Name: ______________________________

4-digit code: ______________________________

- Write your name and your VIP ID in the space provided above.
- The test has five (5) pages, including this one.
- Enter your answer in the box(es) provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses at the right of the problem number.

Page	**Max. points**	**Your points**
2	30	
3	25	
4	25	
5	20	
Total	100	

Problem 1 (15 pts). Find the domain and range of $g(x) = \sqrt{9 - x^2 - y^2}$

domain:

range:

Sketch the level curves for $k = 0, 1, 2, 3$:

Problem 2 (15 pts). If $f(x, y) = \dfrac{xy^2}{x^2 + y^4}$, does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Why?

Problem 3 (15 pts)**.** Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

Problem 4 (10 pts)**.** The dimensions of a rectangular box are measured to be 75 cm, 60 cm, and 40 cm, and each measurement is correct within 0.2 cm. Use differentials to estimate the largest possible error when the volume of the box is calculated from these measurements.

Problem 5 (5 pts)**.** If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find dz/dt when $t = 0$ **without calculating explicitly the derivative from the composition**. Use the chain rule in the form that was explained in class.

$$\frac{dz}{dt} = \boxed{}$$

Problem 6 (10 pts)**.** If $f(x, y, z) = x \sin yz$, find the directional derivative of f at $(1, 3, 0)$ in the direction of $\boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$.

$$D_{\boldsymbol{v}} f(1, 3, 0) = \boxed{}$$

Problem 7 (10 pts)**.** Find the local maxima, minima and saddle points of $f(x) = x^4 + y^4 - 4xy + 1$.

max	min	saddle pts.

Problem 8 (10 pts)**.** Find the absolute maximum and minimum values of the function $f(x,y) = 4x + 6y - x^2 - y^2 + 7$ on the set $D = \{(x,y) : 0 \leq x \leq 4, 0 \leq y \leq 5\}$. Make sure to sketch the set D and indicate the different borders.

Problem 9 (10 pts)**.** Use Lagrange multipliers to find the maximum and minimum values of the function $f(x,y) = x^2y$ subject to the constraint curve $x^2 + 2y^2 = 6$.