Name: _____

VIP ID: _____

- Write your name and VIP ID in the space provided above.
- The test has six (6) pages, including this one.
- Enter your answers in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.
- No books, notes or calculators may be used on this test.

Page	**Max. points**	**Your points**
2	20	
3	20	
4	20	
5	20	
6	20	
Total	100	

Problem 1 (5 pts—all or nothing)**.** Find the domain of $f(x) = \sqrt{(1-x)(2-x)}$.

domain = $\boxed{}$

Problem 2 (5 pts—all or nothing)**.** Let $f(x) = x^2 + 4$, $g(x) = \sqrt{x}$. Find $g \circ f$

$\big(g \circ f\big)(x) = \boxed{}$

Problem 3 (5pts)**.** Evaluate the following limit:

$$\lim_{N\to\infty} \frac{1}{N^2} \sum_{n=1}^{N} 5n$$

$\boxed{}$

Problem 4 (5 pts)**.** Assume y is a function of x given implicitly by $\sin(x+y) = x + y$. Find y'.

$\boxed{}$

Problem 5 (10 pts)**.** Use logarithmic differentiation to find the derivative of the function

$$y = \frac{\tan^2 x \sin^4 x}{e^{3x}(x^2+1)}$$

$y' =$

Problem 6 (10 pts)**.** Find an equation of the tangent line to the curve $y = \ln(xe^x)$ at the point $(1, 1)$.

Problem 7 (20 pts)**.** Sketch the graph of the rational function $f(x) = \dfrac{x^2}{1-x^2}$.

Indicate clearly: Domain; x- and y-intercepts; vertical and horizontal asymptotes; intervals of increase, decrease and different concavity. Indicate also the location of relative extrema, if any.

Problem 8 (10 pts)**.** The volume of a cube is increasing at a rate of 300 cm^3/min. How fast are the edges increasing when the length of an edge is 10 cm?

The edges are increasing at a speed of

Problem 9 (10 pts)**.** A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

Dimensions of most economic fence:

Problem 10 (1/2/3/4 pts)**.** Compute the following limits:

(a) $\lim_{x\to-\infty} \dfrac{x^2-2x-8}{x^2-4} =$

(b) $\lim_{x\to 2} \dfrac{x^2+2x-8}{x^2-4} =$

(c) $\lim_{x\to-2} \dfrac{x^2-2x-8}{x^2-4} =$

(d) $\lim_{x\to 0} x^{1/x} =$

Problem 11 (2/4/4 pts)**.** Evaluate each integral:

(a) $\int \left(\frac{1}{x} - 2^x + 3\sin x\right) dx =$

(b) $\int 4x \sin x^2 \, dx =$

(c) $\int_1^2 \left(5x + \dfrac{2}{3x^5} - \sqrt{2}e^x\right) dx =$