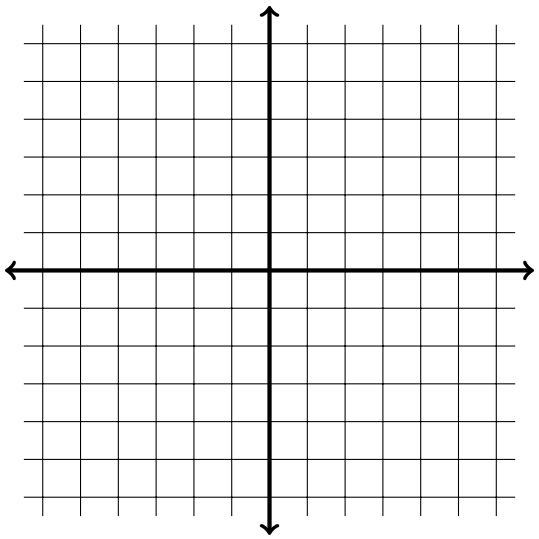
Name: _____

4-digit code: _____

- Write your name and your VIP ID in the space provided above.
- The test has six (6) pages, including this one, and scratch page at the end.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.

Page	**Max. points**	**Your points**
2	20	
3	30	
4	30	
5	20	
Total	100	

Problem 1 (10 pts)**.** Find (and sketch) the domain of $f(x,y) = \dfrac{\sqrt{4-x^2}}{y^2+3}$



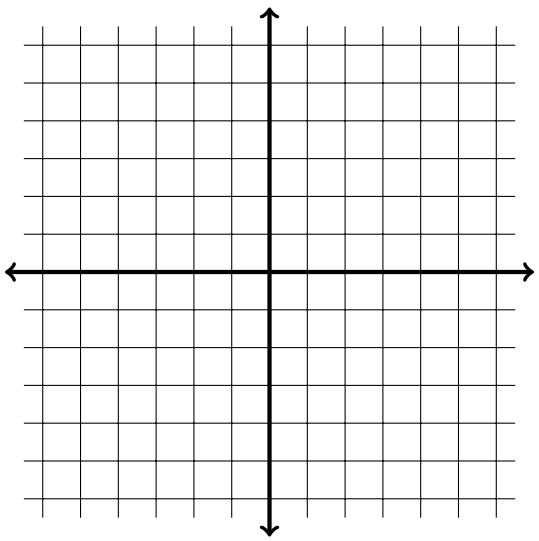
Show work here:

domain:

Problem 2 (10 pts—5 points each part)**.** For the function $f(x,y) = \sqrt{y - x^2 + x}$.

(a) Sketch the level lines $f(x,y) = k$ for $k = -1, 0, 1, 2$
(whenever the equations make sense)

(b) Use the previous information to compute the range of the function f.



Show work here:

range:

Problem 3 (10 pts)**.** Find the tangent plane to the elliptic paraboloid $z = f(x, y) = 2x^2 + y^2$ at the point $(1, 1, 3)$.

Problem 4 (10 pts)**.** Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the function $f(x, y) = \dfrac{xy^2}{y + 2x}$.

$\dfrac{\partial f}{\partial x} =$ $\dfrac{\partial f}{\partial y} =$

Problem 5 (10 pts)**.** Compute the directional derivative $D_{\boldsymbol{v}} f(0, \pi)$ for the function $f(x, y) = e^x \cos(xy^2 - 2y)$ and the direction of the vector $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j}$.

$D_{\boldsymbol{v}} f(0, \pi) =$

Problem 6 (15 pts—5 pts each box)**.** Find the local maxima, minima and saddle points of $f(x) = x^4 + y^4 - 4xy + 1$.

max	min	saddle pts.

Problem 7 (15 pts)**.** Find the absolute maximum and minimum values of the function $f(x, y) = 4x + 6y - x^2 - y^2 + 7$ on the set $D = \{(x, y) : 0 \leq x \leq 4, 0 \leq y \leq 5\}$. Make sure to sketch the set D and indicate the different borders.

Problem 8 (20 pts—5 pts each)**.** Consider the function $z = f(x, y) = e^{xy}$

(a) What is the *maximum value* of any of the directional derivatives of f at the point $(2, 0)$?

Value of maximum directional derivative: $\boxed{}$

(b) Are there any directions $\boldsymbol{u}$ for which the directional derivative $D_{\boldsymbol{u}} f(2, 0) = 0$? If so, find at least one such direction.

$\boldsymbol{u} =$ $\boxed{}$

(c) Are there any directions $\boldsymbol{v}$ for which the directional derivative $D_{\boldsymbol{v}} f(2, 0) = -1$? If so, find at least one such direction.

$\boldsymbol{v} =$ $\boxed{}$

(d) Are there any directions $\boldsymbol{w}$ for which the directional derivative $D_{\boldsymbol{w}} f(2, 0) = -4$? If so, find at least one such direction.

$\boldsymbol{w} =$ $\boxed{}$