# Lesson 7: Power Functions. Basic Operations with Functions

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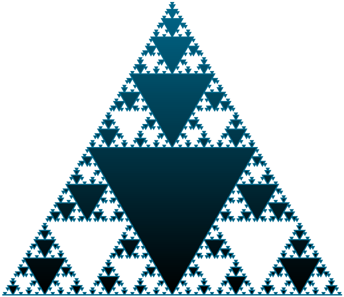

# What do we know?

- Functions
  - $x-$ and $y-$intercepts ($f(x) = 0, f(0)$)
  - Change from $x = a$ to $x = b$

    $$\Delta y = f(b) - f(a)$$

  - Average Rate of Change from $x = a$ to $x = b$

    $$ARC = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

  - Relative Change from $x = a$ to $x = b$

    $$RC = \frac{\Delta y}{f(a)} = \frac{f(b) - f(a)}{f(a)}$$

- Linear Functions:
  $f(x) = b + mx$
- Exponential Functions
  $P_0 a^t = P_0(1 + r)^t = P_0 e^{kt}$

# Power Functions

## Definitions

We say that $Q(x)$ is a power function of $x$ if $Q(x)$ is proportional to a constant power of $x$:

$$Q(x) = kx^p$$

The coefficient $k$ is called *constant of proportionality*, and $p$ is the *power*.

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### Which of these functions are power functions?

$$y = 5\sqrt{x}$$

$$y = 2x$$

$$y = \frac{3}{x}$$

$$y = \left(3x^5\right)^2$$

$$y = \frac{3}{8x}$$

$$y = \frac{5}{2\sqrt{x}}$$

$$y = \pi$$

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$y = 5\sqrt{x} = 5x^{1/2}$ $\quad k = 5 \quad p = \frac{1}{2}$

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| $y = \dfrac{3}{x}$ | | |
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| $y = (3x^5)^2 = 9x^{10}$ | $k = 3^2 = 9$ | $p = 10$ |
| $y = \dfrac{3}{8x}$ | | |
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| $y = \dfrac{5}{2\sqrt{x}} = \frac{5}{2}x^{-1/2}$ | $k = \dfrac{5}{2}$ | $p = -\frac{1}{2}$ |
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| $y = \dfrac{5}{2\sqrt{x}} = \frac{5}{2}x^{-1/2}$ | $k = \dfrac{5}{2}$ | $p = -\frac{1}{2}$ |
| $y = \pi = \pi x^0$ | $k = \pi$ | $p = 0$ |

# Power Functions

## Definitions

Sums of power functions with non-negative integer exponent are called polynomials

$$y = p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

The highest of the powers, $n$, is called the degree of the polynomial. The power function with that highest power, $a_n x^n$, is called the leading term of the polynomial, and the corresponding coefficient $a_n$ is called the leading coefficient.

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### Example

The following are polynomials. What are their degrees and leading coefficients?

$$y = \pi$$

$$y = 1 - 2x + 3x^4 - 5x^6$$

$$y = \frac{1}{2}x^3 - \frac{3}{4}x^5 + \frac{5}{6}x^7$$

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| $y = \pi$ | $n = 0$ | $a_0 = \pi$ |
| $y = 1 - 2x + 3x^4 - 5x^6$ | $n = 6$ | $a_6 = -5$ |
| $y = \frac{1}{2}x^3 - \frac{3}{4}x^5 + \frac{5}{6}x^7$ | | |

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$$y = \pi \qquad n = 0 \qquad a_0 = \pi$$

$$y = 1 - 2x + 3x^4 - 5x^6 \qquad n = 6 \qquad a_6 = -5$$

$$y = \tfrac{1}{2}x^3 - \tfrac{3}{4}x^5 + \tfrac{5}{6}x^7 \qquad n = 7 \qquad a_7 = \tfrac{5}{6}$$

# Operations with functions

## Vertical Operations

| Operation on $f(x)$ | Effect on the graph |
| --- | --- |

# Operations with functions

## Vertical Operations

| **Operation on $f(x)$** | **Effect on the graph** |
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| $f(x) + C \qquad (C > 0)$ | Vertical shift (up) by $C$ units |

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| $af(x) \qquad (a > 1)$ | Vertical stretching by a factor of $a$ |

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| $af(x) \qquad (0 < a < 1)$ | Vertical compression by a factor of $1/a$ |

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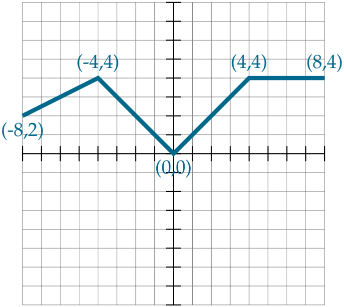
| $af(x) \qquad (a > 1)$ | Vertical stretching by a factor of $a$ |
| $af(x) \qquad (0 < a < 1)$ | Vertical compression by a factor of $1/a$ |
| $-f(x)$ | Reflection with respect to the $x$–axis |

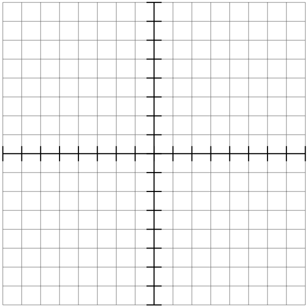
# Operations with functions

## Vertical Operations

**Example**

Given the graph of the function $f(x)$ below, sketch the graph of the function $2f(x) - 3$



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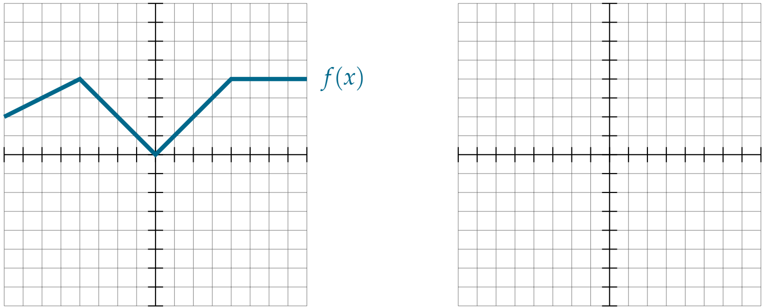


$$f(x) \to 2f(x) \to 2f(x) - 3$$

# OPERATIONS WITH FUNCTIONS

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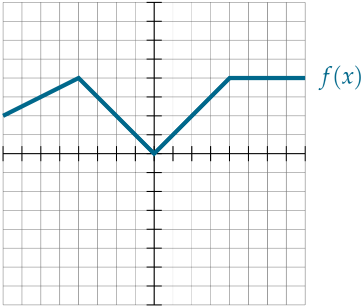


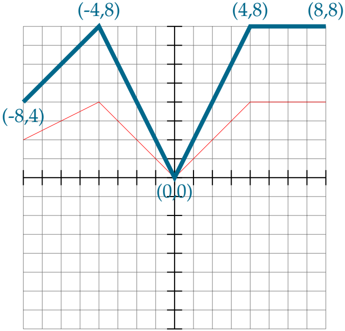


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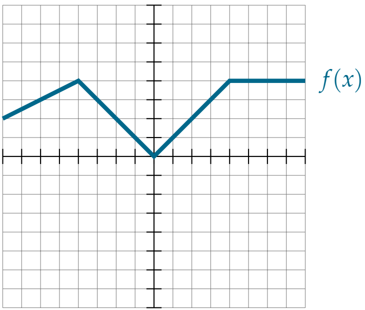


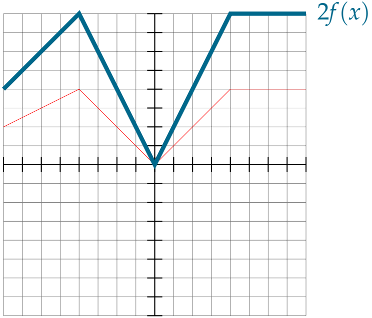


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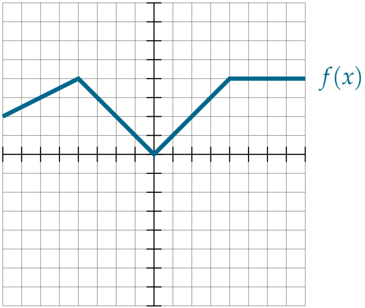


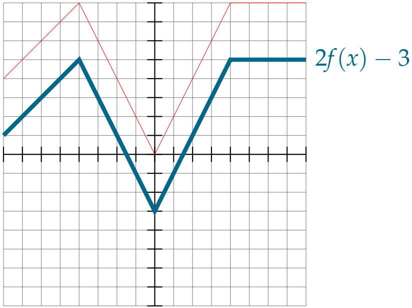


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## Horizontal Operations

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| Operation on $f(x)$ | Effect on the graph |
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| $f(x+C) \qquad (C>0)$ | Horizontal shift (left) by $C$ units |

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| **Operation on $f(x)$** | | **Effect on the graph** |
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| $f(x+C)$ | $(C>0)$ | Horizontal shift (left) by $C$ units |
| $f(x-C)$ | $(C>0)$ | Horizontal shift (right) by $C$ units |

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| $f(ax) \qquad (a>1)$ | Horizontal compression by a factor of $1/a$ |

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| $f(-x)$ | Reflection with respect to the $y$–axis |

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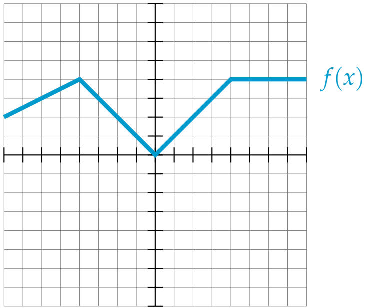
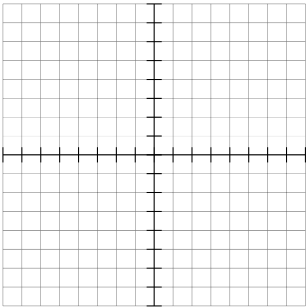
## Horizontal Operations

### Example

Given the graph of the function $f(x)$ below, sketch the graph of the function $2 - f(2x)$



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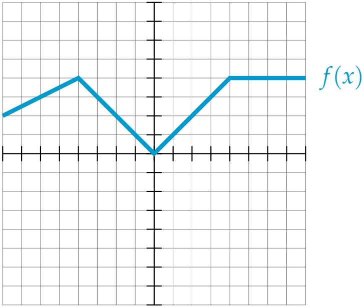


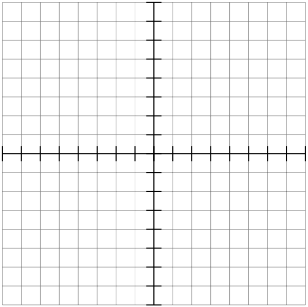

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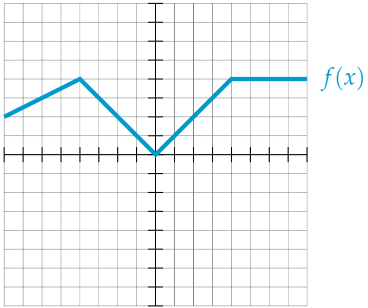


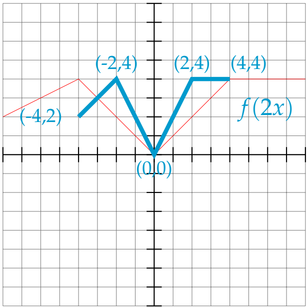


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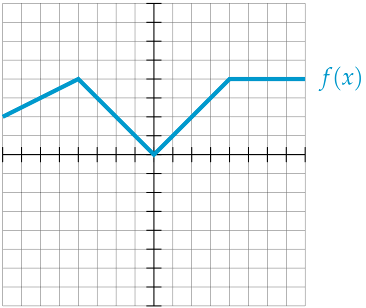


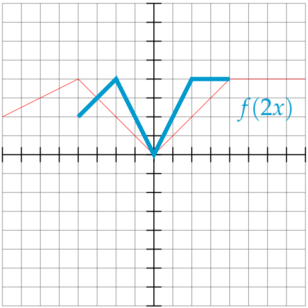


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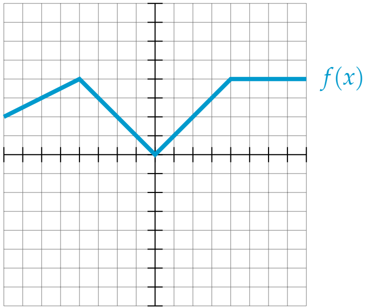


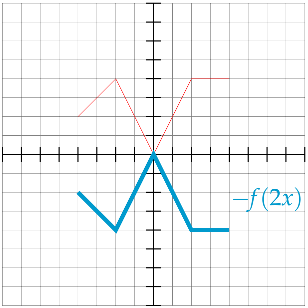


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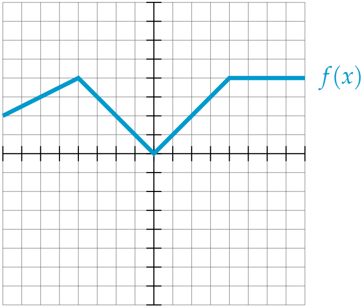


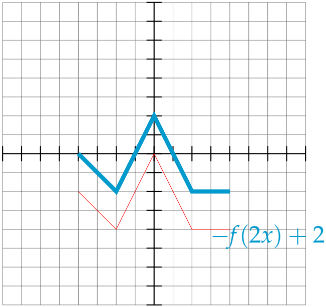


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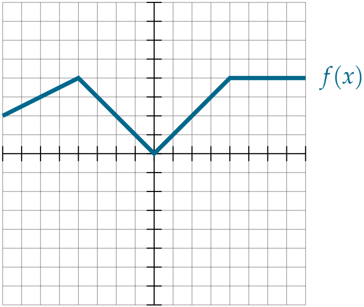
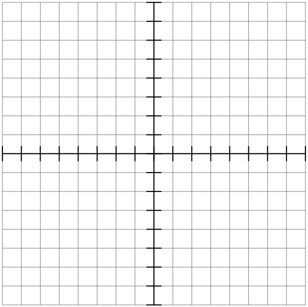
# Operations with functions

## Horizontal Operations

**Example**

Given the graph of the function $f(x)$ below, sketch the graph of the function $f(4x-2)$



# Operations with functions

## Horizontal Operations

### Example

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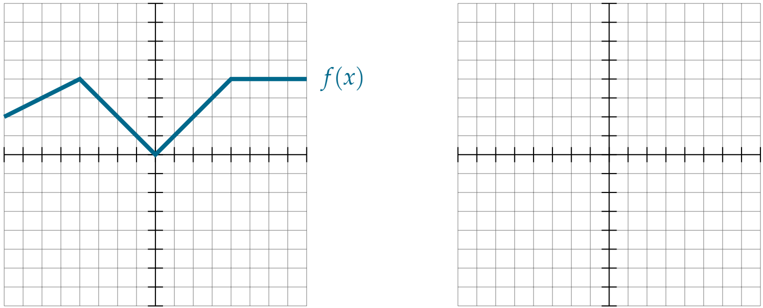


$$f(x) \to f(4x) \to f(4x-2)$$

# Operations with functions

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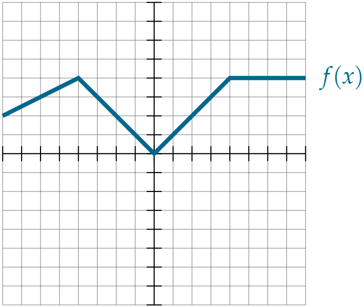


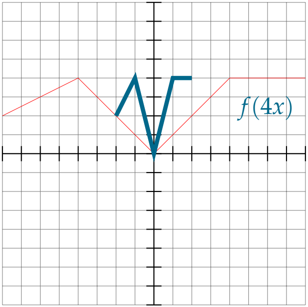


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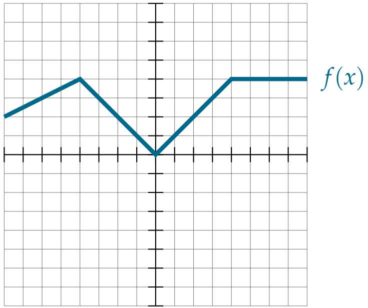


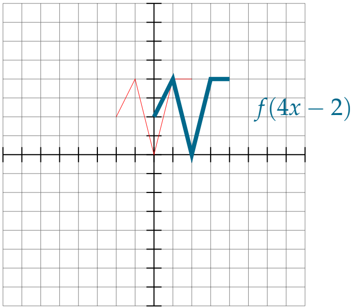


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# Operations with functions

## Examples

### Example

Write an equation for a graph obtained by vertically stretching the graph of $y = x^3$ by a factor of 3, followed by a vertical upward shift of 2 units.

# Operations with functions

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Write an equation for a graph obtained by vertically stretching the graph of $y = x^3$ by a factor of 3, followed by a vertical upward shift of 2 units.

Solution: $y = 3x^3 + 2$

| Original function | After vertical stretch | After shift up |
| --- | --- | --- |
| $x^3$ | | |

# Operations with functions

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# Operations with functions

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**Example**

What is the equation if the order of the transformations is interchanged?

# Operations with functions

## Examples

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### Example

What is the equation if the order of the transformations is interchanged?

Solution: $y = 3(x^3 + 2)$

| Original function | After shift up | After vertical stretch |
|---|---|---|
| $x^3$ | $x^3 + 2$ | $3(x^3 + 2)$ |