Name: ____________________

VIP ID: ____________________

- Write your name and your VIP ID in the space provided above.
- You have 150 minutes (2.5 hours) to complete the exam.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- You must show proficiency solving theoretical questions on differential equations—each of the parts in Problem #1. Each of these differential equations are graded as follows:

Perfect solution	5 pts
Arithmetic errors, everything else correct	4 pts
Correct treatment of the differential equation, poor integration	2 pts
Incorrect treatment of the differential equation, or bad algebra	0 pts

If the combined score of Problem #1 is not at least 35 points, none of the application problems will be graded.

	Points	Score
Theory	50	
Applications	50	
Total	100	

I, ____________________, have chosen to take the final exam for Section 010 of Math 242 in the Spring'18 session. The grade I earn on this final exam will be my grade for the course and once I begin this exam, I must complete it. I am hereby declining my option to take the grade I currently have in the course, and I realize that my final course grade, as determined by this final exam alone, may be lower than my current grade. I realize this decision is final.

Student Signature: ____________________ Date: __________

Problem 1 (50 pts–5 pts each part)**.** Find a general solution to the following differential equations. If you desire to solve any of these equations employing Laplace transforms, assume each necessary initial condition is one: $y(0) = 1$, $y'(0) = 1$, $y''(0) = 1$, etc.

(a) $x^2 y' = xy + x^2 e^{y/x}$

(b) $3xy^2 y' + 3x^4 + y^3 = 0$

(c) $y' + y \tan x = \cos x$. Assume $x \in \left(0, \frac{\pi}{2}\right)$.

(d) $y'' = (x + y')^2$

(e) $yy'' + (y')^2 = yy'$. Assume $2y' < y$.

(f) $y^2(xy' + y)\sqrt{1 + x^4} = x$

(g) $(x - y)y' = x + y$

(h) $y'' + 2y' + 26y = 28 \cos 3x$

(i) $y'' + y = \sec^3 x$

(j) $x^2 y' = 1 - x^2 + y^2 - x^2 y^2$

Problem 2 (10 pts)**.** A spherical bowl with radius 5 ft is full of water at time $t = 0$. At that moment a circular hole with radius 1 in. is opened in the bottom of the tank. How long will it take for all the water to drain from the tank?

Problem 3 (10 pts–5 pts each part)**.** A body with mass 0.5 kg is attached to the end of a spring that is stretched 2 m by a force of 100 N. The mass and spring are attached to a dashpot that provides 1 N of resistance for each meter per second of velocity. This is set in motion one meter to the right, and moving to the left at that time with an initial velocity of 5 m/s.

(a) Find the position function of the body.

(b) Indicate the amplitude, frequency, period of oscillation and time lag of this motion.

Problem 4 (10 pts)**.** Find all curves for which the part of the normal drawn at any point (x, y) between this point and the x–axis is bisected by the y–axis.

Problem 5 (10 pts)**.** Suppose that at time $t = 0$, two thirds of a logistic population of 153,000 persons have heard a certain rumor, and that the number of those who have heard it is then increasing at the rate of 1000 persons per day. How long will it take for this rumor to spread to 75% of the population?

Problem 6 (10 pts)**.** Consider a logistic population $P(t)$ of fish on a lake, measured in hundreds after t years, with $k = 3$ and $M = 6$. Suppose that 450 fish are *harvested* annually (at a constant rate throughout the year). If the lake is initially stocked with 375 fish, when will its population reach 85% of the carrying capacity?

Formula Sheet

$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x)\,dx$				
1	$\frac{1}{s}$	$s > 0$	$cf(x) \pm g(x)$	$cF(s) \pm G(s)$	$s > max(a, b)$
x^n	$\frac{n!}{s^{n+1}}$	$s > 0$	$e^{\alpha x} f(x)$	$F(s - \alpha)$	$s > a + \alpha$
$e^{\alpha x}$	$\frac{1}{s - \alpha}$	$s > \alpha$	$x^n f(x)$	$(-1)^n F^{(n)}(s)$	$s > a$
$\sin \beta x$	$\frac{\beta}{s^2 + \beta^2}$	$s > 0$	$f'(x)$	$sF(s) - f(0)$	
$\cos \beta x$	$\frac{s}{s^2 + \beta^2}$	$s > 0$	$f''(x)$	$s^2F(s) - sf(0) - f'(0)$	

- The slope of the tangent line to the curve at (x_0, y_0) is $f'(x_0)$.
- The slope of the normal line to the cure at (x_0, y_0) is $-1/f'(x_0)$.
- The equation of the tangent line at (x_0, y_0) is $y - y_0 = y'(x - x_0)$.
- The equation of the normal line at (x_0, y_0) is $y - y_0 = (x_0 - x)/f'(x_0)$.
- The x–intercept of the tangent is $x_0 - f(x_0)/f'(x_0)$.
- The y–intercept of the tangent is $f(x_0) - x_0 f'(x_0)$.
- The x–intercept of the normal is $x_0 + f(x_0)f'(x_0)$.
- The y–intercept of the normal is $f(x_0) + x_0/f'(x_0)$.
- The length of the tangent between (x_0, y_0) and the x–axis is $|y_0|\sqrt{1 + 1/f'(x_0)^2}$.
- The length of the tangent between (x_0, y_0) and the y–axis is $|x_0|\sqrt{1 + f'(x_0)^2}$.
- The length of the normal between (x_0, y_0) and the x–axis is $|y_0|\sqrt{1 + f'(x_0)^2}$.
- The length of the normal between (x_0, y_0) and the y–axis is $|x_0|\sqrt{1 + 1/f'(x_0)^2}$.
- The length of the subtangent is $|f(x_0)/f'(x_0)|$.
- The length of the subnormal is $|f(x_0)f'(x_0)|$.