Lesson 13: Using the Derivative I

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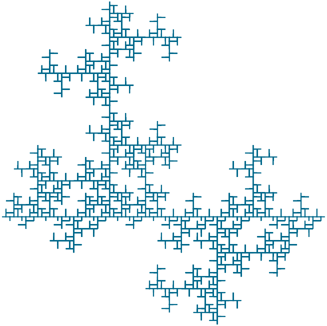

What do we know?
The General Program

- Functions
  - $x-$ and $y-$intercepts ($f(x) = 0, f(0)$)
  - Change from $x = a$ to $x = b$

    $$\Delta y = f(b) - f(a)$$

  - Average Rate of Change from $x = a$ to $x = b$

    $$ARC = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

  - Relative Change from $x = a$ to $x = b$

    $$RC = \frac{\Delta y}{f(a)} = \frac{f(b) - f(a)}{f(a)}$$

  - Instantaneous Rate of Change at $x = a$

    $$f'(a)$$

  - Relative Rate of Change at $x = a$

    $$\frac{f'(a)}{f(a)}$$

- Linear Functions:
  $f(x) = b + mx$
- Exponential Functions
  $P_0a^t = P_0(1 + r)^t = P_0e^{kt}$
- Power Functions

  $$kx^p$$

- Polynomials
  $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$

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Compute the second derivative of the following functions:

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$h(x) = \left(x^5 - x^{1/3}\right)^2$

$$\begin{aligned} h'(x) &= 2\left(x^5 - x^{1/3}\right)\left(5x^4 - \tfrac{1}{3}x^{-2/3}\right) \\ &= \left(2x^5 - 2x^{1/3}\right)\left(5x^4 - \frac{1}{3}x^{-2/3}\right) \\ &= 10x^9 - \frac{32}{3}x^{13/3} + \frac{2}{3}x^{-1/3} \end{aligned}$$
$$h''(x) = 90x^8 - \frac{416}{9}x^{10/3} - \frac{2}{9}x^{-4/3}$$

Using the Derivative

How does the sign of derivatives affect the shape of a function?

Because the derivative is linked to the values of slopes of tangent lines, we have the following results:

If the derivative is...	then, the function is...
$f'(x) > 0$	increasing at x
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The sign of the second derivative also offers a clear interpretation in terms of graph shape.

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$f''(x) > 0$	concave upwards at x ($\smile$)
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We say that f has an inflection point at x if the concavity of f changes at x.

Using the Derivative

Maxima and Minima

Definition

We say that a critical point $x = a$ is

- a (local) minimum of f, if $f(a) \leq f(x)$ for nearby values of x.
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The First Derivative Test

- If $f'(c) = 0$, $f'(x) < 0$ for $x < c$, and $f'(x) > 0$ for $x > c$, then $x = c$ is a local minimum.
- If $f'(c) = 0$, $f'(x) > 0$ for $x < c$, and $f'(x) < 0$ for $x > c$, then $x = c$ is a local maximum.

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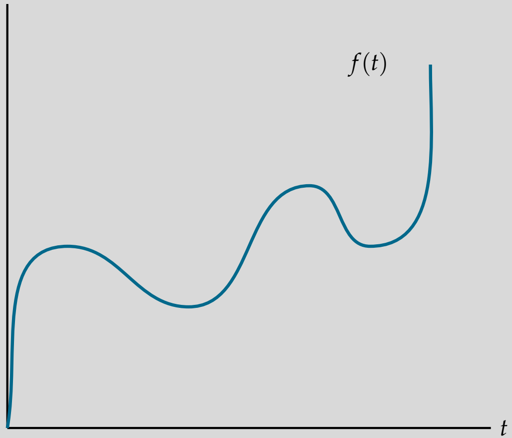
The Second Derivative Test

- If $f'(c) = 0$, and $f''(c) > 0$, then $x = c$ is a local minimum.
- If $f'(c) = 0$, and $f''(c) < 0$, then $x = c$ is a local maximum.

Using the Derivative

Examples

Find all critical points and inflection points of the following function



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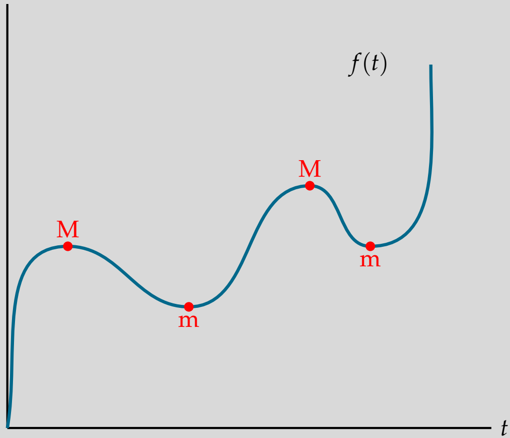

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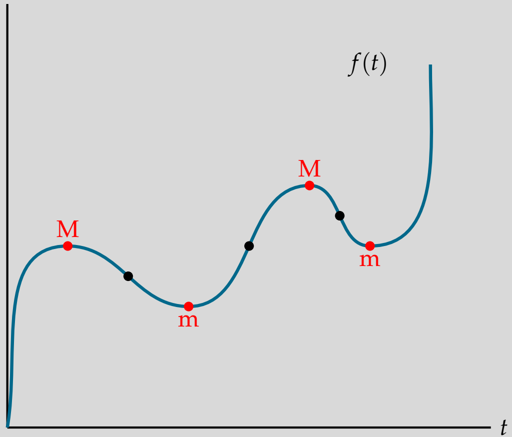

Using the Derivative

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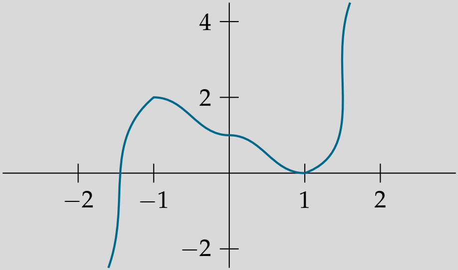
The figure below is a graph of $f'(x)$. Find the x–values that are critical points of the function f itself. Are they local maxima, local minima, or neither?



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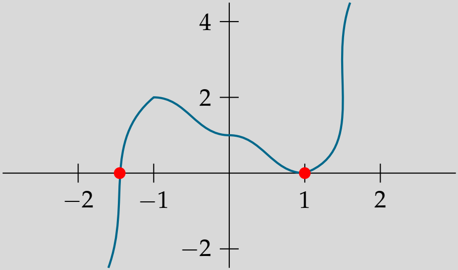

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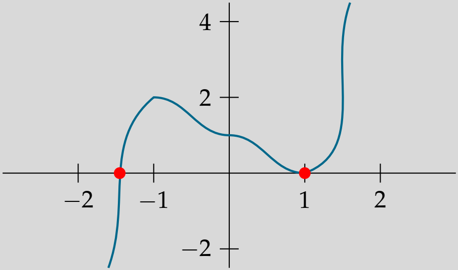

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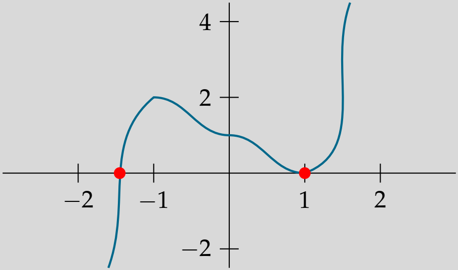

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- Another at $c = 1$. Note how $f'(x) > 0$ both before and after $c = 1$. This point is neither maximum nor minimum.

Using the Derivative

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The function $f(x) = x^4 - 7x^3 + 17x$ has a critical point at $x = 1$. Use the second derivative test to identify it as a local maximum or local minimum.

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Therefore, $x = 1$ is a local maximum of f.

Using the Derivative

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Use the first derivative to find all critical points, and use the second derivative to find all inflection points of the function

$$f(x) = 2x^3 + 3x^2 - 180x + 3.$$

Identify each critical point as a local maximum, a local minimum, or neither.

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We have $f'(x) = 6x^2 + 6x - 180$; therefore, the critical points are

$$x = \frac{-6 \pm \sqrt{36 - 4 \cdot 6 \cdot (-180)}}{12} = \{-6, 5\}$$

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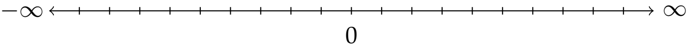

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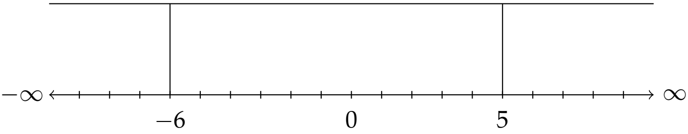

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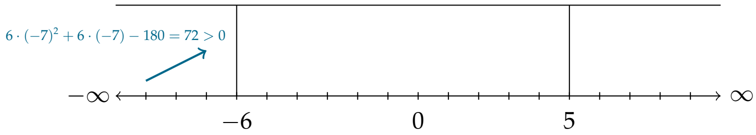

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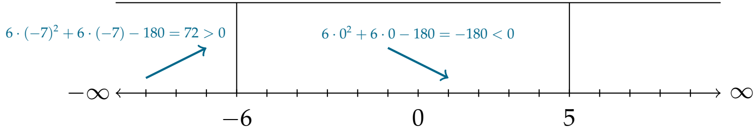

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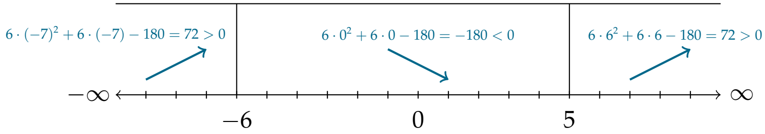

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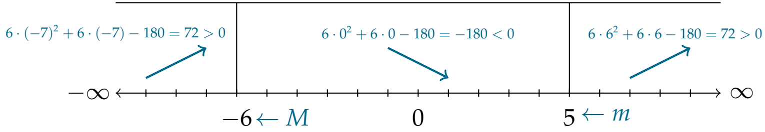

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To look for inflection points, we need to compute the second derivative:

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$$f''(x) = 0$$

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To look for inflection points, we need to compute the second derivative:

$$f(x) = 2x^3 + 3x^2 - 180x + 3, \quad f'(x) = 6x^2 + 6x - 180, \quad f''(x) = 12x + 6$$

The inflection points are among the zeros of the second derivative:

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Using the Derivative

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Use the first derivative to find all critical points, and use the second derivative to find all inflection points of the function

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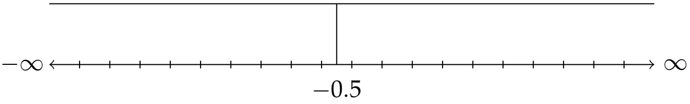

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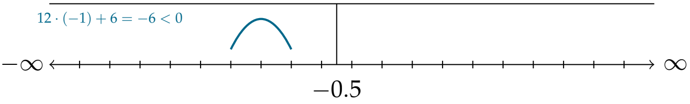

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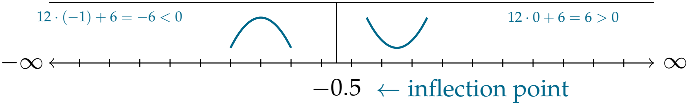

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$$\begin{cases} 6a + b = -34 \\ a = -12 \end{cases} \qquad \begin{cases} a = -12 \\ b = -34 + 6 \cdot 12 = 38 \end{cases}$$

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Find constant a and b so that the minimum of the parabola $f(x) = x^2 + ax + b$ is at the point $(6, 2)$.

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If $(6, 2)$ is to be a minimum, it should be $f''(6) > 0$. This is obviously satisfied. We are good.

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$$a(5 - 5\ln 5) = 8, \qquad a = \frac{8}{5 - 5\ln 5} \approx -2.625369982$$