

Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has five (5) pages, including this one and the table of Laplace transforms at the end.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given at the right of each problem number.
- No books, notes or calculators may be used on this test.

Page	Max	Points
2	35	
3	40	
4	25	
Total	100	

Problem 1 (20 pts). Use techniques based on the Laplace transform to solve the initial value problem $y'' + 3y' + 2y = x$ that satisfies $y(0) = 0, y'(0) = 2$.

Problem 2 (15pts). Find the Laplace transform of $f(x) = \sin 3x \cos 3x$.

Problem 3 (10pts). Find the Laplace transform of the function $f(x) = xe^{2x} \cos(3x)$.

Problem 4 (10pts). Find the Laplace transform of $f(x) = \sin(x)/x$.

Problem 5 (20pts). Find the inverse Laplace transform of $F(s) = (s^2 + \beta^2)^{-2}$

Problem 6 (25pts). Given the system of differential equations below,

$$\begin{cases} x' = 4x - 3y \\ y' = 6x - 7y \end{cases}$$

and initial conditions $x(0) = 0$, $y(0) = 1$,

- (a) Use Euler's method to solve it numerically, with time step size $h = 1$, for $N = 3$ steps.
- (b) Solve the system using the method of elimination.

$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x) dx$	$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x) dx$
1	$\frac{1}{s} \quad s > 0$	$cf(x) \pm g(x)$	$cF(s) \pm G(s) \quad s > \max(a, b)$
x^p	$\frac{\Gamma(p+1)}{s^{p+1}} \quad s > 0$	$x^n f(x)$	$(-1)^n F^{(n)} \quad s > a$
x^n	$\frac{n!}{s^{n+1}} \quad s > 0$	$e^{\alpha x} f(x)$	$F(s - \alpha) \quad s > a + \alpha$
$e^{\alpha x}$	$\frac{1}{s - \alpha} \quad s > \alpha$	$\frac{f(x)}{x}$	$\int_s^\infty F(\sigma) d\sigma \quad s > a$
$\sin \beta x$	$\frac{\beta}{s^2 + \beta^2} \quad s > 0$	$f \star g$	$F(s)G(s) \quad s > \max(a, b)$
$\cos \beta x$	$\frac{s}{s^2 + \beta^2} \quad s > 0$	$f'(x)$	$sF(s) - f(0) \quad s > a$

You may use this as scratch paper. Do not detach from the rest of the test