Math 242 Final Exam, Friday 12 December

Name: **Last 4 digits of SSN:**

Show all **work clearly**, **make sentences**. No work means no credit. The points are: ex1: 10, ex2: 10, ex3: 10, ex4: 15, ex5: 15, ex6: 15, ex7: 15, ex8: 10, ex9: 20, ex10: 15, ex11: 15 (Total=150 pts).

Exercise 1

1. We give a differential equation $y' = f(x, y)$. Write the algorithm of the Euler method. Apply this algorithm to find the first two (y_0 and y_1) value of an approximate solution of the differential equation

$$y' = 3x + 2y, \quad y(0) = 1,$$

and with step size $h = 0.5$.

2. Method of variation of parameters in the case $n = 2$:
We consider the second-order linear differential equation

$$y'' + P(x)y' + Q(x)y = f(x),$$

where P, Q and f are continuous. A general solution is given by:

$$y_c(x) = c_1y_1(x) + c_2y_2(x),$$

where c_1 and c_2 are constants.
What will be the form of a particular solution ? To find this solution, what system of equations, with unknown c_1' and c_2', do we have to solve ?

Exercise 2 Solve the initial value problem:

$$xy' = 2y + x^3 \cos x, \quad y(\pi) = 1.$$

Exercise 3 Find a general solution of the differential equation

$$3xy' = 1 + y^2.$$

Exercise 4 We consider the following differential equation:

$$xy' + 6y = 3xy^{4/3}.$$

1. What kind of equation is it?
2. What substitution do we have to do?
3. What differential equation do we obtain after the substitution?
4. Solve this last differential equation and then find the expression of y.

Exercise 5 Show that the differential equation

$$(1 + ye^{xy})\, dx + (2y + xe^{xy})dy = 0,$$

is exact and then solve it.

Exercise 6 We give the differential equation:

$$\frac{dx}{dt} = 6x - 2x^2.$$

1. What are the critical points ? Use a phase diagram to determine whether each critical point is stable or unstable.

2. Solve this differential equation with $x(0) = 1$.

Exercise 7 Give the form of a particular solution in each case, but do not determine the values of the coefficients:

1. $y^{(114)} + 59y' = x^3 + 46x - 13,$

2. $y^{(3)} + y'' - y' - y = (x^2 + 1)e^{-x},$

3. $y^{(3)} + y'' - y' - y = 5e^{4x}(13x^2 + 101x - 964)\cos(7x).$

Exercise 8 Find the form of a solution of the following differential equation

$$y^{(3)} - 5y'' + 8y' - 4y = 0.$$

Hint: 1 is a root of the characteristic equation.

Exercise 9 Solve the initial value problem without the Laplace transform:

$$y'' - 6y' + 8y = 24xe^{-2x}, \quad y(0) = 5/12,\ y'(0) = 13/6.$$

Exercise 10 1) Find the Laplace transform of the following functions:

$$f_1(t) = t\cos(2t), \quad f_2(t) = \frac{2\sin 3t}{t}.$$

We recall that $\lim_{x\to\infty} \arctan x = \pi/2$ and that $\int \frac{1}{a^2+x^2}dx = 1/a\arctan(x/a)$.
2) Find the inverse Laplace transform of:

$$F(s) = \frac{s+6}{s^2-10s+41}.$$

Exercise 11 Solve the initial value problem using the Laplace transform:

$$y'' - 5y' + 6y = -3e^{2x}, \qquad y(0) = -1,\ y'(0) = 2.$$