The distance between $P_1(2,1,5)$ and $P_2(-2,3,0)$ is ?

A: use distance formula to get $d=\sqrt{45}$

Find the
(a) component form
(b) length of the vector
formed by initial point $P(-3,4,1)$ and terminal Point $Q(-5,2,2)$

$V=\langle -2,-2,1\rangle$

length $=|V|$

$|V|=3$

Find the distance from $S(1,1,3)$ to the Plane $3x+2y+6z=6$

$n=\langle 3,2,6\rangle$

$d=\left|\vec{PS}\cdot\frac{n}{|n|}\right| = \frac{17}{7}$

Find the angle between $U=i-2j-2K$ and $V=6i+3j+2K$

$\theta=\cos^{-1}\left(\frac{U\cdot V}{|U||V|}\right) = 1.76$ rad

Find the volume of the box formed by

$U=i+2j-K$

$V=-2i+3K$

$W=7j-4K$

$V=|(U\times V)\cdot W|$

$V=23$ units^3

Find the distance from point $S(1,1,5)$ to the line

$x=1+t$

$y=3-t$

$z=2t$

$d=\frac{|\vec{PS}\times V|}{|V|}$

$d=\sqrt{5}$

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1. $f(x,y)=5xy-7x^2-y^2+3x-6y+2$

$\frac{\partial f}{\partial x}=5y-14x+3 \quad \frac{\partial f}{\partial y}=5x-2y-6$

2. $\lim\limits_{\substack{(x,y)\to(3,0)\\ 3x-y\neq 9}} \frac{\sqrt{3x-y}-3}{3x-y-9}=\frac{1}{6}$

3. Find the tangent line of $\langle t,-\sin t,\cos t\rangle$ $t\in[-\pi,\pi]$

$\left.\begin{array}{l} x=t\\ y=-t\\ z=1\end{array}\right\} t\in[-\pi,\pi]$

4. Find the intersection of the curve $r(t)=\langle t\cos t, t\sin t, t\rangle$ with $z=\pi/2$ $=\langle 0,\pi/2,\pi/2\rangle$

5. Gradient of $f(x,y)=25-x^2-y^2$ at $(3,4)$

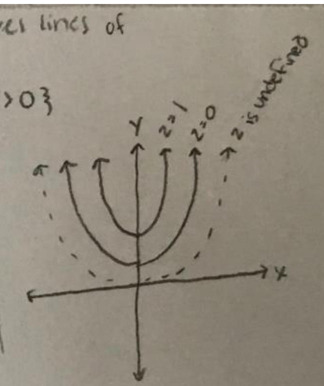
$\nabla f(x,y)=\langle -6,-8\rangle$

6. Find the Domain, Range, and level lines of

$z=f(x,y)=\ln(3y-x^2)$

Domain: $\{f(x,y)\in\mathbb{R}^2,\ 3y-x^2>0\}$

Range: $-\infty<z<\infty$



7. Find the Directional Derivative of

$f(x,y)=e^{xy}+3x^2y$ at $(0,2)$

$=2\cos(\pi/3)$

Tangent Planes

Find equation of tangent plane of $f(x,y,z)=y\cos(xy)$ at $P_0=(1,\pi,1)$

$\boxed{\text{Answer: } -y-z+1+\pi=0}$

Global Extrema

Find absolute extrema for $f(x,y)=x^2+xy+y^2-6x+1$ on $0\le x\le 5$ & $-3\le y\le 0$

Absolute max is @ $(0,-3)\to\boxed{10}$ = value

Absolute min is @ $(5,0)\to\boxed{-34}$ = value

Directional Derivatives

Find the direction in which the function $f(x,y)=2x^2+2y^2$ increases more rapidly at the point $(1,1)$. What is that value?

$\boxed{\begin{array}{l}\text{Direction: } \langle 4,4\rangle\\ \text{Value: } \sqrt{32}\end{array}}$

Finding Critical Points

Find all critical points of $f(x,y)=9-2x+4y-x^2-4y^2$

$x=-1$

$y=\frac{1}{2}$

One critical point: $(-1,\frac{1}{2})$

Using Hessian (continuation of)

$\frac{\partial^2 f}{\partial x^2}=-2 \quad \frac{\partial^2 f}{\partial y^2}=-8$

Hess $f(x,y)=-2(-8)-(0)(0)=16$

$16>0$

$\frac{\partial^2 f}{\partial x^2}(-1,\frac{1}{2})=-2<0$

$(-1,\frac{1}{2})$ is a local max of $f(x,y)$

Line Integrals

$\int x-3y^2+z$ over the line segment C where $(0,0,0)$ joins $(1,1,1)$

$\int_0^1 (t-3t^2+t)\sqrt{3}\,dt$

Substitution

$\iint_R(\sqrt{y/x}+\sqrt{xy})\,dx\,dy$ bounded by $xy=1$, $xy=9$, $y=x$, $y=4x$

$8+52\ln 2$ = answer

Spherical

$\int_0^{\pi}\int_0^{\pi}\int_0^{2\sin\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

π^2 = answer

Cylinderical

$\int_0^{2\pi}\int_0^{1}\int_r^{\sqrt{2-r^2}} dz\,r\,dr\,d\theta$

$\frac{4\pi(\sqrt{2}-1)}{3}$ = answer

$\int_0^{2\pi}\int_0^{\pi}\int_0^{(1-\cos\phi)/2} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$ Spherical

answer $=\frac{\pi}{3}$

$\int_0^{2\pi}\int_0^{1/2\pi}\int_0^{3+24r^2} dz\,r\,dr\,d\theta$

answer $=\frac{17\pi}{5}$ Cylinderical

$|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

distance between points P_1 and P_2

$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = a^2$

std eq. for sphere of radius a and center (x_0, y_0, z_0)

Unit vector in direction of $\vec{V} = \frac{\vec{V}}{|\vec{V}|}$

$U \cdot V = U_1V_1 + U_2V_2 + U_3V_3$

$\cos\theta = \left(\frac{U \cdot V}{|U||V|}\right)$

$\text{Proj}_{\vec{u}}(\vec{V}) = \lambda \cdot \frac{\vec{u}}{|\vec{u}|}$

$\lambda = \text{Component}_{\vec{u}}(\vec{V}) = \vec{V} \cdot \frac{\vec{u}}{|\vec{u}|}$

$|\vec{V}| = \sqrt{V_1^2+V_2^2+V_3^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

angles between Planes: $\cos\theta = \left(\frac{n_1 \cdot n_2}{|n_1||n_2|}\right)$

standard parametrization of the line through $P_0(x_0, y_0, z_0)$ Parallel to $\vec{v} = V_1 i + V_2 j + V_3 K$ is:

$$\left.\begin{aligned} x &= x_0 + tV_1 \\ y &= y_0 + tV_2 \\ z &= z_0 + tV_3 \end{aligned}\right\} t \in (-\infty, \infty)$$

$U \times V = (|U||V| \sin\theta)n$

Area of Parallelogram $= |U \times V|$

$d = \frac{|\vec{PS} \times V|}{|V|}$ dis. from points to a line through P, ∥ to V

Volume of Parallelepiped $= |(U \times V) \cdot W|$

dist. from a point to a plane: $d = \left|\vec{PS} \cdot \frac{\vec{n}}{|\vec{n}|}\right|$ P = Point on plane, n = normal vector

eq. for a plane in space $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$

Ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Elliptical Paraboloid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$

Elliptical Cone: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$

Hyperboloid One sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Hyperboloid of two sheets: $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Hyperbolic Paraboloid: $\frac{y^2}{b^2}-\frac{x^2}{a^2}=\frac{z}{c}, \; c>0$

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$\vec{r}'(t) = \vec{v}(t)$

$|\vec{v}(t)| = \vec{s}(t)$

$\vec{r}''(t) = \vec{a}(t)$

$\text{Length} = \int_a^b \sqrt{x'(t)^2+y'(t)^2+z'(t)^2}$

Gradient: $\nabla f(x_0, y_0) = \langle \frac{\partial f}{\partial x}(x_0, y_0), \frac{\partial f}{\partial y}(x_0, y_0) \rangle$

Tangent: $\langle x(t)+t \cdot x'(t), y(t)+t \cdot y'(t), z(t)+t \cdot z'(t) \rangle$

Directional Derivative: $\nabla f(x_0, y_0) \cdot \frac{\vec{v}}{|\vec{v}|}$

$\vec{T}(t) = \vec{v}(t) / \vec{s}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$

$\vec{N}(t) = \vec{T}'(t) / |\vec{T}'(t)|$

$\vec{B}(t) = \frac{\vec{T}(t) \times \vec{N}(t)}{|\vec{T}(t) \times \vec{N}(t)|}$

$K(t) = \frac{|\vec{T}'(t)|}{|V(t)|} = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$

Curvature

Limits:
1. Plug in points
2. if $\lim = \frac{0}{0}$, Simplify
3. Approach in different ways
   - Set $y=0$ and $y=x$ or $y=mx$
   - lim DNE if different
4. Polar Coordinates

1.) Tangent planes

- to compute tangent of a graph of a function $z=f(x,y)$ at point (x_0, y_0, z_0):

$z = f(x_0, y_0, z_0) =$

$\frac{\partial f}{\partial x}(x_0, y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y-y_0)$

2.) Directional Derivatives

- $D_{\vec{v}} f(x_0, y_0) = \nabla f(x_0, y_0) \cdot \frac{\vec{v}}{|\vec{v}|}$
- $D_{\vec{v}} f(x_0, y_0)$ is always largest in the direction of the gradient $\vec{v} = \frac{\nabla f(x_0, y_0)}{|\nabla f(x_0, y_0)|}$

3.) Finding Critical Points

- a critical point is any point (x,y) for which $\nabla f(x,y)=0$. Solve for (x,y) on equation $\nabla f(x,y)=0$:

$\frac{\partial f}{\partial x}(x,y) \qquad \frac{\partial f}{\partial y}(x,y)$

4.) Classify Local min/max/saddle points using the Hessian.

- To check if a critical point (x_0, y_0) is a max/min/of saddle point use the Hessian:

$$\text{Hess. } f(x_0, y_0) \begin{vmatrix} \frac{\partial^2 f}{\partial^2 x} & \frac{\partial^2 f}{\partial y \partial x} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial^2 y} \end{vmatrix} = \frac{\partial^2 f}{\partial x}\frac{\partial^2 f}{\partial^2 y} - \frac{\partial^2 f}{\partial x \partial y}\frac{\partial^2 f}{\partial y \partial x}$$

- If Hess $f(x_0, y_0) < 0$, then (x_0, y_0) is saddle point
- If Hess $f(x_0, y_0) > 0$, & $\frac{\partial^2 f}{\partial x^2}(x_0, y_0) > 0$, then (x_0, y_0) is local <u>min</u>
- If Hess $f(x_0, y_0) > 0$, & $\frac{\partial^2 f}{\partial x^2}(x_0, y_0) < 0$, then (x_0, y_0) is local <u>max</u>

5) Global/Absolute Extrema

Step 1. Find Candidates

inside: $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$, solve for x & y

segments: make chart using

on border $x = x_0 + t(x_2 - x_1)$, $y = y_0 + t(y_2 - y_1)$, $t \in [0,1]$

Step 2. Evaluate Candidates

$(x_1, y_1) \longleftrightarrow (x_2, y_2)$

$x = x_1 + t(x_2 - x_1)$

$y = y_1 + t(y_2 - y_1)$

$f(x,y) = h(x,y)$

$h(0) =$ max or min $h'(0) =$ max or min

$h(1) =$ max or min $h'(1) =$ max or min

Step 3. Choose Largest/Smallest among candidates

- choose largest <u>max</u> among <u>all</u> <u>max</u> candidates
- choose smallest <u>min</u> among <u>all</u> <u>min</u> candidates

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Cylinderical: r = radius, θ = angle around z-axis, z = height, $\iiint_R r \, dz \, dr \, d\theta$

Spherical: ρ = radius, θ = angle around z-axis, φ = angle from z-axis, $\iiint_R \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

cylinderical	Spherical
$x = \rho\sin\phi\cos\theta$	$x = r\cos\theta$
$y = \rho\sin\phi\sin\theta$	$y = r\sin\theta$
$z = \rho\cos\phi$	$z = z$
$r = \rho\sin\phi$	$r^2 = x^2 + y^2$
$\rho^2 = \sqrt{x^2+y^2+z^2}$	