- Magnitude $= |v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
- Direction $= \frac{v}{|v|}$
- midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$
- $u \cdot v = u_1v_1 + u_2v_2 + u_3v_3$

  $\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$
- orthogonal if $u \cdot v = 0$
- $\text{proj}_v u = \left(\frac{u \cdot v}{|v^2|}\right) v$

- sphere of radius a^2 at (x_0, y_0, z_0)

  $(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = a^2$
- scalar component of u in the direction of $v = u \cdot \frac{v}{|v|}$

  $u \cdot v = |u||v| \cos\theta$
- xy plane $z = 0$
- yz plane $x = 0$
- xz plane $y = 0$

- plane $v_1(x-x_0) + v_2(y-y_0) + v_3(z-z_0) = 0$
- Line $x = x_0 + tv_1$

  $y = y_0 + tv_2$

  $z = z_0 + tv_3$
- $|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

Velocity $= \vec{v}(t) = \vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$

Speed $= |\vec{v}(t)| = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$

Acceleration $= \vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) = \langle x''(t), y''(t), z''(t) \rangle$

Length of curve $= l = \int_a^b S(t)\,dt = \int_a^b |\vec{v}(t)|\,dt$

Curvature $(K) = K(t_0) = \frac{|\vec{v}(t_0) \times \vec{a}(t_0)|}{S(t)^3}$ or $\frac{|T'(t)|}{|\vec{r}\,'(t)|}$

$\vec{T}(t) = \frac{\vec{v}(t)}{S(t)}$

$\vec{N}(t) = \frac{\vec{T}\,'(t)}{|T'(t)|}$ $\quad \vec{B}(t) = \frac{\vec{T}(t) \times \vec{N}(t)}{|\vec{T}(t) \times \vec{N}(t)|}$ $\quad \nabla f(x_0, y_0) = \left\langle \frac{df}{dx}(x_0, y_0), \frac{df}{dy}(x_0, y_0) \right\rangle$

$r^2 = x^2 + y^2 \quad x = r\cos\theta \quad y = r\sin\theta$

Limits: Tech 1 = Evaluate
Tech 2 = Simplify
3 = Different directions $y = mx$ $y = x^2$ e+c
4 = polar

$\frac{\text{blah}}{x^2+y^2}$ $y = x$

Tangent plane to a surface at a point

(x_0, y_0, z_0) $\quad F_x(x-x_0) + F_y(y-y_0) + F_z(z-z_0) = 0$

$F_x = \frac{\partial f}{\partial x}(x_0, y_0, z_0)$ $\quad F_y = \frac{\partial f}{\partial y}(x_0, y_0, z_0)$ $\quad F_z = \frac{\partial f}{\partial z}(x_0, y_0, z_0)$

Gradient

$\nabla f(x_0, y_0, z_0) = \left\langle \frac{\partial f}{\partial x}(x_0, y_0, z_0), \frac{\partial f}{\partial y}(x_0, y_0, z_0), \frac{\partial f}{\partial z}(x_0, y_0, z_0) \right\rangle$

Gradient in Direction of a vector

$= \nabla f(x_0, y_0, z_0) \cdot \frac{\vec{v}}{|\vec{v}|}$

Local Extrema

Given $f(x, y) = \dots$

Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y}$

Set $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

- using two formulas find critical points
- plug critical points into $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y}$

Hessian $\quad (a,b)$ = critical point

$H = \frac{\partial^2 f}{\partial x^2}(a,b) \cdot \frac{\partial^2 f}{\partial y^2}(a,b) - \frac{\partial^2 f}{\partial x \partial y}(a,b)$

- local max at (a,b) if $\frac{\partial^2 f}{\partial x^2} < 0$ and $H > 0$
- local min at (a,b) if $\frac{\partial^2 f}{\partial x^2} > 0$ and $H > 0$
- saddle point at (a,b) if $H < 0$
- inconclusive if $H = 0$

Absolute Maxima of bounded Region

- List critical points, evaluate f at points
- Boundary points where f has local extrema evaluate f at points
- look for lowest and highest value of f since Absolute extrema are also local extrema

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$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$

$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$

$\int \tan^2 ax\,dx = \frac{1}{a}\tan(ax) - x$

$\int \tan ax\,dx = \frac{1}{a}\ln|\sec(ax)|$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan(ax)$

$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)$

common trig subs

$1 - \sin^2\theta = \cos^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

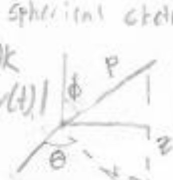
$\sec^2(\theta) - 1 = \tan^2\theta$

Line Integral

$a \le t \le b \quad r(t) = g(t)i + h(t)j + k(t)k$

$\int_a^b f(g(t), h(t), k(t))\,|v(t)|$

Spherical sketch



polar sketch (r, θ)

Conversions

$x = r\cos\theta \quad x = \rho\sin\phi\cos\theta$

$y = r\sin\theta \quad y = \rho\sin\phi\sin\theta$

$z = z \quad z = \rho\cos\phi$

$x^2 + y^2 = r^2 \quad x^2 + y^2 = \rho^2\sin^2\phi$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) \quad x^2 + y^2 + z^2 = \rho^2$

Unit circle

	sin	cos
0	0	1
$\pi/6$	1/2	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$	1/2
$\pi/2$	0	1

2 points in the intersection of the plane?

$x+2y-z=0$

$3(x-1)+2y-(z+4)=0$

- impose $x=0$ plug into 2 equations

$2y-z=0$

$-3+2y-(z+4)=0$

$\left.\begin{matrix}2y-z=0\\2y-z=7\end{matrix}\right\}$ impossible

- Impose $y=0$

$x-z=0$

$3(x-1)-(z+4)=0$

$\left.\begin{matrix}x-z=0\\3x-z=7\end{matrix}\right\}$ works

$x=z$

$3x-x=7$

$2x=7$

$x=\frac{7}{2}$ point $(7/2,0,7/2)$

- Impose $z=0$

$x+2y=0$

$3(x-1)+2y-4=0$

$x+2y=0$

$3x+2y=7$

$x=\frac{7}{2}$ $y=-\frac{7}{4}$ point $(7/2,-7/4,0)$

Find the angle between planes $x+y+z=0$ and $2x-y+3(z-1)=0$

$P=(0,0,0)$ $\vec{v}=\langle 1,1,1\rangle$

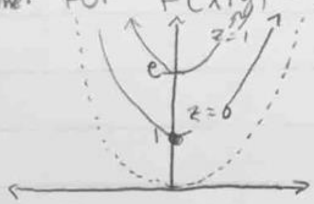
$Q=(0,0,1)$ $\vec{w}=\langle 2,-1,3\rangle$

$\vec{v}\cdot\vec{w}=|\vec{v}|\cdot|\vec{w}|\cos\theta$

$$\lim_{f(x,y)\to(0,0)}\frac{x^2-xy}{\sqrt{x}-\sqrt{y}}=\lim_{f(x,y)\to(0,0)}\frac{x^2-xy}{\sqrt{x}-\sqrt{y}}\cdot\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}\ \text{etc...}=0$$

Find domain, range, + level lines for $F(x,y)=\ln(y-x^2)$

Range: $\mathbb{R}$



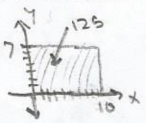
Plug these into $f(x,y)$

$y-x^2=0\to y=x^2$

$y-x^2=1\to y=x^2+1$

$y-x^2=e\to y=x^2+e$

Find Absolute Extrema of $f(x,y)=73+8x+12y-x^2-y^2$ over the Region $D=\{(x,y)\in\mathbb{R}^2\ \ 0\le x\le 10,\ 0\le y\le 7$



① Critical points $\Rightarrow (4,6)$

$\left.\begin{matrix}\frac{\partial f}{\partial x}=8-2x\\\frac{\partial f}{\partial y}=12-2y\end{matrix}\right\}\begin{matrix}x=4\\y=6\end{matrix}$

$F(4,6)=125$

Wall 1

(0,0) (0,10)

$x=0+10t=10t$ $[0,1]$

$y=0+0t=0$

$F(10t,0)=h(t)$

$h(t)=73+80t-100t^2$

$h'(t)=80-200t$ $t=\frac{80}{200}=\frac{2}{5}$

$h(0)=73$

$h(1)=54$ (10,0)

$h\left(\frac{80}{200}\right)=89$

(4,6)

Wall 2

(10,7)

(10,0)

$\left.\begin{matrix}x=10+0t\\y=0+7t\end{matrix}\right\}[0,1]$

$F(10,7t)=h(t)$

$h(t)=73+80+84t-100-49t^2$

$=53+84t-49t^2$

$h(0)=53$ (10,0)

$h(1)=88$ $h'(t)=84-98t$ $t=\frac{84}{98}$

$h\left(\frac{84}{98}\right)=89$ (10,6)

Wall 3

(0,7) (10,7)

$\left.\begin{matrix}x=0+10t=10t\\y=7+0t=7\end{matrix}\right\}[0,1]$

$F(10t,7)=h(t)$

$h(t)=73+80t+84-100t^2-49$

$=108+80t-100t^2$

$h(0)=108$

$h(1)=88$

$h'(t)=80-200t$ $t=\frac{2}{5}$

$h\left(\frac{2}{3}\right)=124$ (4,7)

Wall 4

(0,7)

(0,0)

$\left.\begin{matrix}x=0+0t=0\\y=0+7t=7t\end{matrix}\right\}[0,1]$

$F(0,7t)=h(t)$

$h(t)=73+84t-49t^2$

$h(0)=73$

$h(1)=108$

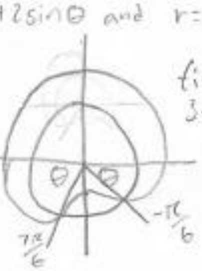
$h'(t)=84-98t$ $t=\frac{84}{98}$

$h\left(\frac{84}{98}\right)=109$

Abs max at $(4,6)=125$

Abs min at $(0,10)=53$

Determine the area of the region $r=3+2\sin\theta$ and $r=2$



find θ

$3+2\sin\theta=2$

$\sin\theta=-1/2$

$\pi/6$

$2\le r<3+2\sin\theta$

$-\pi/6\le\theta\le 7\pi/6$

Polar

Set up the limits for evaluating the triple integral over the shape with coordinates $(0,0,0),(1,1,0),(0,1,0)$

$0\le z\le y-x$

$x\le y\le 1$

$0\le x\le 1$

Find the volume of the solid enclosed by the cylinder $x^2+y^2=4$ bounded by $z=0$ and $z=x^2+y^2$

$0\le z\le r^2$

$0\le r\le 2$

$0\le\theta\le 2\pi$

Find the volume of the shape cut from the sphere $\rho\le 1$ and cone $\phi=\pi/3$

$0\le\rho\le 1$

$0\le\phi\le\pi/3$

$0\le\theta\le 2\pi$