Name: ______________________________

VIP ID: ______________________________

- Write your name and your VIP ID in the space provided above.
- The test has five (5) pages, including this one and a formula sheet at the end.
- **Feel free to detach** the formula sheet from the booklet. You may use the back as scratch paper.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given at the right of each problem number.
- Problems 4 and 5 are geometric applications and will be graded as follows: First I will evaluate if you obtained the equation correctly (10 pts). If the equation is not correct, I will stop grading. However, if the equation is correct, I will evaluate the solution of the equation over 10 points. You may choose to solve these equations with classical methods, or with methods based upon the Laplace transform. The score (over 10 points) that you obtain on this part **will be added as extra credit to the corresponding previous exam.**

Page	**Max**	**Points**
2	30	
3	30	
4	40	
Total	100	

Problem 1 (20 pts—10 pts each part)**.** During the period from 1790 to 1930, the U.S. population $P(t)$ after t years grew from 3.9 million to 123.2 million. Throughout this period, $P(t)$ remained close to the solution of the initial value problem

$$\frac{dP}{dt} = 0.03135P - 0.0001489P^2, \quad P(0) = 3.9.$$

(a) What limiting population does it predict?

(b) What 1930 population does this model predict?

Problem 2 (10 pts)**.** Plot a slope field to indicate the stability of the following population model:

$$\frac{dP}{dt} = (P-1)^3(P-3)^2(2P^2 - 13P + 15)$$

Problem 3 (10 pts each)**.** Suppose that at time $t = 0$, half of a logistic population of 100,000 persons have heard a certain rumor, and that the number of those who have heard it is then increasing at the rate of 1000 persons per day. How long will it take for this rumor to spread to 80% of the population?

Problem 4 (20 pts–10 pts for equation, 10 pts for solution)**.** Find all curves for which the normal at point (x, y) and the line joining the origin with that point form an isosceles triangle having its base on the x–axis.

Problem 5 (20 pts–10 pts for equation, 10 pts for solution)**.** Find the family of curves for which the length of the part of the tangent between the point of contact (x, y) and the y–axis is equal to the y–intercept of the tangent.

Problem 6 (20 pts—10 pts each)**.** Find the orthogonal trajectories of each of the following families of curves:

(a) $3x + 5y = k$

(b) $y = 3x - 4 + ke^{-5x}$

Formula Sheet

$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x)\,dx$				
1	$\frac{1}{s}$	$s > 0$	$cf(x) \pm g(x)$	$cF(s) \pm G(s)$	$s > max(a, b)$
x^n	$\frac{n!}{s^{n+1}}$	$s > 0$	$e^{\alpha x} f(x)$	$F(s-\alpha)$	$s > a + \alpha$
$e^{\alpha x}$	$\frac{1}{s-\alpha}$	$s > \alpha$	$x^n f(x)$	$(-1)^n F^{(n)}(s)$	$s > a$
$\sin \beta x$	$\frac{\beta}{s^2+\beta^2}$	$s > 0$	$f'(x)$	$sF(s) - f(0)$	
$\cos \beta x$	$\frac{s}{s^2+\beta^2}$	$s > 0$	$f''(x)$	$s^2F(s) - sf(0) - f'(0)$	

- The slope of the tangent line to the curve at (x_0, y_0) is $f'(x_0)$.
- The slope of the normal line to the cure at (x_0, y_0) is $-1/f'(x_0)$.
- The equation of the tangent line at (x_0, y_0) is $y - y_0 = y'(x - x_0)$.
- The equation of the normal line at (x_0, y_0) is $y - y_0 = (x_0 - x)/f'(x_0)$.
- The x–intercept of the tangent is $x_0 - f(x_0)/f'(x_0)$.
- The y–intercept of the tangent is $f(x_0) - x_0 f'(x_0)$.
- The x–intercept of the normal is $x_0 + f(x_0)f'(x_0)$.
- The y–intercept of the normal is $f(x_0) + x_0/f'(x_0)$.
- The length of the tangent between (x_0, y_0) and the x–axis is $|y_0|\sqrt{1 + 1/f'(x_0)^2}$.
- The length of the tangent between (x_0, y_0) and the y–axis is $|x_0|\sqrt{1 + f'(x_0)^2}$.
- The length of the normal between (x_0, y_0) and the x–axis is $|y_0|\sqrt{1 + f'(x_0)^2}$.
- The length of the normal between (x_0, y_0) and the y–axis is $|x_0|\sqrt{1 + 1/f'(x_0)^2}$.
- The length of the subtangent is $|f(x_0)/f'(x_0)|$.
- The length of the subnormal is $|f(x_0)f'(x_0)|$.