Lesson 6: Exponential Growth and Decay

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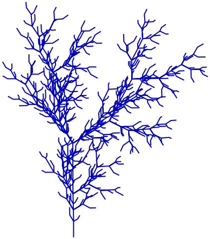

What do we know?

- Functions
  - $x-$ and $y-$intercepts ($f(x) = 0, f(0)$)
  - Change from $x = a$ to $x = b$

    $$\Delta y = f(b) - f(a)$$

  - Average Rate of Change from $x = a$ to $x = b$

    $$ARC = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

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- Linear Functions:
  $f(x) = b + mx$
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$$\begin{array}{ll} a = 1 + r & r = a - 1 \\ k = \ln a & a = e^k \\ r = e^k - 1 & k = \ln(1 + r) \end{array}$$

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The EPA investigated a spill of radioactive iodine. The radiation level at site was 2.4 milirems/hour. The level of radiation decays at a continuous hourly rate of $k = -0.04$.

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$$P = P_0 e^{kt} = 2.4e^{-0.04t} (\text{in milirems/hour}) \qquad t = \text{ hours after}$$

The first question is asking for the radiation level after 24 hours:

$$P(24) = 2.4e^{-0.04\times 24} = 0.9189429264 \text{ milirems/hour}$$

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$$t = \frac{-1.386294361}{-0.04} \approx 34.65735903 \text{ hours.}$$

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$$t = \frac{-\ln 2}{k} = \frac{-\ln 2}{-0.0025} \approx 277.25887224 \text{ years.}$$

Compound Interest

An amount of money, P_0, is deposited in an account paying interest at a rate of r per year. Let P be the balance in the account after t years.

- If the interest is compounded annually, then $P = P_0(1+r)^t$.
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- We use the formula $P(t) = P_0 e^{rt}$ for the same values of r and P_0

$$P(3) = 5000e^{0.08\times 3} = \$6356.24575$$

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Or even better, eight years almost 40 days (39.9218 days)

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You win the lottery and are offered the choice between \$1 million in four yearly installments of \$250,000 each, starting now, or a lump-sum payment of \$920,000 now.

Assuming a 6% interest rate per year, compounded continuously, and ignoring taxes, which should you choose?

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The following table summarizes the two situations:

	2015	2016	2017	2018
First option	250,000	+250,000	+250,000	+250,000
Second option	920,000			

Compound Interest

Examples

	2015	2016	2017	2018
First option	250,000	265459 +250,000 =515459	+250,000	+250,000
Second option	920,000			

Let us examine the first option.

- In one year, the initial amount of \$250,000 gives us $P(1) = 250000e^{0.06\times 1} = 265459.13675$ after compounding at 6%. Then we add \$250,000 more, for a total of $265459.13675 + 250000 = 515459.13675$.

Compound Interest

Examples

	2015	2016	2017	2018
First option	250,000	265459 +250,000 =515459	547333 +250,000 = 797333	 +250,000
Second option	920,000			

Let us examine the first option.

- In one year, the initial amount of \$250,000 gives us $P(1) = 250000e^{0.06\times 1} = 265459.13675$ after compounding at 6%. Then we add \$250,000 more, for a total of $265459.13675 + 250000 = 515459.13675$.
- After another year, this amount increases to $P(1) = 515459.13675e^{0.06} = 547333.34989$. We add \$250,000 again, to obtain $547333.34989 + 250000 = 797333.34989$.

Compound Interest

Examples

	2015	2016	2017	2018
First option	250,000	265459 +250,000 =515459	547333 +250,000 = 797333	846637 +250,000 =1096637
Second option	920,000			

Let us examine the first option.

- In one year, the initial amount of \$250,000 gives us $P(1) = 250000e^{0.06\times 1} = 265459.13675$ after compounding at 6%. Then we add \$250,000 more, for a total of $265459.13675 + 250000 = 515459.13675$.
- After another year, this amount increases to $P(1) = 515459.13675e^{0.06} = 547333.34989$. We add \$250,000 again, to obtain $547333.34989 + 250000 = 797333.34989$.
- After another year, this amounts increases to $P(1) = 797333.34989e^{0.06} = 846637.69106$. At that moment, we add the last \$250,000 to get the final amount of $846637.69106 + 250000 = 1096637.69106$

Compound Interest

Examples

	2015	2016	2017	2018
First option	250,000	265459 +250,000	547333 +250,000	846637 +250,000 =1096637
Second option	920,000			1101439

Let us examine the second option. We only need to worry about how much we have after 3 years:

$$P(3) = 920000e^{0.06\times 3} = 1101439.974$$

Which option do you prefer?