Name: _____

4-digit code: _____

- Write your name and the last 4 digits of your SSN in the space provided above.
- The test has sixteen (10) pages including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	Max	Points	Page	Max	Points
			6	5	
2	10		7	20	
3	10		8	10	
4	10		9	15	
5	10		10	10	
Total	40		**Total**	60	

Problem 1 (5 pts)**.** Find the area of the region that is enclosed between the curves $x = 2y^2$ and $x = 4 + y^2$.

$A =$ ____

Problem 2 (5 pts)**.** Find the volume of the solid that is obtained by rotating the region bounded by the curves $x = 2\sqrt{y}$, $x = 0$ and $y = 9$ about the y-axis.

$V =$ ____

Problem 3 (5 pts)**.** Use the Fundamental Theorem of Calculus to find the derivative of

$$f(x) = \int_{1-3x}^{1} \frac{u^3}{1+u^2}\, du.$$

$f'(x) =$ ____

Problem 4 (5 pts)**.** Find a positive value of k such that the average value of $f(x) = 2 + 6x - 3x^2$ over the interval $[0, k]$ is equal to 3.

$k =$ ____

Problem 5 (5 pts)**.** Use **integration by parts** to evaluate the integral $\int xe^{2x}\,dx$.

$$\int xe^{2x}\,dx = \boxed{}$$

Problem 6 (5 pts)**.** Evaluate the improper integral $\int_1^\infty \frac{dx}{x^3}$.

$$\int_1^\infty \frac{dx}{x^3} = \boxed{}$$

Problem 7 (10 pts)**.** Use a **trigonometric substitution** to evaluate the integral $\int \frac{dx}{\sqrt{x^2-9}}$.

$$\int \frac{dx}{\sqrt{x^2-9}} = \boxed{}$$

Problem 8 (5 pts)**.** Use **partial fractions** to evaluate the integral $\displaystyle\int \frac{dx}{x^2+x-2}$.

$$\int \frac{dx}{x^2+x-2} = \boxed{\phantom{\text{answer}}}$$

Problem 9 (10 pts)**.** Find a formula for the general term of the following sequences:

(a) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$

$x_n =$ [ ]

(b) $1 - \frac{1}{2}, \frac{1}{3} - \frac{1}{2}, \frac{1}{3} - \frac{1}{4}, \frac{1}{5} - \frac{1}{4}, \ldots$

$x_n =$ [ ]

Problem 10 (10pts)**.** Write out the first five terms of the sequence $\left\{ \frac{(-1)^{n+1}}{n^2} \right\}_{n=1}^{\infty}$

Determine whether the sequence converges, and if so find its limit.

First five terms: [ ]

$\lim_{n\to\infty} x_n =$ [ ]

Problem 11 (10 pts)**.** Determine whether the series converge, and if so find their sum:

(a) $\displaystyle\sum_{k=1}^{\infty}\Big(-\frac{3}{4}\Big)^{k-1}$

$$\sum_{k=1}^{\infty}\Big(-\frac{3}{4}\Big)^{k-1} = \boxed{}$$

(b) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)}$

$$\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)} = \boxed{}$$

Problem 12 (5 pts)**.** Apply the **divergence test** and state what it tells you about the series.

$$\sum_{k=1}^{\infty} \frac{k^2+k+3}{2k^2+1}.$$

Problem 13 (5 pts)**.** Use the **integral test** to determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{5k+2}$ converges.

Problem 14 (5 pts)**.** Use the **ratio test** to determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{4^k}{k^2}$ converges. If the test is inconclusive, then say so.

Problem 15 (5 pts)**.** Use the **root test** to determine whether the series $\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$ converges. If the test is inconclusive, then say so.

Problem 16 (5 pts)**.** Classify the series $\sum_{k=1}^{\infty} (-1)^k \frac{4k^2+1}{k^3+2}$ as absolutely convergent, convergent or divergent.