Math 242 Test 3, Thursday 17 April

Name: **Last 4 digits of SSN:**

Show all work **clearly**, **make sentences**. No work means no credit. The points are:
ex1: 15, ex2: 35, ex3: 25, ex4: 25.

Exercise 1 (Laplace Transform of $\sin kt$ and $\cos kt$)

1. *a*) Consider the function $f(t) = \sin kt$. Write the differential equation satisfied by f (you will need to differentiate two times). What are the initial conditions ?
   b) Deduce the Laplace transform of $\sin kt$ (don't use the table of Laplace transform !!).

2. Use the Laplace transform of a derivate to deduce the Laplace transform of $\cos kt$.

Exercise 2
Find the inverse Laplace transform of the following functions:

$a)\ F(s) = \dfrac{3s+5}{s^2+4}$, $b)\ G(s) = \dfrac{3}{s^2(s^2+9)}$ (without partial fractions),

$c)\ H(s) = \dfrac{2s+9}{s^2-6s+36}$, $d)\ K(s) = \dfrac{2s-1}{s^2-s-12}$ (with partial fractions).

Exercise 3
Solve the initial value problem using the Laplace transform:

$$y'' + y = \cos(3t), \quad y(0) = 1, y'(0) = 0.$$

Exercise 4
Solve the initial value problem using the Laplace transform:

$$x^{(3)} + 9x' = 18, \qquad x(0) = x'(0) = 0 \text{ and } x''(0) = 9.$$