$y - y_1 = m(x - x_1)$ $y = a + mx$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ | $a^{x+y} = a^x \cdot a^y$ | $a^{x-y} = a^x / a^y$ | $a^{xy} = (a^x)^y = (a^y)^x$ | $ab^x = a^x \cdot b^x$

$P = P_0 a^t = P_0 (1+r)^t$
$P_0$ = initial quantity (compounded anually)

compounded continu... $P = P_0 e^{rt}$ ; $r = a - 1$ ; $a = 1 + r$

$k = \ln(a)$ ; $a = e^k$ ; $r = e^k - 1$ ; $k = \ln(1+r)$

r is always greater than k!

vertical operations
$f(x) + c$ $(c > 0)$ ↑ v shift up
$f(x) - c$ $(c > 0)$ ↓ v shift down
$af(x)$ $(a > 1)$ v stretch by a
$af(x)$ $(0 < a < 1)$ v compression by $(1/a)$
$-f(x)$ reflection w/ respect to x axis

horizontal oper.
$f(x+c)$ $(c > 0)$ h shift left
$f(x-c)$ $(c > 0)$ h shift right
$f(ax)$ $(a > 1)$ h compress. by $\frac{1}{a}$
$f(ax)$ $(0 < a < 1)$ h stretch by a
$f(-x)$ reflection w/ respect to y axis

change $\Delta y = f(b) - f(a)$ (units = dep. var)
Average ROC $= \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$ (units = dep / ind)
rel. change $= \frac{\Delta y}{f(a)} = \frac{f(b) - f(a)}{f(a)} \times 100\%$ no units

$\ln(ab) = \ln a + \ln b$ | $\ln(a/b) = \ln a - \ln b$
$\ln(a^r) = r \ln a$ | $\ln(e^r) = r$ $e^{\ln r} = r$
$\ln 1 = 0$ | $\ln e = 1$ | $\ln(x)$ undefined $x \le 0$

Doubling time $T = \frac{\ln(2)}{k}$

quad: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\ln(x) = c$ ; $x = e^c$

power function $Q(x) = kx^p$ — power; constant of proportion

polynomials $y = a_n x^n$
$a_n$ = leading coefficient
n = degree

input = independent var
output = dependent

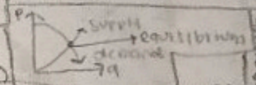
$y = f(x)$ dep ind

y intercept → $f(0)$
x intercept → $f(x) = 0$

$P = P_0 a^t = P_0 (1+r)^t$
$P_0$ = initial quantity
a = factor by which P changes when t increases by 1.
a > 0 expon. growth $0 < a < 1$ exponential decay

r < 0 expo. decay
r > 0 expon. growth

Cost = fixed cost + variable cost
revenue = price x quantity
Profit = Revenue − cost
Breakeven = (Revenue = cost)




half life $= T = \frac{-\ln(2)}{k}$

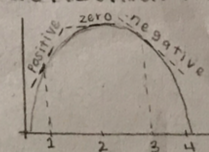
contin. for continuous growth/decay $P = P_0 e^{kt}$

---

D1 : $f(x) = c$ ; $f'(x) = 0$
D2 : $f(x) = x$ ; $f'(x) = 1$
D3 : $h(x) = f(x) + g(x)$ ; $h'(x) = f'(x) + g'(x)$
D4 : $h(x) = f(x) - g(x)$ ; $h'(x) = f'(x) - g'(x)$
D5 : $h(x) = c \cdot f(x)$ ; $h'(x) = c \cdot f'(x)$
D6 : $f(x) = x^n$ ; $f'(x) = nx^{n-1}$
D7 : $f(x) = e^x$ ; $f'(x) = e^x$
D8 : for any $a > 0$. $f(x) = a^x$ ; $f'(x) = a^x \ln a$
D9 : $f(x) = \ln x$ ; $f'(x) = \frac{1}{x}$

D10 : $f(x) = g(x)^n$ ; $f'(x) = ng(x)^{n-1} g'(x)$
chain rules $f(x) = e^{g(x)}$ ; $f'(x) = g'(x) e^{g(x)}$
$f(x) = a^{g(x)}$ ; $f'(x) = g'(x) a^{g(x)} \ln a$
$f(x) = \ln g(x)$ ; $f'(x) = \frac{g'(x)}{g(x)}$

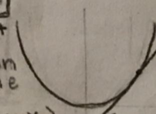
the slope of a horizontal line is ∅.




D11 : $h(x) = f(x) \cdot g(x)$ ; $h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

D12 : $h(x) = \frac{f(x)}{g(x)}$ ; $h'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$

use point slope equation of a line
$y - y_0 = m(x - x_0)$
to find tangent line of a point



Leibnitz notation: $f'(x) = \frac{df}{dx}$ ← derivitive of f ← with respect to x

---

AROC $= \frac{f(b) - f(a)}{b - a}$

relative change $= \frac{f(b) - f(a)}{f(a)}$

instantanious rate of change $= f'(x)$

marginal cost: $C'(q)$
marginal revenue: $R'(q)$

RROC $= \frac{f'(a)}{f(a)}$

1st Derivative test (critical points)
$f'(x) > 0$ = inc @ x
$f'(x) < 0$ = dec @ x
$f'(x) = 0$ = stationary @ x

2nd Derivative Test (inflection points)
$f''(x) > 0$ = concave up @ x (∪)
$f''(x) < 0$ = concave down @ x (∩)
$f''(x) = 0$ = might change @ x

1st derivative on graph. neg → pos = min ; pos → neg = max

Point slope form
$y - y_1 = m(x - x_1)$

finding slope
$\frac{y_2 - y_1}{x_2 - x_1}$

Elasticity of demand
$E = \left| \frac{p}{q} (f'(p)) \right|$

Logistic Growth
$P = f(t) = \frac{L}{1 + Ce^{-kt}}$
(L = carrying capacity)

Global max = highest point on graph
Global min = lowest on graph

Steps for global max/min:
① canidates (find critical points & include intervals)
② Evaluate
③ chose largest & smallest

How to find a tangent line:
find an (x, y), find m (slope) + plug into point slope.
derivative

---

A1 = add a constant if you need to
A2 $= \int f(x) + \int g(x) \to F(x) + G(x)$
A3 $= \int (f(x) - g(x))dx = \int f(x)dx - \int g(x)dx$
A4 = antiderivative of $cF(x)$ is $cF(x)$, $\int cF(x)dx = c \int F(x)dx$
A5 $= \int 1 dx = x$ , $\int dx = x$
A6 $= \int x^n dx = x^{\frac{n+1}{n+1}}$ , $(n \neq -1)$
A7 $= \int e^x dx = e^x$
A8 $= \int a^x dx = \frac{a^x}{\ln a}$ $(a > 0)$
A9 $= \int \frac{1}{x} dx = \ln|x|$ , $\int \frac{dx}{x} = \ln|x|$ , $\int x^{-1} dx = \ln|x|$

A10 $= \int w(x)^n \cdot w'(x) dx = \frac{w(x)^{n+1}}{n+1}$ , $(n \neq -1)$
A11 $= \int a^{f(x)} \cdot f'(x) dx = \frac{a^{f(x)}}{\ln a} + c$ $(a > 0)$
A12 $= \int \frac{F'(x)}{F(x)} dx = \ln|F(x)| + c$

Total area of function $y = f(x)$ between [a, b] is equal to the sum of the 2 integrals in the absolute value of each integral

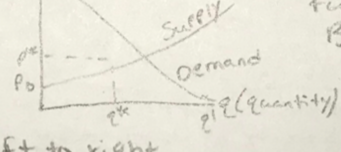
When graphing, $F'(x) = f(x)$
1) If $f > 0$ on an interval, then $F(x)$ is increasing on that interval
2) If $f < 0$ on an interval, then $F(x)$ is decreasing on that interval
3) If $F(x) = 0$ on an interval, then $F(x)$ is a constant on that interval

concavity, $F''(x) = f'(x)$
1) If $f' > 0$ on an interval, then f is increasing, and the graph of F is concave up 2. If $f' < 0$ on an interval, then f is decreasing, F concave down

---

- Variable cost = Items x price per item
- Average of a function: $\frac{1}{b-a} \int_a^b f(x)dx$
- Total change of a rate of change btw x = a and x = b : $\int_a^b |f(x)| dx$
- Logistic growth: $p = f(t) = \frac{L}{1 + Ce^{-kt}}$
- x = quantity and y = price
- cost = fixed cost + variable cost
- future value = (present value)$(e^{rM})$
- present value $= \int_0^m S(t)e^{-rt} dt$
- ReinMan sum = base x height for left to right
- consumer surplus $= \int_0^x f(x) - y dx$ = Area under demand curve from 0 to q

consumer surplus = Demand curve − y ; Producer surplus = y − supply curve




- Elasticity of Demand:
$E = \left| \left( \frac{\text{Percent } \Delta \text{ in price}}{\text{Percent } \Delta \text{ in demand}} \right) \right| = \left| \left( \frac{\Delta q}{q} \right) / \left( \frac{\Delta p}{p} \right) \right| = \left| \frac{\Delta q}{q} \cdot \frac{p}{\Delta p} \right| = \left| \frac{p}{q} \cdot \frac{\Delta q}{\Delta p} \right|$
- present and future
$B = Pe^{rt}$
$PV = \frac{P_0}{e^{rt}}$
- FTC: $\int_a^b f(x)dx = F(b) - F(a)$
- producer surplus → $\int_0^x y - f(x) dx$
- Smaller E in elasisity the better

You recieve 3 equal payments of $2000 each, paid once per year starting now. You can assume a 5% interest rate is compounded continuously. The future value of the payments on the day you recieve your final payment is...

$2000e^{0.05\cdot 2} + 2000e^{0.05\cdot 1} + 2000$

The amt (A) in mg of a drug is 25 when it first enters the system decreases by 12% an hour. A possible formula for this after t hours is...

$A = 25(1-.12)^t$ or $A = 25(.88)^t$

The avg ROC of a herb in the US during 1997–2001 is 9.5 million dollars a year. That means that during those years: The sales increased by an average of 9.5 million dollars each year

Converting the function $P = 750e^{0.04t}$ to the form $P = Pa^t$ gives...

$e^{0.04} = 1.0408 \rightarrow P = 750(1.0408)^t$

$C(500) = 10000 + 8(500)$ $R(9) = 12(9)$

a) Find the cost of $C(500) = 10000 + 8(500) = 10000$ dollars

b) Find the revenue of $R(500) = 12(500) = 6000$ dollars

c) Does the company make money? No

d) What is the breakeven?

$10000 + 8q = 12q$ $q = 1500$

$P = 6.4(1.0126)^t$

$P$ = billions $t$ since 2004

a) yearly % rate of growth? $\rightarrow$ 1.26% per year

b) What was the pop. in 2004? 6.4 billion

c) What is the predicted pop. in the year 2024? $P = 6.4(1.0126)^{20}$ = billion 8.22 people

The number N of a species found on an island is proportional to the 4th root of the area of the island, A.

$N \propto \sqrt[4]{A} \rightarrow N = KA^{1/4}$

Write in Leibnitz notation for the derivitive of the given function - an employee's pay, P, in dollars for a week is a function of number of hours worked. $\frac{dP}{dH}$ dollars hours

$f(x) = 5$ ; $f'(x) = 0$

$f(x) = x$ ; $f'(x) = 1$

$f(x) = 200x$ ; $f'(x) = 200$

$f(x) = 2\pi - x^{2/3}$ ; $f'(x) = 0 - \frac{2}{3}x^{(2/3)-1}$

$h(t) = \frac{4+\sqrt{t}}{t^5} \rightarrow \frac{4}{t^5} + \frac{t^{1/2}}{t^5} \rightarrow 4t^5 + t^{-9/2}$

$h'(t) = 4t\cdot(-5)t^{-5-1} - \frac{9}{2}t^{(-9/2)-1}$

$f(x) = (3x-5)^6$

$f'(x) = 6(3x-5)^{6-1}\cdot(3-0)$

$f(x) = e^{3x^2-4x+7}$

$f'(x) = (6x-4)\cdot e^{3x^2-4x+7}$

$f(x) = 2^{x^6} - 3e^x$

$f'(x) = (6x^{6-1} - 3e^x)2^{x^6-3e^x}\cdot\ln 2$

Find the tangent line to the graph of $y = f(x) = 3x^2 - 5x + 6$ at $x = 1$

use $y - y_0 = m(x - x_0)$

$x_0 = 1$ (given)

$y_0 = f(x_0) = f(1) = 3\cdot 1^2 - 5(1) + 6 = 4$

m = slope is derivitive of function @ 1

$\hookrightarrow f'(x) = 3\cdot 2x^{2-1} - 5\cdot 1 = 6x - 5$

$\hookrightarrow = f'(1) = 6(1) - 5 = 1$

equation: $y - 4 = 1(x-1) \rightarrow y = x + 3$

$f(x) = \frac{1}{\sqrt{x^7}} = \frac{1}{(x^7)^{1/2}} = x^{-7/2}$ ; $f'(x) = -\frac{7}{2}x^{(-7/2)-1}$

$f(x) = (x^2-4)(4x^6-7x+2)$

$f'(x) = u'v + uv'$ substitute the value below for f'

$u = x^2 - 4$ $v = 4x^6 - 7x + 2$

$u' = 2x$ $v' = 24x^5 - 7$

$f(x) = x^5$ ; $f'(x) = 5x^{5-1}$

$f(x) = e^x$ ; $f'(x) = e^x$

$f(x) = \ln(3x^5 - e^x)$

$f'(x) = \frac{15x^4 - e^x}{3x^5 - e^x}$

$f(x) = 2^x$ ; $f'(x) = 2^x \ln 2$

$f(t) = e^t \ln t$

$u = e^t$ $v = \ln t$

$u' = e^t$ $v' = 1/t$ $f'(t) = (e^t/t) + (e^t \ln t)$

$f(x) = \ln x^2$

$\ln x^2 = 2\ln x$ $f'(x) = \frac{2}{x}$; $x > 0$

$f(x) = \frac{\pi + 2\pi x - \ln\pi + \pi^x}{e - e^x + 3x^2}$ $\leftarrow = u$ $\leftarrow = v$

$f'(x) = \frac{u'v - uv'}{v^2}$ substitute the values below for f'

$u' = 0 + 2\pi + \pi^x \ln\pi$ $v' = 0 - e^x + 6x$

Find the relative rate of change if $f(x) = 5x + 4$

$RRC = \frac{f'(x)}{f(x)} = \frac{5}{5x+4}$

Find the tangent line of $f(x) = x^2 + 2x - 3$ at $x = -1$. $(-1, -4)$

1.) $f(x) = (-1)^2 + 2(-1) - 3 = -4$

2.) Slope $= f'(x) = 2x + 2$

$m = 2(-1) + 2 = 0$

$y + 4 = 0(x+1)$

$f(x) = y = -4$

The demand curve for a product is $q = 1100 - 6p^3$. Find the price that max. the Revenue on the interval $[0, 13.54]$

$R(p) = 1100 - 6p^3 \longrightarrow 1100p - 6p^3$

$R'(p) = 1100 - 18p^2$ $p(1100 - 6p^2)$

$1100 - 18p^2 = 0$ $p = 0$ $1100 - 6p^2 = 0$

$p = 7.82$ $p = 13.54$

$R(0) = 0$

$R(7.82) = 5732$

$R(13.54) = 0$

so, $p = 7.82$ max. Revenue

The demand curve of a product is $p = 90 - 10q$. Find the elasticity when $p = 50$.

$p = 50$

$q = f(p) = -\frac{1}{10}p + 9$ $f'(p) = f'(50) = -\frac{1}{10}$

$q = -\frac{1}{10}(50) + 9 = 4$ $E = \left|\frac{50}{4}\cdot -\frac{1}{10}\right|$ $\boxed{E = 1.25}$

If the cost function is $C(q) = 0.08q^3 + 75q + 100$, what is the marginal cost?

$= 0.24q^2 + 75$

If $P = 150(1.4)^t$ at $t = 3$, what is the ARC? $\frac{f(3.00001) - f(3)}{3.00001 - 3} = 138.49$

ex: $\int \frac{dx}{13x+4}$

$w = 13x + 4$

$dw = 13$

$\int \frac{1}{13w}dw = \frac{1}{13}\int\frac{1}{w}dw$

$= \frac{1}{13}\ln|w| = \frac{1}{13}\ln|13x+4|$

ex: $\int \frac{19x}{\sqrt{3x^2+478}}dx$

$w = 3x^2 + 478$

$dw/dx = 6x$

$\frac{1}{6}dw = x\,dx$

$19\int x w^{-1/2}dx$

$\frac{19}{6}\int w^{-1/2}du$

$\frac{19}{6}\cdot\frac{w^{-1/2+1}}{-1/2+1}$

$\frac{19}{6}\cdot\frac{w^{1/2}}{1/2}$

$= \frac{19}{3}(3x^2+478)^{1/2}$

$= \frac{19}{3}\sqrt{3x^2+478}$

ex: $\int \frac{3-x}{x^2}dx = \int\frac{3}{x^2} - \frac{x}{x^2}dx$

$= \int 3x^{-2} - x^{-1} = 3\frac{x^{-2+1}}{-2+1} - \ln|x|$

$= -3x^{-1} - \ln|x| + C$

$= \frac{-3}{x} - \ln|x| + C$

ex: $\int x^2 2^{29x^3+4}dx$

$w = 29x^3 + 4$

$dw/dx = 87x^2$

$\frac{1}{87}dw = x^2 dx$

$\frac{1}{87}\int 2^w dw$

$= \frac{1}{87}\frac{2^w}{\ln(2)}$

$= \frac{1}{87}\frac{2^{29x^3+4}}{\ln(2)}$

$= \frac{2^{29x^3+4}}{87\ln(2)}$

ex: $\int \underbrace{(x^4-4x^2-7)^{12}}_{w}\cdot\underbrace{(4x^3-8x)}_{dw}dx$

$\int w^{12}\frac{dw}{dx} = \frac{w^{12+1}}{12+1}\frac{dw}{dx} = \frac{w^{13}}{13}\frac{dw}{dx} = \frac{(x^4-4x^2-7)^{13}}{13}$

ex: Find an antiderivative of $f(x) = 3x^3 - 7x^2 + 9x - 15$ that satisfies $F(0) = 1234.5$

$\int 3x^3 - 7x^2 + 9x - 15 = 3\frac{x^{3+1}}{3+1} - 7\frac{x^{2+1}}{2+1} + 9\frac{x^{1+1}}{1+1} - 15x$

$= \frac{3}{4}x^4 - \frac{7}{3}x^3 + \frac{9}{2}x^2 - 15x + C = F(x)$

$F(0) = 0 + C = 1234.5$

solution: $F(0) = \frac{3}{4}x^4 - \frac{7}{3}x^3 + \frac{9}{2}x^2 - 15x + 1234.5$

Find antiderivative of $f(x) = 15/x^5 - 21/x$ that satisfies

$F(1) = 130 \rightarrow \int 15x^{-5} - 21x^{-1}dx = -\frac{15}{4}x^{-4} - 21\ln|x| + C = F(x)$

$F(1) = -\frac{15}{4}x^{-4} - 21\ln|x| + 133.75$

Evaluate the following integrals.

$\int_{10}^{103} 9xe^{x^2}dx$

$= 4.5e^{10609} - 4.5e^{100}$

What the average value of $f(x) = \sqrt{9-x^2}$ over the interval $0 \le x \le 3$?

2.36

The demand curve is $p = 53 - q^2$ and The supply curve is $p = 3 + q^2$, q is quantity and p is price. Find the producer surplus to $/unit.

$\int = 83.33 = \$83$ $p = 28$ $q = 5$

After a foreign substance is introduce into the blood, the rate in which antibodies is given by $r(t) = \frac{t}{t^2+1}$ thousand of antibodies per minute, Assume no antibodies are given at $t = 0$, find total quantity at end 7 minutes?

1.96 Antibodies

Compute an approx. to the elasticity of demand for the following?

| Price | 75 | 90 |
|---|---|---|
| room/week | 100 | 90 |

$\frac{90-100}{100}$

$= -0.1 \times 100 = -10\%$

Assume that the total sales of a new app give it introduce obbassed the logistic growth $p = f(t) = \frac{532}{1 + 8669e^{-1.33t}}$ when does the point of diminishing return occur? $\boxed{5.08}$

Find the present and future value of a constant income stream of $1000/year over a 20 year period, 6% compound constinuously.

present $= \int_0^{20} 1000e^{-0.06} = \$11,647$

Future $= \$11,647e^{0.06(20)}$

$= \$38,669$