

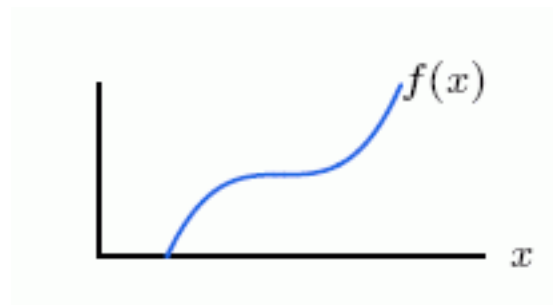
Math 122

Section 003/S03

Part Three: Applications to Derivatives

**Brendan Kelly, Critical Points**

- 1) How many critical points are there? How many are local maxima? How many are local minima?



- 2)  $f(x) = x^4 - 3x^3 + 17x$  Critical point at  $x=2$ . Use the Second Derivative test to identify it as a local max or min.

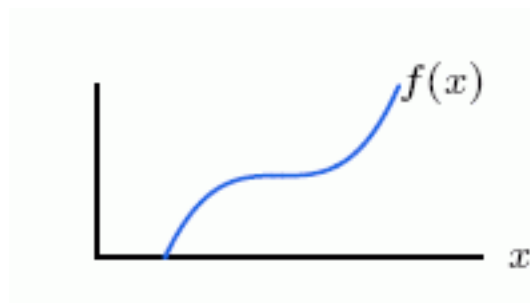
- 3) Use the first derivative to find all critical points.

$$f(x) = (4x^3) + (2x^2) - 90x + 6$$

Identify each critical point as a local maximum, minimum, or neither.

**Jeremy Kleinwaks, Inflection Points**

- 1) How many inflection points are there?



- 2) Use the second derivative test to find all inflection points of the function

$$f(x) = (2x^4) + (x^3) - (35x^2) + 6$$

- 3) Make a sign chart to decide whether this is an actual inflection point.

Find the inflection points of  $f(x) = (4x^3) - (9x^2) + 18$

### Jenn Colter, Logistic Growth

- 1) If  $t$  is the year since 2001, the one model for the population of the world. Population ( $P$ ), in billions, is  $P = 40 / (1 + 11e^{-.09t})$
- What does this model predict for the maximum substance population of the world?
  - Plot this function including  $P$ -intersect, inflection, and carrying concavity.
  - According to this model when will the population of the world reach 20 billion?

### Jenn Colter, Interpretations in terms of the Concavity / 2<sup>nd</sup> Derivative

- Use the second derivative test to find where the function  $f(x) = (x^6) - (24x^5) + 12$  is concave up.
- Given the same function  $f(x) = (x^6) - (24x^5) + 12$ . Use the second derivative to find the point of inflection.

### William Hansen, Global Max/Min

- 1) Find global max and min values of:  $f(x) = (1/3x^2) - (x^2) + 16$  over the interval  $[-1, 1]$ .
- 2) The demand curve for a product is given by  $q = 1100 - 6p^2$ . Find the price that maximizes revenue for sales of this product.
- 3) Plot the graph of  $f(x) = (5x^3) - (4e^x)$  over the interval  $[-1, 5]$ . Find all global max and min.

### **Jared Gulden, Maximizing Revenue**

- 1) At a price of \$80 for a half day trip, a white water rafting company attracts 300 customers. Every \$5 decreases in price attracts an additional 30 customers.
  - a. Find demand equation.
  - b. Show revenue as function of price.
  - c. What price will maximize revenue?

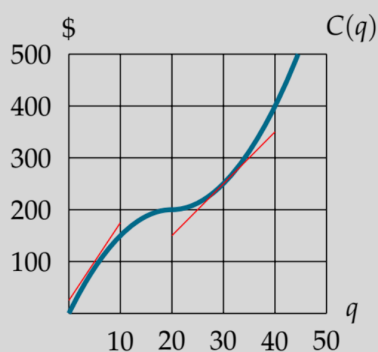
### **Samuel Hanna, Elasticity of Demand**

- 1) The demand curve for a product is given by  $q = 1000 - 2p^2$ , where  $p$  is the price. Find the elasticity of demand at  $p=10$  and  $p=15$ .
- 2) The demand of a product is given by  $p = 90 - 10q$ . Find the elasticity of demand when  $p=80$ .
- 3) Raising the price of a hotel room from \$75-\$80 per night reduces weekly sales from 100 rooms to 90 rooms.
  - a. Should we raise the price?
  - b. Complete (an approximation) to the elasticity of demand in this situation.

### Amanda Murphy, Finding Marginal Cost Functions

#### Example

In the figure below, is marginal cost greater at  $q = 5$  or at  $q = 30$ ?



- 1)
- 2) A company manufactures fuel tanks for automobiles. The total weekly cost (in dollars) of producing  $x$  tanks is given by  $C(x) = 10,000 + 90x - 0.05x^2$ .
  - a. Find the marginal cost function.
  - b. Find the marginal cost at a production level of 500 tanks per week and interpret the results.

- c. Find the exact cost of producing the 501<sup>st</sup> item.
- 3) For a company that sells kids' toys, the total cost of production  $x$  is given by the function
- $$C(x) = 2350 + 80x - 0.04x^2$$
- and that all  $x$  toys are sold when the price is equal to
- $$p(x) = -2x + 35$$
- a. Estimate the marginal cost of producing the 6<sup>th</sup> unit.
- b. Calculate the actual cost of producing the 6<sup>th</sup> unit.

### Jeffrey Hill, Relative Rate of Change

- 1) The annual production of peanuts in the world is represented by  $w = f(t)$ , in million tons, and is a function of  $t$  years since the start of the 1990.
- a. Interpret the statements  $f(15) = 545$  and  $f'(15) = 36$  in terms of peanut production.
- b. Calculate the Relative Rate of Change of  $w$  at  $t = 15$ ; interpret it in terms of peanut production.
- 2) Compute the Relative Rate of Change of the following functions:
- a.  $f(x) = 20x + 37$
- b.  $f(x) = 8x^2 + \sqrt{x}$
- c.  $f(x) = \ln(12x - 3)$
- d.  $f(x) = 7e^{4x}$

### Jeffrey Hill, Marginal Cost Functions

- 1) The cost of producing  $q$  items is given by  $C(q) = 1100 + 140q - 0.2q^2$ .

- a. Find the Marginal Cost Function
  - b. Find  $C(105)$  and  $MC(105)$ . Give units and explain what it means about cost of production.
- 2) Assume that  $C(q)$  and  $R(q)$  represent the cost and revenue in dollars, of producing  $q$  items.
- a. If  $C(100)=8600$  and  $MC(100)=48$ , estimate  $C(104)$ .
  - b. If  $MC(100)=48$  and  $MR(100)=70$ , approximately how much profit is earned by the 101<sup>st</sup> item?
- 3) Assume that  $C(q)$  and  $R(q)$  represent the cost and revenue in dollars, of producing  $(q)$  items.
- a. If  $C(400)=412,800$  and  $MC(400)=211$ , estimate  $C(402)$ .
  - b. If  $MC(400)=211$  and  $MR(400)=251$ , approximate how much profit is earned by the 401<sup>st</sup> item.

**Ryan Chenette, Marginal Cost and Revenue**

- 1) If  $C(35)=4200$  and  $MC(35)=6$ ,
- a. Estimate  $C(40)$
  - b. If  $MC(35)=24$  and  $MR(35)=40$ , approximately how much profit is earned by the 36<sup>th</sup> item?

- 2) An industrial car manufacturer process costs  $c(q)$  in billions to produce  $q$  million cars; these cars then sell for  $R(q)$  billion dollars. If  $C(4.0)=9$ ,  $R(4.0)=11$ ,  $MC(4.0)=3$ , and  $MR(4.0)=4$ , calculate:
- The profit producing 4 billion units.
  - The approximate change in revenue from 4.0 to 7.0 billion units.

**Nicole Bellows, Second Derivatives**

- Find the second derivative of the function  $f(x)=x^3+11x^2-52x$ .
- Use the second derivative test to determine whether there is a local maximum or minimum at  $x=1$  for the function  $f(x)=x^3-3x+10$ .