) : tan(2x)=(2tan(x))/(1-tan^2(x)) **Half angle formulas** sin^2(x)=(1-cos(2x))/2 : cos^2(x)=(1+cos(2x))/2 **Equation of a line through** P(x0,y0,z0) with direction vector <a,b,c> $\frac{x-x0}{a}=\frac{y-y0}{b}=\frac{z-z0}{c}$ **Equation of a line** with perpendicular vector $\vec{v}=<a,b,c>$ ax+by+cz=d **Direction cosines** $\boldsymbol{cos\alpha}=\frac{\vec{v}\cdot\vec{\imath}}{|\vec{v}|}$ $\boldsymbol{cos\beta}=\frac{\vec{v}\cdot\vec{\jmath}}{|\vec{v}|}$ $\boldsymbol{cos}\Gamma=\frac{\vec{v}\cdot\vec{k}}{|\vec{v}|}$

TTAP: $z-z_0=\frac{\partial F}{\partial x}(x_0,y_0)(x-x_0)+\frac{\partial F}{\partial y}(x_0,y_0)(y-y_0)$ LA: $L(x,y)=f(x_0,y_0)+\frac{\partial F}{\partial x}(x_0,y_0)(x-x_0)+\frac{\partial F}{\partial y}(x_0,y_0)(y-y_0)$ Imp. Diff $\frac{\partial z}{\partial x}=\frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$ $\frac{\partial z}{\partial y}=\frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$

Diff Fun: $dz=\frac{\partial f}{\partial x}(x_0,y_0)\,dx+\frac{\partial f}{\partial y}(x_0,y_0)\,dy$; $\Delta z=f(x_0,y_0)-f(x,y)$ Grad: $\nabla f=\frac{\partial F}{\partial x}(x_0,y_0)+\frac{\partial F}{\partial y}(x_0,y_0)$ DirectiDer. $D_u g=\vec{u}\cdot\nabla f$ u= unit vector

Chain Rule $\frac{\partial f}{\partial t}=\frac{\partial f}{\partial x}\cdot\frac{dv}{\partial t}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$ e.g. $z=x^2+y^2+xy$, $x=\sin t$ $y=e^t$ $\frac{\partial z}{\partial t}=(2x+y)\cos(t)+(2y+x)e^t=\sin 2t+2e^{2t}+e^t(\cos t+\sin t)$

Para. equat: Find $\vec{PQ}\to<a,b,c>$ $x=x_0+at$, $y=y_0+bt$, $z=z_0+ct$ Domain: e.g. $\sqrt{1+x-y^2}$ Domain = $x\ge-1+y^2$ Range = $[0,\infty)$: $\ln t\ne0$ $\sqrt{\#}=0$

Contour  Level Curves $f(x,y)=y^2+1$ k=1, 5, 10 Set k to function $y^2+1=1$, $y^2+1=10$ then sketch Limits: $\lim_{(x,y)\to(0,0)}\frac{xy^2}{x^2+y^4}$ Plug in diff value for x or y to come from diff dirs if they all equal same = continuous

Partial Derv. $f(x,y)=x^2+y^2$ $\frac{\partial f}{\partial x}=2x$ $\frac{\partial f}{\partial y}=2y$; for second Partial $F_{xx}=2$ $F_{xy}=0$ $F_{yx}=0$ $F_{yy}=2$ TTAP e.g $z=3x^2-y^2+3y$, $(-3,5,17)$; $f_{(x)}=6x$ $f_{(y)}=-2y+3$ $f_{(x)}(-3,5)=-18$, $F_y(-3,5)=-7$ using equation $z-17=F_{(x)}(-3,5)(x-(-3))+f_y(-3,5)(y-5)$ $z-17=-18(x--3)+-7(y-5)$ or $z=-18x-7y-2$

Diff Fun e.g $m=p^3q^7$ $dm=\frac{\partial m}{\partial p}dp+\frac{\partial m}{\partial q}dq=3p^2q^7dp+7p^3q^6dq$ e.g Find dz & Δz; $z=5x^2+y^2$ changes from (1,1) to (0.95, 1.1) $dx=\Delta x=-0.05$, $dy=\Delta y=0.1$ $Z_x=10x$ $Z_y=2y$ $dz=Z_x(1,1)dx+Z_y(1,1)dy=10(-0.05)+2(0.1)=-0.3$ $\Delta z=f(0.95,1.1)-f(1,1)=5(0.95)^2+(1.1)^2-5-1\approx-0.278$

Lagrange Find max rate change & Dir $f(p,q)=5qe^{-p}+6pe^{-q}$, (0,0) $\nabla f(p,q)=<-5qe^{-p}+6e^{-q}, 5e^{-p}-6pe^{-q}>$ $\nabla F(0,0)=(6,5)$ or = to $<\frac{6}{\sqrt{61}},\frac{5}{\sqrt{61}}>$ = direction Max rate of change = $\sqrt{61}$

Abs. Max & min

LA e.g. $f(x,y)=\sqrt{y+(\cos(x))^2}$ $f_x(x,y)=\frac{1}{2}(y+\cos^2x)^{-1/2}(2\cos x)(-\sin x)=\frac{-\cos x\sin x}{\sqrt{y+\cos^2x}}$ $f_y(x,y)=\frac{1}{2}(y+\cos^2(x))^{-1/2}(1)=\frac{1}{2\sqrt{y+\cos^2x}}$

Derivatives $e^x=e^x$ $b^x=b^x\ln(b)$ $\ln(x)=\frac{1}{x}$ $\sin x=\cos x$ $\cos x=-\sin x$ $\tan x=\sec^2x$ $\csc x=-\csc x\cot x$ $\arcsin(x)=\frac{1}{\sqrt{1-x^2}}$ $\arccos(x)=\frac{-1}{\sqrt{1-x^2}}$ $\arctan=\frac{1}{1+x^2}$

Imp. Diff eg.; $F(x,y)=\sin(x,y)-xe^y=0$ $\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{\cos(x-y)-e^y}{\cos(x-y)(-1)-xe^y}=\frac{\cos(x-y)-e^y}{\cos(x-y)+xe^y}$

Dif. Der. e.g. $F(x,y)=2x\sin(xy)\to$ $f_x(x,y)=2x\cos(xy)\cdot y+2\sin(xy)$ $=2xy\cos(xy)+2\sin(x,y)$ $f_y(x,y)=2x\cos(xy)\cdot x$ $=2x^2\cos(xy)$

local M & Min step #1 find critical points of derivative, then take second partial derivative and do cross product $\frac{\partial^2f}{\partial x^2}(x_0,y_0)$ $\frac{\partial^2f}{\partial x\partial y}(x_0,y_0)$ $\frac{\partial^2f}{\partial x\partial y}(x_0,y_0)$ $\frac{\partial^2f}{\partial y^2}(x_0,y_0)$

step #1 Find critical points step #2 Parametrize border of D step #3 Evaluate $f(x,y)$ for all canadidates (x,y) largest is abs. Max smallest is abs. min

if cross product = >0 then saddle point, if cross product is <0 and $\frac{\partial^2f}{\partial x^2}>0$ = min
" " " " " <0 = max
Then plug in critical points

$D_uf(x,y)=f_x(x,y)\cos\theta+f_y(x,y)\sin\theta$ $\theta=\pi/6$ - given $D_uf(2,0)=f_x(2,0)\cos\frac{\pi}{6}+f_y(2,0)\sin\frac{\pi}{6}=0+8(\frac{1}{2})=4$

Vector Fields: 2-D: $f(x,y)=P(x,y)i+Q(x,y)j=\langle P(x,y),Q(x,y)\rangle$
Cyl Coord $x=r\cos(\theta)$, $y=r\sin(\theta)$, z=z $r^2=x^2+y^2$, Jacobian = r
Sph Coord $x=\rho\sin(\varphi)\cos(\theta)$ $y=\rho\sin(\varphi)\sin(\theta)$ $z=\rho\cos(\varphi)$ Jacobian = $\rho^2\sin(\varphi)$ $0\le\theta\le2\pi$ $0\le\varphi\le\pi$
Center of Mass: $m=\iint_D\rho(x,y)\,dA$, $M_y=\iint_D x\rho(x,y)\,dA$, $M_x=\iint_D y\rho(x,y)\,dA$, $(\bar{x},\bar{y})=\left(\frac{M_y}{m},\frac{M_x}{m}\right)$
Change of variables: $J(u,v)=\left(\frac{\partial x}{\partial u}\frac{\partial y}{\partial v}\right)-\left(\frac{\partial y}{\partial v}\frac{\partial x}{\partial u}\right)$ **Green theorem** $\oint P\,dx+Q\,dy=\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dA$
Line Integral $\int Pdx+Qdy=\int P(x,y)dx+\int Q(x,y)dy$, $\int f(x,y)dx=\int f(x(t),y(t))x'(t)dt+\int f(x(t),y(t))y'(t)dt$
FTVec Calc: If F(x,y) =<P(x,y), Q(x,y)> is conservative, f(x,y) satisfies ∇f(x,y) = F(x,y), and C is parameterized by $\vec{r}(t)=\langle x(t),y(t)\rangle$ $a\le t\le b$, then $\int_c p\,dx-Q\,dy=f(\vec{r}(b))-f(\vec{r}(a))$
$\sin^{-1}(x)=\frac{1}{\sqrt{1-x^2}}$, $\cos^{-1}(x)=\frac{-1}{\sqrt{1-x^2}}$, $\tan^{-1}(x)=\frac{1}{1+x^2}$, $\cot^{-1}(x)=\frac{-1}{1+x^2}$, $\sec^{-1}(x)=\frac{1}{x\sqrt{x^2-1}}$, $csc^{-1}(x)=\frac{-1}{x\sqrt{x^2-1}}$
$\int\sin^2(x)\,dx=\frac{1}{2}x-\frac{1}{4}\sin(2x)+C$, $\int\cos^2(x)\,dx=\frac{1}{2}x+\frac{1}{4}\sin(2x)+C$