Math 242 Test 2, Tuesday 1 April

Name: **Last 4 digits of SSN:**

Show all work **clearly**, **make sentences**. No work means no credit. The points are:
ex1: 15, ex2: 15, ex3: 15, ex4: 15, ex5: 10, ex6: 15, ex7: 20.

Exercise 1 We give the differential equation:

$$\frac{dx}{dt} = x^2 - 5x + 4.$$

1. What are the critical points ? Use a phase diagram to determine whether each critical point is stable or unstable.

2. Solve this differential equation with $x_0 = 2$.

Exercise 2 We give an initial value problem and its exact solution $y(x)$:

$$y' = -3x^2 y, \quad y(0) = 3, \quad y(x) = 3e^{-x^3}.$$

Apply Euler's method to approximate the solution on the interval $[0, 1]$ with step size $h = 0.25$. Write the formula you use for the computation. Then compare the four-decimal-place values of the approximate solution with the values of the exact solution using the following array. Does this step size look good ?

x	0	0.25	0.5	0.75	1
approx solution					
exact solution					

Exercise 3 Solve the differential equation:

$$y^{(3)} - 14y'' + 49y' - 36y = 0,$$

using the fact that the function $x \mapsto e^{9x}$ is solution of this differential equation. Then find the unique solution satisfying the initial conditions:

$$y(0) = 1, \ y'(0) = 0, \ y''(0) = 6.$$

Exercise 4 Give the form of a particular solution in each case, but do not determine the values of the coefficients:

1. $y^{(13)} + 2y' = (x^5 + x^2 + 1)$,
2. $y^{(3)} - y'' - y' + y = (x^3 - 2x)e^x$,
3. $y^{(3)} - y'' - y' + y = 5e^x(x^2 + 5x - 8)\cos(27x)$.

Exercise 5 Find a linear homogeneous constant-coefficient equation with the general solution:

$$y(x) = Ae^{-x} + B\cos(3x) + C\sin(3x) + x(D\cos(3x) + E\sin(3x)).$$

Exercise 6 Solve the differential equation :

$$y'' + 9y = 2\sec(3x).$$

To find a particular solution you will use the method of variation of parameters. We recall that $\sec x = \dfrac{1}{\cos x}$ and that $\displaystyle\int \tan(3x)dx = -\frac{1}{3}\ln|\cos(3x)|$.

Exercise 7 Solve the initial value problem:

$$y'' + 2y' - 8y = 8x - 6 + 12e^{2x}, \quad y(0) = -1,\ y'(0) = 7.$$