

Name: _____

VIP ID: _____

- Write your name and your VIP ID in the space provided above.
- The test has eight (8) pages, including this one and one page of scratch paper at the end with a table of Laplace transforms.
- **Do not answer** any problem in the scratch paper. All solutions must be provided on pages 2–7 where it proceeds.
- Show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given at the right of each problem number.

Page	Max	Points
2	15	
3	15	
4	20	
5	20	
6	10	
7	20	
Total	100	

Problem 1 (15 pts—5 pts each part). A body with mass 0.5 kg is attached to the end of a spring that is stretched 2 m by a force of 100 N. It is set in motion one meter to the right, and moving to the left at that time with an initial velocity of 5 m/s.

(a) Find the position function of the body.

$$x(t) =$$

(b) Indicate the amplitude, frequency, period of oscillation and time lag of this motion.

Amplitude:

Frequency:

Period:

Time lag:

(c) Sketch the solution curve. Make sure to label all relevant information (amplitude, time lag and period).

(b) Sketch the solution curve for $t \in [0.20]$. Indicate clearly how far the mass moves to the right before starting back toward the origin (show all necessary work to find this value)

Problem 4 (20 pts—10 pts each). Consider an undamped forced motion with equation

$$x'' + 9x = 32 \cos 5t.$$

(a) Assume $m = 1$. Find the values of k , F_0 and ω .

$$k = \quad F_0 = \quad \omega =$$

(b) Find $x(t)$ if $x(0) = x'(0) = 0$. Sketch the motion for $t \in [0, 2\pi]$.

$x(t) =$

Problem 5 (10 pts). Consider a damped forced motion with equation

$$x'' + 2x' + 9x = 20 \cos 5t.$$

Find $x(t)$ if $x(0) = x'(0) = 0$. Sketch the motion for $t \in [0, 2\pi]$.

$x(t) =$

Problem 6 (20 pts—10 pts each). Consider an undamped forced motion with equation

$$x'' + 25x = 30 \cos 5t.$$

- (a) Assume $m = 1$. Find the values of k , F_0 and ω .

$$k = \quad F_0 = \quad \omega =$$

- (b) Find $x(t)$ if $x(0) = x'(0) = 0$. Sketch the motion for $t \in [0, 2\pi]$.

$$x(t) =$$

$f(x)$	$\mathcal{L}\{f\} = \int_0^\infty e^{-sx} f(x) dx$		
1	$\frac{1}{s} \quad s > 0$	$cf(x) \pm g(x)$	$cF(s) \pm G(s) \quad s > \max(a, b)$
x^n	$\frac{n!}{s^{n+1}} \quad s > 0$	$e^{\alpha x} f(x)$	$F(s - \alpha) \quad s > a + \alpha$
$e^{\alpha x}$	$\frac{1}{s - \alpha} \quad s > \alpha$	$x^n f(x)$	$(-1)^n F^{(n)}(s) \quad s > a$
$\sin \beta x$	$\frac{\beta}{s^2 + \beta^2} \quad s > 0$	$f'(x)$	$sF(s) - f(0)$
$\cos \beta x$	$\frac{s}{s^2 + \beta^2} \quad s > 0$	$f''(x)$	$s^2 F(s) - sf(0) - f'(0)$