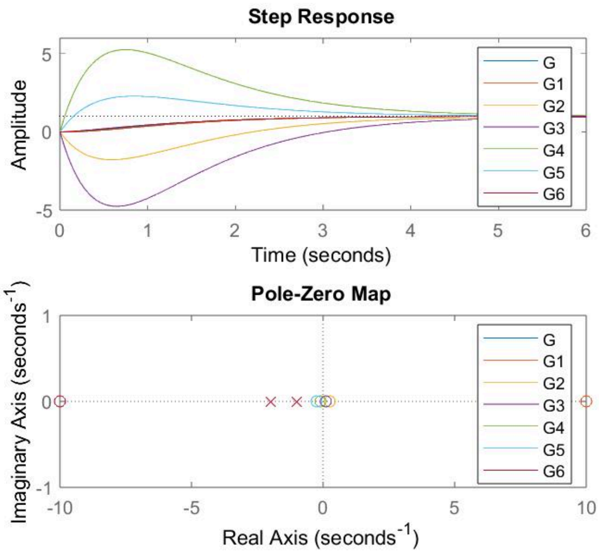
Classical Control Lab Report (M1 CORO)

Problem 3.1 - Influence of zeros on Step response

The step responses were simulated for the seven transfer functions : $G(s) = \frac{1}{(s+1)(0.5s+1)}$ and $G_c(s) = \frac{-s+a}{a(s+1)(0.5s+1)}$ for a = -10, -0.25, -0.1, +0.1, +0.25, +10. They are presented in fig 1. The transfer function G(s) is represented as G and the other transfer functions G_c are represented as [G1, G2, G3, G4, G5, G6].



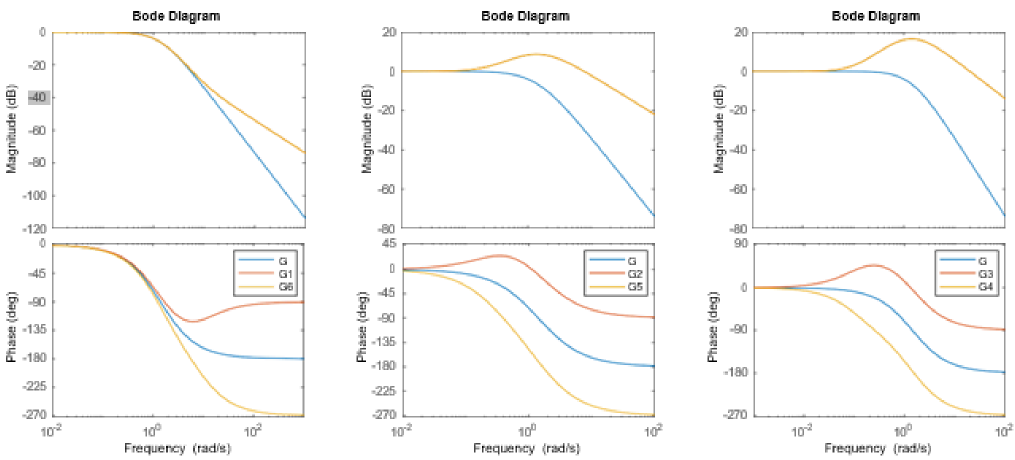
As seen from the table, the largest over/undershoots exist when the zeros exist and they lie very close to the origin of the complex plane. This issue seems to vanish when they are comparatively far from the origin or they don't exist at all.
The behaviour of (undershoot and overshoot) is dictated by the sign of the stable zeros, according to the following equations :

$M_u > \frac{1}{ct_s}$ for $c \in R^+$ and $M_p y_\infty > \frac{1}{-ct_s}$ for $c \in R^-$ when $ct_s << 1$

Where M_p = Magnitude of overshoot, M_u = Magnitude of undershoot, c = System zero, t_s = Settling time.

Problem 3.2 - Influence of zeros on frequency response

Figure 2 shows the Bode diagrams for the transfer functions G through G6 in triples for comparison.



It can be observed that the system with higher under/overshoot tends to exhibit higher peaks in the magnitude plot of frequency response. The frequency of these oscillations ω_r and damping ratio ζ (=1) are the same for all the systems. Hence it can be said that the oscillatory behaviour is exclusively due to the zeros of the system.

The slope of the high frequency asymptote is same for the transfer functions, shown in the figure below. It can be said that the slope is influenced by the existence of the zeros, rather than a or the magnitude of zeros i.e. |c|.

In the phase plot, the high frequency asymptote is shifted by $\pm$ 90 degrees, depending on the sign of the zero c. Where $c \in R_-$, there is a shift of $+90^o$, and $c \in R_+$, results in a shift of -90^o.

A stable system can be described as a ”non-minimum phase system” when the inverse of the system is unstable. In other words, there exists a zero with positive real part, which creates a response with an initial undershoot. Therefore, amongst the above transfer functions, only those which have undershoot [G4, G5, G6] can be described as non-minimum phase systems.

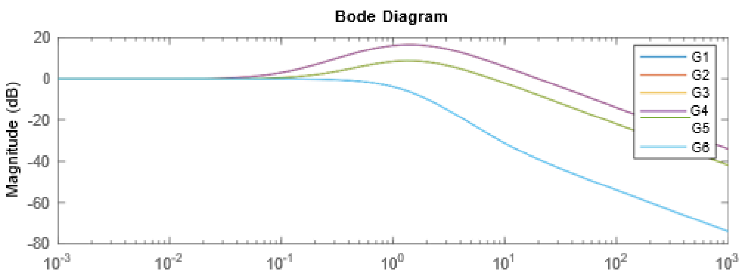


Figure 3: Bode diagram showing the same high frequency slope for transfer functions G1 through G6

TF	Undershoot / Overshoot	Magnitude (%)	Zeros
G	-	-	-
G1	-	-	-10
G2	Overshoot	128.5	$-\frac{1}{4}$
G3	Overshoot	426.3	$-\frac{1}{10}$
G4	Undershoot	476.2	$\frac{1}{10}$
G5	Undershoot	177.7	$\frac{1}{4}$
G6	Undershoot (Small)	0.83	10