Math 242 Test 1, Friday 21 September

Name: **Last 4 digits of SSN:**

Show all work clearly. No work means no credit. The points are:
ex1: 7, ex2: 15, ex3: 9, ex4: 9, ex5: 5 plus 5 point for the writing.

Exercise 1 A spacecraft is in free fall towards the surface of the moon at a speed of
1000 mi/h. Its retrorockets, when fired, provide a **constant deceleration** of 20,000
mi/h^2.

1. Find the expression of the motion of the spacecraft when the retrorockets start
   (ignore the moon's gravitational field). We take y_0 for the initial height.

2. At what height above the lunar surface should the astronauts fire the retrorockets
   to insure a soft touchdown ($v = 0$ at impact) ?

Exercise 2 We considere the following differential equation:

$$y^2(xy' + y)(1 + x^4)^{1/2} = x.$$

1. Write this differential equation as a Bernouilli's equation.

2. What substitution do we have to do?

3. What differential equation do we obtain after the substitution?

4. Solve this last differential equation and then find the expression of y.

Exercise 3 A pitcher of buttermilk initially at 25 °C is to be cooled by setting it on the front porch, where the temperature is 5 °C. Suppose that the temperature of the buttermilk has dropped to 15 °C after 20 min.

1. Using the Newton's law of cooling, determine the temperature of the buttermilk at a time t (t in minutes).

2. When will the temperature of buttermilk be 10 °C (you can use that $\ln 4 = 2\ln 2$)?

Exercise 4 We consider the following differential equation:

$$(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0.$$

1. Show that this equation is exact.

2. Then solve this differential equation.

Exercise 5 Solve the differential equation:

$$2y\frac{dy}{dx} = \frac{x}{\sqrt{x^2 - 16}}, \quad y(5) = 2.$$

(you can use a formula of type $\int f'f^n = \ldots$)