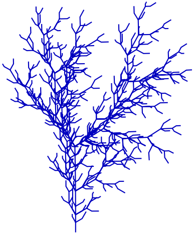
Lesson 3: Relative Change. Applications to Economics

Francisco Blanco-Silva

University of South Carolina



What do we know?

The General Program

- Functions
  - $x-$ and $y-$intercepts ($f(x) = 0, f(0)$)
  - Change from $x = a$ to $x = b$

    $$\Delta y = f(b) - f(a)$$

  - Average Rate of Change from $x = a$ to $x = b$

    $$ARC = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

What do we know?

The General Program

- Functions
  - $x-$ and $y-$intercepts $(f(x) = 0, f(0))$
  - Change from $x = a$ to $x = b$

$$\Delta y = f(b) - f(a)$$

  - Average Rate of Change from $x = a$ to $x = b$

$$ARC = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

- Kinds of functions:
  - Linear
    $f(x) = a + mx$

Relative Change

Given a function $y = f(x)$, the relative change between $x = a$ and $x = b$ is

$$RC = \frac{\Delta y}{f(a)} = \frac{f(b) - f(a)}{f(a)}.$$

It does not carry units. Instead, if we multiply it by 100, we may express it as a percentage.

Relative Change

Examples

Example

If the population increases by 1000 people, find the relative change in the population of:

- Coyote, NM (pop. 1,559)
- NYC (pop. 8,250,000)

Relative Change

Examples

Example

If the population increases by 1000 people, find the relative change in the population of:

- Coyote, NM (pop. 1,559)
- NYC (pop. 8,250,000)

For Coyote: $f(a) = 1559, f(b) = 1559 + 1000 = 2559$; therefore,

$$\text{RC} = \frac{f(b) - f(a)}{f(a)} = \frac{1000}{1559} \approx 0.6414, \text{ an increase of about } 64.14\%$$

Relative Change

Examples

Example

If the population increases by 1000 people, find the relative change in the population of:

- Coyote, NM (pop. 1,559)
- NYC (pop. 8,250,000)

For Coyote: $f(a) = 1559, f(b) = 1559 + 1000 = 2559$; therefore,

$$\text{RC} = \frac{f(b) - f(a)}{f(a)} = \frac{1000}{1559} \approx 0.6414, \text{ an increase of about } 64.14\%$$

For NYC:

$$\text{RC} = \frac{1000}{8250000} \approx 0.00012, \text{ a (very small!) increase of about } 0.012\%$$

Relative Change

Examples

Example

Find the relative change in the price of a \$75.99 pair of jeans, if the sale price is \$52.99.

Relative Change

Examples

Example

Find the relative change in the price of a \$75.99 pair of jeans, if the sale price is \$52.99.

$$\mathrm{RC} = \frac{f(b) - f(a)}{f(a)}$$

Relative Change

Examples

Example

Find the relative change in the price of a \$75.99 pair of jeans, if the sale price is \$52.99.

$$\mathrm{RC} = \frac{f(b) - f(a)}{f(a)} = \frac{52.99 - 75.99}{75.99} = \frac{-23}{75.99} \approx -0.303$$

Relative Change

Examples

Example

Find the relative change in the price of a \$75.99 pair of jeans, if the sale price is \$52.99.

$$\text{RC} = \frac{f(b) - f(a)}{f(a)} = \frac{52.99 - 75.99}{75.99} = \frac{-23}{75.99} \approx -0.303$$

The price has been reduced by 30% for this sale.

Applications to Economics

Some of the functions of interest to decision-making in a firm, industry of government are

- The Cost function.

$$\textit{Total Cost} = \textit{Fixed Cost} + \textit{Variable Cost}$$

Applications to Economics

Some of the functions of interest to decision-making in a firm, industry of government are

- The Cost function.

$$\textit{Total Cost} = \textit{Fixed Cost} + \textit{Variable Cost}$$

- The Revenue function.

$$\textit{Revenue} = \textit{Price} \times \textit{Quantity}$$

Applications to Economics

Some of the functions of interest to decision-making in a firm, industry of government are

- The Cost function.

$$\textit{Total Cost} = \textit{Fixed Cost} + \textit{Variable Cost}$$

- The Revenue function.

$$\textit{Revenue} = \textit{Price} \times \textit{Quantity}$$

- The Profit function.

$$\textit{Profit} = \textit{Revenue} - \textit{Cost}$$

Applications to Economics

Some of the functions of interest to decision-making in a firm, industry of government are

- The Cost function.

$$\textit{Total Cost} = \textit{Fixed Cost} + \textit{Variable Cost}$$

- The Revenue function.

$$\textit{Revenue} = \textit{Price} \times \textit{Quantity}$$

- The Profit function.

$$\textit{Profit} = \textit{Revenue} - \textit{Cost}$$

- The Depreciation function.

Applications to Economics

Some of the functions of interest to decision-making in a firm, industry of government are

- The Cost function.
$$\textit{Total Cost} = \textit{Fixed Cost} + \textit{Variable Cost}$$
- The Revenue function.
$$\textit{Revenue} = \textit{Price} \times \textit{Quantity}$$
- The Profit function.
$$\textit{Profit} = \textit{Revenue} - \textit{Cost}$$
- The Depreciation function.
- Supply and Demand curves. Equilibria.

Cost, Revenue, Profit

Examples

Example (Making smartphones)

The factory and machinery needed to begin production are fixed costs, which are incurred even if no phones are made. The cost of labor and materials are variable costs, since these quantities depend on how many are made. The fixed costs for this company are \$24,000 and the variable costs are \$37 per phone.

1. What is the Total Cost function for this company?
2. If phones sell for \$250 each, find the manufacturer's revenue function.
3. Sketch these functions, and identify their $y-$intercept and slope.
4. Find a formula for the profit function of the smartphone manufacturer. Find the *break-even point*.

Cost, Revenue, Profit

Examples

1. $C = f(q)$

   (q is the quantity, in number of items, C is the total cost, in dollars)

Cost, Revenue, Profit

Examples

1. $C = f(q) = \underbrace{24000}_{\text{fixed}} + \underbrace{37q}_{\text{variable}}$

   (q is the quantity, in number of items, C is the total cost, in dollars)

Cost, Revenue, Profit

Examples

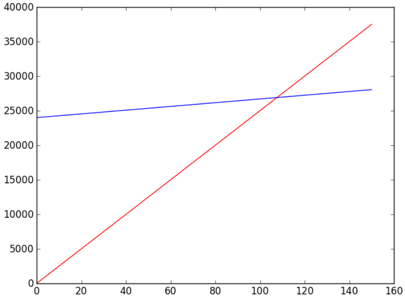
1. $C = f(q) = \underbrace{24000}_{\text{fixed}} + \underbrace{37q}_{\text{variable}}$
   (q is the quantity, in number of items, C is the total cost, in dollars)
2. The Revenue is $R = f(q) = 250q$.

Cost, Revenue, Profit

Examples

1. $C = f(q) = \underbrace{24000}_{\text{fixed}} + \underbrace{37q}_{\text{variable}}$

   (q is the quantity, in number of items, C is the total cost, in dollars)
2. The Revenue is $R = f(q) = 250q$.
3. The graph of the Cost function is a line with slope 37 and $y-$intercept $24,000$. The graph of the Revenue function is another line, with slope 250 and $y-$intercept equal to zero.



Cost, Revenue, Profit

Examples

4. The Profit is $\Pi = f(q) = \underbrace{250q}_{\text{Revenue}} - \underbrace{(24000 + 37q)}_{Cost}$

Cost, Revenue, Profit

Examples

4. The Profit is $\Pi = f(q) = \underbrace{250q}_{\text{Revenue}} - \underbrace{(24000 + 37q)}_{Cost} = 213q - 24000.$

Cost, Revenue, Profit

Examples

4. The Profit is $\Pi = f(q) = \underbrace{250q}_{\text{Revenue}} - \underbrace{(24000 + 37q)}_{Cost} = 213q - 24000.$

   The break-even point happens when Revenue equals Cost:

$$250q = 24000 + 37q$$

Cost, Revenue, Profit

Examples

4. The Profit is $\Pi = f(q) = \underbrace{250q}_{\text{Revenue}} - \underbrace{(24000 + 37q)}_{\textit{Cost}} = 213q - 24000.$

   The break-even point happens when Revenue equals Cost:

$$250q = 24000 + 37q$$

$$q = \frac{24000}{213} \approx 112 \text{ cellphones}$$

Depreciation

Examples

Example

Suppose that the smartphone manufacturer has a machine that costs \$20,000 and is sold ten years later for \$3,000.

> *We say that the value of the machine depreciates from \$20,000 to a resale value of \$3,000 in ten years.*

The depreciation formula gives the value $V = f(t)$ in dollars of the machine as a function of t in years. Assume that the value depreciates linearly. Find that function.

Depreciation

Examples

Example

Suppose that the smartphone manufacturer has a machine that costs \$20,000 and is sold ten years later for \$3,000.

> *We say that the value of the machine depreciates from \$20,000 to a resale value of \$3,000 in ten years.*

The depreciation formula gives the value $V = f(t)$ in dollars of the machine as a function of t in years. Assume that the value depreciates linearly. Find that function.

Hint: Find two points on this line by gathering info from the body of the problem: $(0, 20000)$ is one, $(10, 3000)$ is another. Note one is a $y-$intercept, and with the other you may compute the slope. Use the slope-intercept equation of a line.

DEPRECIATION

EXAMPLES

Example

Suppose that the smartphone manufacturer has a machine that costs \$20,000 and is sold ten years later for \$3,000.

> *We say that the value of the machine depreciates from \$20,000 to a resale value of \$3,000 in ten years.*

The depreciation formula gives the value $V = f(t)$ in dollars of the machine as a function of t in years. Assume that the value depreciates linearly. Find that function.

Hint: Find two points on this line by gathering info from the body of the problem: $(0, 20000)$ is one, $(10, 3000)$ is another. Note one is a $y-$intercept, and with the other you may compute the slope. Use the slope-intercept equation of a line.

$$V = f(t) = 20000 - 1700q$$

Applications to Economics

Supply and Demand

The quantity of an item q that is manufactured and sold depends on its price. Higher prices make manufacturers supply more, but consumers demand less.

- A supply curve relates the quantity of items that manufacturers are willing to make compared to the price per unit.
- A demand curve relates the quantity of items that buyers are willing to purchase compared to the price per unit.
- The equilibrium price and quantity, $(p^\star, q^\star)$, is found where both curves cross.

Applications to Economics

Supply and Demand

The quantity of an item q that is manufactured and sold depends on its price. Higher prices make manufacturers supply more, but consumers demand less.

- A supply curve relates the quantity of items that manufacturers are willing to make compared to the price per unit.
- A demand curve relates the quantity of items that buyers are willing to purchase compared to the price per unit.
- The equilibrium price and quantity, $(p^\star, q^\star)$, is found where both curves cross.

Hint: The higher the price, the more units would the manufacturer want to sell. The lower the price, the more units would the buyer want to purchase!

Supply and Demand

Examples

Example

One of the tables below represents a supply curve; the other represents a demand curve.

p	182	167	153	143	133	125	118
q	5	10	15	20	25	30	35

p	6	35	66	110	166	235	316
q	5	10	15	20	25	30	35

- Which one is which?
- At a price of $155, approximately how many items would consumers purchase?
- What would the price have to be if you wanted consumers to buy at least 20 items?

Supply and Demand

Examples

Example

One of the tables below represents a supply curve; the other represents a demand curve.

p	182	167	153	143	133	125	118
q	5	10	15	20	25	30	35

p	6	35	66	110	166	235	316
q	5	10	15	20	25	30	35

- Which one is which? TOP: demand
- At a price of \$155, approximately how many items would consumers purchase?
- What would the price have to be if you wanted consumers to buy at least 20 items?

Supply and Demand

Examples

Example

One of the tables below represents a supply curve; the other represents a demand curve.

p	182	167	153	143	133	125	118
q	5	10	15	20	25	30	35

p	6	35	66	110	166	235	316
q	5	10	15	20	25	30	35

- Which one is which? TOP: demand
- At a price of \$155, approximately how many items would consumers purchase? between 10 and 15 items
- What would the price have to be if you wanted consumers to buy at least 20 items?

Supply and Demand

Examples

Example

One of the tables below represents a supply curve; the other represents a demand curve.

p	182	167	153	143	133	125	118
q	5	10	15	20	25	30	35

p	6	35	66	110	166	235	316
q	5	10	15	20	25	30	35

- Which one is which? TOP: demand
- At a price of \$155, approximately how many items would consumers purchase? between 10 and 15 items
- What would the price have to be if you wanted consumers to buy at least 20 items? \$143