Problem 1. State DeMorgan's Laws for set complements $(A \cup B)^{\complement}$ and $(A \cap B)^{\complement}$.

Solution: $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$, $(A \cap B)^{\complement} = A^{\complement} \cup B^{\complement}$. $\square$

Problem 2. Let $A = \{a, b\}$, $B = \{b, 1, 2\}$. Give the elements of $(A \times B) \setminus (A \times \{b\})$ by listing them within braces.

Solution: $(A \times B) \setminus (A \times \{b\}) = \big\{(a, 1), (a, 2), (b, 1), (b, 2)\big\}$ $\square$

Problem 3. For each $n \in \mathbb{N}$, let I_n be the closed interval $\big[-1 + \frac{1}{n}, 1 - \frac{1}{n}\big]$. Describe the set $\bigcup_{n \in \mathbb{N}} I_n$ in either interval or set-builder notation.

Solution: $\displaystyle\bigcup_{n \in \mathbb{N}} I_n = (-1, 1)$. $\square$

Problem 4. For each $n \in \mathbb{N}$, let J_n be the closed interval $\big[1 + \frac{1}{n}, 2 - \frac{1}{n}\big]$. Describe the set $\bigcup_{n=2}^{\infty} J_n$ in either interval or set-builder notation.

Solution: $\displaystyle\bigcup_{n=2}^{\infty} J_n = (1, 2)$. $\square$

Problem 5. Let $X = \{a, b, c, d\}$ and $S = \big\{Y \in \mathscr{P}(X) : b \notin Y, |Y| \le 2\big\}$. Give the elements of S.

Solution: $S = \big\{\emptyset, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}\big\}$. $\square$

Problem 6. Let P and Q be statements. Are the following statements equivalent? Justify your answers.

(a) $P \wedge (Q \vee \neg Q)$ and $(\neg P) \implies (Q \wedge \neg Q)$.

(b) $(\neg P) \wedge (P \implies Q)$ and $\neg(Q \vee P)$.

Solution: (a) They are equivalent.

P	Q	$Q \vee \neg Q$	$\neg P$	$Q \wedge \neg Q$	$P \vee (Q \vee \neg Q)$	$(\neg P) \implies (Q \wedge \neg Q)$
T	T	T	F	F	T	T
T	F	T	F	F	T	T
F	T	T	T	F	F	F
F	F	T	T	F	F	F

(b) They are not. If P is false and Q is true, then $\neg(Q \vee P)$ is false, and $(\neg P) \wedge (P \implies Q)$ is true. $\square$

Problem 7. Consider the following statement S:

"All foreign cars are well made."

Which of the following statements (there may be more than one) correctly negate S?

(a) "All foreign cars are badly made."

(b) "All domestic (non-foreign) cars are well made."

(c) "There are domestic (non-foreign) cars that are well made."

(d) "Some foreign cars are badly made."

(e) "If a car is not foreign, then it is not well made."

Solution: The statement (d) is the only one that correctly negates S. $\square$

Problem 8. Consider the following statement P:

$$\forall X \subset \mathbb{N}, \exists n \in \mathbb{Z}, |X| = n$$

(a) Rewrite $\neg P$ as an affirmative statement (i.e. the symbol $\neg$ should not appear anywhere)

(b) What is $\neg P$ saying in plain English? Is it true or false?

Solution: The negation of P can be written as follows:

$$\exists X \subset \mathbb{N}, \forall n \in \mathbb{Z}, |X| \neq n$$

In plain English, this statement indicates that there is a subset of the natural numbers whose cardinality is not an integer (in other words, that there exist infinite subsets of the natural numbers). This is clearly true. $\square$

Problem 9. Consider the following statement R:

"An integer n is divisible by 15 only if it is divisible by 5."

(a) Rewrite R in the form $P \implies Q$.

(b) Use the word *necessary* or *sufficient* as appropriate:

"For an integer n to be divisible by 5 it is __________ that n be divisible by 15."

(c) Use the word *necessary* or *sufficient* as appropriate:

"For an integer n to be divisible by 15 it is __________ that n be divisible by 5."

(d) State the converse of R.

(e) State the contrapositive of R.

Solution: (a) $P(n) \implies Q(n)$, where $P(n)$ and $Q(n)$ are respectively "15 divides n" and "5 divides n."

(b) "For an integer n to be divisible by 5 it is *sufficient* that n be divisible by 15."

(c) "For an integer n to be divisible by 15 it is *necessary* that n be divisible by 5."

(d) $Q(n) \implies P(n)$: "An integer n is divisible by 5 only if it is divisible by 15."

(e) $\neg Q(n) \implies \neg P(n)$: "If an integer is not divisible by 5, then it is not divisible by 15." $\square$

Problem 10. Let $A = [-1, 0) \cup (0, 1]$, and consider $U = \mathbb{R}$ as the universal set. Describe the set $A^{\complement}$.

Solution: $A^{\complement} = (-\infty, -1) \cup \{0\} \cup (1, \infty)$. $\square$