Lesson 9: Rules of Differentiation—Power functions and Polynomials

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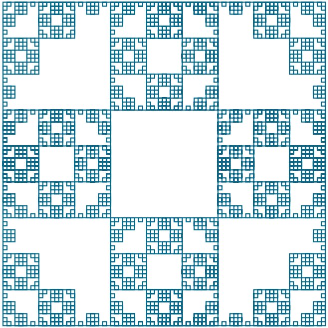

What do we know?

- Functions
  - $x-$ and $y-$intercepts $(f(x)=0, f(0))$
  - Change from $x=a$ to $x=b$
    $$\Delta y = f(b) - f(a)$$
  - Average Rate of Change from $x=a$ to $x=b$
    $$ARC = \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$
  - Relative Change from $x=a$ to $x=b$
    $$RC = \frac{\Delta y}{f(a)} = \frac{f(b)-f(a)}{f(a)}$$
  - Instantaneous Rate of Change at $x=a$
    $$f'(a)$$
- Linear Functions: $f(x) = b + mx$
- Exponential Functions $P_0 a^t = P_0(1+r)^t = P_0 e^{kt}$
- Power Functions
  $$kx^p$$
- Polynomials $a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

The Derivative as a Function

In the previous lesson we have seen that the derivative of a function $y = f(x)$ can be seen as a function itself: One that assigns, to each x, the value of the slope of the tangent line at the graph of f at the point $\big(x, f(x)\big)$.
We denote this new function in two ways

$$f'(x) = \underbrace{\frac{df}{dx}}_{\text{Leibnitz notation}} \quad \begin{array}{l} \longleftarrow \text{derivative of } f \\ \longleftarrow \text{with respect to the variable } x \end{array}$$

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$$\frac{dP}{dH} \quad \begin{matrix} \longleftarrow \text{dollars} \\ \longleftarrow \text{hours} \end{matrix}$$

Therefore, the units are dollars/hour.

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An economist is interested in how the price of a certain item affects its sales. At a price of p dollars, a quantity q of the item is sold.

$$q = f(p)$$

- What does it mean $f(160) = 2050$?
- What does it mean $f'(160) = -25$?
- What does it mean $f'(30) = 49$?

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  If the price goes up from \$160 by \$1 per item, about 25 fewer items will be sold.
- What does it mean $f'(30) = 49$?
  If the price goes up from \$30 by \$1 per item, about 49 more items will be sold.

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The average value of corn production in Kenya, in Kenyan shillings, of the yearly maize production from an average plot of land is a function $y = f(x)$ of the quantity x of fertilizer used (in kilograms).

- Interpret the statements $f(5) = 11,500$ and $f'(5) = 350$.
- Use the statements to estimate $f(6)$ and $f(10)$.
- Which estimate in the previous part is more reliable?

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- $f(6) \approx 11,850$KSh

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D8 For any $a > 0$, the derivative of $f(x) = a^x$ is $f'(x) = a^x \ln a$.

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$$f(x) = x$$
$$f(x) = x + \pi$$
$$f(x) = 200x$$
$$f(x) = 45 - 5x$$
$$f(x) = x^5$$
$$f(x) = 45x^7$$
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$f(x) = 2\pi - x^{3/2}$ $\quad f'(x) = 0 - \frac{3}{2}x^{3/2-1} = -\frac{3}{2}x^{1/2}$

$f(x) = 3\sqrt{x} = 3x^{1/2}$ $\quad f'(x) = 3 \cdot \frac{1}{2}x^{1/2-1} = \frac{3}{2}x^{-1/2}$

$f(x) = \frac{1}{\sqrt{x^7}} = x^{-7/2}$ $\quad f'(x) = -\frac{7}{2}x^{-7/2-1} = -\frac{7}{2}x^{-9/2}$

$f(x) = e^x$

$f(x) = 2^x$

Rules of Differentiation

Basic Examples

Find the derivative of the following functions:

$$f(x) = 5 \qquad f'(x) = 0$$

$$f(x) = x \qquad f'(x) = 1$$

$$f(x) = x + \pi \qquad f'(x) = 1 + 0 = 1$$

$$f(x) = 200x \qquad f'(x) = 200 \cdot 1 = 200$$

$$f(x) = 45 - 5x \qquad f'(x) = 0 - 5 \cdot 1 = -5$$

$$f(x) = x^5 \qquad f'(x) = 5x^{5-1} = 5x^4$$

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Rules of Differentiation

Basic Examples

Find the derivative of the following functions:

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\begin{array}{ll}
f(x) = 5 & f'(x) = 0 \\
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f(x) = 3\sqrt{x} = 3x^{1/2} & f'(x) = 3 \cdot \frac{1}{2}x^{1/2-1} = \frac{3}{2}x^{-1/2} \\
f(x) = \dfrac{1}{\sqrt{x^7}} = x^{-7/2} & f'(x) = -\frac{7}{2}x^{-7/2-1} = -\frac{7}{2}x^{-9/2} \\
f(x) = e^x & f'(x) = e^x \\
f(x) = 2^x & f'(x) = 2^x \ln 2
\end{array}
$$

Rules of Differentiation

Advanced Examples

Find the derivative of the following functions:

$$f(t) = t^2 - 3t^6 + 5e^t$$

$$h(t) = (t - 3t^2)(\sqrt{t} + 4)$$

$$h(t) = \frac{4 + \sqrt{t}}{t^5}$$

Rules of Differentiation

Advanced Examples

Find the derivative of the following functions:

$f(t) = t^2 - 3t^6 + 5e^t$ $\quad f'(t) = 2t^{2-1} - 3 \cdot 6t^{6-1} + 5 \cdot e^t$

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$$h(t) = \frac{4 + \sqrt{t}}{t^5}$$

Rules of Differentiation

Advanced Examples

Find the derivative of the following functions:

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Rules of Differentiation

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Rules of Differentiation

Advanced Examples

Find the derivative of the following functions:

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Rules of Differentiation

Advanced Examples

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$$= \frac{4}{t^5} + \frac{t^{1/2}}{t^5}$$

Rules of Differentiation

Advanced Examples

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Rules of Differentiation

Advanced Examples

Find the derivative of the following functions:

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Rules of Differentiation

Advanced Examples

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$$\begin{aligned} h'(t) &= 4 \cdot (-5)t^{-5-1} - \tfrac{9}{2}t^{-9/2-1} \\ &= -20t^{-6} - \tfrac{9}{2}t^{-11/2} \end{aligned}$$