Math 242 Test 2, Friday 2 November

Name: **Last 4 digits of SSN:**

Show all work clearly, **MAKE SENTENCES**. No work means no credit. The points are:
ex1: 10, ex2: 20, ex3: 25, ex4: 15, ex5: 15 and the course questions are over 15 points.

Course Questions

1. Write the logistic equation and precise what is the carrying capacity. What tool do you use to solve this kind of differential equation ?

2. For what kind of differential equation do we speak about critical point ? Then give the definition of this point and precise what is an equilibrium solution.

3. For a second-order linear differential equation with constant coefficients, give the characteristic equation and the form of solutions depending on the roots of this equation (give the three cases).

Exercise 1 We give the differential equation:

$$\frac{dx}{dt} = 3x - x^2.$$

Find the critical points of this equation and use a phase diagram to determine wether each critical point is stable or unstable.

Exercise 2 We give an initial value problem and its exact solution $y(x)$:

$$y' = x - y, \quad y(0) = 1, \quad y(x) = 2e^{-x} + x - 1.$$

Apply Euler's method to approximate the solution on the interval $[0, 1]$ with step size $h = 0.25$. Write the formula you use for the computation. Then compare the four-decimal-place values of the approximate solution with the values of the exact solution using the following array. Does this step size look good ?

x	0	0.25	0.5	0.75	1
approx solution					
exact solution					

Exercise 3 Solve the differential equation:

$$y^{(3)} - 9y'' + 24y' - 16y = 0.$$

You will first find a small integral root of the characteristic equation by inspection. Then find the unique solution satisfying the initial conditions:

$$y(0) = 1, \; y'(0) = 2, \; y''(0) = 0.$$

Exercise 4 Solve the initial value problem:

$$y'' + 2y' + 10y = 0, \quad y(0) = 2, \; y'(0) = 1.$$

Exercise 5 Find a linear homogeneous constant-coefficient equation with the general solution:

$$y(x) = Ae^{3x} + B\cos(2x) + C\sin(2x) + x(D\cos(2x) + E\sin(2x)).$$

You can use that $16^2 = 256$.