Math 242 Test 1, Tuesday 10 February

Name: **Last 4 digits of SSN:**

Show all work clearly. No work means no credit. The points are:
ex1: 10, ex2: 10, ex3: 15, ex4: 15, ex5: 20, ex6: 15, ex7: 15.

Exercise 1 (10 points) A ball is thrown straight downward from the top of a tall building. The initial speed of the ball is 10 m/s. It strikes the ground with a speed of 60 m/s. How tall is the building ?
You will take $g = 10\ m/s^2$.

Exercise 2 (10 points) Solve the differential equation :

$$3x^5y^2 + x^3y' = 2y^2.$$

Exercise 3 (15 points) A pitcher of buttermilk initially at 25°C is to be cooled by setting it on the front of the porch, where the temperature is 0°C. Suppose that the temperature of the buttermilk has dropped to 15°C after 20 min. When will it be at 5°C ?
You will use that $\ln(1/5)/\ln(3/5) \approx 3.15$.

Exercise 4 (15 points) We are considering the following differential equation:

$$xy' = 4y + x^4 \cos x.$$

Solve this equation with the initial value $y(\pi) = 0$.

Exercise 5 (20 points) We consider the following differential equation:

$$x^3y' = x^2y - y^3,\ x > 0.$$

1. What kind of equation is it?

2. What substitution do we have to do?

3. What kind of differential equation do we obtain after the substitution?

4. Solve this last differential equation and then find the expression of y.

Exercise 6 (15 points) We consider the following differential equation:

$$e^y + y\cos x + (xe^y + \sin x)\, y' = 0.$$

1. Show that this equation is exact.

2. Then solve this differential equation.

Exercise 7 (15 points) We consider the following differential equation:

$$-xy + x^2y' = y^2 + x^2.$$

1. Write this differential equation as a homogeneous one.

2. Then solve this differential equation.