Classical Control (M1 CORO / M1 JEMARO)
Exercises 2 (Lab.)

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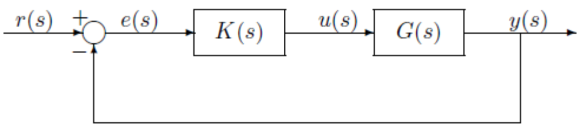
The aim of this study is to synthetize different PID type controllers, $K(s)$, with the code called ASTA for the system

$$G(s) = \frac{1}{(1+s)^2(1+10s)} \tag{1}$$

in the following closed loop architecture



1 Proportional controller

Tune a proportional control $K(s) = P$ such that the maximum of the complementary sensitivity function is $2.3\,db$. In order to compare the performances of the different closed loop that will be designed, note the characteristics of the step response (M_p, t_p, $t_{r5\%}$, $t_{r2\%}$, ϵ_r) and of the frequency response (M_r, ω_r). Note also the stability margins. (Note that $M_r \neq 2.3\,db$ and that M_p is far from 23%. What is the value of P which gives $M_r = 2.3\,db$?)

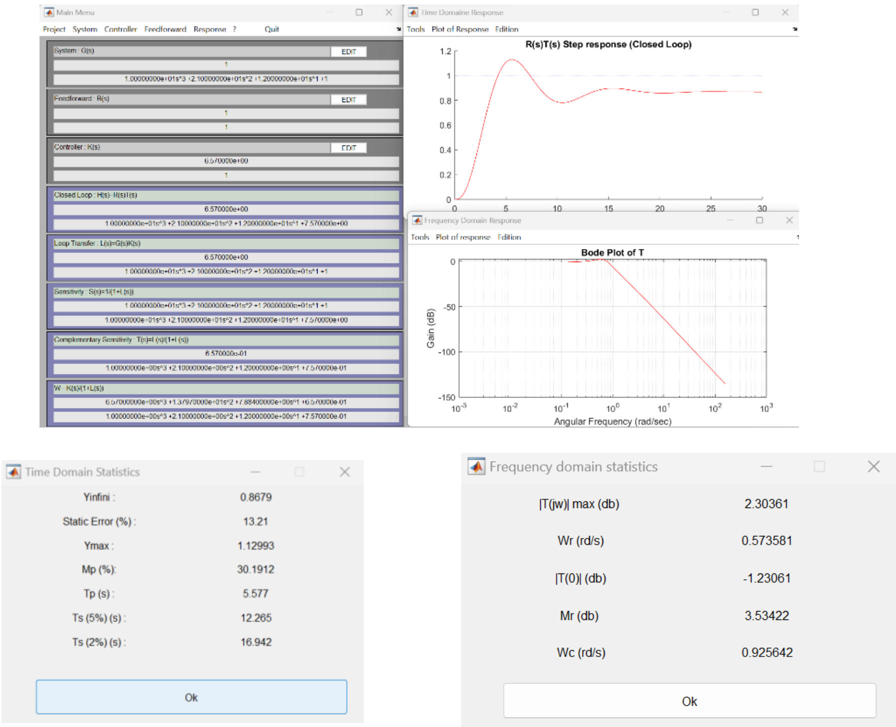


Figure 1 : Plots for Proportional gain 6.57

In this part, a proportional control is applied into the model. We tune K(s)= P, such as P is the maximum of the complementary sensitivity function with 2.3 db. After several trial and error we find P= 6.57, so that the plot yields minimum tangent line to the 2.3 db circle at the Nicols graph. By observing the time domain statistics, it shows that this control has still an error in the output. Therefore, there is needed more tuning by adding integral control.

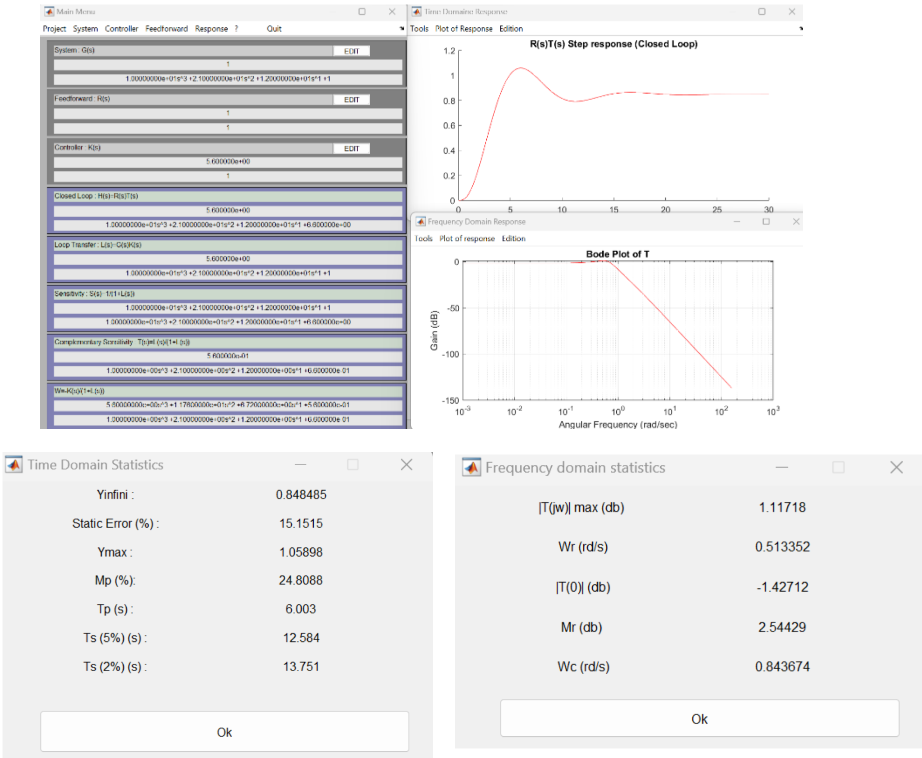


Figure 2 : Plots for Proportional gain 5.36

Key Observation (from P = 6.57 → P =5.36) :

1. Both the motions are more or less oscillatory. However the former system has a larger overshoot as compared to the later one.
2. Cutoff frequency has reduced for the later system, implying slower response (reflected in saturation times).
3. The steady state error has increased.

2 Proportional and Integral controller

Tune a PI controller $\left(K(s) = P(1 + \frac{1}{sT_i})\right)$ which gives the same maximum of the complementary sensitivity function (2.3 db).
Compare the performances of the closed loop with the performances of the previous loop.

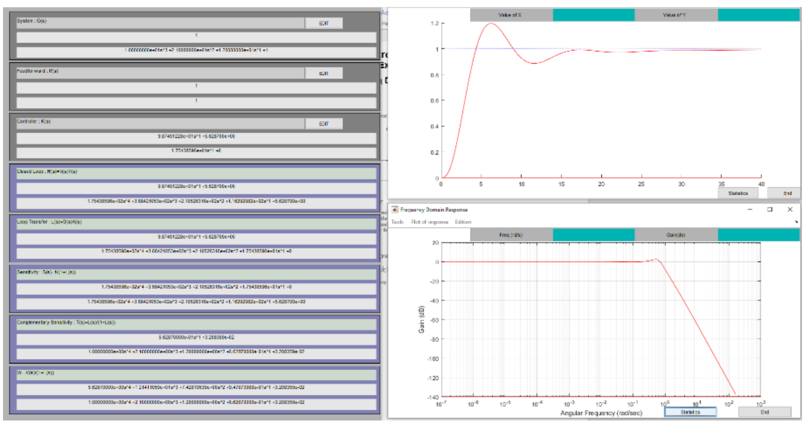
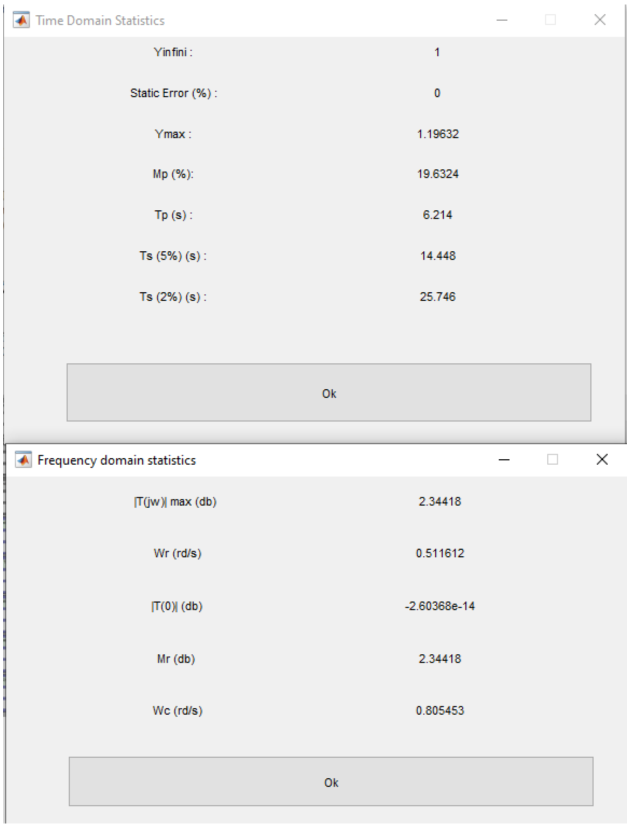


Figure 3 : Plots for Proportional gain 5.6 and Ti 17.54



By adding Proportional and integral control $K(s) = P(1 + 1/s+Ti)$ we can eliminate the output error. By the same performance that want to be achieve which is has sensitivity function with 2.3 db, first we can find Ti using classical advice which is 10/ωr, therefore Ti=10/0.57 = 17.54. While on the other hand, the P could be found by tuning. We found P=5.6287 in which the plot has maximum (db) = 2.3 db. Thanks to adding an integral controller it has no output error at the final value. Nevertheless, this result in the time domain response is slower by comparing settling time Ts from the previous part which achieve 16.94 s, while in this first tuning we only achieve 25.746. Then, we chose Ti=8.77(=5/ωr =5/0.57) for the second tuning. By using the same method to achieve maximum (db) 2.3 db we find P=4.48. In particular, we got small settling time Ts(5%) and Ts(2%) in this tuning part, which is better than previous one.

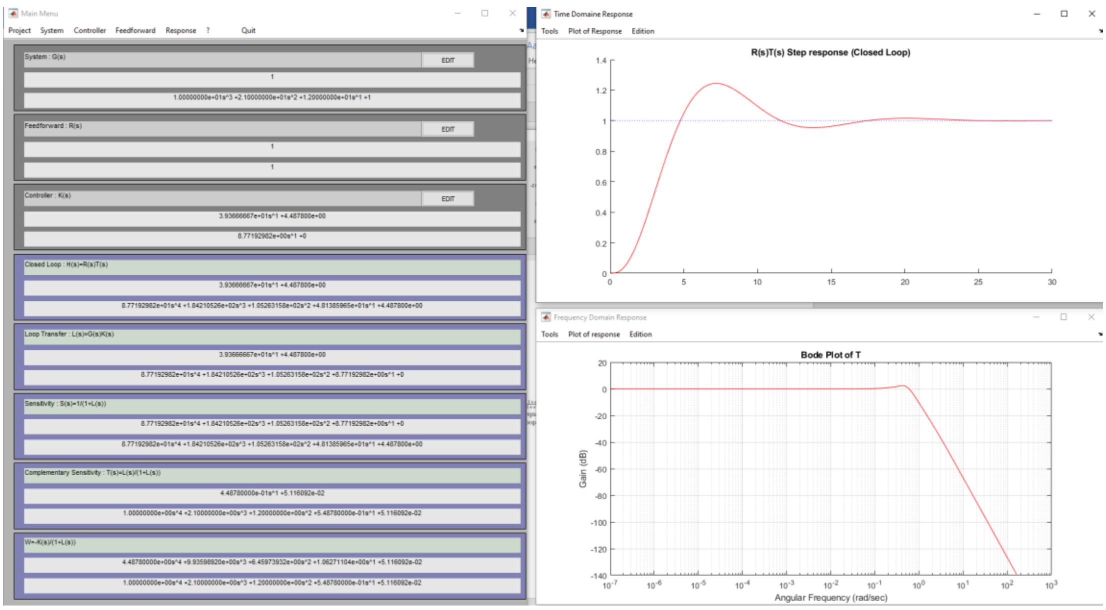


Figure 4 : Plots for Proportional gain 4.48 and Ti 8.77

Time Domain Statistics

Yinfini :	1
Static Error (%) :	0
Ymax :	1.24473
Mp (%):	24.4728
Tp (s) :	7.185
Ts (5%) (s) :	10.658
Ts (2%) (s) :	16.111

Ok

Frequency domain statistics

|T(jw)| max (db)	2.31998
Wr (rd/s)	0.415684
|T(0)| (db)	6.55743e-14
Mr (db)	2.31998
Wc (rd/s)	0.70087

Ok

Nevertheless, the last tuning produce high maximum output with a peak at 1.244. On the next tuning, we will try to find faster tuning but with lower overshoot. Because often in practice, we want to avoid high overshoot due to limited process or actuator limitation/saturation for example.

3 Lag controller

Tune a lag compensator $\left(K(s) = P\left(\frac{1+sT}{1+sTb}\right)\right)$ which gives the same maximum of the complementary sensitivity function ($2.3\,db$) but a steady state error equals to 5% for the step response.
Compare the performances of the closed loop with the performances of the previous proportional loops.

Tuning	Time Domain Statistics		Frequency domain statistics	
1st Tuning T=17.54 P=19 B=3.21	Yinfini :	0.95	|T(jw)| max (db)	2.33102
	Static Error (%) :	5	Wr (rd/s)	0.542632
	Ymax :	1.17219	|T(0)| (db)	-0.445528
	Mp (%):	23.3888	Mr (db)	2.77655
	Tp (s) :	5.987	Wc (rd/s)	0.843674
	Ts (5%) (s) :	13.592		
	Ts (2%) (s) :	22.961		
	Ok		Ok	
2nd Tuning P=19 T=8.77 B=3.74	Yinfini :	0.95	|T(jw)| max (db)	2.31205
	Static Error (%) :	5	Wr (rd/s)	0.485653
	Ymax :	1.20407	|T(0)| (db)	-0.445528
	Mp (%):	26.7442	Mr (db)	2.75758
	Tp (s) :	6.568	Wc (rd/s)	0.768964
	Ts (5%) (s) :	13.743		
	Ts (2%) (s) :	19.066		
	Ok		Ok	

In this part, we tuned a lag compensator in the same condition as the previous one but except for one which had a steady state error 5%. We put P for 19, because steady state error equals to 5% (0.05) and we calculated by using the below given technique :

$$e_{\infty} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} se(s) = \lim_{s\to 0} s\frac{1}{1+G(s)K(s)}\frac{1}{s} = \frac{1}{1+G(0)K(0)} = \frac{1}{1+P}$$

$$e_{\infty} = 0.05$$

$$P = 20 - 1 = 19$$

From the closed loop architecture, we could find the steady state error e(infinite). T is obtained the same way with the PI controller. We tuned the case which is T=17.54 and T=8.77. P=19 is much higher than previous one, but we tuned b, and its performance is as almost same as previous one. this result is not changed in the case b=3.21 or b=3.74. Both of these tuned controllers has dc-gain which is -0.44, it is different from the previous one and its error correctly became 5%. But the second tuning has a slighter better with db maximum 2,31205.

N:B.: It can be seen that in the lag controller, the maximum overshoot could be reduced yet we can still achieve the steady state performance we need (5% error) and with a fast enough settling time.