Name: _____

VIP ID: _____

- Write your name and VIP ID in the space provided above.
- The test has eight (8) pages, including this one, not counting the formula sheet attached at the end.
- You may remove the formula sheet as soon as the proctor instructs so.
- Credit for each problem is given in parentheses at the right of the problem number.
- You must show sufficient work to justify all answers except on multiple-choice questions. Correct answers with inconsistent or no work will not be given credit.
- No books or notes may be used on this test.
- No scratch paper is allowed. You may use the last page of this booklet for that purpose.
- An approved calculator may be used on this test.

Page	**Max. points**	**Your points**
2	16	
3	12	
4	28	
5	20	
6	12	
7	12	
Total	100	

Problem 1 (4 pts). The graph below is a representation of which of the following functions?

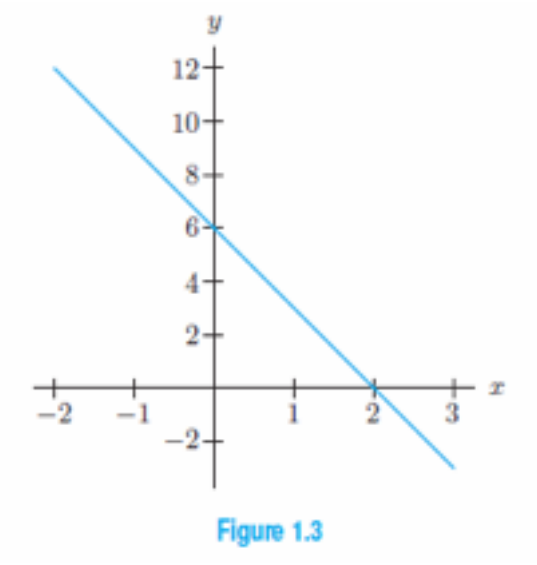

Figure 1.3

- ○ $y = 6x + 6$
- ○ $y = -3x + 6$
- ○ $y = -3x + 2$
- ○ $y = 6x - 2$

Problem 2 (4 pts). You are to receive three equal payments of \$2000 each, paid once per year starting now. You can assume a 5% interest rate, compounded continuously. The future value of the payments, on the day you receive the final payment, is:

- ○ $6000e^{0.05 \cdot 3}$
- ○ $6000e^{0.05 \cdot 2}$
- ○ $2000e^{0.05 \cdot 3} + 2000e^{0.05 \cdot 2} + 2000e^{0.05 \cdot 1}$
- ○ $2000e^{0.05 \cdot 2} + 2000e^{0.05 \cdot 1} + 2000$

Problem 3 (4 pts each). Evaluate the following integrals.

(a) $\displaystyle\int_0^4 \ln(y^2 + 1)\, dy =$

(b) $\displaystyle\int_{10}^{103} 9xe^{30x^2}\, dx =$

Problem 4 (4 pts). If the graph below is that of $f'(x)$, which of the following statements is true concerning the function $f(x)$?

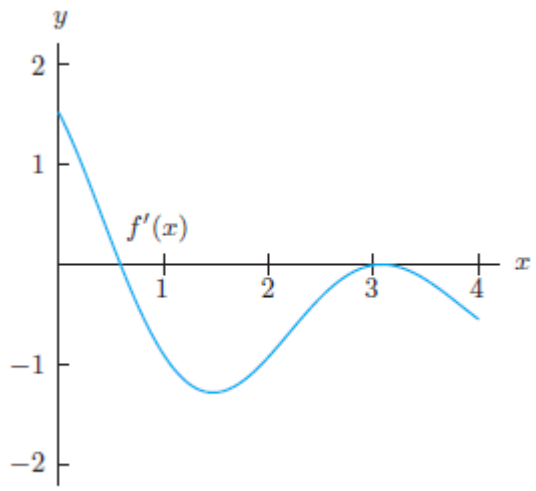

Figure 4.2

- ○ The derivative is zero at two values of x, both being local maxima.
- ○ The derivative is zero at two values of x, one is a local maximum, while the other is a local minimum.
- ○ The derivative is zero at two values of x, one is a local maximum on the interval, while the other is neither a local maximum nor a minimum.
- ○ The derivative is zero at two values of x, one is a local minimum on the interval, while the other is neither a local maximum nor a minimum.
- ○ The derivative is zero only at one value of x, where it is a local minimum.

Problem 5 (4 pts). Find all local max, min, and inflection points of $f(x) = 2x^3 + 3x^2 - 180x + 9$.

Problem 6 (4 pts). Find the global maximum and the global minimum of $f(x) = 2x^3 - 9x^2$ over the interval $-1 \leq x \leq 6$.

Problem 7 (4 pts each)**.** Find the derivative of the following functions:

(a) $f(x) = \sqrt{\dfrac{1}{x^{39}}}$

$f'(x) =$ ☐

(b) $y = 6t^5 - 10\sqrt{t} + \frac{9}{t}$

$y'(t) =$ ☐

(c) $f(x) = (2^x + x^5)(3 - \ln x)$

$f'(x) =$ ☐

(d) $f(x) = \dfrac{x^8 + 2}{x}$

$f'(x) =$ ☐

(e) $f(x) = \ln\left(8 - e^{-x}\right)$

$f'(x) =$ ☐

(f) $f(x) = \left(6 + \ln x\right)^{0.6}$

$f'(x) =$ ☐

(g) $f(x) = 2e^{7x} + e^{-x^6}$

$f'(x) =$ ☐

Problem 8 (4 pts each)**.** Compute the antiderivative of the following functions:

$\int x^5(5-3x^6)^{12}\,dx =$

$\int 6xe^{x^2}\,dx =$

$\int \frac{3x^2}{(8x^3-5)^3}dx =$

$\int \frac{dx}{x-4} =$

$\int 3x^2 4^{5x^3}\,dx =$

Problem 9 (4 pts)**.** The number of acres in a region cleared for farming follows the formula $A = f(t) = 2t^2$, where t is the number of months since the region started to be farmed, and t ranges from $t = 0$ to $t = 10$. Find the **average rate of change** in the number of acres cleared for farming between $t = 1$ and $t = 4$.

- ○ 10 acres/month
- ○ 30 acres
- ○ 10 months/acre
- ○ 30 months
- ○ 0.10 months/acre

Problem 10 (4 pts)**.** What is the **average value** of $f(x) = \sqrt{9 - x^2}$ over the interval $0 \leq x \leq 3$?

Problem 11 (4 pts)**.** For a product, the demand curve is $p = 53 - q^2$ and the supply curve is $p = 3 + q^2$, where q is quantity and p is price in dollars per unit. Find the **producer surplus** when the market is in equilibrium (round your anwer to the nearest dollar).

Problem 12 (4 pts)**.** If t is years since 1990, one model of the population of the worlds, P, in billions, is

$$P = \frac{40}{1 + 11e^{-0.08t}}$$

(a) What does this model predict for the maximum sustainable population of the world?

(b) According to this model, when will the earth's population reach 20 million?

Problem 13 (4 pts)**.** At a price of \$80 for a half-day trip, a white-water rafting company attracts 300 customers. Every \$5 decrease in price attracts an additional 30 customers. What price should the company charge per trip to maximize revenue?

Problem 14 (4 pts)**.** The marginal cost of drilling an oil well depends on the depth at which you are drilling; drilling becomes more expensive, per meter, as you dig deeper into the earth. The fixed costs are one million dollars and, if x is the depth in meters, the marginal costs are $MC(x) = 500 + 12x$ dollars per meter. Find the **total cost** of drilling a 400-meter well.