Name: _____

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1. Let $f(x, y) = -\log(1 - x - y) - \log x - \log y$.
   (a) Find the domain D of f.
   (b) Prove that D is a convex set.
   (c) Prove that f is strictly convex on D.
   (d) Find the strict global minimum.

2. Consider the equation $x = \cos x$.
   (a) Show graphically that there is a unique positive root $x^\star$. Indicate approximately its value.
   (b) Show that Newton-Raphson's method applied to $f(x) = x - \cos x$ converges for any initial guess $x_0 \in \left[0, \frac{\pi}{2}\right]$

3. Compute the first two iterations of Broyden method with initial guess $(1, 4)$ to search for the critical points of the function $f(x, y) = 2x^2 + y^2 - xy$
   (a) Using $\boldsymbol{A}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
   (b) Using $\boldsymbol{A}_0 = \operatorname{Hess} f(1, 4)$.

4. Let $f(x, y) = (x - 9/4)^2 + (y - 2)^2$. Consider the program (P) to find the global minimum of f on the set $S = \{(x, y) \in \mathbb{R}^2 : y \geq x^2, x + y \leq 6, x \geq 0, y \geq 0\}$.
   (a) Write down the KKT optimality conditions and verify that these conditions are satisfied at the point $(3/2, 9/4)$.
   (b) Present a graphical interpretations of the KKT conditions at $(3/2, 9/4)$.
   (c) Show that this point is the optimal solution to the program (P).

5. Compute one iteration of the Frank-Wolfe method to find the minimum value of the function $f(x, y) = (x - 1)^2 + (y + 5)^2$ over the square with vertices at $(2, 2)$, $(3, 3)$, $(4, 2)$ and $(3, 1)$. Use the center of the square as initial guess. Illustrate graphically this step.