

Name: \_\_\_\_\_

VIP ID: \_\_\_\_\_

- Write your name and your VIP ID in the space provided above.
- The test has eight (8) pages, including this one, and one scratch paper (page 8).
- It also has a formula sheet attached. **You may detach this formula sheet only when prompted to do so. Failure to comply will result in losing the privilege to use it.**
- Enter your answers in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, or notes may be used on this test.
- Calculators without a Computer Algebra System (CAS) are allowed.

Page	Max. points	Your points
2	20	
3	10	
4	15	
5	20	
6	20	
7	15	
<b>Total</b>	100	

**Problem 1** (5 pts). Evaluate the integral  $\int x e^x dx$ .

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**Problem 2** (5 pts). Evaluate the integral  $\int (x - 3)\sqrt{x^2 - 6x + 5} dx$ .

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**Problem 3** (5 pts). Evaluate the integral  $\int \sin^2 x \cos^3 x dx$ .

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**Problem 4** (5 pts). Evaluate the integral  $\int \frac{x^3 + 1}{(x + 1)^2(x^2 + 4)} dx$ .

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**Problem 5** (5 pts). Evaluate the following integral, or indicate if it is divergent:  $\int_0^\infty \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$ .

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**Problem 6** (5 pts). Find the volume of the solid obtained by rotating the region bounded by  $y = e^{-x}$ ,  $y = 1/e$ , and  $x = 0$  around the line  $y = 0$ .

**Problem 7** (5 pts). Find the general term of the sequence  $\{3, 2, \frac{5}{3}, \frac{3}{2}, \frac{7}{5}, \frac{4}{3}, \dots\}$ , and compute its limit.

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**Problem 8** (5 pts—all or nothing). Compute the limit of the sequence  $\left\{ \frac{n^2 + 5n + 2}{n^2 + 2n} \right\}_{n=1}^{\infty}$

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**Problem 9** (5 pts). Find a power series representation of the function  $f(x) = x^2 \cos(2x)$

**Problem 10** (10 pts). Study the convergence of the series  $\sum_{n=2}^{\infty} \frac{3^n + 4^n}{5^n}$ . If convergent, evaluate the sum.

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**Problem 11** (10 pts). Classify the series  $\sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$  as absolutely convergent, conditionally convergent, or divergent.

**Problem 12** (10 pts). Find the interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$ .

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**Problem 13** (10 pts). Express the function  $f(x) = \frac{2x}{x^3 + 8}$  as a power series.

**Problem 14** (5 pts). Consider the curve with polar equation  $r = 2 \cos \theta - 2 \sin \theta$ .

- (a) Find a cartesian equation for this curve.
- (b) Sketch its graph. Make sure that the sketch carries all relevant information.

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**Problem 15** (5 pts). Compute the length of the cardioid  $r = 1 + \cos \theta$ .

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**Problem 16** (5 pts). Find the area enclosed by one loop of the four-leaved rose  $r = \cos 2\theta$ .

Scratch paper