Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.

- The test has five (5) pages, including this one.

- Enter your answer in the box(es) provided.

- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.

- Credit for each problem is given in parentheses at the right of the problem number.

- No books, notes or calculators may be used on this test.

Page	Max. points	Your points
2	30	
3	25	
4	25	
5	20	
Total	100	

Problem 1 (15 pts). Find the domain and range of $g(x) = \sqrt{9 - x^2 - y^2}$

domain:

range:

Sketch the level curves for $k = 0, 1, 2, 3$:

Problem 2 (15 pts). If $f(x, y) = \dfrac{xy^2}{x^2 + y^4}$, does $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Why?

Problem 3 (15 pts). Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

Problem 4 (10 pts). The dimensions of a rectangular box are measured to be 75 cm, 60 cm, and 40 cm, and each measurement is correct within 0.2 cm. Use differentials to estimate the largest possible error when the volume of the box is calculated from these measurements.

Problem 5 (5 pts). If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find dz/dt when $t = 0$ **without calculating explicitly the derivative from the composition**. Use the chain rule in the form that was explained in class.

$$\frac{dz}{dt} = \boxed{}$$

Problem 6 (10 pts). If $f(x, y, z) = x \sin yz$, find the directional derivative of f at $(1, 3, 0)$ in the direction of $\boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$. (do **not** normalize $\boldsymbol{v}$ prior to the computation)

$$D_{\boldsymbol{v}}f(1, 3, 0) = \boxed{}$$

Problem 7 (10 pts). Find the local maxima, minima and saddle points of $f(x) = x^4 + y^4 - 4xy + 1$.

max	min	saddle pts.

Problem 8 (10 pts). Find the length ℓ of the curve $\boldsymbol{r}(t) = \boldsymbol{i} + t^2\boldsymbol{j} + t^3\boldsymbol{k}$ for $0 \le t \le 1$.

$\ell =$

Problem 9 (10 pts). At what points does the helix $\boldsymbol{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

points: