## Math 242 Final Exam, Friday 14 December

Name:

Last 4 digits of SSN:

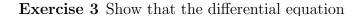
Show all **work clearly**, **make sentences**. No work means no credit. The points are: Ex1: 10, Ex2: 10, Ex3: 15, Ex4: 10, Ex5: 15 and Problem: 40.

Exercise 1 Solve the initial value problem:

$$xy' + 3y = 3x^{-\frac{3}{2}}, \quad y(1) = 0.$$

Exercise 2 Find a general solution of the differential equation

$$y' = 1 + x + y + xy.$$



$$(1 + ye^{xy}) dx + (2y + xe^{xy}) dy = 0,$$

is exact and then solve it.

**Exercise 4** We give an initial value problem and its exact solution y(x):

$$y' = 16x^3 + 2 - 2y$$
,  $y(0) = 1$ ,  $y(x) = -5 + 12x - 12x^2 + 8x^3 + 6e^{-2x}$ .

Apply Euler's method to approximate the solution first on the interval [0,1] with step size h=0.25, and on the interval [0,0.5] with the step size h=0.1. Write the formula you use for the computation. Then compare the four-decimal-place values of the approximate solution with the values of the exact solution using the following arrays. What do you think of this two different cases?

## step size h = 0.25

x	0	0.25	0.5	0.75	1
approx solution					
exact solution					

# step size h = 0.1

X	0	0.1	0.2	0.3	0.4	0.5
approx solution						
exact solution						

## Exercise 5 Solve the differential equation

$$y^{(3)} - 6y'' + 3y' - 18y = 0,$$

using the fact that the function  $x \mapsto e^{6x}$  is solution of this differential equation. Then find the unique solution satisfying the initial conditions:

$$y(0) = 0, y'(0) = 3, y''(0) = 90.$$

#### Problem

We consider the initial value problem

$$y'' - y' - 6y = 169x\cos(3x),\tag{1}$$

with the initial values y(0) = 0, y'(0) = 1.

We want to solve it by two different ways.

## I: Laplace transform

1) Use the theorem of differentiation of transforms to find the Laplace transform of  $x \cos(3x)$ .

2) Let 
$$K(s) = 169 \frac{s+3}{(s+2)(s^2+9)^2}$$
. Show that

$$K(s) = \frac{1}{s+2} + \frac{-13s+195}{(s^2+9)^2} + \frac{-s+2}{s^2+9}.$$

3) Then solve the initial value problem using Laplace transform.
<ul><li>II : Classical way</li><li>1) Find the complementary solution of (1).</li></ul>
1) I ma the complementary solution of (1).
2) Use the sheet about the particular solution to find one for (1).

