Exam 2 Study Guide:

Related Rates:

1) A trough is 10 ft. long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?

2) A particle moves along the curve below:

$$y = \sqrt{1 + x^3}$$

As it reaches the point (2, 3), the *y*-coordinate is increasing at a rate of 4 cm/s. How fast is the *x*-coordinate of the point changing at that instant?

3) A man starts walking north at 4 ft/s from a point *P*. Five minutes later a woman starts walking south at 5 ft/s from a point 500 ft due east of *P*. At what rate are the people moving apart 15 min after the woman starts walking?

4) A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

Max and Mins /Mean Value Theorem/ Rolle's Theorem

5) Use calculus to find the absolute maximum and minimum values of the function:

$f(x) = x - 2\cos(x)$
$[-2, 0]$

6) What are the three Hypotheses of Rolle's Theorem?

7) Find the number c that satisfies the conclusion of Rolle's Theorem.

$f(x) = 6 - 18x + 3x^2$
$[2, 4]$

8) What does the Mean Value Theorem State?

9) Find the number c that satisfies the conclusion of the Mean Value Theorem.

$f(x) = x^3 + x - 5$
$[0, 2]$

10) If $f(3) = 11$ and $f'(x) \geq 1$ for $3 \leq x \leq 5$, how small can $f(5)$ possibly be?

L'Hopital's Rule:

11) Find the limit.

$\lim_{x \to -\infty} \text{sqrt}(9x^6\text{-}x)/x^3+8$

12) Find the limit.

$$\lim_{x \to \infty} \left(\sqrt{81x^2 + x} - 9x \right)$$

Application of Dervitivies/Sketching:

13)

a. Find the vertical and horizontal asymptotes
b. Find the intervals of increase and decrease
c. Find the local maxs and mins
d. Find the intervals of concavity and inflection points
e. Sketch the graph

$F(x)= x^2/x^2-1$

14)

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$F(x)= x/\text{sqrt}(x^2+1)$

Optimization

15) A farmer wants to fence an area of 24 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. What should the lengths of the sides of the rectangular field be so as to minimize the cost of the fence?

16) If 1,200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

17) Find the point on the line $6x + y = 9$ that is closest to the point (-3, 1).

18) Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 4 cm and 6 cm if two sides of the rectangle lie along the legs.

19) A cone-shaped paper drinking cup is to be made to hold $27\ cm^3$ of water. Find the height and radius of the cup that will use the smallest amount of paper.