Name: ______________________________

VIP ID: ______________________________

- Write your name and your VIP ID in the space provided above.
- The test has six (6) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	Max. points	Your points
2	20	
3	20	
4	30	
5	20	
6	10	
Total	100	

Sequences $\{x_n\}_{n=1}^{\infty} = \{f(n)\}_{n=1}^{\infty}$ $\lim_{n\to\infty} x_n = \cdots$

Partial sum sequences.

$S_N = x_1 + x_2 + \cdots + x_N$ $\{s_n\}_{n=1}^{\infty}$ $\lim_{n\to\infty} s_n = \cdots$

Series / Infinite sums.

$\lim_{n\to\infty} S_n = \sum_{n=1}^{\infty} x_n$.

Rules $\sum x_n, \sum y_n$ convergent

(1) $\sum x_n \pm c y_n = \sum x_n \pm c \sum y_n$

(2) $\sum_{n=a}^{\infty} x_{n+k} = \sum_{n=a+k}^{\infty} x_n$

(3) absolutely convergent / conditionally convergent / divergent.

Tricks

(1) zero-limit test aka. n-term test

(2) Integral test. $\boxed{x_n > 0}$

(3) Comparison test $\boxed{x_n > 0}$

(4) Alternating series test.

(5) Ratio test

(6) Root test

Examples.

(1) Alternating series. $\sum_{n=a}^{\infty} (-1)^n x_n$ $\boxed{x_n > 0}$

(2) geometric series $\sum_{n=0}^{\infty} r^n$ $|r| \geq 1$ diverge; $|r| < 1$ converge $\to \frac{1}{1-r}$

(3) p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $p > 1$ convergence; $p \leq 1$ divergence.

(4) telescopic.

Problem 1 (10 pts—5 pts each part)**.** Find a formula for the general term of the following sequences:

(a) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$

$x_n = \boxed{}$

(b) $1 - \frac{1}{2}, \frac{1}{3} - \frac{1}{2}, \frac{1}{3} - \frac{1}{4}, \frac{1}{5} - \frac{1}{4}, \ldots$

$x_n = \boxed{}$

Problem 2 (10 pts—5 pts each part)**.** Write out the first five terms of the sequence $\left\{ \frac{(-1)^{n+1}}{n^2} \right\}_{n=1}^{\infty}$ Determine whether the sequence converges, and if so find its limit.

First five terms: $\boxed{}$

$\lim_{n\to\infty} x_n = \boxed{}$

Problem 3 (20 pts—10 pts each)**.** Determine whether the series converge, and if so find their sum:

(a) $\displaystyle\sum_{n=1}^{\infty} 5\left(\frac{3}{4}\right)^{n-1}$

(**Hint:** This looks like a geometric series)

$$\sum_{n=1}^{\infty} 5\left(\frac{3}{4}\right)^{n-1} = \boxed{}$$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$

(**Hint:** This is a telescopic series. Partial fraction decomposition is your friend here)

$$\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} = \boxed{}$$

Problem 4 (10 pts)**.** Apply the **zero-limit test** (also known as the divergence, or the n-term test) and state what it tells you about the series.

$$\sum_{n=1}^{\infty} \frac{n^2+n+3}{2n^2+1}.$$

Problem 5 (10 pts)**.** Use the **integral test** to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n+2}$ converges.

Problem 6 (10 pts)**.** Use the **ratio test** to determine whether the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{4^n}{n^2}$ converges. If the test is inconclusive, then say so.

Problem 7 (10 pts)**.** Use the **root test** to determine whether the series $\sum_{n=1}^{\infty}\left(\frac{3n+2}{2n-1}\right)^n$ converges. If the test is inconclusive, then say so.

Problem 8 (10 pts)**.** Classify the series $\sum_{n=1}^{\infty}(-1)^n\,\frac{4n^2+1}{n!}$ as absolutely convergent, conditionally convergent, or divergent.

Problem 9 (10 pts)**.** Use any of the **comparison tests** to determine the convergence of the series

$$\sum_{n=0}^{\infty} \frac{3^n}{5^{n+1}+4}$$