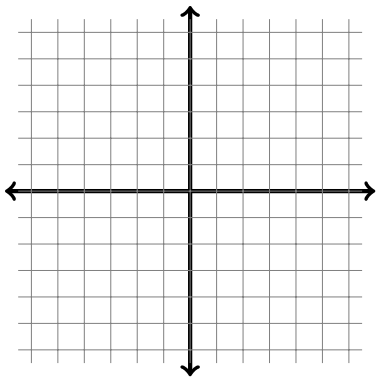
Name: _____

VIP ID: _____

- Write your name and VIP ID in the space provided above.
- The test has six (6) pages, including this one.
- The test is seventy-five (75) minutes long.
- Enter your answer in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	**Max. points**	**Your points**
2	20	
3	10	
4	10	
5	30	
6	30	
Total	100	

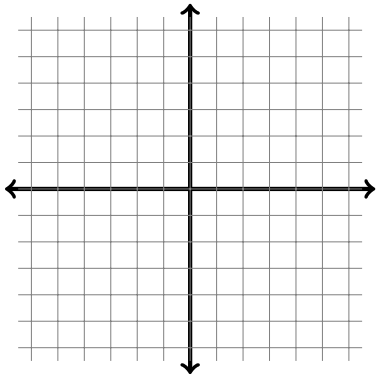
Problem 1 (10 pts)**.** Evaluate $\int_R (3x + 4y^2)\, dA$, where R is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. Sketch the region of integration.



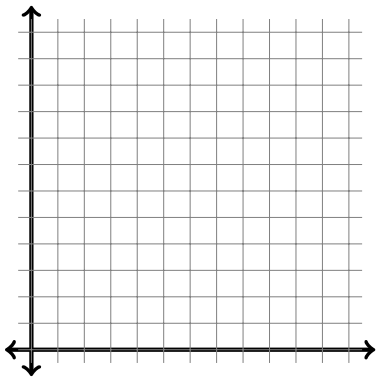
$$\int_R (3x + 4y^2)\, dA = \boxed{}$$

Problem 2 (10 pts)**.** Sketch the domain of integration, and convert the following integrals to polar coordinates. **Do not evaluate.**

$$\int_0^1 \int_0^y x\, dx\, dy = \boxed{}$$



Problem 3 (10 pts)**.** Evaluate the integral $\int_R \sin(y^2)\,dA$ where R is the triangle with vertices $(0,0)$, $(1,1)$ and $(0,1)$. Sketch the domain of integration.
Hint: The order of integration really matters for this problem.



$$\int_R \sin(y^2)\,dA = \boxed{}$$

Problem 4 (10 pts)**.** Use a double or a triple integral (your choice!) to compute the volume under the graph of $f(x, y) = xy$ and above the region bounded by $x = y^2$ and $x = y$.

$$V = \iint_D f(x, y)\, dA = \iiint_R dV = \boxed{}$$

Problem 5 (30 pts—10 pts each part)**.** Let's assume that we are using the spherical coordinates from the textbook: For $r \geq 0$, $0 \leq \phi \leq 2\pi$ and $0 \leq \psi \leq \pi$,

$$\begin{cases} x &= r \sin\psi \cos\phi \\ y &= r \sin\psi \sin\phi \\ z &= r \cos\psi \end{cases}$$

We want to compute the volume of the solid bounded below by the xy–plane, on the sides by the sphere $x^2 + y^2 + z^2 = 4$, and above by the cone $\psi = \pi/3$.

(a) Sketch the object described above to the best of your ability.

(b) Express the volume of the object as a triple integral in either cylindrical or spherical coordinates (your choice).

$$V(D) = \iiint_D \mathbf{1}\, dV = \boxed{}$$

(c) Evaluate that integral to obtain the volume of the object.

$$V(D) = \iiint_D \mathbf{1}\, dV = \boxed{}$$

Problem 6 (30 pts—10 pts each part)**.** Integrate the function $f(x, y, z) = 3xy$ on the solid bounded above by the paraboloid $z = 5 - x^2 - y^2$ and below by the paraboloid $z = 4x^2 + 4y^2$.

(a) Sketch the object to the best of your ability.

(b) Express as a triple integral in either cylindrical or spherical coordinates (your choice).

$$\iiint_D f(x, y, z)\, dV = \boxed{}$$

(c) Evaluate that integral to obtain the volume of the object.

$$\iiint_D f(x, y, z)\, dV = \boxed{}$$