Math 242 Test 1, Tuesday 12 February

Name: **Last 4 digits of SSN:**

Show all work clearly. No work means no credit. The points are:
ex1: 10, ex2: 10, ex3: 15, ex4: 15, ex5: 20, ex6: 15, ex7: 15.

Exercise 1 The skid marks made by an automobile indicated that its brakes were fully applied for a distance of 75 m before it came to a stop. The car in question is known to have a ***constant deceleration*** of 20 m/s^2 under these conditions.

1. Find the expression of the motion of the automobile when the brakes started (take v_0 for initial velocity).

2. How fast - in m/s - was the car traveling when the brakes were first applied ? (You will use that $\sqrt{40 * 75} = 10\sqrt{30}$)

Exercise 2 Solve the differential equation :

$$y' = e^x + 2 + 2y^2 + y^2e^x.$$

Hint: Factor the right-hand side.

Exercise 3 We are considering the following differential equation:

$$xy' = 2y + x^3 \cos x.$$

1. On which intervals does there exist a unique solution?

2. Solve the equation with the initial value $y(\pi) = 2\pi^2$.

Exercise 4 Just before midday, the body of an apparent homicide victim is found in a room that is kept at a constant temperature of 70°F. At noon, the temperature of the body is 80°F and at 1pm it is 75°F.
Assume that the temperature of the body at the time of death was 100°F (in fact the natural temperature of a body is 98.6°F, but to simplify the computations we take 100°F instead of).

1. Using the Newton's law of cooling, determine the temperature of the body at a time t (we take $t = 0$ at noon). You will be carefull about the absolute value.

2. What was the time of death (you can use that $\frac{\ln(3)}{\ln(2)} \approx 1.59$) ?

Exercise 5 We considere the following differential equation:

$$x^2y' + 2xy = 5y^4.$$

1. What kind of equation is it?

2. What substitution do we have to do?

3. What kind of differential equation do we obtain after the substitution?

4. Solve this last differential equation and then find the expression of y.

Exercise 6 We consider the following differential equation:

$$e^y + y\cos x + (xe^y + \sin x)\, y' = 0.$$

1. Show that this equation is exact.

2. Then solve this differential equation.

Exercise 7 We consider the following differential equation:

$$x^3 y' = x^2 y - y^3.$$

1. Write this differential equation as a homogeneous one.

2. Then solve this differential equation.