Math 242 Final Exam, Saturday 3 May

Name: **Last 4 digits of SSN:**

Show all **work clearly**, **make sentences**. No work means no credit. The points are: ex1: 7, ex2: 7, ex3: 11, ex4: 11, ex5: 10, ex6: 15, ex7: 10, ex8: 9, ex9: 15, ex10: 15, ex11: 15 (Total=125 pts).

Exercise 1 Solve the initial value problem:

$$xy' + 3y = 3x^{-\frac{3}{2}}, \quad y(1) = 0.$$

Exercise 2 Find a general solution of the differential equation

$$y' = 1 + x + y + xy.$$

Exercise 3 We considere the following differential equation:

$$xy' = 6y + 12x^4y^{2/3}.$$

1. What kind of equation is it?

2. What substitution do we have to do?

3. What differential equation do we obtain after the substitution?

4. Solve this last differential equation and then find the expression of y.

Exercise 4 We consider the following differential equation:

$$y' = \frac{-3x^2 + 4y^2}{4xy}.$$

1. Write this differential equation as a homogeneous one.

2. Then solve this differential equation.

Exercise 5 Show that the differential equation

$$(1 + ye^{xy})\, dx + (2y + xe^{xy})dy = 0,$$

is exact and then solve it.

Exercise 6 We give the differential equation:

$$\frac{dx}{dt} = x^2 + 5x + 6.$$

1. What are the critical points ? Use a phase diagram to determine whether each critical point is stable or unstable.

2. Solve this differential equation with $x(0) = -4$.

Exercise 7 Solve the differential equation

$$y^{(3)} - 6y'' + 9y' - 54y = 0,$$

using the fact that the function $x \mapsto e^{6x}$ is solution of this differential equation. Then find the unique solution satisfying the initial conditions:

$$y(0) = 0,\ y'(0) = 3,\ y''(0) = 90.$$

Exercise 8 Give the form of a particular solution in each case, but do not determine the values of the coefficients:

1. $y^{(114)} + 59y' = x^4 + x^3 + x$,
2. $y^{(3)} + y'' - y' - y = (x^2 - 2)e^{-x}$,
3. $y^{(3)} + y'' - y' - y = 5e^{4x}(x^2 + 5x - 8)\cos(7x)$.

Exercise 9 Solve the initial value problem without the Laplace transform:

$$y'' - 2y' + 10y = 9xe^x, \quad y(0) = 2, \; y'(0) = 0.$$

Exercise 10 1) Find the Laplace transform of the following functions:

$$f_1(t) = t\sin(2t), \quad f_2(t) = \frac{\sin t}{t}.$$

We recall that $\lim_{x\to\infty} \arctan x = \pi/2$.
2) Find the inverse Laplace transform of:

$$F_1(s) = \frac{s+6}{s^2+4s+8}, \quad F_2(s) = \ln\left(\frac{s^2+1}{s^2+4}\right).$$

Exercise 11 Solve the initial value problem using the Laplace transform:

$$x^{(3)} - x'' - 8x' + 12x = 0, \qquad x(0) = -3,\ x'(0) = 22 \text{ and } x''(0) = -25.$$

Hint: 2 is a root of the characteristic equation.