Name: ______________________________

VIP ID: ______________________________

- Write your name and VIP ID in the space provided above.
- The test has eleven (11) pages, including this one.
- Credit for each problem is given at the right of each problem number.
- Make sure to **box** your proofs, to differentiate them from your exploration and planning. I will only grade for boxed content on each submission.
- Make sure to provide only **direct proofs**. I will test your skill with other techniques in a different test.
- No books, notes or calculators are allowed.

Page	**Max**	**Points**	**Page**	**Max**	**Points**
2	10		7	10	
3	10		8	10	
4	10		9	10	
5	10		10	10	
6	10		11	10	
Total					

Problem 1 (10 pts)**.** Prove the following result:

Theorem. *If x and y are odd integers, then xy is an odd integer.*

Problem 2 (10 pts)**.** Prove the following result:

Theorem. *If b and c are odd integers and a is any integer, then $ab + ac$ is an even integer.*

Problem 3 (10 pts)**.** Prove the following result:

> **Theorem.** *If two integers have opposite parity, then their sum is odd.*

Problem 4 (10 pts)**.** Prove the following result:

Theorem. *Let x and y be positive numbers. If $x \leq y$, then $\sqrt{x} \leq \sqrt{y}$.*

Problem 5 (10 pts–5 pts each part)**.** Prove the following result:

Theorem. *If the equation* $ax^2 + bx + c = 0$ *has two different real-valued solutions, then*

(a) The sum of the two solutions is equal to $-b/a$*.*

(b) The product of the two solutions is equal to c/a*.*

Problem 6 (10 pts–1,1,4,4)**.** The first two steps will help you with the Theorem in this page.

(a) Apply polynomial division to compute $\dfrac{x^2-1}{x-1}$. Or if you prefer, simply *factor* x^2-1.

(b) Apply polynomial division to compute $\dfrac{x^3+1}{x+1}$. Or if you prefer, simply *factor* x^3+1.

(c) Prove the following result:

Theorem. *For each integer a, if 4 divides $a+1$, then 4 also divides a^3+1.*

(d) Prove the following result:

Theorem. *For each integer a, if 5 divides $a+2$, then 5 also divides $2a^3+7a^2+6a$.*

Problem 7 (10 pts)**.** Prove the following result:

> **Theorem.** *If the **greatest common divisor** of two natural numbers a, b is greater than 1, then $b \mid a$ or b is not prime.*

Problem 8 (10 pts)**.** Prove the following result:

Theorem. *If* $x \in \mathbb{R}$ *and* $0 < x < 3/2$, *then* $8x(3 - 2x) \leq 9$.

Problem 9 (10 pts–2.5 pts each part). The following steps will help you with the Theorem in Problem 10

(a) Sketch the region A of the plane given by the inequality $y > 2$. Write A in set-builder notation.

$A =$

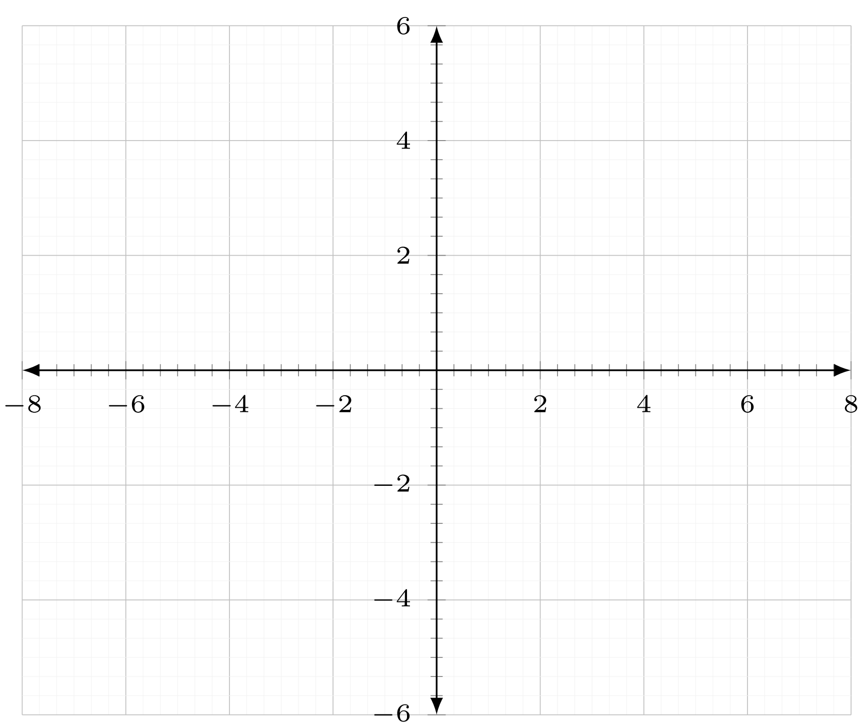
(b) Sketch the region B of the plane given by the inequality $x < -4$. Write B in set-builder notation. B.

$B =$

(c) Write down the formula for the distance d from any point (x, y) to the point $(1, -2)$. Draw the curves with implicit equation $(x-1)^2 + (y+2)^2 = R^2$, for $R = 2, 3, 4, 5, 6, 7$.

$d =$

(d) How far is the point $(1, -2)$ from the vertical line $x = -4$? How far is the point $(1, -2)$ from the horizontal line $y = 2$? What is the closest point from $A \cap B$ to the point $(1, -2)$?



Problem 10 (10 pts). Prove the following result:

Theorem. *If $x < -4$ and $y > 2$, then the distance from (x, y) to $(1, -2)$ is at least 6.*